Foundation of Quantum Theory: Relativistic Approach

Matter-Field interaction 1.3

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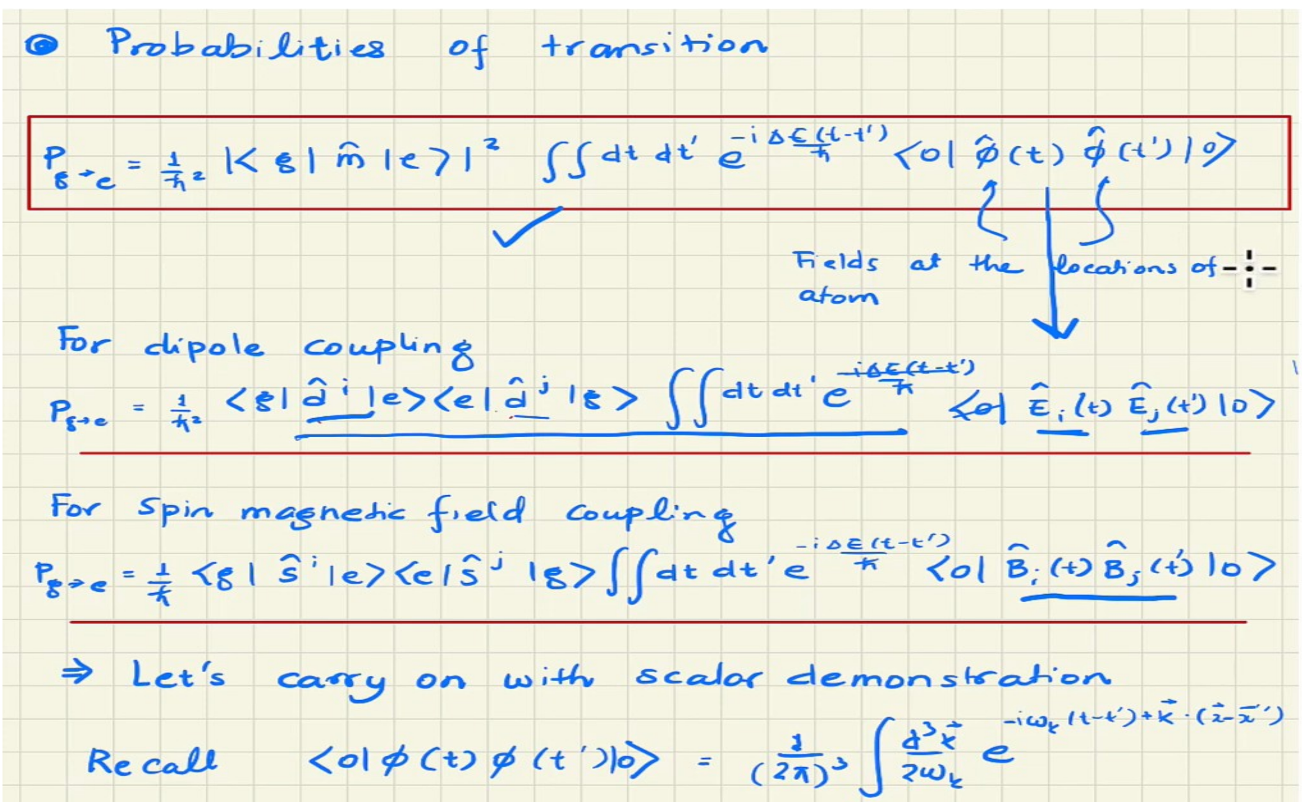
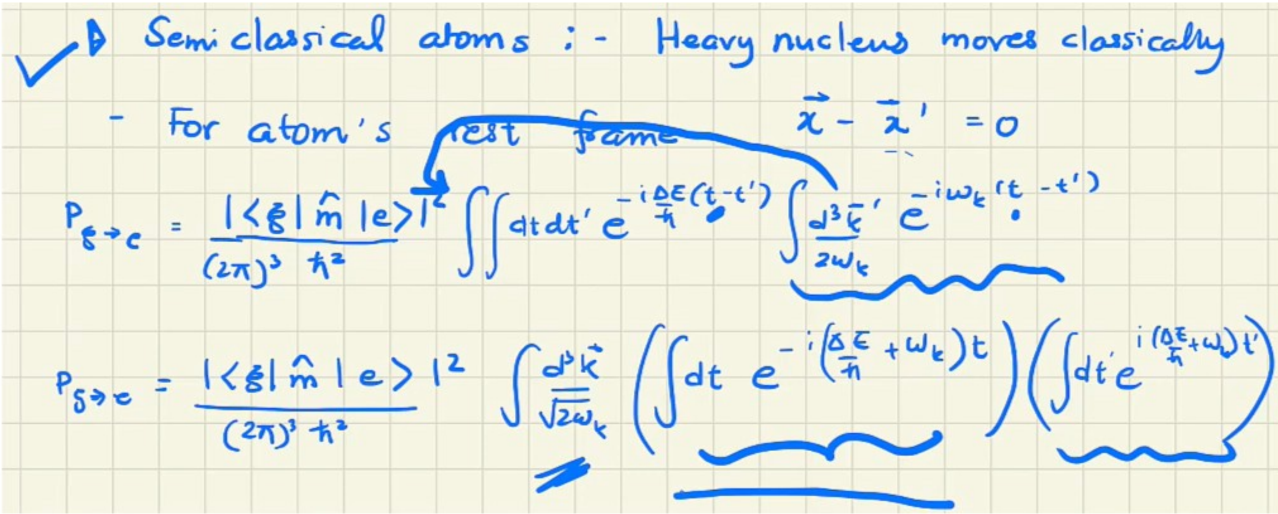
Unruh DeWitt Detectors : Transition probbabilities

Lecture- 35

So, having learnt about the Unruh-Debye detector and how it shifts up the transitions across the atom when it is talking to the background scalar field let us say. We landed an equation of transition across the atomic lines which is probability of going from ground state to excited state. That we obtained is some integral transform of the two-point function of the background field. So, recall when we did this under first order perturbation theory, we obtain as the transition amplitude as a single derivative of the $\int dte^{-i\Delta(E)\phi(t)}$ squared between the initial states and the projected final state.




Semi classical atoms :- Heavy nucleus moves classically

- For atom's rest frame $\vec{x} - \vec{x}' = 0$

$$P_{g\to e} = \frac{|\langle g|\hat{m}|e\rangle|^2}{(2\pi)^3\hbar^2}\iint dt\,dt'\, e^{-i\frac{\Delta E}{\hbar}(t-t')}\int\frac{d^3\vec{k}'}{2\omega_k}e^{-i\omega_k(t-t')}$$

$$P_{g\to e} = \frac{|\langle g|\hat{m}|e\rangle|^2}{(2\pi)^3\hbar^2}\int\frac{d^3\vec{k}}{\sqrt{2\omega_k}}\left(\int dt\, e^{-i\left(\frac{\Delta E}{\hbar}+\omega_k\right)t}\right)\left(\int dt'\, e^{i\left(\frac{\Delta E}{\hbar}+\omega_k\right)t'}\right)$$

● Probabilities of transition

$$P_{g\to e} = \frac{1}{\hbar^2}|\langle g|\hat{m}|e\rangle|^2\iint dt\,dt'\,e^{\frac{-i\Delta E(t-t')}{\hbar}}\langle 0|\hat{\phi}(t)\hat{\phi}(t')|0\rangle$$

For dipole coupling

$$P_{g\to e} = \frac{1}{\hbar^2}\langle g|d^i|e\rangle|\langle e|\hat{d}^j|g\rangle|\iint dt\,dt'\,e^{\frac{-i\Delta E(t-t')}{\hbar}}\langle 0|\hat{\hat{E}}_i(t)\hat{E}_j(t')|0\rangle$$

For spin magnetic field coupling

$$P_{g\to e} = \frac{1}{\hbar^2}\langle g|s^i|e\rangle|\langle e|\hat{s}^j|g\rangle|\iint dt\,dt'\,e^{\frac{-i\Delta E(t-t')}{\hbar}}\langle 0|\hat{\hat{E}}_i(t)\hat{E}_j(t')|0\rangle$$

⇒ Let's carry with scalar demonstration
Recall

$$\langle 0|\phi(t)\phi(t')|0\rangle = \frac{1}{(2\pi)^3}\int\frac{d^3\vec{k}}{\sqrt{2\omega_{\vec{k}}}}e^{-i\omega_k(t-t')\vec{k}(\vec{x}-\vec{x}')}$$

►Semicalssical atoms: Heavy nucleus moves classically

- For atom's rest frame $\vec{x}-\vec{x}'=0$

$$P_{g\to e} = \frac{-\langle g|\hat{m}|e\rangle^2}{(2\pi)^3\hbar^2}\iint dt\,dt'\,e^{\frac{-\Delta E(t-t')}{\hbar}}\int\frac{d^3\vec{k}}{2\omega_k}e^{-i\omega_k(t-t')}$$

$$P_{g\rightarrow e}=\frac{-\langle g|\hat{m}|e\rangle^2}{(2\pi)^3\hbar^2}\int\frac{d^3\vec{k}}{\sqrt{2\omega_k}}\left(\int dt\, e^{-(\frac{\Delta E}{\hbar}+\omega_k)t}\right)\left(\int dt'\, e^{-(\frac{\Delta E}{\hbar}+\omega_k)t'}\right)$$

Now, once we summed over all the possible Fock basis elements of the final state of the field, we ended up getting the transition probability of atom going from ground state to excited state, which is as a Fourier kind of transform of the two-point function of the field. So, having learnt about the Unruh-Debye detector and how it shifts up the transitions across the atom when it is talking to the background scalar field let us say. Now this expression we had obtained for ideal or a demonstration case where the monopole operator of an atom was talking to the field through monopole coupling of the term $\hat{m}\,\phi$ that was the interaction Hamiltonian.

If the transition is being looked between time

$$0 < t < T$$

$$\int_0^T dt\, e^{-i\left(\frac{\Delta E}{\hbar}+\omega_k\right)t} = \frac{e^{-i\left(\frac{\Delta E}{\hbar}+\omega_k\right)T} - 1}{-i\left(\frac{\Delta E}{\hbar}+\omega_k\right)}$$

$$= e^{-i\left(\frac{\Delta E}{\hbar}+\omega_k\right)\frac{T}{2}} \left(\frac{2\sin\left(\frac{\Delta E}{\hbar}+\omega_k\right)\frac{T}{2}}{\left(\frac{\Delta E}{\hbar}+\omega_k\right)} \right)$$

$$\therefore \quad P_{e\to g} = \frac{|\langle g|\hat{m}|e\rangle|^2}{(2\pi)^3\hbar^2} \int \frac{d^3\vec{k}}{2\omega_k} \left(\frac{\sin^2\left(\frac{\Delta E}{\hbar}+\omega_k\right)T/2}{\left\{\frac{\left(\frac{\Delta E}{\hbar}+\omega_k\right)}{2}\right\}^2} \right)$$

Long time value $T \to \infty$

$$\lim_{T\to\infty} \frac{\sin\left(\frac{\Delta E}{\hbar}+\omega_k\right)\frac{T}{2}}{\left(\frac{\left(\frac{\Delta E}{\hbar}+\omega_k\right)}{2}\right)} = \pi\,\delta\left(\frac{\frac{\Delta E}{\hbar}+\omega_k}{2}\right)$$

$$P_{e\to g} = 0 \qquad \left\{ \begin{array}{l} \text{If the field was in} \\ \text{vacuum state initially} \end{array} \right\}$$

Thus, the atom does not get excited on its own!

On the other hand, if the field state initially were $|1_{k_0}\rangle$

Then

$$P_{|g\rangle \to |e\rangle} = \sum_{\psi \in \text{Fock basis}} P_{|g, 1_{k_0}\rangle \to |e, \psi\rangle}$$

$$= \frac{1}{\hbar^2} |\langle g|\hat{m}|e\rangle|^2 \iint dt\, dt'\, e^{-i\Delta E\frac{(t'-t)}{\hbar}} \langle 1_{k_0}|\hat{\phi}(t') \sum_\psi |\psi\rangle\langle\psi| \hat{\phi}(t)|1_{k_0}\rangle$$

$$= \frac{1}{\hbar^2} |\langle g|\hat{m}|e\rangle|^2 \iint dt\, dt'\, e^{-i\Delta E\frac{(t-t')}{\hbar}} \langle 1_{k_0}|\hat{\phi}(t)\hat{\phi}(t')|1_{k_0}\rangle$$

This time we need to evaluate

$$\langle 1_{k_0}|\hat{\phi}(t)\hat{\phi}(t')|1_{k_0}\rangle$$

$$\frac{1}{(2\pi)^3} \langle 0|\hat{a}_{k_0} \int \frac{d^3k_1}{\sqrt{2\omega_{k_1}}} \left(\hat{a}_{k_1} e^{ik_1\cdot x} + \hat{a}^\dagger_{k_1} e^{-ik_1\cdot x}\right) \times \int \frac{d^3k_2}{\sqrt{2\omega_{k_2}}} \left(\hat{a}_{k_2} e^{ik_2\cdot x'} + \hat{a}^\dagger_{k_2} e^{-ik_2\cdot x'}\right) \hat{a}^\dagger_{k_0}|0\rangle$$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3k_1}{\sqrt{2\omega_{k_1}}} \int \frac{d^3k_2}{\sqrt{2\omega_{k_2}}} \Big[\langle 0|\hat{a}_{k_0}\hat{a}_{k_1}\hat{a}^\dagger_{k_2}\hat{a}^\dagger_{k_0}|0\rangle e^{i(k_1\cdot x - k_2\cdot x')} + \langle 0|\hat{a}_{k_0}\hat{a}^\dagger_{k_1}\hat{a}_{k_2}\hat{a}^\dagger_{k_0}|0\rangle e^{-i(k_1\cdot x - k_2\cdot x')} \Big]$$

Since

$$\langle 0|\hat{a}_{k_0}\hat{a}_{k_1}\hat{a}^\dagger_{k_2}\hat{a}^\dagger_{k_0}|0\rangle = \langle 0|\hat{a}_{k_0}[\delta(k_1-k_2) + \hat{a}^\dagger_{k_2}\hat{a}_{k_1}]\hat{a}^\dagger_{k_0}|0\rangle$$

$$= \delta(k_1-k_2)\langle 0|\hat{a}_{k_0}\hat{a}^\dagger_{k_0}|0\rangle + \langle 0|\hat{a}_{k_0}\hat{a}^\dagger_{k_2}\hat{a}_{k_1}\hat{a}^\dagger_{k_0}|0\rangle$$

$$= \delta(0)\delta(k_1-k_2) + \langle 0|[\delta(k_0-k_2) + a^\dagger_{k_2}\hat{a}_{k_0}][\delta(k_1-k_0) + \hat{a}^\dagger_{k_0}\hat{a}_{k_1}]|0\rangle$$

$$= \delta(0)\delta(k_1-k_2) + \delta(k_0-k_2)\delta(k_1-k_0)$$

$$\langle 0|\hat{a}_{k_0}\hat{a}^\dagger_{k_1}\hat{a}_{k_2}\hat{a}^\dagger_{k_0}|0\rangle = \delta(k_0-k_1)\delta(k_2-k_0)$$

$$\therefore \quad \langle 1_{k_0} | \hat{\phi}(t)\, \hat{\phi}(t') | 1_{k_0} \rangle$$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3k_1}{\sqrt{2\omega_k}} \int \frac{d^3k_2}{\sqrt{2\omega_k}} \Big[\Big(\delta(k_1 - k_2)\, e^{i(k_1\cdot x - k_2\cdot x')}\, \delta(0)$$

$$+ \delta(k_0 - k_2)\, \delta(k_1 - k_0)\, e^{i(k_1\cdot x - k_2 x')}$$

$$+ \delta(k_0 - k_1)\, \delta(k_2 - k_0)\, e^{-i(k_1\cdot x - k_2\cdot x')}$$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3k_1}{2\omega_{k_1}} e^{-ik_1\cdot(x - x')}\, \delta(0)$$

$$+ \frac{1}{(2\pi)^3} \left[\frac{e^{ik_0\cdot(x - x')}}{2\omega_{k_0}} + \frac{e^{-ik_0\cdot(x - x')}}{2\omega_{k_0}} \right]$$

Thus,

$$\frac{P_{g\to e}}{\langle 1_{k_0} | 1_{k_0} \rangle} = \frac{|\langle e | \hat{m} | g \rangle|^2}{(2\pi)^3 \hbar^2} \iint dt\, dt'\, e^{-i\frac{\Delta E}{\hbar}(t - t')}$$

$$\frac{1}{\delta(0)} \left[\int \frac{d^3k_1}{2\omega_{k_1}} e^{-ik_1\cdot(x - x')}\, \delta(0) + \frac{e^{ik_0\cdot(x - x')} + e^{-ik_0\cdot(x - x')}}{2\omega_{k_0}} \right.$$

$$e^{ik_0\cdot(x - x')} = e^{-i\omega_{k_0}(t - t') + i\vec{k}_0\cdot(\vec{x} - \vec{x}')}$$

$$= e^{-i\omega_{k_0}(t - t')} \quad \text{for atom at rest}$$

$$\frac{P_{g\to e}}{\langle 1_{k_0} | 1_{k_0} \rangle} = \frac{|\langle e | \hat{m} | g \rangle|^2}{(2\pi)^3 \delta(0)\, 2\hbar^2\, (2\omega_{k_0})} \int dt\, dt' \Big\{ e^{-i(\frac{\Delta E}{\hbar} + \omega_{k_0})(t - t')} + e^{-i(\frac{\Delta E}{\hbar} - \omega_{k_0})(t - t')} \Big\}$$

$T \to \infty$ $\quad\quad$ $T \to \infty$

If the transition is being looked between time $0 < t$

$$\int_0^T dt\, e^{-i(\frac{\Delta E}{\hbar}+\omega_k)t} = \frac{e^{-i(\frac{\Delta E}{\hbar}+\omega_k)T}}{-i(\frac{\Delta E}{\hbar}+\omega_k)}$$

$$= \quad e^{-i(\frac{\Delta E}{\hbar}+\omega_k)} \frac{(2\sin(\frac{\Delta E}{\hbar}+\omega_k)T/2)}{\frac{-(\frac{\Delta E}{\hbar}+\omega_k)}{2}}$$

$$\therefore P_{e\to g} = \frac{\langle g|\hat{m}|e\rangle^2}{(2\pi)^3\hbar^2}\frac{\int d^3\vec{k}}{2\omega_k}\left(\frac{\sin^2-(\frac{\Delta E}{\hbar}+\omega_k)\frac{T}{2}}{-(\frac{\Delta E}{\hbar}+\omega_k)^2}\right)$$

For Long time $t \to \infty$

$$\frac{\lim_{T\to\infty} \sin-(\frac{\Delta E}{\hbar}+\omega_k)T/2}{-(\frac{\Delta E}{\hbar}+\omega_k)} = \pi\delta(\frac{-(\frac{\Delta E}{\hbar}+\omega_k)}{2})$$

$$P_{e\to g}=0$$

{If the field was in vacuum state initially}

Thus, the atom does not get excited on it's own!

On the other hand, if the field state initially were $|1k_0>$

Then

$$P_{|g\rangle\to|e\rangle} = \sum_{\psi\,\epsilon\, flock\, basis} P_{g,0_L\to e_\psi}$$

$$= \frac{-1}{\hbar^2}|\langle g|m|e\rangle|^2\int\int dt\,dt'\,e^2 - i\Delta E\frac{(t-t')}{\hbar}\langle 1_0\hat{\phi}(t')\sum_\psi ||\psi\rangle\langle\psi|\hat{\phi}(t)|t_k\rangle$$

$$= \frac{-1}{\hbar^2}|\langle g|m|e\rangle|^2\int\int dt\,dt'\,e^2 - i\Delta E\frac{(t-t')}{\hbar}\langle 1_{k_0}|\hat{\phi}(t)\hat{\phi}(t')|1_{k_0}\rangle$$

This time we need to evaluate

$$\langle 1_{k_0}|\hat{\phi}(t)\hat{\phi}(t')|1_{k_0}\rangle \frac{1}{(2\pi)^3}\langle 0|\hat{a}_0\int\frac{d^3\vec{k}_1}{\sqrt{2\omega_{\vec{k}_1}}}(\hat{a}_{k_1}e^{ik_1x}+\hat{a}^\dagger_{k_1}e^{-ik_1x})\int\frac{d^3\vec{k}_2}{\sqrt{2\omega_{\vec{k}_2}}}(\hat{a}_{k_2}e^{ik_2x}+\hat{a}^\dagger_{k_2}e^{-ik_2x})\hat{a}^\dagger_0|0\rangle$$

{If the field was in vacuum state initially}

Thus, the atom does not get excited on it's own!

$$\frac{1}{(2\pi)^3}\int\frac{d^3\vec{k}_1}{\sqrt{2\omega_{\vec{k}_1}}}\int\frac{d^3\vec{k}_2}{\sqrt{2\omega_{\vec{k}_2}}}\left[\langle 0|\hat{a}_{k_0}\hat{a}_{k_1}\hat{a}^{\dagger}_{k_2}\hat{a}^{\dagger}_{k_0}|0\rangle e^{i(k_1\cdot x-k_2\cdot x')}+\langle 0|\hat{a}_{k_0}\hat{a}^{\dagger}_{k_1}\hat{a}_{k_2}\hat{a}^{\dagger}_{k_0}e^{-(k_1\cdot x-k_2\cdot x')}|0\rangle\right]$$

Since

$$\langle 0|\hat{a}_{k_0}\hat{a}_{k_1}\hat{a}^{\dagger}_{k_2}\hat{a}^{\dagger}_{k_0}|0\rangle=\langle 0|\delta(k_1-k_2)+a^{\dagger}_{k_2}\hat{a}_{k_1}|0\rangle$$

$$\delta(k_1-k_2)+\langle 0|\hat{a}_{k_0}\hat{a}^{\dagger}_{k_1}|0\rangle+\langle 0|\hat{a}_{k_0}\hat{a}^{\dagger}_{k_2}\hat{a}_{k_1}\hat{a}^{\dagger}_{k_0}|0\rangle$$

$=$

$$\delta(0)\delta(k_1-k_2)+\langle 0|[\delta(k_0-k_2)+\hat{a}^{\dagger}_{k_2}\hat{a}_{k_0}][\delta(k_1-k_0)+\hat{a}^{\dagger}_{k_0}\hat{a}_{k_1}]|0\rangle$$

$$=\delta\ (0)\delta(k_1-k_2)=\delta(k_0-k_2)\delta(k_1-k_0)$$

$$\langle 0|\hat{a}_{k_0}\hat{a}^{\dagger}_{k_1}\hat{a}_{k_2}\hat{a}^{\dagger}_{k_0}|0\rangle=\delta(k_0-k_1)\delta(k_2-k_0)$$

$$\therefore\langle 1_{k_1}|\phi(t)\phi(t')|1_{k_1}\rangle=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3\vec{k}_1}{\sqrt{2\omega_{\vec{k}_1}}}\int\frac{d^3\vec{k}_2}{\sqrt{2\omega_{\vec{k}_2}}}\left[\delta(k_1-k_2)e^{i(k_1x-k_2x')}\delta(0)+\delta(k_2-k_1)\delta(k_1-k_2)e^{i(k_1x-k_2x')}+\delta(k_0-k_1)\delta(k_2-k\right.$$

$$\frac{1}{(2\pi)^3}\int\frac{d^3\vec{k}_1}{\sqrt{2\omega_{\vec{k}_1}}}e^{-ik_0(x-x')}\delta(0)+\frac{1}{(2\pi)^3}\left[\frac{e^{ik_0(x-x')}}{2\omega_k}+\frac{e^{-ik_0(x-x')}}{2\omega_k}\right]$$

$$\frac{P_{g\rightarrow e}}{\langle 1_{k_0}|1_{k_0}\rangle}=\frac{|\langle e|\hat{m}|g\rangle|^2}{(2\pi)^3(\hbar^2)}\int\int dt\,dt'\,e^{\frac{-i\Delta E(t-t')}{\hbar}}$$

$$\frac{1}{\delta(0)}\left[\int\frac{d^3\vec{k}}{\sqrt{2\omega_{\vec{k}}}}e^{-ik_i(x-x')}\delta(0)+\frac{e^{ik_0(x-x')}+e^{-ik_0(x-x')}}{2\omega_k}\right]$$

$$e^{-ik_i(x-x')}=e^{-i\omega_{k_0}(t-t')+I\vec{k}_0(\vec{x}-\vec{x}')}=e^{-i\omega_{k_0}(t-t')}$$

$$\frac{P_{g\rightarrow e}}{\langle 1_{k_0}|1_{k_0}\rangle}=\frac{|\langle e|\hat{m}|g\rangle|^2}{(2\pi)^3\delta(0)(2\hbar^2)}\{\int\int dt\,dt'\,e^{\frac{-i(\frac{\Delta E}{\hbar}+\omega_{k_0})(t-t')}{\hbar}}+e^{\frac{-i(\frac{\Delta E}{\hbar}-\omega_{k_0})(t-t')}{\hbar}}\}$$

Now this exercise what we have performed is just a toy model towards a more realistic setups where indeed realistic operators from atomic side do talk to background fields in this manner. A case in point could be the electromagnetic interactions of fields with atom. A case in point could be the electromagnetic interactions of fields with atom. Now see there is a label i appearing across the two point functions i and j and similarly in the dipole operators d I and d j are appearing. This is because of their vector nature. In the monopole coupling term the interaction Hamiltonian was $\hat{m}\ \phi$. In the dipole coupling the interaction will Hamiltonian will have term like $\vec{d}\cdot\vec{E}$ that is $\sum_i \vec{d}_i \cdot E_I$. And when I do a mod square, I will have a twice of these terms. So I would have a di ei for one and dj ej for the other. So, this *Di, Ei and Dj, Ej* appear as two separate interaction amplitude and whose product complex conjugate of each other constitutes a very similar looking transition probability expression which we have just opted for a scalar field interaction. Similarly, when a spin of an atom talk to the background magnetic field, there also you would obtain a very similar looking expression for probability of transition across the atom, where the role of D will be taken over by the spin and the role of electric field will be taken over by the magnetic field. So, you will have a SIBI and SJBJ kind of terms. And when I sum over all the possible states of the field, that means I do not care what happens to the field after all. I just care about what is the probability of the excitation from the ground state of the atom. I will end up getting expression of this kind, where you see the two point function of magnetic field this time and its integral transforms dictates the transition probability across the atom. So, despite we are doing a academic exercise in some sense of discussing of monopole operator coupled with the field scalar field ϕ. all the realistic physically significant cases will be just replicable from this structure what we are talking about. So, it is a good idea to keep pushing ahead with scalar feed consideration and more or less most of the ideas which we have discussed across this will be very similarly replicated across more realistic setting. So, it is a good idea to keep pushing ahead with scalar feed consideration and more or less most of the ideas which we have discussed across this will be very similarly replicated across more realistic setting.

So, as you see the two-point function *$\phi(t)$* and *$\phi(t')$*s its expectation value across the vacuum state is required in order to undergo this integral transform. Now we know from our previous classes for scalar field, I can compute the two point functions across t and t' through this integration. This is the mode functions representation of the two point function. This we have done previously in the class as well. Now, here is an interesting assumption we have to make in order to do the computation forward. That in this exercise, the two-point function is not only being computed at two different times t/t', we need to provide information at what locations because field is a function of not only time, but different locations fields values are different. So, here we have to use the spatial locations as the locations of an atom. So, we are interested about not fields correlated across two arbitrarily separated spatial points, but only those spatial points where the atoms are at two different times t and P'. So, we are interested about not fields correlated across two arbitrarily separated spatial points, but only those spatial points where the atoms are at two different times t and p'. So, how do I put, how do I compute the locations of the atom? At this stage we make an answer of something called a semi-classical analysis for atoms. So atoms has two mono, so let us say mono electron atoms like hydrogen have two degrees of freedom. One is for the electron and one for its nucleus. Nucleus are typically multifold heavier than the electrons. And we presume that analysis of nucleus would be done classically. They are so massive that we did not consider their quantum properties for up to this accuracy where we are interested about. So, heavy nucleus therefore would be assumed moving classically and it is their locations where the field correlators will be obtained. And the electron around the nucleus is to be treated quantum mechanically and therefore the operators of Positions of electrons we are not putting it over here. The positions over here what we are putting are for the atoms. So atoms trajectories location are being used for x and x'. And ultimately at these locations the field values correlations we are trying to get. Those correlations ultimately go under the integral transform and tell us what is the transition probability. For instance, let us think of an atom which is at rest with respect to us in the lab frame. So, that means our lab frame becomes atom's rest frame as well. In that case, the spatial displacement would be 0 across two different times for atom. So, x would be the same as x' for a future time as well as for the past time. So, I would have a $x - x'$ is equal to 0 which will take off the second argument in the exponent of exponential. So, the two-point function will just become the integral over $e^{-i\hbar\omega kt - E'}$. And that will go and sit in the two-point function representation which we had, which we needed in order to get, in order to get the, in order to get the transition probability. Now, what we can do from here, we can flip the orders of integration. There are three integrations which are involved. Two integrations in t and t' and one integration in d^3k. Actually in that since there are six integrals, but I am collectively calling five integrals, d^3 integrals in k and two integrals in t and t', but I am collectively calling d^3k as one integral. What I do, I take the d^3k integral at the last and I separate out the two integrals in t and t'. That means I will collect all the t dependency at one place integrated with respect to dt. I will collect all the t' dependency at another place and do the integration with respect to dt'. t dependency comes through in the exponentials argument of the first exponential as well as in the second exponential over here. I can collect these together, you will get overall exponential $-i$and then $\Delta(E)/\hbar$ from the first exponential and ω_k from the second exponential. So, this becomes the integral of dt that all t dependency comes over here. So, that integration we need to do. Similarly, if I collect all the t' dependency in the system and collect it at one place, I will get this dt' integral which is over here. Notice one fact that The integral dt with this exponential and integral dt' with this exponential, they are just complex conjugate of each other. So, I need to just compute one of the integrals to its mod square and then perform the d^3k integral and I will be done with getting to know the transition probability. So, that is what we move ahead to and let us compute the two integrals or the one integrals and its mod square and find out what is the transition probability for an inertial atom, let us say which is at rest. For those atoms, we can compute things, the probability of transitions if the interaction is happening for a time duration up to capital T. So let us look at the transition probability between time small t varying from small 0 to capital T. So that means the two integrals which are appearing over here, they all are running from time 0 to capital T. So let us go ahead and perform these two integrals and try to see what results do we get. I have a simple exponential in the, as the integrand which is a fairly simple exercise to do. You can see that it will, you will directly get the exponential back divided by $-i\Delta(E)/\hbar + \omega_k$. So, that would be just the integral. When I put the two limits, for the upper limit I will get this term over here and for the lower limit I will get a $-$ 1 just, okay. Small t would be put to value 0 and the denominator is common for both the terms. So now you can see I have a structure that I have an exponential of something $-$ 1 divided by $-i$. This thing together can be written in terms of something some sine function. How do I do that?I pulled out $e^{-iT/2}$. So this if I pull out. I will be left with $e^{-i\text{the same thing over here } \delta\, t/2 - e + i - + i\Delta(E)/\hbar + \omega kt/2}$. So, whatever I have pulled out as a factor will be its opposite will be put at the second place to reproduce the one. And downstairs I have a $-i$and you can see that according to that analysis I will get twice of sine $\Delta(E)/\hbar + \omega_k$ times $t/2$. So, what this pulling out of this exponential with half the phase factor gives me a sine in the numerator and the denominators $-i$goes into the definition of a sine. So, all I am left with in the denominator is $\Delta(E)/\hbar + \omega$ Q. So, therefore, this is the integral which I would get for inertially non-moving atom in live frame. The first integral will give me just this exponential phase factor and sign as we just saw. Now, I have to take this and mod square this the second integral is just complex conjugate of that. So, if I do just the mod square this overall extra phase will just vanish and I will get a mod square of the whole thing that means twice sin $\Delta(E) + \omega_k T/2$ divided by this mod square of this which is our real number so squares of individually that. What I do, I take this 2 and bring it down to the denominator. So, overall I have this thing $\Delta(E) + \omega_k$, $\Delta(E)/\hbar + \omega_k$ /2. This 2 can be supplied downstairs. So, when I do mod square, I will get a sine square in the top and the denominator by *2* whole square in the bottom. So, I would have this as the whole probability of transition expression because I have to perform the d^3k integral still. However, I have this function which was the mod square of the two time integrals dt and dt′ together both of them gave me the sign/this factor common with exactly cancelling phases. So, I am left with this integral last integral to perform. So, this is a final answer for any arbitrary time capital T. Now suppose we take a time scale which is much larger than any of the time scales of the atom. The time scale of the atom is $\Delta(E)/\hbar$. $\Delta(E)$ is a typical energy gap in atom divided/$\hbar$ will be the typical photon frequency which would be needed. But suppose I am looking at time scales which are much larger than 1 over this frequency. That means I am looking at I am waiting too long compared to the internal time scales. Internal time scales are 1 over energy gap/$\hbar$. That is the 1 over inverse of this frequency is the internal time scale of atom. So, if my time is much, much larger than that time scales, I can approximate this expression to its infinite value limit. So, I take this expression $\sin^2$ *by* this and take capital *t* tending to infinite limit of that in anticipation of a very long time value of probability. At late time, what happens to the probability? Again, if you have seen it before, you would know that the expression which is sin times something times *x* divided by the $\hbar$, call it the whole thing as a, $\Delta(E)/\hbar + \omega_k/2$ divided by $\Delta(E)$ by $\hbar + \omega_k/2$ and factor *t* is there. So, it has a structure sin A T/A and then *t* tending to infinity that gives you $\pi\ \delta(a)$. Therefore, our A was this object $\Delta(E) + \hbar + \omega\ \Delta(E)$ by $\hbar + \omega/2$ that will go and sit in the argument of δ function. So, I will get the limiting value of the sin^2 integral would be just mod square or just a square of this π times a δ function. So, π^2 times δ^2, two δ functions I will get with the same arguments. I will look at the expression which is inside the argument of the δ function. $\Delta(E)/\hbar$ which is the transition gap of the atom. ω_k, ω_k came from the field mode. Remember when we write, we were writing down the two point function, the ω_k appeared here for the first time. And this ω_k was running, it will take different values corresponding to different different k's. So therefore, this ω_k is a continuously running value, but it is positive. ω_k is $\sqrt{(k^2 + \hat{m}^2)}$, if you remember for massless field it is just ω is equal to k, kc actually, but ω is in c is equal to 1 unit, ω is equal to k. That means this is also a positive quantity. So, both $\Delta(E)/\hbar$ and ω_k are positive quantity, that means for any value of ω_k in this integral, the argument of a δ function is always going to be positive and argument of a positive, positive argument of a δ function kills the δ, the function. So, δ of any positive argument is 0. That means the ground state, sorry, this is oppositely written, it should be g to e. So, ground state to excited state transition probability, somehow I have written it the opposite way, this is a typo over here, which I would correct in the notes. So, the excitation probability is the excitation probability is 0 if I wait for long, long enough time. That means the inertial atom at rest does not get excited on its own. So, this should be G to E, so that would become 0. So, ground to excited state transition would not happen automatically if the field was initially in the vacuum. Here we have assumed the two-point function is being computed in the vacuum state of the field. So, if field has no photon to begin with, there is no likelihood in long time that the atom will automatically become excited, that probability turns out to be 0. In order to excite the atom, the field should initiate the discussion, the initial state of the field should be non-vacuum. In vacuum state, there is no probability of atom getting excited. On the other hand, if the field's initial state was non-vacuous for instance, suppose initial state of the field was single photon in momentum k_0. So, my in state would be replaced by the initial So, recall the previous step where we had computed the probability. We had done this thing that initially the state was round for the atom and vacuum for the field. This time it will become 1_{k_0} for the field. This will become 1_{k_0} over here for the field part. And then if I compute the probability of atom going to excited and field going to any of the possible state ψ, the probability of such a process will be obtainable from the amplitude of the transition which will this time have a ϕ squeezed between 1_{k_0} and ψ. This is the same computation which we did for vacuum. You can go back to the previous class and check Only thing happening here differently is this previously the left hand side was vacuum now it has become 1_{k_0} because we are starting with a non-vacuum state. Similarly over here the outermost

state is again 1_{k_0} instead of vacuum. So, previously ψ was the field ϕ was squeezed between 0 and ψ and this side it was ψ and 0 and the summation over ψ Summation over all Fock basis element created an identity here and therefore the two-point function of the field in the vacuum state was obtained. This time the two-point function of the field in the 1_{k_0} excited state of the field would be obtained. So therefore, if I try to ask for the probability of a transition for the atom from ground to excited had the field initially been in the excited state $1k_0$. that probability will be just two point function this time computed in 1_{k_0} state and the same integral transform which we did previously. So that means we need to evaluate for this time what is the two point function expression in 1_{k_0} state. Recall previously we had obtained the two point functions in the vacuum state and obtained that as a simple integral of a $d^3k/2\omega_k$ and exponential phase This time also very similar kind of expressions will come about, but we have to correct for that the state is not vacuum. So, previously ψ was the field ϕ was squeezed between 0 and ψ and this side it was ψ and 0 and the summation over ψ Summation over all Fock basis element created an identity here and therefore the two-point function of the field in the vacuum state was obtained. This time the two-point function of the field in the 1_{k_0} excited state of the field would be obtained. So therefore, if I try to ask for the probability of a transition for the atom from ground to excited had the field initially been in the excited state $1k_0$. That probability will be just two point function this time computed in 1_{k_0} state and the same integral transform which we did previously. So that means we need to evaluate for this time what is the two point function expression in 1_{k_0} state. Recall previously we had obtained the two point functions in the vacuum state and obtained that as a simple integral of a $d^3k/2\omega_k$ and exponential phase This time also very similar kind of expressions will come about, but we have to correct for that the state is not vacuum. So, let us write down the state which is $1k_0$. The state 1_{k_0} can be written as $a^\dagger_{k_0}$ acting on vacuum. So, $a^\dagger_{k_0}$ acting on the vacuum will become my state.
So, this is just the Hermitian conjugate, this is just the bra 0 and this is just the, this is left hand side is the bra 1_{k_0} and right hand side is the ket $1k_0$. In between there are two field operators at location t and location t', position x and position x'. So, therefore I write down the fields as this at location tx and the second integral is just giving me the field expression at location $t'\ x'$. Now again you collect all the operators, all the operators are a_{k_0} here, a_{k_1} here, $a^\dagger_{k_1}$ here, $a^\dagger_{k2}$ here, $a^\dagger_{k2}$ here. Recall when I am writing the two fields as integrals, I am going to keep the labels of the integrals different so that I can differentiate between tx and field at location $t'\ x'$. So the first field which is for ϕ I am calling its Fourier decomposition as d^3k_1 and the second field $\phi(t')$ its Fourier decomposition is being called as d^3k_2 and the labels of exponentials are accordingly adjusted. So all these exponential all these integrals are just real quantities or complex quantities but not operators. So, I can take all these things out of the operator action, only states or operators will act on each other. So, I will collect all the operators between the states. So, all the state which we are looking at is the bra 0 on the extreme left and ket 0 on the extreme right and there are operators in between. So, let us start collecting the operators which will survive. This operator $a^\dagger_{k2}$ And I have a $a^\dagger_{k_0}$ here. And similarly left hand side I have a a_{k_0} operator hitting on the bra 0. And then I have a two operators $a^\dagger_{k_1}$ $a^\dagger_{k2}$, $a^\dagger_{k_1}$ here. Now we know from our previous exercises such an expression will survive if I have a same state from left and right. They survive under operations only if we have same number of a_k and $a^\dagger_k$ hitting each other. So that means I can have one a_{k_0} is from the left and one $a^\dagger_{k_0}$ from the right are already in the game. So that means I have only luxury of arranging one a_k and one $a^\dagger_k$ from these two operators. The two fields operator can only supply me one a_k and one $a^\dagger_k$ from each such that I can write a surviving operator structure. One possible surviving term will be when this a_{k_0} And this a_{k_1} are one hand together while this $a^\dagger_{k_0}$ and this $a^\dagger_{k2}$ are on the other hand together. So, this will be the first term which will be contributing their phases have to be adjusted. Adjusted meaning written in the way they appear. So, I have a $e^{ik1\dot{x}}$ from the first term and $e^{-ik2\dot{x}'}$ from the second term which I have collected and put aside. Similarly, one more offset of operator which will be surviving will be a_{k_0} combining with $a^\dagger_{k_1}$ then $a^\dagger_{k2}$ and that combining with $a^\dagger_{k_0}$ here. So, a_{k_0} $a^\dagger_{k_1}$ $a^\dagger_{k2}$ $a^\dagger_{k_0}$ and their exponentials again will be $e^{-ik1\dot{x}}$ from the first $a^\dagger_{k_1}$ and $e^{+ik2\dot{x}'}$ from the second term. So, collectively this the second term is this. Now, you can verify all other terms which will appear will in

this expression will vanish. For example, a_{k_0}, a_{k_1}, $a^\dagger_{k2}$ and then $a^\dagger_{k_0}$ its expectation across vacuum state is 0. Similarly, a_{k_0}, $a^\dagger_{k_1}$, $a^\dagger_{k2}$, $a^\dagger_{k_0}$, its expectation value across vacuum is 0. So, only two terms will be surviving exercise. One is a_{k_0}, $a^\dagger_{k1}$, $a^\dagger_{k2}$, $a^\dagger_{k_0}$. So, that is what we have to compute. And another term is a_{k_0}, $a^\dagger_{k_1}$, $a^\dagger_{k2}$, $a^\dagger_{k_0}$, which is whose expectation across vacuum. So, let us go ahead and compute these two operators. The first one is a_{k_0}, $a^\dagger_{k_1}$, $a^\dagger_{k2}$, $a^\dagger_{k_0}$. In between I see $a^\dagger_{k_1}$, $a^\dagger_{k2}$ has appeared. That I am going to write as their commutator. I already know $a^\dagger_{k_1}$, $a^\dagger_{k2}$ commutator is *is* $\delta(k_1 – k_2)$. That means a_{k_1}, $a^\dagger_{k2}$ will be *is* $\delta(k_1) – k_2 + a^\dagger_{k2}$ ak1. So, the commutator is *is* $\delta(k_1 – k_2\ )$the commutator has two terms a_{k_1} $a^\dagger_{k2}$ – $a^\dagger_{k2}$ $a^\dagger_{k_1}$ that – $a^\dagger_{k2}$ $a^\dagger_{k_1}$ I bring it on the right hand side of the commutator, therefore the ak1 $a^\dagger_{k2}$ becomes this. And as before a_{k_0} and $a^\dagger_{k_0}$ are appearing on the left and the right respectively. So, then I open it up, the square bracket I open it up, I will get two terms, first term *is* $\delta(k_1) – k_2)$ which is coming with identity which is I am not writing over here but there is identity multiplying *is* $\delta(k_1) – k_2$. But ultimately that *is* $\delta(k_1 – k_2)$ will come out and only a_{k_0} identity $a^\dagger_{k_0}$ will survive inside which is equivalent to a_{k_0} $a^\dagger_{k_0}$ squeezed between the vacuum. That will be the first term.

The second term will be a_{k_0} $a^\dagger_{k2}$ a_{k1} $a^\dagger_{k2}$ $a^\dagger_{k_0}$ squeezed between the vacuum. $a^\dagger_{k_0}$ and this a_{k_0} and this $a^\dagger_{k_0}$ squeezing the $a^\dagger_{k2}$ and ak1 in between. Again I have a structure where I have a a_{k_0} and $a^\dagger_{k2}$ coming along and $a^\dagger_{k_1}$ $a^\dagger_{k2}$ coming along. These two can also be written in terms of their respective commutators. The first pair which is appearing over here can be written as a δ key $k_0 – k_2 + a^\dagger_{k2}$ a_{k_0}. Use the commutator between a_{k_0} and $a^\dagger_{k2}$ to write the first square bracket here. Similarly, ak1 $a^\dagger_{k_0}$ can be written as the second square bracket over here. And now you can see I will have many terms coming about, first term is just $\delta(0)$, the first term which I am going to write over there here is $\delta(k_1 -k_2)$ which is coming from here and then a_{k_0} $a^\dagger_{k_0}$. If I try to compute you can see that this in this object it is *1k* not *1k* inner product. which will just give me $\delta(0)$. Remember *1k* not state has a normalization of a $\delta(0)$. So this squeezing is going to give me a $\delta(0)$ and I will have a $\delta(k_1 -k_2)$ multiplying it. Now in this square bracket I have many terms this $\delta(k_0 -k_2)$ can multiply this $\delta(k_1 -k_0)$ and then the vacuum vacuum squeezed. Vacuum is normalized so I will get first term as $\delta(k_0 -k_2)$ times $\delta(k_1 -k_0)$ and the normalization of vacuum state which is 1. All other terms you can see are going to be 0 because on all other terms one a_k operator is going to hit/0. So, this operator hitting the 0 will be 0. Similarly, this a_{k_0} hitting the 0 will be 0 or $a^\dagger_{k2}$ hitting this 0 will be 0. So, you can verify apart from the two δ functions multiplying each other, all the other three terms which get generated in the cross multiplication, their expectations are 0. So, that means the first term which has appeared over here is just products of δ functions in funny way. The first term is $\delta(0)$, $\delta(k_1 -k_2)$, the second term is $\delta(k_0 -k_2)$ and $\delta(k_1 -k_0)$. So, this is true, this is for this whole term. Now I know the whole term because I have completely computed the first expectation value. You can see the second expectation value is not much different, we have indeed computed something very similar already. We have computed the expectation value of this operator which appeared in between which has a very similar structure like what we wanted to compute for the second term. So, therefore we can write down the a_{k_0} $a^\dagger_{k_1}$ and $a^\dagger_{k2}$ $a^\dagger_{k_0}$ would be the same almost the expectation of this term which appropriate identification what was k_1, what was k_2 and what was k_0. And you can see that even this term expectation value is just a product of δ functions. Only thing changing with this exponential phase will become the same thing, but the complex conjugate of each other. So you see, once I have taken care of the k_2 integral in the first term, and while both k_1 and k_2 in the second and the third term over here, I get this structure. So let us go ahead and see what are we up to under this. In the transition probability expression, what we needed to compute was the double integral in dt dt′ with this exponential weightage of the two-point function. Now, since I have taken the initial state to be $1_{k_0}$and not ground state or the vacuum state of the field, that state $1_{k_0}V_0$ we know is not normalizable. That means it is not automatically normalized. $1_{k_0}1_{k_0}$inner product is not 1. So, it is good idea to define things in terms of normalized state. That means I divide the whole expression by normalization of the state $1k_0$. So, I take this divide by the expression of the probability divided by $1_{k_0 1k_0}$ that will be normalized probability. When I do that normalization of 1_{k_0} , $1_{k_0}$is just $\delta(0)$. So, 1 over $\delta(0)$

extra factor I have to compute, I have to put in getting a normalized probability in the whole expression. The whole expression before was just this and this square box put together. That was the two point function and the double integral in the times that tells me what would be the total transition probability. Since the state was not normalized, I should divide this thing with the normalization of the state, I get this 1 over $\delta(0)$ extra vector. Now spot one thing, this $\delta(0)$ is here, for a moment just forget it, then look at this exponential thing which is multiplying that. This is exactly the two-point structure which we just dealt with. Remember, when we were dealing with initial state being the two-point, initial state being the vacuum, the two-point function was just this integral $d^3k\ \omega_k(t - t') + k(\dot{x} - x')$. Together it is just to the $e^{ik(\dot{x} - x')}$. So, the same thing has appeared multiplying $\delta(0)$ this time. This is the two-point function of the vacuum. This gets multiplied with $\delta(0)$ because this integral itself was coming with the $\delta(0)$. But fortunately since we are looking about the normalized probability distribution, normalized probability of transition 1 over $\delta(0)$ also gets multiplied. So these $\delta(0)$ and that $\delta(0)$ exactly cancel each other. And I am left with the first term which is just the vacuum transition probability which we know for long time average was 0. So, therefore, the horrendous term which was a $\delta(0)$ kind of thing which was somewhat problematic does not contribute anything to the probability transition. But still there are two more terms which are left. These are the terms which will try to see if there can be a transition or not. Again for inertial atoms I will open up this dot product $k_0(\dot{x} - x')$ will be $\omega_{k0}\ (t - t') - + ik_0(\dot{x} - x')$. Again for inertial atom at rest this portion will be just washed out, there $x - x'$ is 0 and only the temporal integral would be the temporal part of the exponential will be surviving. That should go and sit in the two places here and here two terms appearing over here they have to undergo the integrals and they individually become $e^{-i\omega k0(t - t')}$and the second one will become $e^{+i\omega k(t - t')}$. And outside there is an exponential $-i\Delta(E)/\hbar\ (t - t')$.

So, ultimately or the first term is sum of two products of δ function, the second term is just one product of δ function, you can verify this by playing around with the computation. So, therefore, the Correlator of interest, the two-point correlator of interest between $1_{k_0}$and 1_{k_0}just becomes integral transforms over various δ functions multiplying each other and then the exponential phase. So, the first set of operators which was getting squeezed between the vacuum state was this a_{k_0}, $a^\dagger_{k_1}$ $a^\dagger_{k2}$ and $a^\dagger_{k_0}$ with exponential phase $k_1(x) - k_1(x')$ So, therefore, these two δ functions which are appearing over here, they overall will get multiplied with the same exponential phase which is with the first term. while the second set of δ function product will just get multiplied with the complex conjugate of the first one. So, that is what happens, I just write down the two terms separately of the δ functions, meaning I will have a $\delta(0)$, $\delta(k_1 -k_2)$ coming with this exponential phase δ $\delta(k_0 -k_2)\delta(k_1 -k_0)$ with the same exponential phase like first term, while the second term $\delta(k_0 -k_1)$ $\delta(k_2 -k_0)$ comes with a complex conjugate of the exponential phase. Now, since I have the integrations under control for the at least the two point correlator, I will do away with the all the d^3k integrals. So, when I do the d^3k_2 integral, This δ function here will set that all the k_2 s in this term should be replaced by k_1. When I do the k_2 integral in this term, it will just set me that all the k_2 integrals, all the k_2 values should be put to value k_0. And similarly, in the last term, I do all the k_2 s should be put to value k_0 under the integration. So the d^3k_2 integrals can successfully be done using all the δ functions. I will get the first term will become e^{ik1} now will become common thing. It will be dotted with $x - x'$ and the $\delta(0)$. So this integral will contribute that. The second integral k_2 will be put to value k_0. So in the exponential and in the exponential k_2 will take the value k_0. But you see even the k_1 integral has the similar thing that k_1 integral will be put to value k_0 as well. So both the integrations can be done in this middle term and it will just put all the k_1 and k_2 to value k_0. So, therefore, the second term I will get is $e^{-ik0(\dot{x} - x')}/2\ k_0$. $\sqrt{2\omega_k}$ coming from will become $2\omega_k$ $\sqrt{2\omega_{k0}}$ from the k_2 integral and similarly √b3 k_1 integral will also convert square $\sqrt{2\omega_k}$ as $\sqrt{2\omega_{k0}}$. Together they become $2\omega_{k0}$. Similarly, you can just do the last integral. Also, you will see that you will still get 1 over $2\omega_{k0}$ from these things

So, you can just write these two terms, combine these exponentials, you will get the first exponential is $e^{-i\Delta(E)/\hbar + \omega k0}$ coming from the first term. And the second thing will be just $e - i\Delta(E)/\hbar\ (t - t') - \omega_{k_0}$ this time from the complex conjugate term which is the second term, which is over here. These integrals we have already evaluated in the vacuum exercise as well, vacuum two-point correlator exercise as well. And we know this, the first term is again going to give me $\pi(\Delta(E)/\hbar + \omega_{k_0}/2)^2$. This integrals we have computed before and the second term will similarly give me δ of $\Delta(E)/\hbar - \omega_k$ not this time. Because of this complex conjugate term comes with a negative time, negative sign, the two exponential combine each other with a relative negative sign in between. So, I will get a δ of, δ function of $\Delta(E)/\hbar - \omega_{k0}$ divided$/2$ this time in long time average. So, this integral we had computed before and just using this argument once more and writing these two terms like that. So, now this first term which has δ function with a positive argument, anyway this is going to become 0. So, this will not contribute in the transition probability. But the second term might, the second term will survive because there is a probability that ω_{k0} and $\Delta(E)/\hbar$ are of the same value, then this term will become non-zero. Even if ω_{k0} and $\Delta(E)/\hbar$ are magnitude wise same, the first δ function does not survive. The second one survive only when the photon frequency k_0 was the state which was vacuum, which is replacing the vacuum of the field. If the photon frequency is the same as the energy gap, then the transition might happen. The statement is that the probability will become non-zero only if the δ function is saturated. . Only thing changing with this exponential phase will become the same thing, but the complex conjugate of each other. So you see, once I have taken care of the k_2 integral in the first term, and while both k_1 and k_2 in the second and the third term over here, I get this structure. So let us go ahead and see what are we up to under this. In the transition probability expression, what we needed to compute was the double integral in $dtdt'$ with this exponential weightage of the two-point function. Now, since I have taken the initial state to be 1_{k_0} and not ground state or the vacuum state of the field, that state $1_{k_0} V_0$ we know is not normalizable. That means it is not automatically normalized. inner product is not 1. So, it is good idea to define things in terms of normalized state. That means I divide the whole expression by normalization of the state 1_{k_0}. So, I take this divide by the expression of the probability divided by 1_{k_0} 1_{k_0} that will be normalized probability. When I do that normalization of $1k_0$, 1_{k_0} is just $\delta(0)$. So, 1 over $\delta(0)$ extra factor I have to compute, I have to put in getting a normalized probability in the whole expression. The whole expression before was just this and this square box put together. That was the two point function and the double integral in the times that tells me what would be the total transition probability. Since the state was not normalized, I should divide this thing with the normalization of the state, I get this 1 over $\delta(0)$ extra vector. Now spot one thing, this $\delta(0)$ is here, for a moment just forget it, then look at this exponential thing which is multiplying that. This is exactly the two-point structure which we just dealt with. Remember, when we were dealing with initial state being the two-point, initial state being the vacuum, the two-point function was just this integral $d^3k\ \omega_k(t - t') + k(\dot{x} - x')$. Together it is just $e^{ik(\dot{x} - x')}$. So, the same thing has appeared multiplying $\delta(0)$ this time. This is the two-point function of the vacuum. This gets multiplied with $\delta(0)$ because this integral itself was coming with the $\delta(0)$. But fortunately since we are looking about the normalized probability distribution, normalized probability of transition 1 over $\delta(0)$ also gets multiplied. So these $\delta(0)$ and that $\delta(0)$ exactly cancel each other. And I am left with the first term which is just the vacuum transition probability which we know for long time average was 0. So, therefore, the horrendous term which was a $\delta(0)$ kind of thing which was somewhat problematic does not contribute anything to the probability transition. But still there are two more terms which are left. These are the terms which will try to see if there can be a transition or not. Again for inertial atoms I will open up this dot product $k_0 \cdot (\dot{x} - x')$ will be $\omega_{k0}\ (t - t') + ik_0 \cdot (\dot{x} - x')$that should go and sit in the two places here and here two terms appearing over here they have to undergo the integrals and they individually become $e^{-i\omega k0(t - t')}$ and the second one will become $e^{+i\omega k(t - t')}$. And outside there is an exponential $- i\Delta(E)/\hbar\ (t - t')$. So, you can just write these two terms, combine these exponentials, you will get the first exponential is $e^{-i\Delta(E)/\hbar + \omega k0}$ coming from the first term. And the second thing will be just $e^{-i\Delta(E)/\hbar\,(t-t')-\omega k0}$ this time from the complex conjugate term which is the second term, which is over here. These integrals we have already evaluated in the vacuum exercise as well, vacuum two-point correlator exercise as well. And we know this, the first term is again going to give me π times $(\Delta(E)/\hbar + \omega_{k_0}/2)^2$. This integrals we have computed before and the second term will similarly give me δ of $\Delta(E)/\hbar - \omega_{k0}$ this time. Because of this complex conjugate term comes with a negative time, negative sign, the two exponential combine each other with a relative negative sign in between. So, I will get a δ of, δ function of $\Delta(E)/\hbar - \omega_{k0}/2$ this time in long time average. So, this integral we had computed before and just using this argument once more and writing these two terms like that. So, now this first term which has δ function with a positive argument, anyway this is going to become 0. So, this will not contribute in the transition probability. But the second term might, the second term will survive because there is a probability that ω_{k0} and $\Delta(E)/\hbar$ are of the same value, then this term will become non-zero. Even if ω_{k0} and $\Delta(E)/\hbar$ are magnitude wise same, the first δ function does not survive. The second one survive only when the photon frequency k_0 was the state which was vacuum, which is replacing the vacuum of the field. If the photon frequency is the same as the energy gap, then the transition might happen. The statement is that the probability will become non-zero only if the δ function is saturated. That means the photon's frequency is the same as the energy gap. If you excite the atom, if you excite the field, the initial state is 1_{k_0} where 1_{k_0} is very different from $\Delta(E)/\hbar$. Then even the second δ function will become zero. That means this probability only survives when the initial state has the same energy as that of the energy gap of the atom. And what is the probability? Again there was a normalization division. The square of a δ function can be written as the first δ function times second argument beginning 0. $(\delta(x))^2$ can be written as a $\delta(x)$ times $\delta(0)$. So that is what the trick I have used and division by $\delta(0)$ coming from the norm of one k_0 states, they cancel each other. So, $\delta(0)$, $\delta(0)$ cancel each other and then the survival probability is this. The probability of transition is just the part which is remaining, just the part which survives is this. This has given a wrong sign, wrong box formation. So, this is the expression which will survive. This tells only survive, only transition which is allowed through interaction of the field with the atom is if the field has one photon in itself which has the same energy as that of energy gap of the atom. Then only the transition will happen, otherwise it will not. This is very much reasonably expected. So, Unruh-David detector rule at least is telling us the results are consistent with our expectation. If there is no photon in the field, the atom should not get excited. If there is a photon in the field, the atom may get excited depending upon the photon in the field has the same energy as that of atom's transition lines or not. So, that is very well reasonable approach. So, therefore the learning is that resonance condition, this is called the resonance condition that the photon and the energy gap of the atom are the same. However, we have computed this thing for long time average t tending to infinity. However, in long time average, we cannot just rely on the leading order perturbation theory. Remember, when we started doing the business, the time ordered exponential, we had truncated the higher order contribution, saying that the leading order contribution is the dominant one. But you can check for yourself that higher order terms which we had thrown away, they are small at small time steps. If I wait for large and large and large time, they will not remain small enough. So they will become, they have a tendency to grow for longer times. That means if I literally take t tending to infinity, I have to account for all order terms of the time ordered exponential, not just the leading order. So therefore, this result which we have obtained the photons cannot get excited. In if it is in vacuum or photons get excited if their frequency if atoms get excited if the frequency of the photon in the field is same as its energy gap all these results cannot be fully trusted or one can reverse the game and say that t tending to infinity by that I really do not mean infinity I just mean t is much much greater than the internal time scale which we just discussed in today's set of, today's discussion, that t much, much greater than δH, $\delta\ \hbar$ upon, $\hbar/\Delta(E)$ is as good as t tending to infinity. So, I do not really mean t tending to infinity, I just mean t *is* much, much greater than the internal time scale. But this time scale should be smaller than the time scale at which the second or higher order terms in the perturbation theory start becoming important. So, remember what we had thrown out? In the time ordered exponential if when we were doing the interaction picture we had retained only the leading order term which was this integral we had thrown out the second order term which was the this term when these two terms become starting to compete against each other they that means when they become of the same order from that point onwards I cannot do perturbation theory up to leading order that means perturbation theory breaks down over there so therefore there should be the upper limit of integral t_0 after which leading order perturbation theory is not good enough. So if I take my evaluation probability of transition time capital *t* much larger than the internal time scale of the atom which is $\hbar/\Delta(E)$, but much shorter than the time scale at which perturbation theory breaks down. Then whatever we have computed is good enough, only thing you have to keep in mind the δ functions and other things which are really true for *t* tending to infinity. They will not appear but their cousins will appear. That means *t* for very very big function for very very big value of *t* but smaller than a particular number which is t_0 which is really very high. So *t* much much greater than $\hbar/\Delta(E)$, I will get very close to a δ function, not exact the δ function which we are getting here, but some function which is very good approximation to a δ function. So, therefore, in this time scale where I am much larger than the internal time scale of the atom, but much smaller than the time scale at which perturbation theory breaks down. Till that time scale our analysis is trustworthy enough. It is not exactly correct, but trustworthy enough. So, whatever we have obtained that the photons will not able to excite, the vacuum will not be able to excite the atom, will definitely will not be able to excite the atom at least for a long time period capital T, which is larger than its internal time, but shorter than the perturbation theory's limit. So, whatever we have obtained that the photons will not able to excite, the vacuum will not be able to excite the atom, will definitely will not be able to excite the atom at least for a long time period capital T, which is larger than its internal time, but shorter than the perturbation theory's limit. That is trustworthy enough for a sufficiently large time period. Fine, so then we stop over here and these are the natural generalizations which we see which are intuitively correct. Only thing of interest is that our intuition runs within this time window larger than $\hbar/\Delta(E)$ and shorter than t_0.