Fiber Optics
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Lecture - 17
Electromagnetic Analysis of Waveguides – VII

Till now we have carried out the analysis of symmetric planar wave guides where the high index region is sandwiched between 2 lower index regions of same refractive index. Now in this lecture we will extend the analysis to asymmetric planar waveguide asymmetric planar waveguides are more practical wave guides. So, we will extend this analysis to asymmetric planar waveguide and see how the waveguide hence is done in these kind of waveguides.

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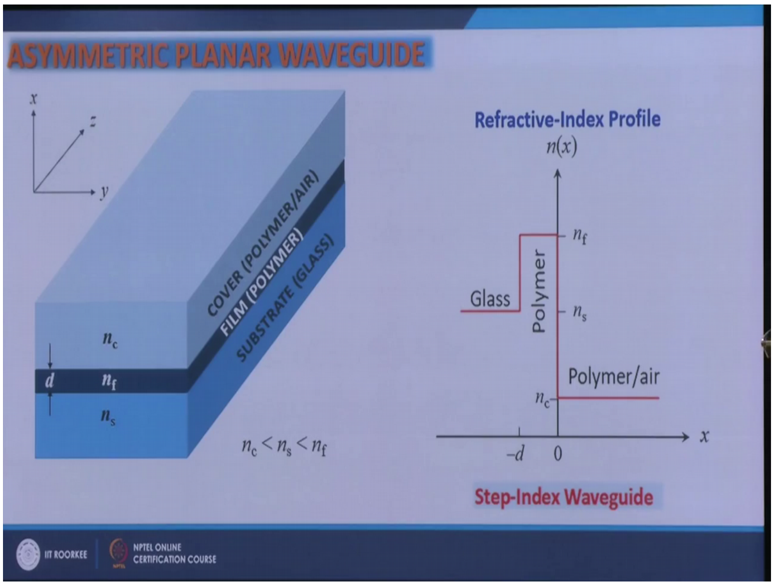

So, what is an asymmetric planar waveguide you for example, take a glass of substrate of refractive index n s, and you deposit a film of refractive index n f of thickness d, this film can be for example, of a polymer material, the refractive index n f is higher than the refractive index n s and then it is surrounded by air. So, air works as cover; also you can put a cover of polymer material itself of refractive index n c, where n c is smaller than n s and n s is smaller than n f.

So, this kind of waveguide we can see that is has asymmetric structure, because n s is not equal to n c here. If I look at the refractive index profile of this waveguide, then this is the substrate an example is glass, this is the guiding film the example is polymer, and this is the cover region which can be air or another polymer of lower refractive index. Here at x is equal to 0, I have made the interface between the film and the cover, and at x is equal to minus d represents the interface between the substrate and the film.

Since the refractive indices of the individual layers are uniform. So, this waveguide is step index waveguide. I can also have a waveguide in which the refractive index in the guiding film is not uniform, but it varies with x.

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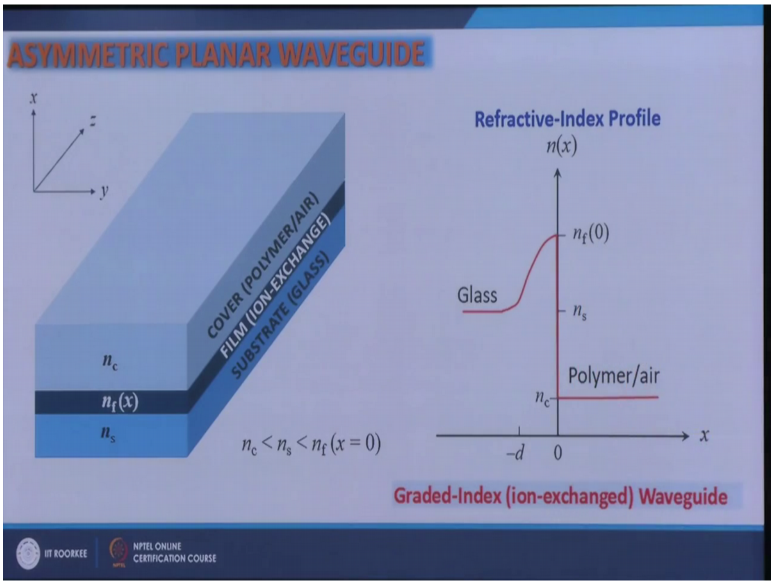

So, how I can make this waveguide well I take a substrate of material glass, and then I diffuse certain ions into this pay over a certain distance. So, I will have a diffusion profile for example, I can diffuse silver ions into glass and that will change the refractive index of the material near the surface.

So, now this n f would be a function of x now, and then I can have cover as air or cover of a polymer layer. So, now, this n c is smaller than n s, and then the refractive index of the film at the surface of the film is n f at f is equal to 0. So, n s is smaller than that. If I look at the refractive index profile of this structure now, so, I have a substrate and this is the guiding film. This guiding film now has refractive index with varies which varies with x and ultimately it merges into the refractive index of glass.

So, this is the surface index n f 0, and this n f 0 is larger than n s and which is larger than n c.

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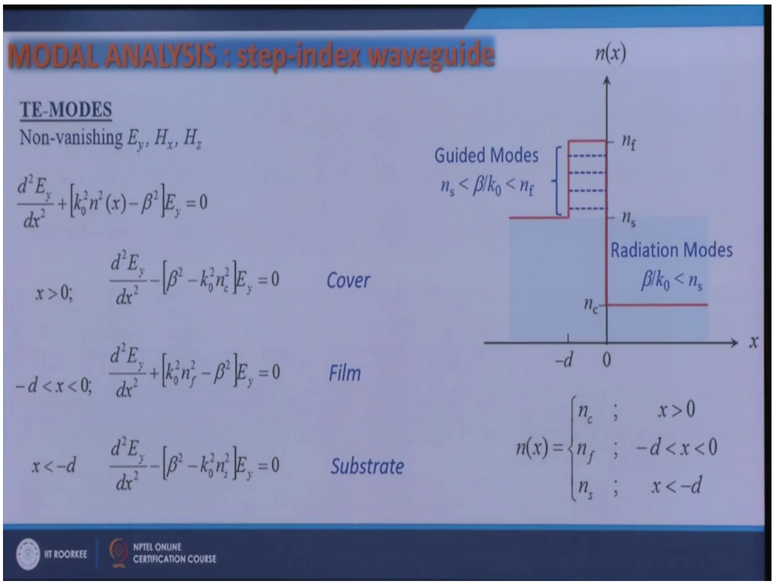

So, this is a typical graded index ion exchange waveguide. Let us do the modal analysis of this step index waveguide, the step index planar asymmetric waveguide. So, this is the refractive index profile and I can write down the refractive index in various regions as n x is equal to n c, which is defined by the region x greater than 0 which is the cover, then in the region minus d less than x less than 0 the refractive index is n f which is the film region, and for x smaller than minus d which represents the substrate region, the refractive index is n s.

Now, the modes of this waveguide are here guided modes, whose refractive index is lie between n s and n f. So, these are some representative guided modes, and for beta over k dot smaller than n s, the field would radiate out in the substrate region and for beta over k naught is smaller than n c the field would radiate out in the cover region as well. So, in this region you will have radiation modes.

We are interested in the guided modes of the structure, which are defined in this range, and let us first do the TE mode analysis of the waveguide, and as I can see that the refractive index variation is in x direction, and I am considering propagation in z direction. So, the non vanishing components corresponding to TE modes are E y, H x and H z.

I know from my previous analysis that the wave equation satisfied by TE modes of such a waveguide is given by d 2 E y over d x square, plus k naught square n square of x, minus beta square E y is equal to 0. So, the procedure is again the same I write down this equation in all the three regions, I write down the solutions of these three equations and then apply boundary conditions to match the solutions.

So, this is the equation in cover region x greater than 0, which is d 2 E y over d x square, minus beta square minus k naught square n c square E y is equal to 0, because in this region n x is equal to n c, and I have taken minus sign outside and represented it in this form because beta over k naught is greater than n c, so that this quantity is positive for guided modes.

In the film region which is defined by minus d is less than x is less than 0, the equation becomes d 2 E y over d x square, plus k naught square n f square minus beta square E y is equal to 0, again for guided modes this quantity in the square brackets is positive. In the substrate region defined by x less than minus d, the equation is d 2 E y over d x square minus beta square minus k naught square n s square, E y is equal to 0 and again for guided modes this quantity is positive.

So, now I can represent this as gamma c square, this as kappa f square and this as gamma s square and then find out what are the solutions of these three equations.

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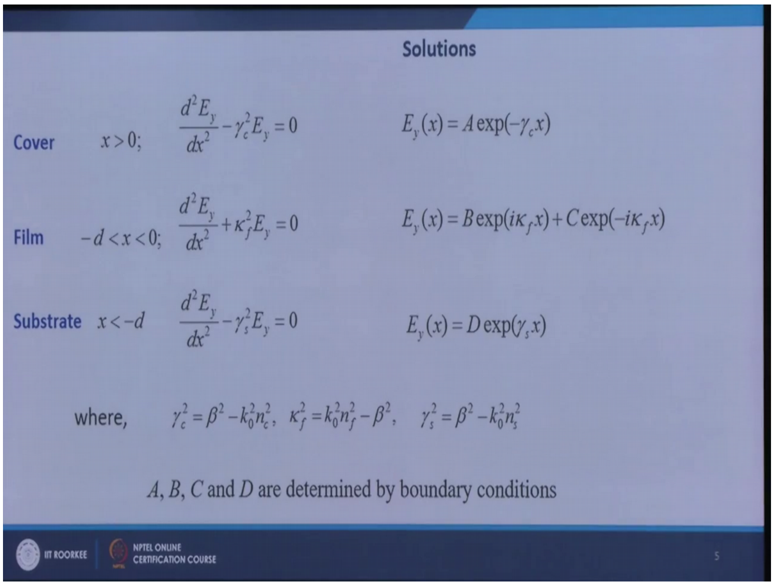

So, these are the three equations now, where gamma c square is equal to beta square minus k naught square n c square, kappa f square is k naught square n f square minus beta square, and gamma s square is beta square minus k naught square n s square.

So, what are the solutions? The solution of this differential equation is A e to the power minus gamma c x, and e to the power plus gamma c x. And as you know that we are interested in guided modes. So, we cannot have exponentially amplifying solution. So, I am only considering the exponentially decaying solutions in cover regions and substrate region.

Now, in the film this equation will give you the oscillatory solutions, which I write as B e to the power i kappa f x plus C e to the power minus i kappa f x, and in the substrate region it is D e to the power gamma s x because x is a smaller than minus d. So, it is negative. So, this again gives you exponentially decaying solutions. Now a b c and d are constants and these constants are determined by the boundary conditions.

So, let us apply boundary conditions and obtain relationships between a b c d and obtain the Eigen value equation or characteristic equation which is satisfied by the propagation constants of the guided modes.

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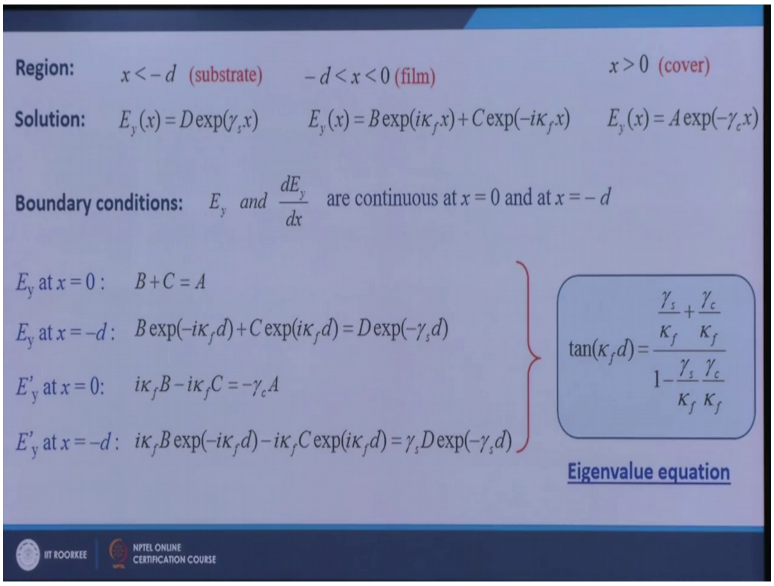

So, these are the solutions in different regions in substrate in the film and in the cover and what are the boundary conditions? Boundary conditions are again the tangential components of e and h are continuous, and the tangential components here are E y and h z. So, they give me that E y and d E y over d x because h z is related to E y as d E y over d x. So, they are continuous at the interfaces x is equal to 0 and at x is equal to minus t.

So, let us apply these boundary conditions. So, if I look at the continuity of E y at x is equal to 0. So, I apply it here. So, if I approach from the film and if I approach from the cover towards x is equal to 0. So, I will get B plus C is equal to A. Now continuity of E y at x is equal to minus d which is the substrate film interface, this gives me B e to the power x is equal to minus d. So, B e to the power minus I kappa f d plus C e to the power I kappa f d is equal to D, D e to the power minus gamma s d.

Now, let us see the continuity of the derivatives at x is equal to 0 and at x is equal to minus t. So, E y prime at x is equal to 0 gives me I kappa f B minus I kappa f C is equal to minus gamma c A and continuity of E y prime at x is equal to minus d interface gives me I kappa f B e to the power minus I kappa f d minus I kappa f C, e to the power I kappa f d is equal to gamma s d e to the power minus gamma s d.

So, I have now four equations relating these four constants a b c and d, and by mathematical manipulation of these four equations I can do 2 things one is obtain b c and d in terms of a, and then eliminating a b c d all together and form a transcendental equation in beta which is also known as Eigen value equation or characteristic equation. So, that equation comes out to be tan kappa f d is equal to gamma s over kappa f plus gamma c over kappa f divided by 1 minus gamma c over kappa f times gamma c over kappa f.

So, this is the equation which is satisfied by beta because the only unknown here is beta, beta appears in kappa f gamma s and gamma c. So, if I solve this equation and find out the roots of this equation then I can know what are the propagation constants of the modes, what are the modes, which are supported by this asymmetric step index planar waveguide.

As we have done earlier also that it is always a good idea to obtain everything in normalized parameters. So, that we are more or less independent of waveguide parameters, here we cannot have complete independence from waveguide parameters, but to certain extent it is still we can draw some universal curve. So, how do we define normalized parameters here? First is normalized frequency v.

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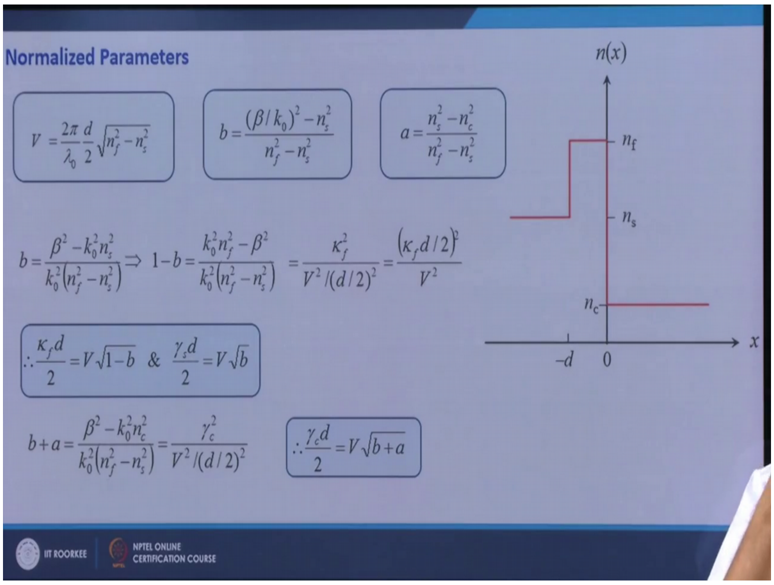

If you remember that in symmetric planar waveguide we had only 2 refractive indices n 1 and n 2, n 1 is greater than n 2. So, there we had defined it in terms of n 1 square minus n 2 square.

Now, now in place of n 1 I have n f, but low refractive indices are now up to n c and n s which one we should take in order to define V. So, it is very simple to choose because n s defines the cut-off of guided modes. So, I should choose n s because as soon as the effective index of a mode falls below n s it is no more guided. So, I choose n s to define the normalized frequency v here, and similarly for defining the normalized propagation constant b I choose n s.

So, b is defined as beta over k naught square minus n s square over n f square minus n s square. There is another parameter here in this structure, which tells you how asymmetric this waveguide is. The asymmetry in the structure is introduced by the different values of n s and n c. So, what is the difference between n s and n c? So, let us define by asymmetry parameter a, which is given as n s square minus n c square divided by n f square minus n s square.

So, now I have here three normalized parameters in place, and now I can represent my transcendental equation or Eigen value equation in terms of these normalized parameters. If you remember that in the transcendental equation the three terms appear which are kappa f, gamma s and gamma c, which contain beta.

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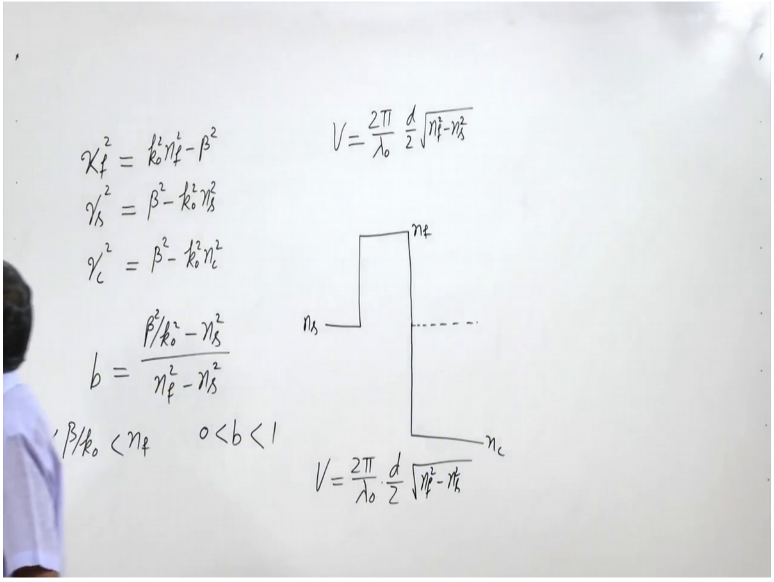

So, now I will have to represent these kappa f gamma s and gamma c in terms of v b and a. And we can see that kappa f square is given by k naught square n f square, minus beta square. Gamma s square is given by beta square minus k naught square and s square, and gamma c square is given by beta square minus k naught square and c square.

So, keeping these in mind and looking at the expressions here I would now try to obtain kappa f gamma c and gamma s in terms of v b and a. So, let us first try for kappa f, kappa f has k naught square n f square minus beta square, and I can see that if I do 1 minus b then I can obtain this k naught square n f square minus beta square in the numerator of this. So, b is given by this. So, if I do 1 minus b it becomes k naught square n f square minus beta square, divided by k naught square n f square minus n s square, and 2 pi over lambda naught is k naught.

So, if I multiple here by d by 2 and d by 2, then I can have kappa f square here or I have kappa f square directly here and this is nothing, but v square divided by d by 2 square. So, this becomes kappa f d by 2 whole square divided by v square. So, this gives me kappa f d by 2 kappa f d by 2 is equal to v times square root of 1 minus b.

If I look at gamma s square, gamma s square is b dash square minus k naught square n s square. So, it is directly appearing here beta square minus k naught square n s square divided by k naught square this thing. So, this will immediately give me gamma as d by 2 is equal to v times square root of b. What about now gamma c square, which is defined as beta square minus k naught square n c square. So, I can see from here that if I do b plus a then I can only obtain gamma c square in the numerator. So, I do b plus a. So, it becomes beta square minus k naught square n c square divided by this. So, which is gamma c square over v square divided by d by 2 whole square, and this gives me gamma c d by 2 is equal to v times square root of b plus a.

So, in this way I have now obtained kappa f gamma c and gamma s in terms of v b and a. So, I can now represent my Eigen value equation in normalized parameters. So, this equation simply becomes tan 2 v square root of 1 minus b, is equal to square root of b divided by 1 minus b. Which is coming from gamma s term, plus b plus a divided by 1 minus b square root which is coming from the cover term gamma c term. Divided by 1 minus square root of b divided by 1 minus b and the square root of b plus a divided by 1 minus b.

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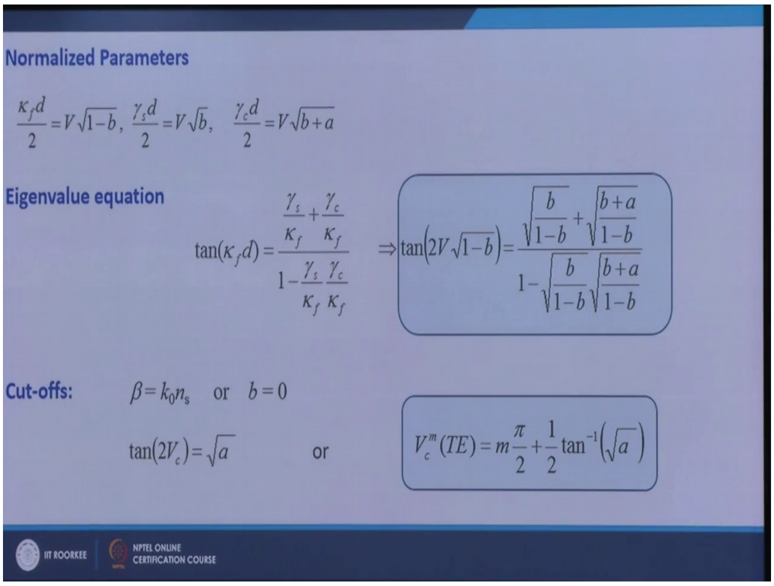

$$\frac{\kappa_f d}{2} = V\sqrt{1-b}, \quad \frac{\gamma_s d}{2} = V\sqrt{b}, \quad \frac{\gamma_c d}{2} = V\sqrt{b+a}$$

$$\tan(\kappa_f d) = \frac{\frac{\gamma_s}{\kappa_f} + \frac{\gamma_c}{\kappa_f}}{1 - \frac{\gamma_s}{\kappa_f}\frac{\gamma_c}{\kappa_f}} \Rightarrow \tan\left(2V\sqrt{1-b}\right) = \frac{\sqrt{\frac{b}{1-b}} + \sqrt{\frac{b+a}{1-b}}}{1 - \sqrt{\frac{b}{1-b}}\sqrt{\frac{b+a}{1-b}}}$$

$$\beta = k_0 n_s \quad \text{or} \quad b = 0$$

$$\tan(2V_c) = \sqrt{a} \quad \text{or} \quad V_c^m(TE) = m\frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\sqrt{a}\right)$$

And of course, since b is equal to b dash square over k naught square, minus n s square divided by n f square, minus n s square and for cut off beta over k naught is equal to n s or for guided volts, beta over k naught lies between n s and n f. So, b would lie as usual between 0 and 1. So, I solve this equation for a given value of v, v is if you remember v is defined as 2 pi over lambda naught times d by 2, times square root of n f square minus n s square. So, this v contains all the waveguide parameters and the wavelength. So, for a given waveguide and wavelength I have v and for that value of v I can solve this equation to obtain the normalized propagation constant b.

What are the cut offs? Cut offs are defined by beta is equal to k naught n s or b is equal to 0. So, if I put it there then the cut off equation becomes tan 2 V c is equal to square root of a or cut off for TE mode m th TE mode is now given by V c m is equal to m pi by 2 plus half tan inverse a square root of a. You can see here that the cut off of even TE 0 mode is finite.

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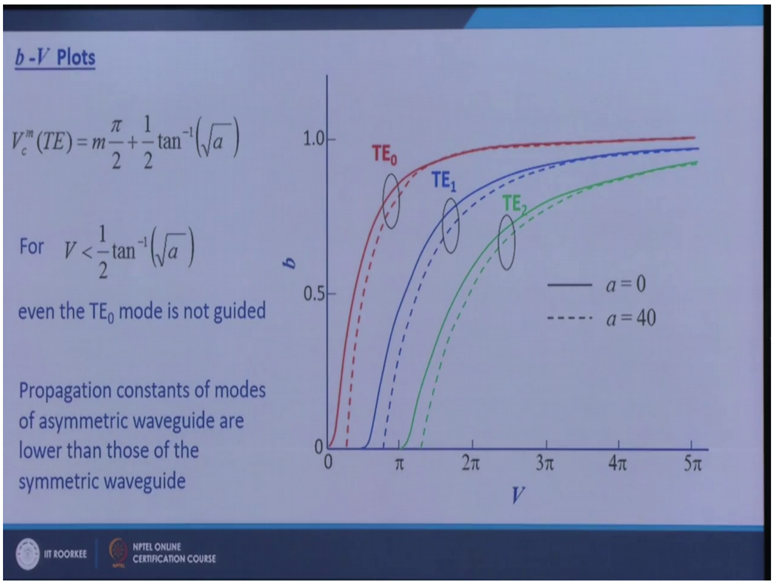

So, in a particular range of V which is defined by V less than half tan inverse square root of a even the TE 0 mode is not guided.

This is the difference as compared to the symmetric planar waveguide. In symmetric planar waveguide t 0 mode has 0 cut off. So, t 0 mode was always guided, but here it is not the case. Now let us solve this equation the characteristic equation or Eigen value equation for different values of V, and plot the roots as obtained as a function of v. So, here I have plotted the roots for both the cases a is equal to 0 which represents the symmetric planar waveguide, and then for a very high value of a which is 40 for asymmetric planar waveguide. These solid lines are corresponding to symmetric planar waveguide a is equal to 0, and these dash lines are corresponding to asymmetric planar waveguide

And what I can see that t 0 mode has cut off here, t 1 mode has cut off here, t 2 mode has cut off here. In case of symmetric planar waveguide this was the cut off v c m is equal to m pi by 2 was the corresponding cut off. Now all the cut offs have been shown shifted by this much amount which is half tan inverse of square root of a. So, this is one thing another thing that I see here is that if I take a particular value of v then the propagation constant of the mode is now smaller than the propagation constant of the corresponding symmetric waveguide mode.

And it is it is understandable that because asymmetric is introduced by introducing the cover region. So, if this is the symmetric waveguide, this is the symmetric waveguide. So, this is n s this is n f this is again the level n s, but now in asymmetric planar waveguide I have n c. So, I am reducing the refractive index in the cover region. So, the effect is to pull down the effective indices of the modes towards lower side so that we can see from here itself that the propagation constants of asymmetric planar waveguides are smaller than those corresponding to symmetric planar waveguide.

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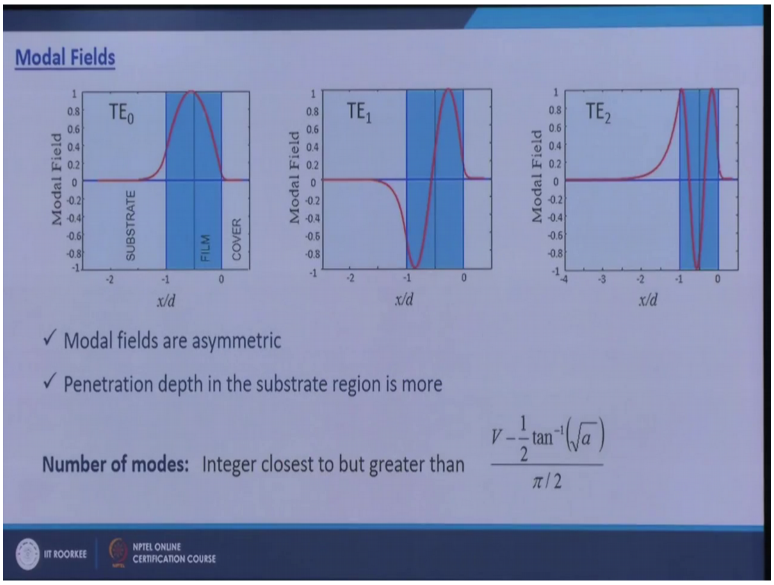

Let us look at modal fields. So, these are the modal fields of TE 0 TE 1 and TE 2 modes of a typical asymmetric planar waveguide. So, I can see that the modal fields are asymmetric and you can see that the penetration gap is more in the substrate and less in the cover which is understandable, because here at this interface the index contrast is smaller as compared to the index contrast at this interface ok.

So, the field extends more into the substrate region as compared to in the cover region. How many modes are supported? If you go back to symmetric planar waveguide the number of modes are the integer which is closest to, but greater than v over v over pi by 2, but now all the cut offs are shifted by this much amount. So, the number of modes would now be v minus half tan inverse square root of a divided by pi by 2. So, you obtain this number and find out the integer closest to, but greater than this number.

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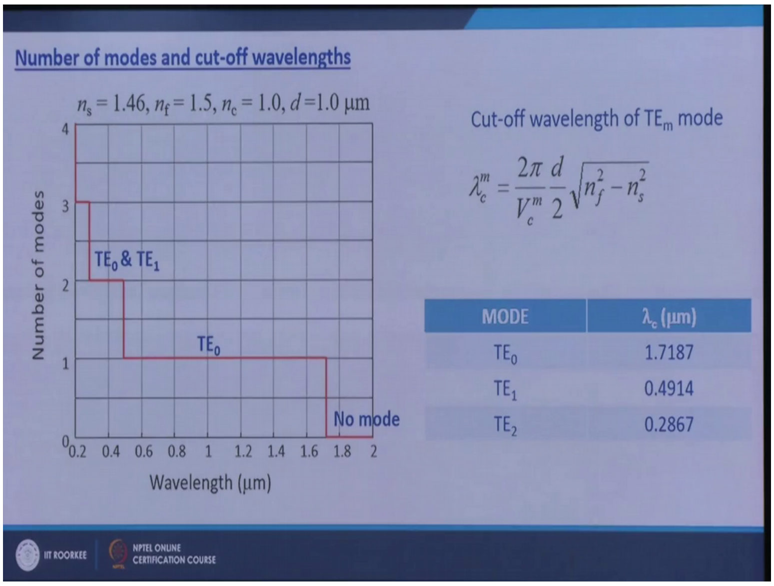

What are the cut off wavelengths of various modes and if I change the wavelength how the number of modes would change. So, I can see from here that the cut off wavelength for m th TE mode would be given by lamda c m is equal to 2 pi over V c m, d by 2 square root of n f square minus n s square which comes directly from the definition of normalized frequency v. So, I can write it down also which we will quite often use.

V is equal to 2 pi over lambda naught times d by 2, times square root of n f square minus n s square. So, from here I get these cut off wavelengths, now if I find out the cut off wavelengths corresponding to various modes then I see that for TE 0 mode the cut off wavelength is 1.7 187 for these parameters of waveguide, and for TE 1 mode it is 0.4914 and for TE 2 mode it is 0.2867.

So, if I start from a longer wavelength let us say 2 micro meter, then until I cross this t 0 mode is not guided. So, from here to here there is no mode guided by the structure, and as soon as I cross this go below this wavelength, then t 0 mode starts appearing and t 0 mode would be guided now for all the wavelengths is smaller than this.

If I further reduce the wavelength and as soon as I go below 0.49 micrometer TE 1 mode starts appearing and below these this wavelength I will have both TE 0 and TE 1 and so on. So, this is how the waveguide would guide different modes if I change the wavelength. How the thickness affects the guided modes. So, from here I can find out the cut off thickness of mth TE mode from here itself. So, if I find out d in terms of v now and put the cut off of various modes in terms of v, then I can find out the cut off of thickness ok.

So, if I have 0 thickness it means no waveguide no mode. If I start increasing the thickness then up to 0.58 micron there is no mode guided because t 0 mode has finite cut off and as soon as I cross this then TE 0 mode starts appearing, and when I cross 2.03 micron then TE 1 mode starts appearing and so on. So, as I increase the waveguide thickness the number of modes start increasing I should have the label here which is d in micron. So, the label here is t in micron.

Let us work out few examples. So, I take a dielectric step index asymmetric planar waveguide.

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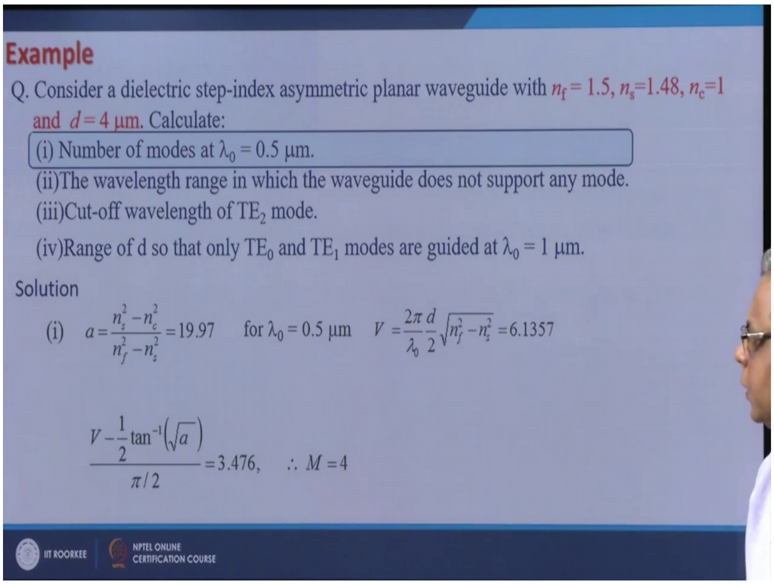

Defined by n f is equal to 1.5 and n s is equal to 1.48 and n c is equal to 1, and d is equal to 4 micrometer. Now the first thing is to calculate the number of modes at lambda naught is equal to 0.5 micron. So, I first calculate the value of asymmetric parameter a which comes out to be about 20, then I find out what is the value of v at lambda naught is equal to 0.5 micrometer, and this comes out to be about 6.13.

Then I find out this number v minus half tan inverse of square root a, divided by pi by 2. So, this comes out to be 3.4. So, the number of modes are 4.

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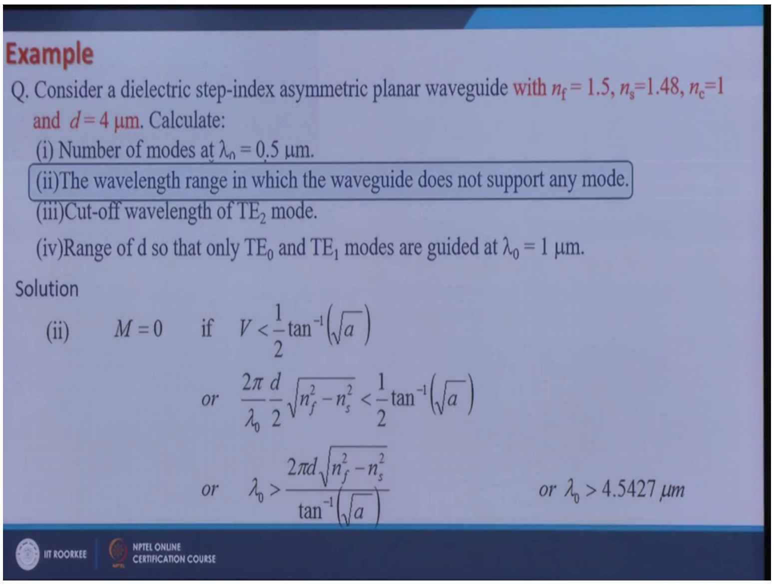

The second is wavelength range in which the waveguide does not support anymore. So, I know there would not be any more support it if v is less than half tan inverse of square root a, which if I put the value V here the expression for V here. Then this gives me the condition in terms of lambda naught as lambda naught should be greater than this, and if I plug in all these numbers I find out for lambda naught greater than 4.5427 micrometer the waveguide would not support anymore.

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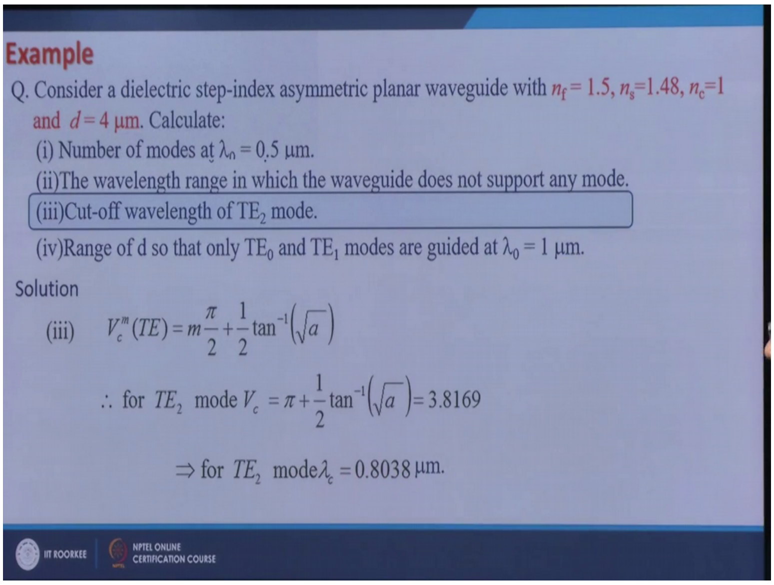

Third is what is the cut off wavelength of TE 2 mode? So, I find out what is the V value for cut off of TE 2 mode. So, which is given by this with m is equal to 2. So, if I put m is equal to 2 then v c is equal to this, and the cut off V c for cut off v for TE 2 mode is this if I translate it to the wavelength it comes out to be 0.8038 micrometer.

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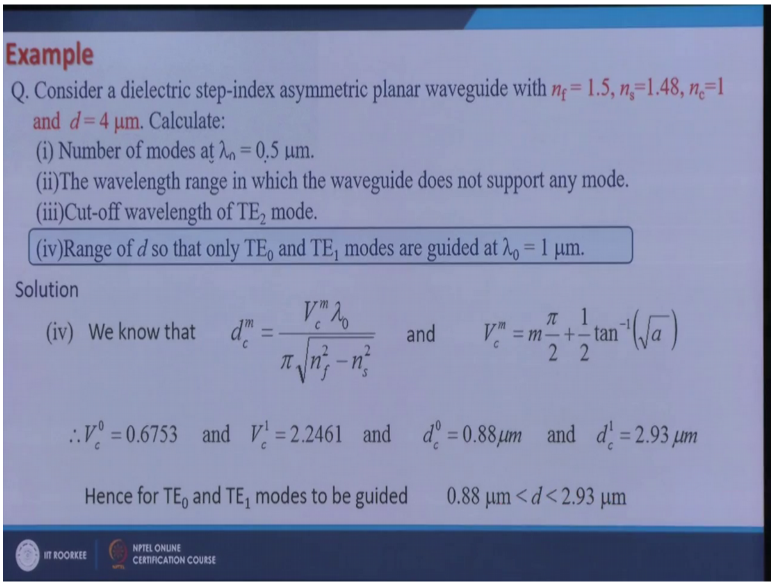

What is the range of d so, that only TE 0 and TE 1 modes are guided at lambda naught is equal to 1 micrometer.

So, I know that for mth mode the cut off thickness is this, and I want TE 0 and TE 1 both the modes guided. So, I find out this d c for both the modes, and I also know this V c m for mth mode is given by this. So, I find out V c 0 for t 0 mode, V c is this for TE 1 mode V c is this correspondingly if I now find out d c for these 2 modes, then for TE 0 mode cut off thickness is 0.88 micrometer, and for this is 2.93 micrometer. So, if TE 0 and TE 1 modes are to be guided then thickness should be between this and this.

So, this is all in this lecture in the next lecture, we will extend the analysis to TM modes.

Thank you.