Foundation of Quantum Theory: Relativistic Approach
Change in atomic characteristics via field interactions
Perturbation theory 1.6
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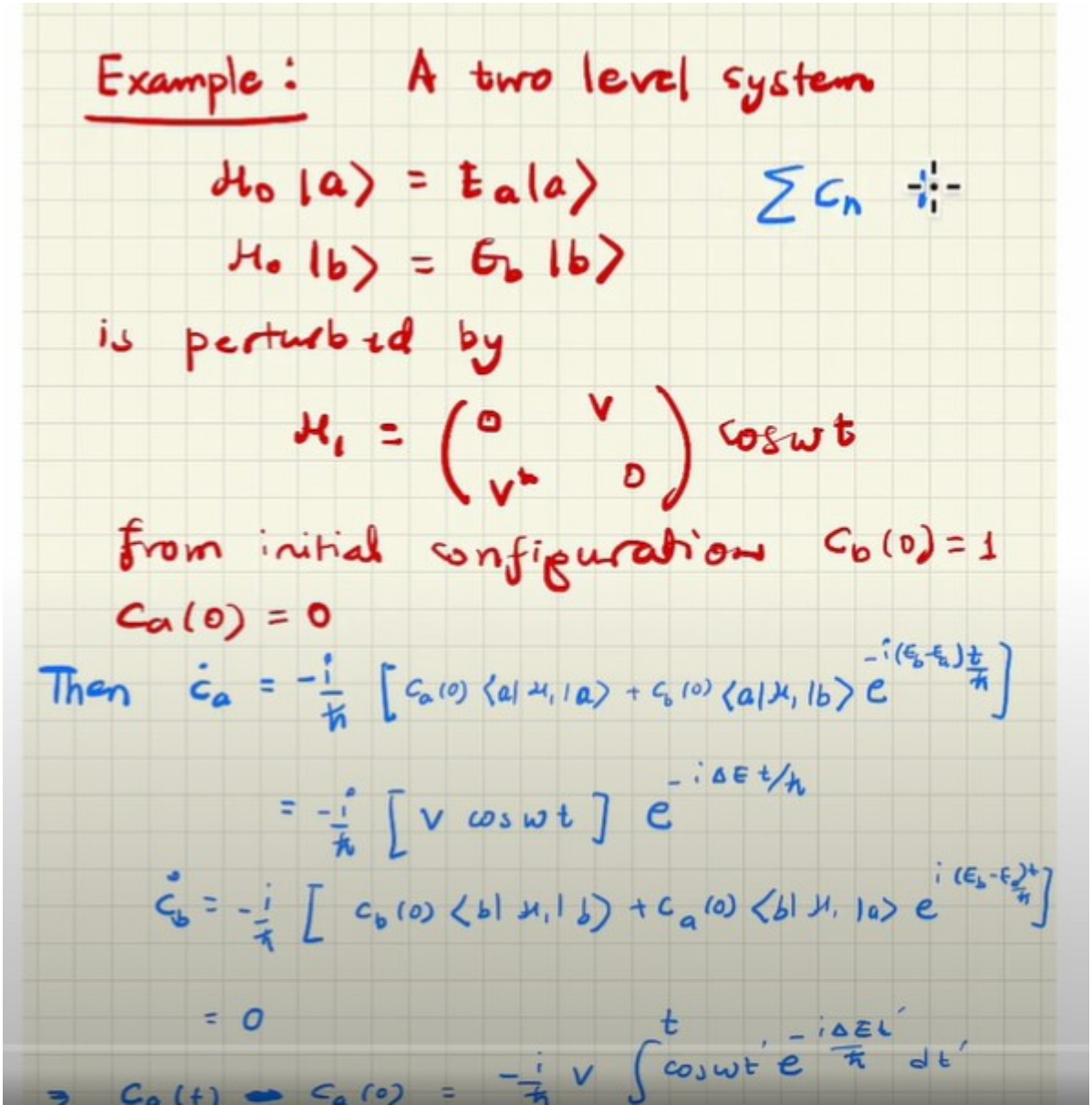
Lecture- 06

So let us resume our discussion of time dependent perturbation theory. In that discussion in the previous class what we learned that if the Hamiltonian develops a time dependency the effect of that is felt across the coefficients of superposition $c_n(t)$. Typically we decompose any state like this and in time dependent perturbation theory these function in themselves become time dependent, so that after evaluation in time, they not only remain $c_n(t)$, but they become $c_n(t)\, e^{-iEn(t)\,\hbar}|n>$. So, there is an explicit time dependence these $c_n(t)$'s are supposed to come up with in time dependent perturbation. For a particular case of a two-level system where only two eigenstates of the free Hamiltonians are there a and b with eigenvalues E_a and E_b , let us perturb the system and add a Hamiltonian which is a function of time in the sense of oscillatory function $cos\omega t$, ω being the frequency of oscillation and then some operator whose matrix representation in this two-level basis looks like this off diagonal matrix.
There is no diagonal component and the off diagonal components are complex conjugate to each



other.And now in this setting if we want to study the process in which a system was initially in the excited state, that means $c_b(t)$ is equal to 0 was 1, while $c_a(t)$ at time t is equal to 0 was 0.

Example: A two level system

$\mathcal{H}_0 |a\rangle = E_a |a\rangle$

$\mathcal{H}_0 |b\rangle = E_b |b\rangle$

$\mathcal{H}_0 |a\rangle = E_a |a\rangle$

H0 $|b\rangle = E_b |b\rangle$ *is perturbed by*

$$\begin{pmatrix} 0 & V \\ V^2 & 0 \end{pmatrix} cost\ \omega t$$

From initial configuration of $c_b(0) = 1$

$c_a(0) = 0$

$$\dot{c}_a = \frac{-i}{\hbar} \quad [c_a(t)\ (0) < a|\ \mathcal{H}_I|a\rangle + c_b\ (0) < a|\ \mathcal{H}_I|b\rangle \quad e^{\frac{-i(E_b - E_a)t}{\hbar}}$$

$$= \quad \frac{-i}{\hbar}[V \cos\omega t]\, e^{-\Delta E \frac{t}{\hbar}}$$

$$\dot{c}_b = \frac{-i}{\hbar} \quad [c_b(o) < b|\ \mathcal{H}_I|b\rangle + c_a(t)\ (0) < a|\ \mathcal{H}_I|a\rangle \quad e^{\frac{-i(E_b - E_a)t}{\hbar}} \quad] = 0$$

$$= \quad -i\hbar\frac{V}{2}\left[\frac{e^{i(\omega - \frac{\Delta E}{\hbar})t} - 1}{i(\omega - \frac{\Delta E}{\hbar})} - -i\hbar\frac{V}{2}\frac{e^{i(\omega + \frac{\Delta E}{\hbar})t} - 1}{i(\omega + \frac{\Delta E}{\hbar})}\right]$$

Defining $\frac{\Delta E}{\hbar} = \boldsymbol{\omega}$ and using the fact that $\frac{e^{ix} - 1}{ix} = \frac{e^{ix/2}(e^{ix/2} - e^{-ix/2})}{\frac{2ix}{2}} = \frac{e^{ix/2}\sin x/2}{x/2}$

$$c_a(t) = -i\hbar\frac{V}{2}\left[e^{i(\omega - \Omega)\frac{t}{2}}\frac{\sin(\omega - \Omega)b/2}{(\omega - \Omega)/2} - e^{i(\omega + \Omega)\frac{t}{2}}\frac{\sin(\omega + \Omega)b/2}{(\omega + \Omega)/2}\right]$$

On the other hand $\dot{c}_b(t) = c_b(0) = 1$

(upto linear order of $\mathcal{H}_I$)

So, that is to say initially the state was, the system was in the excited state.

And we want to know if the system will remain in excited state throughout or due to the fact that Hamiltonian will change over time, the system would develop some probability of being found in the ground state as well. So in order to look at that, we look at the differential equation which will govern the growth or the decay of the two coefficients $c_a(t)$ and cb. Since due to the fact that the Hamiltonian is completely off diagonal, its diagonal entries are 0, so the first term will vanish. So, this does not survive because the diagonal entry is 0. So, all I am left with is the right hand side, in the right hand side is the second term. The second term is $V\cos\omega t$, so that $V\cos\omega t$ appears over here. The exponential is $e^{-iEt/\hbar}$ which is this and then cb was supposed to come up. The exponential is $e^{-\Delta E/\hbar}$ which is this and then $c_b(t)$ was supposed to come up. So, since I am looking at a $\dot{c}_a(t)$ very close to initial time, then I should approximately put $c_b(t)$ at a time just away from 0 which is slightly away from them, this can be approximated as 1. So, therefore, this $c_b(0)$ has been put to value 1. So, therefore, the growth of $c_a(t)$, $\dot{c}_a(t)$ actually controlled by the $c_b(t)$ which was supposed to be, which we have put it to 1. But know for the fact that if I do not do this exercise, I do not put it to 1, then in the full generality, the term which is

coming over here is just $c_b(t)$. Similarly, if I look at the growth rate of the other coefficient $c_b(t)$, $\dot{c}_b(t)$, then $-i/\hbar$ term will be common, then I will have a $c_b(t)$, let us say this will be at time t, this will be at time t as well. But any general time, this equation they have to satisfy. So, I would have this differential equation, the first term comes with $c_b(t)$ and the second diagonal entry this part and then $c_a(t)$ and the second of diagonal entry this part, they will come along. Now, in this $\dot{c}_b(t)$ equation the first term becomes identically 0 because the second diagonal element is 0. So, this fact this object is 0 while $c_a(t)$ is supposed to be here but at a time very close to initial time I can safely put $c_a(t)$ to value 0 meaning it looks like that it is very small number if I am very close to time t is equal to 0. So $c_a(t)$ of 0 is 0 and of $c_a(t)$ some t which is very close to 0 should be very close to 0 as well. So therefore this is almost equal to 0. So therefore, it looks like that $c_b(t)$, the second coefficient, the coefficient of the second eigenstate does not change much. This wavy 0 is that it does change because $c_b(t)$, $c_b(t)$ is not identically 0. $c_a(t)$ of 0 was 0, but $c_a(t)$ of some small time will be very small. So therefore, I am putting it to wiggly 0. If I look at the first equation which we just derived, $\dot{c}_a(t)$ is just V cosωt. Remember, V cosωt is this matrix element, which is $<a|\mathcal{H}|b>$. If I look at the first equation which we just derived, $\dot{c}_a(t)$ is just V cosωt. Remember, V cosωt is this matrix element, which is $<a|\mathcal{H}|b>$. This is $<b|\mathcal{H}|a>$. So, $<b|\mathcal{H}|a>$. So, $\dot{c}_a(t)$ is the off-diagonal matrix element here, multiplied with the exponential, which is $e^{-i\,\Delta E t/\hbar}$ and $c_b(t)$. Now, if I have to integrate this equation, then I integrate left hand side I will get the $c_a(t)$ -$c_a(t)$ of 0, right hand side I will get this equation where a $c_b(t)$ should be there. And again here also a $c_b(t)$should be there. So, this integration I will do. You look for the fact that if I am doing an exercise for t very close to 0, all the c', b', t' can be put to approximately value 1. Because c, b start with value 1 and it will remain at value 1 up to somewhat smallest time, just after 0 time, it should not drop down to something very different from 1 meaning at the leading order of the perturbation theory it is this can be this $c_b(t')$ during the time duration 0 to t where t is very small can be approximated to 1 and therefore this expression can be used to compute for small times this expression is good. So, just to note down for small times this box equation can be used. For large times, I cannot use that because $c_b(t)$ at large time, I do not know whether it is close to 1 or not. So, one can compute this integration which we had done in the previous discussion session and we end up in getting these sinc functions, the two sinc functions which we have derived in the last class as well. So, what we can do, we can formally go ahead and try to see what happens to their respective probabilities. So, to do that let us look at the case when I was looking at the differential equation for $\dot{c}_b(t)$ once more. So, let me write it down here. $\dot{c}_b(t)$ is supposed to be $-i/\hbar$, then $c_a(t)$ at time, $c_b(t)$ and then b perturbation terms b the diagonal element + $c_a(t)$ and then the
$<b|\mathcal{H}|a>$ and $e^{-i\omega t/\hbar}$. This is what we will get for a general expression which we have written upstairs as well. So, this equation I have just reproduced, the second equation here. So, that is what we have reproduced downstairs. Now, we know that this matrix element, diagonal matrix element, this is always 0. Therefore, I will put this quantity to 0 value, this will go away. I will be only left with $-i/\hbar$ times $c_a(t)$ and then this matrix element which is $<b|\mathcal{H}|a>$ and $e^{i\omega t}$. This equation I have to integrate. So, let us integrate on the both sides, do this dt integral from 0 to t, let us say $dt'\dot{c}_b(t)$.

$$\dot{c}_b = -\frac{i}{\hbar}\left[c_b(t) \langle b|H_1|b\rangle + c_a(t)\langle b|H_1|a\rangle e^{i\frac{\Omega}{2}t}\right]$$

$$\int_0^t dt' \dot{c}_b = -\frac{i}{\hbar}\int_0^t c_a(t') \langle b|H_1|a\rangle e^{i\Omega t'} dt'$$

So, at a subsequent time

$$c_b(t) - c_b(0) = -\frac{i}{\hbar}\int_0^t dt'\, c_a(t') \langle b|H_1|a\rangle$$

$$= \frac{-i}{\hbar}\int_0^t dt' \langle b|H_1(t')|a\rangle e^{i\Omega t'}$$

$$\left(\frac{-i}{\hbar}\right)\int_0^{t'} dt'' \langle a|H_1(t'')|b\rangle e^{-i\Omega t''}$$

$$= -\frac{1}{\hbar^2}\left(\int_0^t dt' H'_{ba}(t') e^{i\Omega t'} \int_0^{t'} dt'' H'_{ab}(t'') e^{-i\Omega t''}\right.$$

Upto leading order

$$c_a(t) = \frac{-i}{2\hbar} V\left[e^{i\left(\frac{\omega-\Omega}{2}\right)t} \frac{\sin\left(\frac{\omega-\Omega}{2}\right)t}{\left(\frac{\omega-\Omega}{2}\right)} - e^{i\left(\frac{\omega+\Omega}{2}\right)t} \frac{\sin\left(\frac{\omega+\Omega}{2}\right)t}{\left(\frac{\omega+\Omega}{2}\right)}\right]$$

Thus, de-excitation probability

$$|c_a(t)|^2 = \frac{|V|^2}{4\hbar^2} \frac{\sin^2\left(\frac{\omega-\Omega}{2}\right)t}{\left(\frac{\omega-\Omega}{2}\right)^2} \times$$

$$\left| \left\{ 1 - e^{i\Omega t} \frac{(\omega-\Omega)}{(\omega+\Omega)} \frac{\sin\left(\frac{\omega+\Omega}{2}\right)t}{\sin\left(\frac{\omega-\Omega}{2}\right)t} \right\} \right|^2$$

In the limit $\omega \to \Omega$

$$|c_a(t)|^2 = \frac{|V|^2}{4\hbar^2} t^2 \left| \left\{ 1 - \frac{e^{i\Omega t}}{t\left(\frac{\omega+\Omega}{2}\right)} \sin\left(\frac{\omega+\Omega}{2}\right)t \right\} \right|^2$$

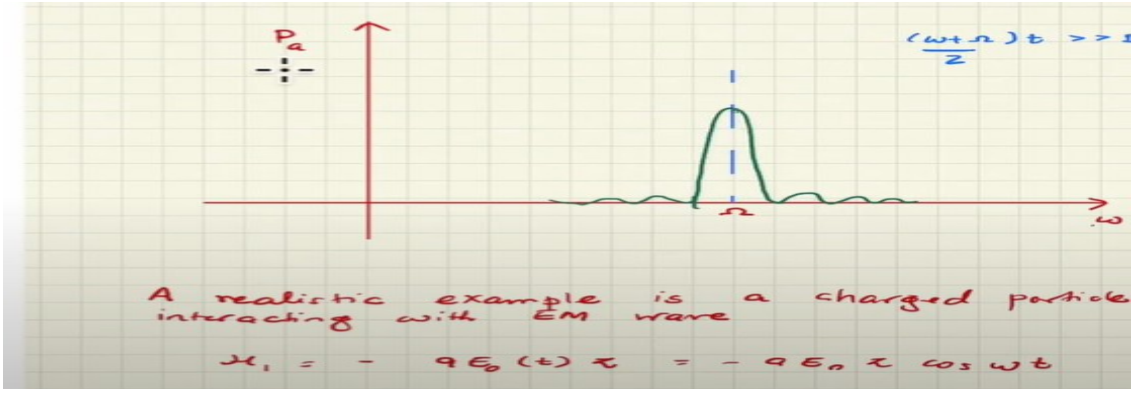
For $\left(\frac{\omega+\Omega}{2}\right)t >> 1$ we can drop the second term, and

If I do that, this thing whole will also undergo time integration from 0 to t. Left hand side I will get $c_b(t)$ - $c_b(0)$. The time derivative integration over time will just give me the functions at the boundaries and right hand side will be the integration I just wrote down over here. Notice that in the right hand side ket primed is appearing as well. So, in order to know what is the $c_b(t)$ I have to know what were the ca's in the meanwhile, 0 to t, what were their values. So, they need to get integrated. Just knowing the matrix element of the perturbation Hamiltonian is not sufficient. I need to know what were the coefficients $c_a(t)$ as well. So, $c_a(t)$ can be obtained from the equations which we just wrote down here. True equation was $c_b(t)$ was here, but for small times I could ignore $c_b(t)$ to be 1. But for all general times, $c_b(t)$ is supposed to come over there. So, let me write down the equation for $\dot{c}_a(t)$ as well. So, let me use this space. So, $\dot{c}_a(t)$ was supposed to be $-i/\hbar\ c_a(t)$ then this $\mathcal{H}|a\rangle + c_b(t)\langle a|\mathcal{H}|b\rangle$ and then $e^{i\omega t}$. So, this would be the equation for $\dot{c}_a(t)$. Again due to the fact that the diagonal element is 0, $\dot{c}_a(t)$ would be just $-i/\hbar$ and $c_b(t)\ \langle a|\mathcal{H}|b\rangle$ and $e^{-\omega t}$. That means if I integrate this equation left hand side I will get $c_a(t)$ at time t - $c_a(t)$ at time 0 and right hand side would be just integration of $-i/\hbar$ as the integration of $c_b(t')$ then the matrix element $\langle a|\mathcal{H}|b\rangle$ and $e^{-i\omega t'}$. This is what I will get. And since $c_a(0)$ is 0, the left hand side only survives with one entry which is $c_a(t)$. This $c_a(t)$ over here is required to be fed in here in order to know what is $c_b(t)$. So, I just used this integral form which is $-i/\hbar$, this integral with a $c_b(t)$ here coming along and matrix element between a, $\mathcal{H}$, b and $e^{-i\omega t'}$. All these things I have written here. So, this is my $c_a(t')$. Remember, $c_a(t)$ if I wanted to compute the integration has a range from 0 to t. So, the upper limit of this appears as a functional dependency here. So, therefore, in this expression I want t primed appearing in the function $c_a(t)$ t prime. So, the integral representation will also have integration from 0 to t prime since I am writing for $c_a(t)$ t prime. So, ultimately you see the $c_b(t)$ - $c_b(0)$ which was 1 is this expression of a double integral. In a double integral, matrix element $\langle b|\mathcal{H}|a\rangle$ has appeared or $\mathcal{H}$, call it, $\langle b|\mathcal{H}|a\rangle$ has appeared and $\langle a|\mathcal{H}|b\rangle$ has also appeared. And they are at two different times, but they are complex conjugate of each other. two matrix elements of H perturbation Hamiltonian has appeared in product to determine the deviation of $c_b(t)$ from being 1. In the integration of $c_a(t)$, this equation here, only one matrix element had appeared. While in the definition of $c_b(t)$, T's deviation from being 1, two matrix elements have appeared, double second order object while $c_a(t)$ is a first order object. $c_b(t)$ as a structure that it is 1, so I can take this - 1 on the right hand side, $(1 - 1/\hbar)^2$ and then this double integral which I am calling hash. So, this is the structure I will get that it tells me that $c_b(t)$ becomes different from 1. in second order of matrix element. Two matrix elements put together will decide how much different it is from one. Now, we had both equations in hand, $c_a(t)$ is just single integration of matrix element $\mathcal{H}$, and $e^{-i\omega t}$, while $c_b(t)$ is different from one in two powers of $\mathcal{H}$as well as double integrals. One for dt running from 0 to t and other for dt double primed running from 0 to t primed. And then inside I have $\langle a|\mathcal{H}(t')|b\rangle$as ' and $e^{i\omega t'}$and as well $\langle a|\mathcal{H}(t'')|b\rangle e^{-i\omega t''}$ So, these are the two guiding equations which we will have. Now let us put it in our problem. I know what is the off diagonal element for my system. My Hamiltonian was the off diagonal matrix this times cosωt. So therefore I know my matrix components, matrix elements that I will feed into the problem and I will get this sinc function and another sinc function as we derived earlier.

Now what I can do I can from the square bracket I can pull out the first term. I can just pull out the first term then inside the bracket I will be left with 1 - the ratio of the second term upon the first term. That ratio simplifies various things. So you will see you will get just $e^{+i\omega t}$ and then the ratio of the summation of frequency to the difference of the frequency and the two sine functions you will get inside. And if I want to know what is the probability of the particle to be found in the ground state after a time t, I will just mod square this $c_a(t)$. So, what I have done, I pulled it out the bracket, inside I am left with 1 - this whole quantity, let us call it something like beta, so I will get 1 - β and then When I mod square it the

outside quantity here will just become $V^{2,}$ $|V|^{2/4}/4\hbar^2$. And then this term which I had pulled out that mod square will just give me the sinc whole square, sinc whole square this exponential is total phase under mod square it will become 1. And then inside $1 - |\beta|^2$ which I am writing like this only. Now here the ωis the energy gap between the two levels a and b while ωis the frequency of the hamiltonian's perturbation so hamiltonian is changing at a frequency ω while energy gap across two eigenstates of the hamiltonian of free part was a and b was Ωthese are two different parameters and in principle they can have different values However if I choose my perturbation, if I select my perturbation, make my perturbation in a way that its frequency almost matches this frequency gap, then I am looking at a limit ω tending to Ω. Then I can use this expression, but I can take the limit ωgoing to Ω. So you see when ω goes to Ω, ω - Ω goes to 0. So this quantity goes to 0, while this quantity also goes to 0. I am getting a structure of 0 upon 0, but you know from your calculus or algebra that this sinc function *sinax/x* in the limit x tending to 0 gives you a. So, therefore if you identify this part as x. Then t will behave as a and when you take the limit extending to 0 which is small frequency ω goes to capital energy gap capital frequency Ωin that limit the sinc function will become t. So it will become t square in this limit the outside sinc function will become t and sinc square function will become t square. Similarly in the interior as well there was a sinc coming in the inverse. So, that will give you 1 over t. So, 1 over t will appear over here and all the remaining work parameters exponential here, ω+ ωhere and sine function which is appearing over here they will survive. So, in the limit when I am talking about small frequency goes to the energy gap, this function simplifies to this dependency. Apart from this first limit which we looked at, if I also look at a particular limit where the summation of the frequency times the time at which I am trying to find the probability, that product is much much greater than 1. This can happen in realistic physical systems, this is very reasonable case because frequency of typical in frequency gap across hydrogen atom optical lines is like 10 to the power 15 per second, 15 hertz. And if I observe after let us say 1 millisecond, 10 to the power - 3 second, the product is roughly 10 to the power 12, which is much much greater than 1. So this is a realistic and reasonable assumption. If I take in the limit, first limit is the frequency of the Hamiltonian matches the frequency of the energy gap. And second limit is that I am looking for a time scale which is longer than 1 over frequency of the energy gap. This $(\omega + \Omega)/2$ in this limit becomes just ω. In the limit small frequency is equal to the larger frequency larger Ω. So in the limit ωt much much greater than 1 You can see this number becomes much larger than 1. This product becomes much, much larger than 1. While numerator everything is bounded by value 1. This product of $e^{i\omega t}$ and $\sin(\omega + \Omega)/2t$ can never exceed value 1. So denominator becomes very large, numerator remains at 1. So I can neglect the second term co mpletely. So this can be ignored. In the limit, two limits, first limit was this, second limit was this. Under these two approximations, the second term inside the bracket can be forgotten. And all I am left with is the first term. First term was 1.

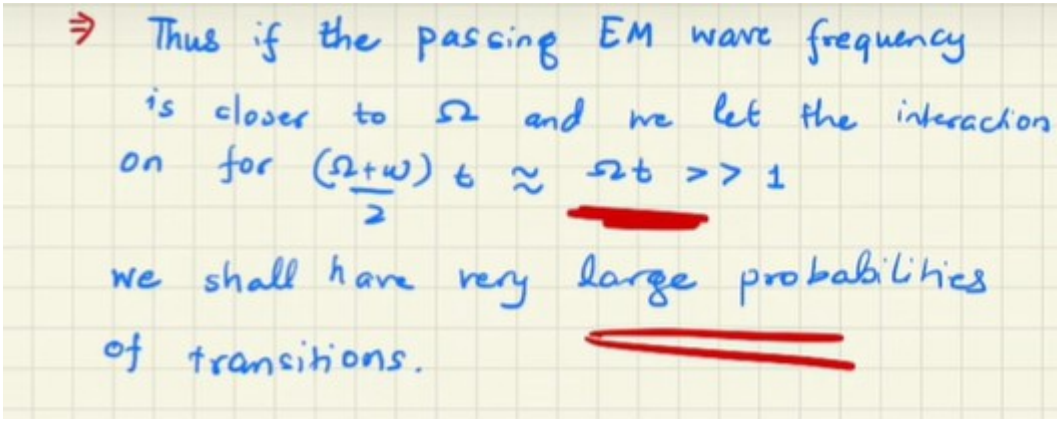
So I see in these two limits, the transition probability or probability to be found in the ground state scales linearly with time squared. So therefore this is a result for large time. Previously we had written small time result for large time this result is correct. Large time as well as resonance. Resonance in the sense that two frequencies match. So large time + resonance leads to a quadratic rise in the probability. So if I write the probability plot $|c_a(t)|^2$ is equal to 0, it was 0. Then it starts rising like this and saturates to 1 at a very long time.



Now this was the story when you use the resonance.

Resonance meaning the frequency is matched. The sync function game like this happened. If the frequency do not match if your ωand Ωare not the same thing but two different things then it is usual sinc function and usual sinc function looks like this sinc to square looks like this and that is what the transition probability or $c_a(t)$ mod square will do. It will look like a curve which is something like this over here. When your driving frequency, driving frequency meaning Hamiltonian perturbation frequency matches the energy gap then you see a sharp rise in transition at a large time.

At large time t will be large therefore the probability will also become very large because t square has become very large. But at all other frequency things are not very large because sync function is not very large around of resonance. If you go away from resonance, sync function quickly dilutes to oscillatory and decaying function. It will not be very large. And if you are really really far away, for example, your driving frequency somewhere here, while your energy gap is somewhere here, you will almost get no transition whatsoever. So therefore, in order to get a reasonable number of transitions, you should perturb the system with the frequency which is the internal time scale, internal frequency of the atom or the system as well. So that is the resonance condition.

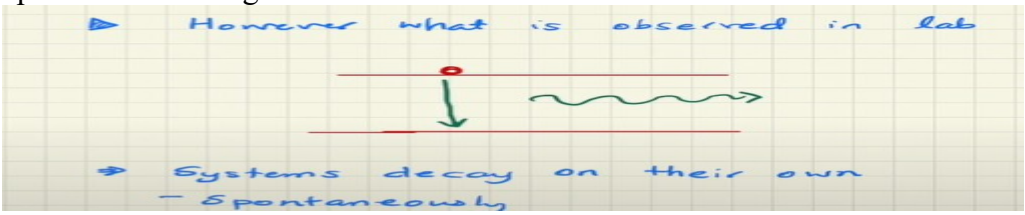


A typical example of a Hamiltonian which we considered would be a charge talking to time dependent electric field. So, if I talk of an electron in a hydrogen atom and suppose I apply electric field from outside or shine electromagnetic wave of photon on this system, then the charge of the electron will talk to the electric field of the electromagnetic wave and you will get this kind of Hamiltonian extra. Where E0 of t would be oscillatory function which is E0 times cosωt, so therefore This becomes exact the exercise which we have performed and now we know if the electromagnetic wave has a frequency ωwhich is also the energy of the atomic lines meaning the hydrogen atom is suppose it is between ground state and first excited state that energy level then we will see rapid transitions between these two states. If your frequency is much away from ωthen you will not expect anything to happen if you are here you do not expect at all anything to happen if you are here something small will happen if you

are here something larger will happen and if you meet the resonance then it will most prominent transitions will happen okay. So, idea is for resonance as well as for long time put together large probabilities of transitions are to be observed. And if you do the computation for the other problem where I started from the ground state $c_a(t)$ of 0 was 1 and $c_b(0)$ was 0 that means the system was in the ground state. Then also if you would do the computation and you will realize that exact same expression will now come for $c_b(t)$. $|c_b(t)|^2$ exactly become the same expression we have just derived this. That means if you are excited in the beginning then your de-excitation rate is linearly proportional to de-excitation probability becomes directly proportional to t square. And if you are in the ground state to begin with your excitation probability becomes proportional to t square with exactly the same coefficient. So, the two processes are possible that if you shine a light with a frequency matching the frequency of the energy gap.

Then if you are in excited state, you will come down to the ground state. A high probability of that is witnessed and therefore you will expect some photons to be emitted by this de-excitation. If you enter with a frequency which is not close to Ω, then most likely nothing will happen. There is small likelihood of something happening, but most likely nothing will happen. This photon of a different frequency will just pass through. Atom will remain where it was and nothing significant will happen. Similarly, if you are in the ground state, if you shine a light which is way different from its energy gap, then again the likelihood of any transition is very small. But if you shine a light whose frequency is very close to the frequency of the energy gap, then the atom can absorb this and go up and become excited. Both these processes have equal probability.

So one thing is called absorption and other is called stimulated emission, okay. So this is for cosωt kind of thing even for sinωt the results will go through you can verify that and actually if you have any arbitrary time dependence that can also be broken in terms of $e^{i\omega t}$ basis and once you do that you will always find some support of frequency ωis equal to Ωwith weightage $\mathcal{H}(\omega)$. So any arbitrary function can be thought of as made up from sin or cos kind of decomposition and one of the decomposition frequency will be exactly the same frequency which energy gap of the atom is coming with. So effectively the coefficient of that frequency which is $\mathcal{H}(\omega)$ plays the role of your V in this case, the problem which we discussed. The role of V will be played by $\mathcal{H}_I(\Omega)$. At all different frequency $\mathcal{H}(\omega)$s will be the Vs for those different frequency but I know that all other different frequency do not cause much of transition. So light can be thought of of made up of different frequencies out of which most of the frequencies like ω1, ω2, ω4, ω5, ω6 and so on they do not cause much of a transition. Only one frequency is responsible for most of the transition and this is the resonance frequency. And its weightage is decided by $\mathcal{H}(\omega)$. Alright, so that tells me suppose I do not have a time any $\mathcal{H}_I$, suppose $\mathcal{H}(\omega)$ is 0. And suppose for more general case, suppose I do not apply any time dependent Hamiltonian. If $\mathcal{H}_I|\ t>$ is 0 all these matrix elements will also be 0 which we just derived and therefore all transition probabilities will go to 0.



So therefore we will expect that if there is no externally applied electric field no photon is shined excited state will remain excited ground state will remain ground only when photons are sent in with a matching frequency this will come down and this will go up if you send photons of a different frequency most likely nothing will happen and if you do not send anything at all then there is no likelihood of any of these processes happening. Systems will remain wherever they are. So they will not undergo any transitions. So it is expected that systems remains in this configuration eternally if

there is no $\mathscr{H}(t)$. However, this is not what we see in lab. In the lab what we see is an excited atom automatically goes down to the ground state without any externally applied field. And this is called a process of spontaneous emission. That nothing is causing it to come to the ground state. Previously when the externally applied photon was shined and then the atom came to the ground state it was called stimulated emission because this photon has stimulated that downward movement. But in labs it is seen that excited atom even without shining any external light undergoes a decay. and sends out a photon.
That means you are sending in 0 photon and you are getting 0 + 1 photon out. This is called spontaneous emission case the question is why it is happening because $c_a(t)$ $\dot{c}_b(t)$ whatever equations which we had written previously, they should identically be 0. Because all the terms we are depending on the matrix elements of perturbation. So, how does this transition happen? Why this does not remain 1 for all time and why this does not remain 0 for all time? Because our equations are telling me that if there are no perturbations, they should remain 1 and 0 for all time. There is an explanation of this spontaneous emission which we will see in the coming weeks which goes by the name of spontaneous emission through vacuum fluctuations of quantum field. So the idea we will learn about when we do not apply anything. What we think we have not applied anything still there are some quantum fields which doing zigzag in the vacuum mat itself. So therefore the atom is seeing those things as a time dependent perturbations and those behave, those things cause the transitions as in the perturbation theory which we computed. So, what we think that we have not applied is only a classical picture, quantum mechanically always there is some vacuum fluctuations which tries to bring that down. So, the terminology vacuum fluctuations and other things that we will deal in the coming weeks and most likely in week 7 we will see the quantum field theory + this atomic systems perturbation theory which will be very relevant for our discussions of matter field intervention.
So, we stop over here for this. And we will move on to another topic of a time dependent perturbation theory in the next class. All right, I stop here.