Foundation of Quantum Theory: Relativistic Approach

Thermal Fields 1.2

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Scalar field thermal state

Lecture- 32

In the previous class, we had obtained this result that in the thermal state, since the overlap between n_k and n_k+1 or n_k-1 supposed to be 0. In the previous class, we had obtained this result that in the thermal state, since the overlap between n_k and n_{k+1} or n_k-1 supposed to be 0.

$$+ \int \frac{d^3 k}{\sqrt{2\omega_k}} e^{-ik \cdot x} \sqrt{n_k+1} \, |\{n_v, \dots n_k+1, \dots n_{k_n} \dots\}\rangle$$

Prove: $\langle n_{k\pm1} | n_k \rangle$

$\therefore \quad \langle \phi(x) \rangle_\beta = 0$

The thermal correlation function

$$\langle \phi(x)\phi(y) \rangle_\beta = Tr\left(\frac{\phi(x)\phi(y)e^{-\beta H}}{Z(\beta)} \right)$$

$$= \frac{\sum_{[n]} \langle \{n_k\}|\phi(x)\phi(y)e^{-\beta H}|\{n_K\}\rangle}{Z(\beta)} \phi(y)\phi(x)|\{n_k\}\rangle$$

$$=$$

$$\phi(y)\frac{1}{(2\pi)^{3/2}}\left[\phi(x)\phi(y)\int \frac{d^3\vec{k}}{\sqrt{2\omega_{\vec{k}}}}\sqrt{n_k}\, e^{ik \cdot x}|\{n_1, \dots n_{k-1}, \dots n_{kn}\dots\}\rangle + \int \frac{d^3\vec{k}}{\sqrt{2\omega_k}}\sqrt{n_k+1}\, e^{-ik \cdot x}|\{n_1, \dots n_{k+1}, \dots r\right.$$

$$=$$

$$\frac{1}{(2\pi)^3}\int \frac{d^3\vec{k}}{\sqrt{2\omega_{\vec{k}}}}(\hat{a}_k e^{+ik'y}+\hat{a}_k^\dagger e^{-ik'y}) \times \phi(y)\frac{1}{(2\pi)^{3/2}}\left[\phi(x)\phi(y)\int \frac{d^3\vec{k}}{\sqrt{2\omega_{\vec{k}}}}\sqrt{n_k}\, e^{ik \cdot x}|\{n_1, \dots n_{k-1}, \dots n_{kn}\dots\}\rangle + \int \frac{d^3\vec{k}}{\sqrt{2\omega_k}}\sqrt{n_k+1}\, e^{-ik \cdot x}\right.$$

Therefore, thermal states also share the same property as far as the expectation value is concerned with the vacuum state. Unlike the coherent state where the field expectation was non-zero for coherent state, for vacuum state this was zero, for thermal state also it turns out to be zero. But now let us check compared to the thermal state, compared to the vacuum state what is the structure of correlations in the thermal state. F We realized, we remember that the correlation structure in the vacuum state was something $1-1/\pi^2$ divided by the invariant distance d^2 or something. For coherent state, there were extra terms which were like classical location x expectation times location y expectation. Let us try to find out whether since expectation of ϕ and expectation of $\phi(y)$ at location x and y are identically 0.4 for the thermal state, whether such a result is true for thermal state as well that I do get total correlation in the thermal state as vacuum par t+ an extra term which is individually expectation product. And if that happens, then the correlation function will share, the two-point function of the thermal state will share the same value as compared to the vacuum two-point correlation as well, because individual expectation happens to be zero. So, first we need to check do we have a structure like this which was to the coherent state or not. In order to obtain that again we have to do the same computation that I have to now put two operators alongside with the density matrix in the trace divided by $Z(\beta)$. And therefore,

that would be the definition of $\phi(x)\phi(y)$ expectation value the two point correlation function. Like before, I have to write it in the proper basis. So I will squeeze the operators appearing in set n_k and set n_k bra and ket, sum over all the possible n_k set which is appearing over there and divide it with $Z(\beta)$. $Z(\beta)$ we know what it is. This piece we have already computed so many times and now we have to find out what is this. So the same trick I will play, first I will compute the operation of two ϕs on the n_k, set n_k and take the dagger of that to convert things into like this. However, this time I have to be careful, when I do the dagger of this thing, the operators order flips. So I have a $\phi(x)$, $\phi(y)$. Then if I take the dagger I would get a n_k ket will become n_k bra and ϕ_1 operator and ϕ_2 operators the products dagger will appear which will be equal to $\phi_2{}^\dagger\phi_1{}^\dagger$. So, it would become $\phi_2{}^\dagger\phi_1{}^\dagger$ and since they are Hermitian operators real scalar field the † is equal to themselves. So, I will get a ϕ_2 ϕ_1 when I do the dagger the operator order flips ϕ_1 ϕ_2 † will become $\phi_2\phi_1$. So, therefore, I should obtain first the operation of $\phi(y)$ $\phi(x)$ take the dagger of the whole thing then $\phi(x)$ will come on the left $\phi(y)$ will come on the right and the ket will become a bra. But up to that care, the straightforward algebra is fine. We have already a result of how $\phi(x)$ acts/n_k in the previous discussion session.   We will realize that it is plane wave times $\sqrt{n_k}$ with one particle down in kth basis and which kth basis is that will be summed over. Similarly, the second term comes with a plane with a–sign in the exponential. kth mode has one extra particle excitation and the normalization corresponding to a dagger section is n_k+1. So, one extra particle in mode k and that mode will be changing as we sum over the integration. So, that is the operation of the first $\phi(x)$ on n_k. Now I should do the second $\phi(y)$ operation to that. So I will write down $\phi(y)$ also in the similar kind of expansion basis that we have seen before and only here I have to put that this is being done at location y. So everywhere with k exponential will be made up of function of y. And that is going to hit the $\phi(x)$ n_k, which already I have a result for this as the whole summation over here. So, these two things, two operators will hit these two states sum. There are effectively four types of terms going to appear. This $\hat{a}_k$ hits this whole term.   Remember, this whole term is nothing but $\hat{a}_k$ acting on the set n_k. So this $\hat{a}_{k'}$ will hit $\hat{a}_k n_k$ or this $\hat{a}_{k'}$ can hit this state. Remember again this whole state is $\hat{a}^\dagger{}_k$ hitting the set n_k. So the possibility is $\hat{a}_k$ combines with this $\hat{a}_k$ or $\hat{a}_{k'}$ this combines with this $\hat{a}^\dagger{}_k$. Or that $\hat{a}_k$ † combines with this $\hat{a}_k$ or that $\hat{a}_k$ † combines with this $\hat{a}^\dagger{}_k$. So there are four possible sets of operation and every time these pair of operators are supposed to hit this n_k set. So that is what the structure is. So what will happen if this $\hat{a}_k$' goes ahead and hits this n_k, n_{k-1} and n_k, n_{kn}. So this $\hat{a}_k$' is supposed to bring down number of particle by one unit in which mode? k' mode and later on all the k' will be summed over meaning one by one it will suppress particle content in each mode by one. Not simultaneously in all of them, but it would be summation over first, the first state will have one particle, k_1 will be going down and all other things will be like before. In between there is a n_{k-1}, then the second term would be n_{k1} remains the same, the second state gets a one particle deduction and n_{k-1} is already there and so on. So this $\hat{a}_{k'}$ will one by one gradually shift down the number contained in each mode one by one as a summation that is equivalent to integration by one unit down.

So therefore, the $\hat{a}_{k'}$ will keep making things one by one down. So it will either create k', if k' is equal to k, then this n_k–one state will go down further by one unit, it will become n_k–two. Or otherwise when k' is not equal to k, then the other k' mode which was initially $n_{k'}$ will go down with one unit.

$\hat{a}_{k'}$ acting on $|\{n_1 \ldots n_k - 1 \cdot n_{\star_n} \ldots\}\rangle$ will convert it

to $\quad |\{n_1 \ldots n_{k-2} \ldots n_{\star_n} \ldots\}\rangle$ ✓

or $\quad |\{n_1 \ldots n_k - 1, \, n_{k'} - 1, \ldots n_{\star_n} \ldots\}\rangle$ ✓

Or, $\quad \hat{a}_{k'}^{\dagger}$ acting on $\quad |\{n_1 \ldots n_k + 1 \cdot n_{\star_n} \ldots\}\rangle$

$|\{n_1 \ldots n_{k+2} \ldots n_{\star_n} \ldots\}\rangle$ ✓

$|\{n_1 \ldots n_k + 1, \, n_{k'} + 1, \ldots n_{\star_n} \ldots\}\rangle$ ✓

None of these will survive under overlap with

$$\boxed{e^{-\beta H} |\{n_k\}\rangle} = e^{-\beta \int d^3\vec{x} \, n_k \omega_k} |\{n_k\}\rangle$$

Thus surviving overlap will be

$\hat{a}_{\textcircled{k}}$ acting on $\quad |\{n_1 \ldots n_k + 1 \cdot n_{\star_n} \ldots\}\rangle = |\{n_{\star_n}$

and $\quad \hat{a}_k^{\dagger}$ acting on $\quad |\{n_1 \ldots n_k - 1 \cdot n_{\star_n} \ldots\}\rangle$

Formally surviving terms of expansion are

$$\langle \{n_k\} | \left[e^{i(k \cdot y - k \cdot x)} \, \hat{a}_k \cdot \hat{a}_k^{\dagger} + e^{i(k \cdot x - k'y)} \hat{a}_k^{\dagger} \cdot \hat{a}_k \right] |\{n_k\}\rangle$$

$$= \delta(\bar{k} - \bar{k}') \left[(n_k + 1) \, e^{ik \cdot (y - x)} + n_k e^{ik \cdot (x - y)} \right]$$

$\hat{a}_k{}'$ acting on $|\{n_1, \ldots n_{k-1}, \ldots n_{kn} \ldots\}\rangle$ will convert or to $|\{n_1, \ldots n_{k-2}, \ldots n_{kn} \ldots\}\rangle$
or $|\{n_1, \ldots n_{k-1}, n_{k'-1} \ldots n_{kn} \ldots\}\rangle$

Or $a_k^{\dagger}{}'$ acting on $|\{n_1, \ldots n_{k+1}, \ldots n_{kn} \ldots\}\rangle$ will convert or to $|\{n_1, \ldots n_{k+2}, \ldots n_{kn} \ldots\}\rangle$
or $|\{n_1, \ldots n_{k+1}, n_{k'+1} \ldots n_{kn} \ldots\}\rangle$

None of these will survive under overlap with

$$e^{-\beta H}|\{n_k\}\rangle = e^{\beta \int d^3 \vec{k} n_k \omega_k}|\{n_k\}\rangle$$

Thus overlapping will be

$\hat{a}_k$ acting on $\quad |\{n_1, \dots n_{k+1}, \dots n_{kn} \dots\}\rangle$

and $\hat{a}_k^{\dagger}$ acting on $\quad |\{n_1, \dots n_{k-1}, \dots n_{kn} \dots\}\rangle$

Formally surviving terms of expansion are

$$\langle\{n_k\}|\left[e^{i(k \cdot y - k \cdot x)}\hat{a}_k{}' \hat{a}_k^{\dagger} + e^{i(k \cdot x - k \cdot y)}\hat{a}_k^{\dagger}{}' \hat{a}_k\right]|\{n_k\}\rangle$$

$$= \quad \delta(k - k')[(n_k + 1)e^{i(k \cdot y - k \cdot x)} + n_k e^{i(k \cdot x - k \cdot y)}]$$

So, it can convert the state which it is hitting, the $\hat{a}_k{}'$ which is appearing over here is supposed to hit this state. So, there are two possibilities can generate, a state of this kind, sorry after action on this state of this kind. or state of that kind. This is for k is equal to k and this is for k not equal to k. The same logic is there when ak, the same logic is there when the $\hat{a}_k{}'^{\dagger}$, let me make it clean again so that we can see with clarity that $\hat{a}_k{}^{\dagger}$ goes and hits this one or that one. So, first we will consider its action on this one. So if $\hat{a}_k{}'^{\dagger}$ goes and hits the second ket which has a n_{k+1} particle, if $k' = k$, it will increase the particle content in kth mode by 1 more, so it will become n_{k+2}. Or if k' is not equal to k, so somewhere else it will increase the particle content with 1 negative k. So again the action of $\hat{a}_k{}'$ will either raise it by 2 or it make it something like that. $\hat{a}_k$ is' action on the first kit was either break it down by 2 or somewhere two particles one and one each in the machine.So, action of this converts it into –2 particle in k mode or –1 and –1 in k and k mode. Similarly, action of this object on this converts either +2 in k mode or +1+1 in k and k mode. However, action of these things results in states like this.   But these are meaningless states in the sense that when I take the overlap of the $e^{-\beta H}|n_k\rangle$. Remember, when we are doing this, we are computing ϕ by $\phi(x)$. Ultimately, the whole thing at the end of the day, the dagger of that has to be taken and overlap with $e^{-\beta H}|n_k\rangle$ has to be done. So, if the whole thing we do, then $e^{-\beta H}|n_k\rangle$ is the eigen state equation. Get n_k is eigen state of $e^{-\beta H_k}$. So, I will have a overlap of these kind of states with n_k and we have seen before. If n_k, particle content in n_k does not match from the particle content of the number of particles in the bra, the overlap is zero. So whatever overlap action of this does onto this, its action of this does onto this state, its overlap with $e^{-\beta H}$ times this will be zero. Similarly, whatever action this does on the second kit, its overlap with $e^{-\beta H}|n_k\rangle$ would also be zero. So, only thing which will survive under this action is when this $\hat{a}_k$ goes and hits on this and when $\hat{a}_k^{\dagger}$ from this side goes and hits the first hit. So, if you do that, If $\hat{a}_k$ goes and hits the second term n_k+1, so this is already one particle extra, k will search for this thing and find out if it is the same, it will bring down it to be n_k back. So, it will become similar to n_k again, then the overlap will survive. Similarly, when $\hat{a}_k^{\dagger}$ and goes on and hits the n_{k-1}, the first ket the first ket. Remember both operators were supposed to act on both the kets appearing in the expansion. So when this term goes and hits this one, then already it was n_{k-1} to begin with. $\hat{a}_k^{\dagger}$s action will make it one particle extra in n_k, the kth mode, it will become back to n_k. So you see out of the four possibilities we listed down above, Let me again for one more time clean it up. There are four possibilities when $\hat{a}_k$ over here hits this state or $\hat{a}_{k'}$ over here hits this state or other possibilities that $\hat{a}_k{}^{\dagger}$

hits this state or $\hat{a}_{k'}^{\dagger}$ hits this state. Out of these four possibilities, two possibilities create a state different than ak. So, state different than state n_k will under overlap with $e^{-\beta H}|n_k>$ coming from the right hand side will vanish.So, two states of the four possibilities will vanish. Only two states will generate again the state n_k back and these are these operations when $\hat{a}_k$ acts on n_k plus one state and $\hat{a}_k^{\dagger}$ acts on $n_k n_k$–one state. So you do that, you write down what survives, so ultimately as repeatedly we are discussing, action of this onto this state and action of this onto this state survives.

You can see now this was already $\hat{a}_k$ and $\hat{a}_k^{\dagger}$ has to act on this. So I will have a structure like $\hat{a}_k^{\dagger}$ ak. or $\hat{a}_k$ $\hat{a}_k^{\dagger}$ this state is $\hat{a}_k^{\dagger}$ ket n_k. So, ultimately what is going to survive is the plane waves coming from both sides. So, you see surviving terms plane waves this should combine with sorry this should combine with that and this should combine with this. So, if that happens you can be slightly attentive the first $\hat{a}_k$ $\hat{a}_k^{\dagger}$ will come with this plane wave combination and the second term where the $\hat{a}_k^{\dagger}$ comes with $\hat{a}_k$ will come with this plane wave combination that you can verify from here that you can verify from the combination which is going and hitting the state. So, let me clean it up and you can see what we have written. Four possibilities, two of them are different states than state n_k, two of them are same states as n_k and only those two states will survive the overlap with $e^{-\beta H_k}$ and $e^{-\beta H}$ acting on n_k. So, ultimately the inner product which is going to survive is things like this. Again for this to survive exactly the kshould be equal to k, otherwise this will create an extra particle in n_k mode and this will decrease somewhere else. So, they should compensate each other's action. Similarly, here the dagger and this thing should not decrease and increase in separate modes, otherwise number contents will change. So, they should do the operations in the same mode. So, therefore, there should be a $\delta(k-k)$. That can be verified from the algebra itself that the commutator structure if you implement to write it down neatly, you will get that the k and k have to be the same.

So, we have the expectation of operators which will appear in the whole computation of the expectation of $\phi(x)\phi(y)$ $e^{-\beta H}$. where this became $e^{-\beta\omega_k n_k}$ and the ket n_k so overall what we have to get is the squeezing of $\phi(x)\phi(y)$ between n_k kets. So, that is what we have done over here. You can see ultimately it is the surviving part of $\phi(x)\phi(y)$ in the states ket n_ks from both sides and this is giving me this result. So, ultimately this is what will go and sit in this expression, expression which we wrote down over here. We needed to write the expectation of the whole thing in the numerator. Out of this, this trivially converted itself into $e^{-\beta H}$. β summation or integration $d^3k n_k \omega_k$ as real number and then the set n_k back. So, this can go out of this whole thing here and ultimately $\phi(x)\phi(y)$ will get squared between n_k. And out of this as we discussed in the preceding few minutes that only two terms will survive and these two terms are exactly this. and$k$plus one with plane wave structure like $y-x$ and n_k with plane wave structure like that. So, you go over here, you go into this and compute So just a second, I have to, yeah, so I had to take, I had to take the dagger of this thing first. So I had to take the whole things result here, dagger of these things first, and then take the inner product with respect to this. Remember we wanted $\phi(x)\phi(y)$ over here. What we have obtained over here is in this case is $\phi(y)$. $\phi(x)$. Remember, $\phi(y)$ is appearing over here and $\phi(x)$ is appearing over here. So, what we have to do? We have to take the conjugate of this. The conjugate of this will flip the negatives here and positives over here. So, ultimately in the interesting result, a–should appear here and+should appear here. So, therefore, in the computation of the expectation of $\phi(x)\phi(y)$ a – will appear here and a+will appear here.
 So, this will be the full computation e $n_k \omega_k$ is coming from $e^{-\beta H}$ acting on nk so this is a real number which has come out and the remaining thing which is surviving inside is $\phi(x)\phi(y)$ operator squeezed between n_ks and this is the result for that and everything was supposed to be $Z(\beta)$ divided by $Z(\beta)$ Now, this $Z(\beta)$ computation we have done so many times, it would have all those n_k, n_k overlap which will appear in this and that computation as well. So, in the numerator again overlap between n_k, n_k is coming

through as we have discussed and in the downstairs in *Z(β)* also the same thing is appearing. So, overlap like before *Z(β)* cancels out all this *nω* a *nω* factors in up and down. What more? We had previously realized that *Z(βH)* as exactly this structure that it has all *k* is possible $1/(1-e^{-\beta\omega_k})$. And here I have a n_k times this summation or 1 times this summation or again n_k times that summation, both the summations which we have done. This computation for all other *k* different than all other*k*print different than*k*again will ride along with n_k here and get me the factors which we had seen in the previous class in the expectation of the expectation of the field operator themselves so remember $n_k e^{-\beta\omega k n_k}$ out to be $e^{-\beta\omega}$ divided by *(1–e^{-βω})* times *1/(1–e^{-βω})* if you recall. This was the structure we had obtained.

$$\therefore \quad \langle \phi(x)\,\phi(y)\rangle_\beta \;=$$

$$\sum_{\{n_k\}} \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\omega_k} \left(e^{-\beta\int d^3\xi\, n_\xi \omega_\xi} \right) \frac{\left[(n_k+1)\,e^{-ik\cdot(y-x)} + n_k\,e^{+ik\cdot(x-y)}\right]}{Z(\beta)}$$

$$= \frac{1}{Z(\beta)}\frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega_k} \prod_k \sum_{n} \left(e^{-\beta n_\xi \omega_\xi} \left((n)+1)e^{ik\cdot(y-x)} + n_k e^{ik\cdot(x-y)} \right) \right)$$

- Like before $Z(\beta)$ cancels all of the

$$\left(\frac{1}{(1-e^{-\beta\omega_k})}\right) \quad \text{factors !} \quad \left(\prod_{k'\neq k}\sum_{n_{k'}} e^{-\beta\omega_{k'}n_{k'}}\right)\left(\sum_{n_k}\right)$$

$$\langle \phi(x)\phi(y)\rangle_\beta = \frac{1}{2\pi^3}\sum_{\{n_k\}}\int\frac{d^3k}{2\omega_k}\frac{e^{-\beta\int d^3\vec{k}\, n_k\omega_k}\left[(n_k+1)e^{-ik(y-x)}+n_k e^{ik(x-y)}\right]}{Z(\beta)}$$

$$= \frac{1}{Z(\beta)}\frac{1}{(2\pi)^3}\int\frac{d^3k}{2\omega_k}\prod_k\sum_n e^{-\beta n_k\omega_k}\left((n_{k+1})e^{ik(y-x)}+(n_k)e^{ik(x-y)}\right)$$

cancells all of the term $\dfrac{1}{1-e^{-\beta\omega_k}}$ factors !

$$\boxed{\langle \phi(x)\phi(y)\rangle_\beta = \frac{1}{(2\pi)^3}\sum_{\{n_k\}}\int\frac{d^3k}{2\omega_k}\frac{\left(\left(\langle n_k\rangle_\beta+1\right)e^{-ik(y-x)}+\langle n_k\rangle_\beta e^{ik(x-y)}\right)}{Z(\beta)}}$$

$$\langle n_k\rangle_\beta = \frac{1}{e^{\beta\omega_k}-1}$$

$\star$ Clearly $\beta\to\infty$ $(T\to 0)$ we have
langle n_k rangle_$\beta\to 0$

$$\langle \phi(x)\phi(y)\rangle_\beta \rightarrow \langle 0|\phi(x)\phi(y)|0\rangle$$

$$\langle \phi(x)\phi(y)\rangle - \langle\langle 0|\phi(x)\phi(y)|0\rangle\rangle_\beta = \frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega_k}\langle n_k\rangle_\beta \left(e^{ik\cdot(x-y)}+c.c.\right)$$

Now, this thing along with the product k, all the possible ks, then this, all the possible k over here, then this k value, together they cancel this kind of factor coming from the $Z(\beta)$. So, I will be left with this only as far as this n_k kind of summation is concerned. For one kind of summation, it is exactly cancelled again with this $Z(\beta)$ factor. So, ultimately what I am saying is, the product and the sum here are two kinds, n_k times this or just $e^{-\beta n_k}$ with 1. So, ultimately I am going to get from nk $e^{-\beta\omega_k}$ for all k this thing, this is for a single k and sorry let me write it more clearly that for all k not equal to k, I have just $e^{-\beta\omega kn_k}$ where $n_k\omega_kn_k$ and for just for a single k I have summation n_k then n_k time $e^{-\beta\omega_k n_k}$. So, this product is for all possible k out of which all but one has this plane exponential character, only one term has an extra n_k which is appearing over here. This divided by $Z(\beta)$ leaves me the Planckian factor $1/(e^{\beta\omega}-1)$, the expectation n. So, this is nothing but expectation of n. This term, this times the partition function divided by this times the exponential divided by the partition function is nothing but expectation value of n which is $1/(e^{\beta\omega}-1)$. 1 times this factor divided by $Z(\beta)$ is 1 because 1 times this factor itself is $Z(\beta)$ and divided by $Z(\beta)$ is 1, so they cancel out. Similarly, here n_k times $e^{-\beta\omega}$ and k summation over all the possible ks and product over all possible ks also $Z(\beta)$ also expectation of n. So, this becomes expectation of N in the thermal state, this also becomes expectation in the thermal state and this becomes $Z(\beta)$ divided by $Z(\beta)$ which is 1.   So, ultimately you can see the computation which we performed for the expectation value of field, very similar kind of computations arrive in this case as well. And ultimately this whole computation, this n_k gets replaced after the summation through its thermal expectation value. This n_k also gets replaced by its thermal value and 1 becomes 1 at the post of removal of $Z(\beta)$. $Z(\beta)$ goes into defining the expectation n_k, defining expectation of 1. Expectation of 1 remains 1. Expectation of n_k becomes the Planckian factor $1/(e^{\beta\omega}-1)$ over here.

So, you see now we have the two poincorrelation function in the thermal state and it has a very nice structure that I have a $e^{ik(x-y)}$. So, remember here we had written a–sign over here should be which I erased out again and +sign here. So, I can write this as y–x with a–sign as x–y and here x–y with a +sign should become a + over here. Okay, so you see this is the structure you have, sorry, sorry, sorry, we had written it wrongly. So, $+x$–y is coming from here, this was –, this was y–x and this is x–y and a – of this thing was also there. So,–of $ik(x-y)$ this becomes and this becomes $+ik(\dot{x}-\dot{y})$.   So, just fix this errors which I had made because I had to take a dagger remember and that I have written in y–x and x–y one of them will have an extra–sign. So, we will get the structure. So, this–of y–x I am just calling at x–y that is all. So, this $n(\beta)$ expectation appears and $n(\beta)$ expectation appears here as well. So, you see we have a two-point function in a thermal field completely evaluated in the sense of I know what this function is, I know what this is, in principle I should do this integration and get done with this and ultimately it give me some non-zero survival number. But you see, when I take β tending to infinity, which is temperature tending to 0, if I take β tending to infinity, then this exponential factor becomes e^{∞}, $1/e^{\infty}$ approaches to 0. That means this will vanish, this will vanish and only thing I will be surviving with will be this $e^{ik(x-y)}$, this $d^3k/(2\omega_k)$ integral $1/(2\pi)^3$. So if you recall, this was exactly the vacuum piece. Vacuum two-point function was exactly this. So if I take infinite β or zero temperature, I am left with only the vacuum contribution.So the one standing here is the vacuum part. So I can therefore separate out the vacuum part. I can take this 1 times this into this integration on the left hand side. That will become just the vacuum two point function. So now I have a result at hand that the two point

function of a thermal field–two point function of a thermal field, two point function of a field in vacuum is equivalent to first term and the last term, n_k with $e^{+ikx-y}+$ its complex conjugate and n_k factor is common. So, you see unlike the coherent state, it did not separate out into the expectation of ϕ at x and expectation of ϕ at y because those things were identically zero. This is not this quantity this surviving piece on the right hand side is not that so it is not like a coherent state it survives with a definite number which depends on your excitation across different different modes put together and you have a left over residual thermal correlations at hand vacuum correlations and thermal field correlations are different but still the thermal correlation contains a part which is vacuum and on top of that there is a non-vacuous thermal kind of fluctuation left over so this again is an important result which will become handy if when we start discussing particles put in thermal environment or thermal field so therefore. You see vacuum coherent and any other state we can in principle compute correlation functions in that. And these correlation functions are estimate of how much of quantum fluctuations are there even when the field is empty. That means expectation is 0, not empty but expectation is 0. So, in those cases where field expectation is 0 even then quantum fluctuations will do something to our atomic systems which in the next class onwards we will start seeing or if field expectation is non-zero like if it is a classical kind of field which is mimicked good enough by a coherent state then also there are inherent quantum fluctuations coming from the vacuum part. The vacuum part The fluctuations part in coherent state are exactly similar like vacuum state because otherwise it is like a classical system. So, you see vacuum, thermal and coherent all come with certain quantum fluctuations, some with larger, some with smaller. And those quantum fluctuations are supposed to do something to the atomic system when they start talking to the electric field or scalar field or spinal field. And those quantum fluctuations are supposed to do something to the atomic system when they start talking to the electric field or scalar field or spinal field. We have done exercises for scalars, but you can see now mathematical structure of even a final field or vector field are very similar. Oscillator story is common, only extra indices are there. Apart from k, there should be spin sums or polarization sums, that is all. So, more or less everything will go through like before, only thing extra polarization direction or extra spin direction will come about in the game, which you have to be just be aware of. Otherwise, all the things which we have generally obtained for scalars is supposed to go through for any other free field as well. So, I stop over here and in the next class onwards, we will try seeing how quantum field starts affecting the statements of atomic systems which we used to discuss with the perturbation theory previously. Now, we will try to see how much of perturbation quantum fluctuations and quantum fields bring about to an atomic system.