Radio Astronomy

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Radio Astronomy Fundamentals Part - 2

Hello everyone and welcome to the lecture 3 of this week 2 and we are continuing on discussion the basic radiation fundamentals. Quantitating from lecture number 2. So in lecture number 2 we discussed about different quantities like flux density, luminosity, intensity where little bit of a difficulty in understanding about the which one depends on the distance and which one does not. But we tried to resolve the issues in the basic concepts of different properties of what is a flux density, intensity and luminosity and I hope it helped. We had also given couple of shown examples worked out and few also will be there in the subsequent assignment to help you. But even if you are having problem we can even have some follow up lectures discussing more on some problems which will clear out the concept.

So now as we started from the lecture number 2 of this week that the different cosmic bodies do radiate emit at different parts of the deterministic spectrum and the closest what matches is with something called a black body radiation which kind of is works out for most of the EM radiation that we observe at the radio wavelengths. So a black body is actually an idealized physical object that absorbs all incident radiation across the entire electromagnetic spectrum and emits radiation based solely on its temperature. It does not reflect or transmit any radiation. A black body is often considered as a perfect radiator and a perfect absorber of electromagnetic radiation.

The temperature of the lamp blank block rises faster than the silver block because the black surface absorbs radiant energy from the sun at a greater rate. So if you have seen these two examples of a silver coated body and a lamp black coated body this temperature rises faster because it absorbs most of the if not all of the solar incident solar radiation than the other silver coated one which reflects significant amount. So the temperature does not rise as significantly as the black body. This is just a close example to show you what black body means. Because absorption and emission are balanced a material that is a good absorber like lamp black is also a good emitter and a material that is a poor absorber like silver is also a poor emitter.

The spectral distribution of black body radiation is described by Planck's law. It is a very important contribution of Max Planck in 1900. This law accurately predicts the intensity of radiation emitted by a black body at different wavelengths and temperatures taking into account the quantum nature of the energy levels. What is my quantum nature? We have here in quantum mechanics that you know that energies are quantized okay. And it does not necessarily have a continuous flow but they are quantized in terms of h nu.

So each the minimal packet of energy is h nu where h is the Planck's constant and nu is the frequency of the emission. So Max Planck proposed that the energy of electromagnetic radiation inside a black body is not a continuous but instead comes in discrete packets of quanta of energy. The spectrum of the radiation emitted by hot opaque objects can be fit by a model that assumes quantized units of energy. The primary law governing the black body radiation is the Planck radiation law. Planck used quantization of energy En is equal to n h nu where n is the level.

Planck used this quantization. It gives the intensity of radiation emitted as a function of wavelength for a fixed temperature. So we have these two formula the b nu the spectral radiance as a function of time temperature the capital T is given by 2 h nu cube over c square then 1 over e to the power h nu kt minus 1 that is the Planck function. Nu is of course the frequency of radiation and T is the temperature of the body okay. If we write it in terms of the wavelength we get b lambda t is equal to h c squared over lambda to the power 5 then 1 over e to the power h c over lambda kt minus 1 okay.

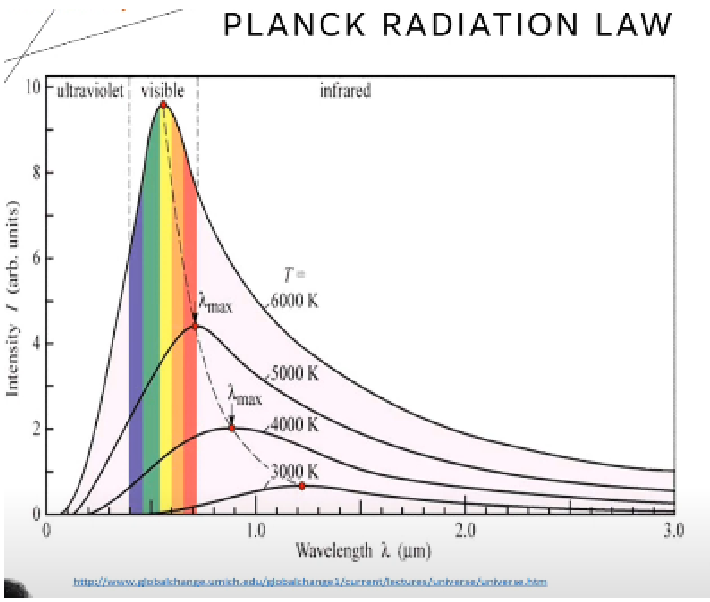
$$B_\lambda(T)=\frac{2hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}$$

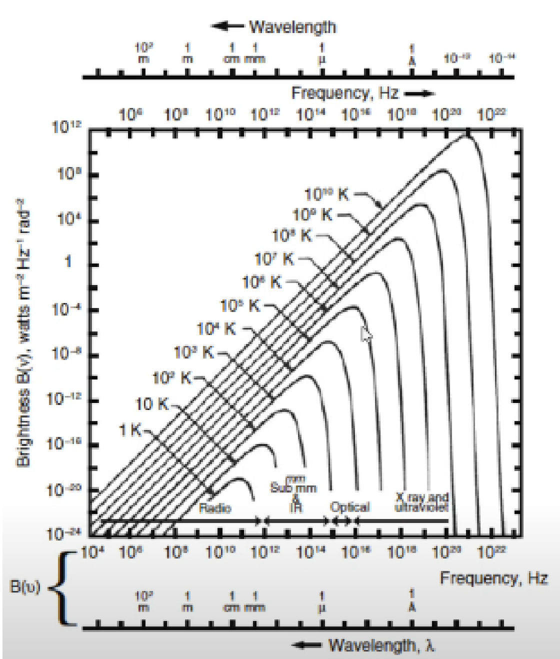
h: Planck's constant.
c: the speed of light
k: Boltzmann's constant.

$$B_\nu(T)=\frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$$

K is the Boltzmann constant h is the Planck's constant and c is the speed of light. The T is the absolute temperature of the black body in terms of Kelvin and lambda is the wavelength of radiation so does nu. So that is the Planck's law. Now how do we see this? We plot this look at the right hand side curve first. If we look at the brightness the b nu in the units of watts per meter square per hertz per radian square you see that this is kind of a continuous curve as a function of different frequency.

The wavelength decreases in this level and the frequency is increasing towards the right and wavelength is increasing towards the left. So you see as the temperature of the black body actually rises from 1 Kelvin to the power 10 Kelvin what significantly you can see the peak is moving towards lower and lower wavelength or higher and higher frequency okay that is quite significant actually. So if as the temperature rises the peak in this emission spectra actually shifts from radio to sub millimeter to ultraviolet optical ultraviolet and so and so forth it goes to higher and higher frequency so it moves from radio to gamma rays in some sense as temperature goes higher. The same is actually taken into this particular plot on the left hand side if you see there is ultraviolet visible and infrared and as the temperature goes the peak of this location goes from infrared to visible to an ultraviolet slowly and slowly. So that is a very significant result of this Planck's law.

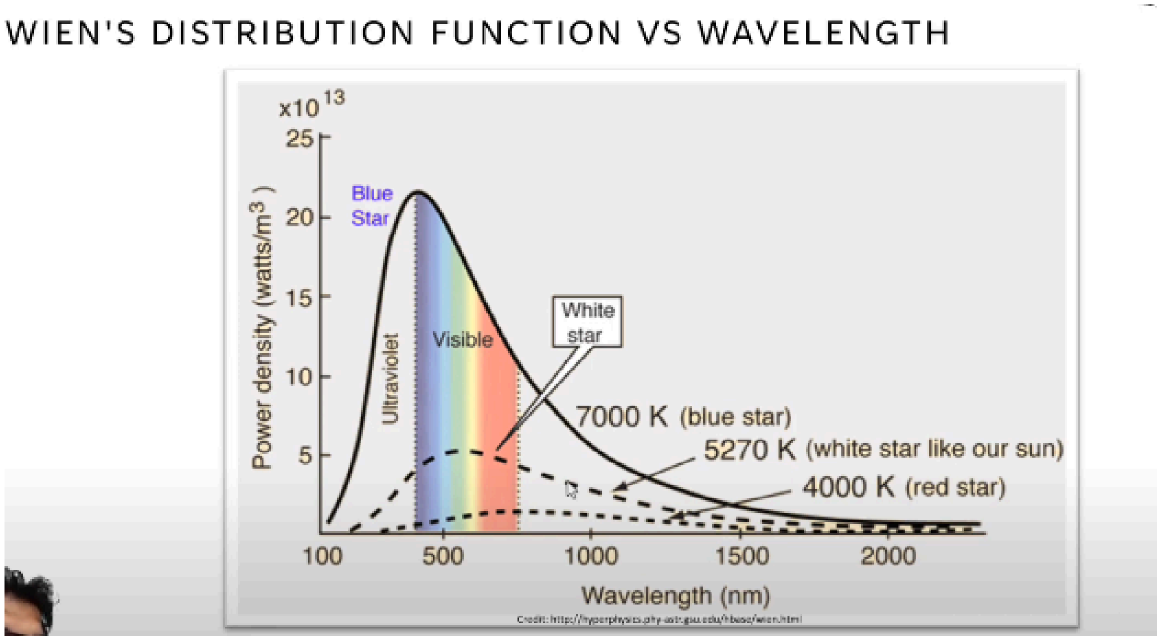




That kind of gives you an understanding that you know the maximum value the maximum wavelength the wavelength at where this spectra actually peaks that and the temperature kinds of have a inverse relationship as the wavelength decreases as temperature increases the wavelength at which the spectrum peaks actually decreases. This actually is given by this famous law called Wien's Testament law will be coming in a bit. So that is the whole power of the thing so that also gives like as you move to the more higher and higher frequency the temperature also increases so the spectrum peaks at a higher and higher frequency with the temperature of the blackbody constantly increasing. Python has become very famous so we have given a small code for you to play with it will be in this particular slide that we share with you thanks to TA we have put in a short to this thing you can basically play with this Planck function which we have defined over here and if you can just keep plotting with the constant of temperature so you can do it by yourself and see how this thing matches and also kind of you can connect the maximum value of lambda, lambda max for each temperature and you can see yourself the development of Wien's Testament law. So Wien's Testament law we know that this lambda max or the value of the wavelength where there is a peak for the blackbody spectrum is called lambda max the wavelength at which the blackbody radiation becomes more intense or the most intense is the lambda max.

$$\lambda_{max} = b/T$$

T is the absolute value of temperature for the blackbody and B is the Wien's Testament constant which the value of that is 2.898 10 to the minus 3 meters Kelvin. Wien's Testament law that is linked by this lambda max is equal to B over T or lambda max times temperature is a constant. So Wien's law is essentially an understanding in understanding the spectrum of electromagnetic radiation emitted by objects at different temperatures such as stars or other astronomical bodies it played a crucial role in the development of quantum mechanics and our understanding of thermal radiation. So this is again a bit of explanation as say for a star which has a red star of 4000 Kelvin the peak is near you know the infrared for a white star like our Sun with 5270 Kelvin the peak value occurs in the visible range for a blue star sorry peak value yeah for a blue star of much higher temperature peak value is almost it's peaking near about from visible to ultraviolet and so as the temperature of the star becomes hotter and hotter the peak slowly shifts from infrared to visible to ultraviolet in this way.



So I think this is another short snippet of python code you can start playing with and it will be available in the in the in the handout. Another important law in this context is the Stefan's Boltzmann law it is actually a fundamental principle in physics that describes the total radiant energy emitted by a perfect blackbody per unit surface area. It quantifies the relationship between temperature of a blackbody and the rate at which it radiates the energy. Stefan Boltzmann's law states that a blackbody radiates electromagnetic waves with the total energy flux E directly proportional to the fourth power of absolute temperature of the blackbody or E is proportional to the power 4 but T is the absolute temperature and if you put on the proportionality constant the Sigma is called Stefan's constant or it is given by 5.67 to the power minus 8 watts per meter square temperature to the power 4.

$$E = \sigma T^4$$

So that's the entire Stefan Boltzmann law and we can also derive it from the Planck's law itself. So we go back to the Planck's spectral radiance function B nu function of temperature T is 2 h nu cube over c squared and 1 over e to the power h nu k over kt minus 1. This is the Planck's law so if you integrate over all values of frequency then you essentially end up with this integral you replace the h nu over kt by x by substitution and then you essentially end up with the integral which is x cube over e to the power x minus 1 integral of that from 0 to infinity which comes out to be pi to the power 4 over 15. If you replace that you finally get something like the B T is equal to Sigma T to the power 4 where Sigma is given by this 2 pi to the power 4 k to the power 4 over 15 c squared h cube. This is Stefan's constant which comes out to be 1.

8 10 to the power minus 5 ergs per centimeter square per second per temperature k Kelvin inverse to the power 4. So this is the another way to derive Stefan Boltzmann from the Planck's law.

The intensity of the light emitted from the blackbody surface is given by Planck's law :

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Integrating over all values of frequencies, we get

$$B(T) = \frac{2h}{c^2} \int_0^\infty \frac{\nu^3}{e^{h\nu/kT} - 1} \mathrm{d}\nu \, .$$

$$x = \frac{h\nu}{kT}$$

$$B(T) = \frac{2h}{c^2} \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} \mathrm{d}x \, .$$

$$\int_0^\infty \frac{x^3}{e^x - 1} \mathrm{d}x = \pi^4/15$$

$$B(T) = \sigma T^4 \, , \quad \sigma = \frac{2\pi^4 k^4}{15 c^2 h^3} = 1.8047 \times 10^{-5} \, \mathrm{erg\,cm^{-2}\,s^{-1}\,K^{-4}}$$

What are the applications of this law in astronomy? Luminosity of the star the Stefan Boltzmann law allows astronomers to estimate the total luminosity of stars on their observed radiation by measuring the flux energy which is energy per unit area per unit time received from a star and knowing its distance from the earth astronomers can estimate the total radiant power or luminosity. So if you know the temperature you know the flux. And characterizing exoplanets when studying exoplanets the Stefan Boltzmann law is used to estimate their temperatures and hence classify them as hot Jupiters warm Neptune's rocky planets or other types of planets based on the emission characteristics.

So this is exoplanets are basically the planets which are not in our solar system but outside so they're a huge kind of an expedition to understand are we the only intelligent you know people on in the universe they're only inhabited by the earth or out there there are other intelligent form of life existing in other parts of the universe. You know intuitively it is it is not absurd that to imagine that there should be multiple such cases just earth cannot be just the only one and so there is hunt for other exoplanets maybe the same distance from their star and having been able to retain their atmosphere so it's conducive to have life in a in a normal form like single cell organism to up to intelligent form as like as good as human beings. So that that particular quest ended up trying to choose the right kind of exoplanets for which in which life is possible and that's a that's a good quest and so Stefan Boltzmann law even as basic as this has a role to play in understanding those results. In stellar radii by combining the Stefan Boltzmann law with other observational data such as stars distance apparent brightness spectroscopic information astronomers can estimate radii of the stars providing insights into their sizes and evolutionary states. Cosmology and early universe Stefan Boltzmann law plays a crucial role in understanding CMB radiation which is the afterglow of the Big Bang.

The law helps in determining the temperature of its radiation which is crucial for understanding the early stages of the universe. There are lots of lots and lots of applications. Next in the line is the Rayleigh-Jans law. It's actually a limit to the Planck's law and we will show you how it works out but it is a classical theory that describes spectral radiance of blackbody radiation at a given wavelength for low frequencies. That was proposed by independently by Rod Rayleigh and James Jans in 1905.

Rayleigh-Jans law attempted to explain the spectral distribution of thermal radiation emitted by a blackbody which is an idealized object that perfectly absorbs and emits all radiation incident upon it. According to classical physics it was assumed that the energy radiated by a blackbody is proportional to the temperature of the body and the frequency of the radiation. It highlighted the limitations of classical theories and the need for a quantum mechanical approach to accurately describe the behavior of the blackbody radiation at all wavelengths. Now how we derive Rayleigh-Jans law from the Planck's law. So Planck's law we go again back to the spectral radiance function which is given by this by the one in the bracket and we note that if you have h nu over kT very very less than 1 or h nu is very very less than k times T then actually you can expand this in terms of series, exponential series and you can retain the only the first term instead of all the terms.

RAYLEIGH-JEANS LAW

The intensity of the light emitted from the blackbody surface is given by Planck's law :

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

This is the classical limit of the Planck law since it does not contain Planck's constant.

$h\nu \ll kT$: *Rayleigh-Jeans Law.* An expansion of the exponential

$$e^{h\nu/kT} \cong 1 + \frac{h\nu}{kT} + \ldots \quad \Longrightarrow \quad B_{RJ}(\nu, T) = \frac{2\nu^2}{c^2}kT$$

The Rayleigh-Jeans law can also be expressed as

$$B(\lambda, T) = \frac{8\pi kT}{\lambda^4}$$

B(λ, T) is the spectral radiance of blackbody radiation.
k is Boltzmann's constant ($\sim 1.380649 \times 10^{-23}\, J\, K^{-1}$)
is the absolute temperature of the blackbody in kelvin.

$$h\nu \ll kT \Longrightarrow \frac{\nu}{\text{GHz}} \ll 21\left(\frac{T}{\text{K}}\right)$$

So e to the power h nu over kT will become 1 plus h nu over kT and there is a denominator there is a minus 1 so 1 goes away with the other one and h nu over kT remains only in the denominator in the Planck's law. So if you combine put this into there you finally get the spectral radiance function under the R-J limit or the Rayleigh-Jans limit as 2 nu squared over c squared times kT. That's a very powerful derivation because now you have a quantity spectral radiance which is almost like an intensity is proportional to just the temperature of the blackbody. And so that's which slowly slowly will give rise something called a brightness temperature concept. Anyway before we come to that we just show the other form of the Rayleigh-Jans law in terms of lambda so that is 8 pi kT over lambda to the power 4 and just to know how this translates in actual reality in terms of number.

So what is the limit like for example what wavelengths or what frequencies qualify to be in the Rayleigh-Jans limit and what doesn't. So in order to do that understand that because that's a very powerful number for observers h nu over kT will finally translate into nu in terms of gigahertz which is less than less than 21 times T over k in Kelvin. Okay that's the that's the number you look for. Basically you can say it works you know below 5 gigahertz as you go above 5 gigahertz we have to check and see if it is still applicable it may be some secondary order correction on top of that. So B this in this entire thing B is the spectral radiance of blackbody radiation, k is the Boltzmann temperature, T is the absolute temperature of the blackbody and lambda is the wavelength of radiation.

So yes I think there is another interesting plot where you can plot both Rayleigh-Jans limit and the next Plank's function in at the same time it is a snippet of the Python code which you can use. So that brings us to this definition of something called brightness temperature which is extremely important there is lots of temperatures will be coming in our discussion throughout this course this is the first one which is the brightness temperature after the blackbody temperature and we will have antenna temperature, sky temperature, noise temperature etcetera etcetera this is a whole course where we will talk at one end about intensity, flux density on the other side different temperatures qualifying different objects. Sometimes the sky, sometimes the physical objects, sometimes some parameters and system. Okay so Rayleigh-Jans limit said that is the Brj and which is 2 by 2 nu square over c square times kT.

$$B_{RJ}(\nu, T) = \frac{2\nu^2}{c^2} kT$$

So if you just keep rearranging those matters you will finally see that the T which is now casted as the brightness temperature so T sub B can be written in terms of lambda square over 2k times I nu which is the specific intensity okay.

$$T_B = \frac{c^2}{2k}\frac{1}{\nu^2} I_\nu = \frac{\lambda^2}{2k} I_\nu \Rightarrow T_B = \left(\frac{\lambda^2}{2k}\right) I_\nu$$

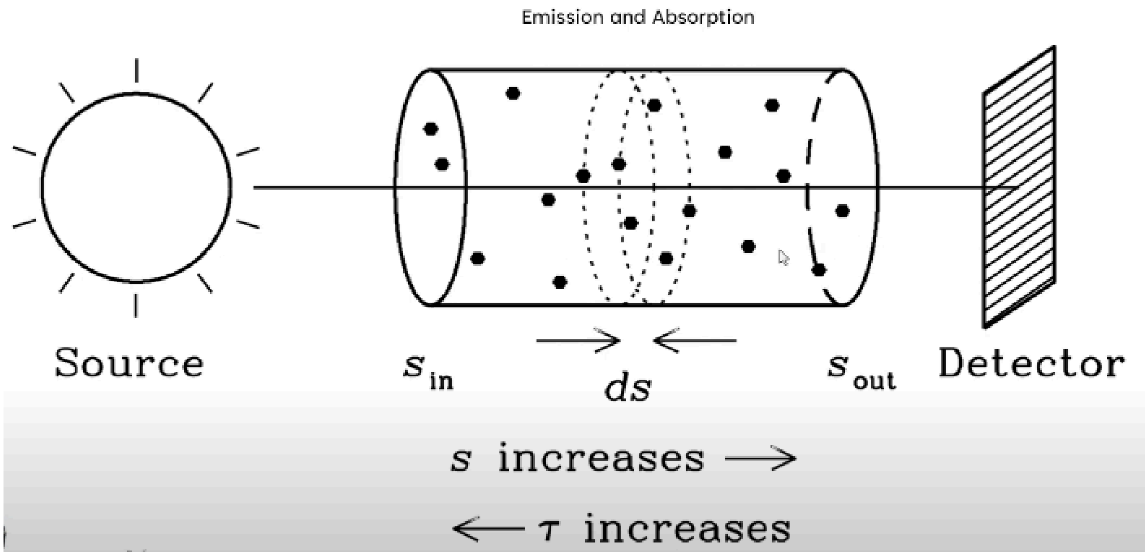
So that is also can be mentioned here in terms of nu and the lambda both. For a source with a flux density S nu this can be written in terms of the brightness temperature as well as the size of the object. So if you keep writing this in terms of the flux density so flux density in Jansky can be expressed as 2.65 times brightness temperature in Kelvin of course over this extension of the source in theta in R minutes square and then lambda over in lambda to the power minus 2 in terms of centimeter okay.

$$S_\nu = \frac{2k\,\nu^2}{c^2} T_B\, \Delta\Omega \Rightarrow \left(\frac{S_\nu}{\mathrm{Jy}}\right) = 2.65\, T_B \left(\frac{\theta}{\text{arc minutes}}\right)^2 \left(\frac{\lambda}{\mathrm{cm}}\right)^{-2}$$

So this is the very useful tilt so we have the spectral radiance in the Rj limit, Rayleigh-Jans limit that is proportional to the temperature and nu square and from there you will define the brightness temperature in terms of specific intensity I nu and specifically of both in terms of lambda as well as nu and then from there derive the what the flux density will look like for an extended source.

So that is the final formula given over here which is very very useful to if you have a extent then what is the relationship between the source the brightness temperature of the source and the flux density. This will keep coming as we move forward.

So the next topic of today's discussion is radiative transfer and this is how the electromagnetic radiation propagates through a medium and so suppose it is a source towards the left it is a star for example it passes through our intervening medium before it reaches a detector which can be also telescope or your eyes. So while it goes through this through this extended medium it can be interstellar medium it can be the proprietary medium it can be the galactic medium inter cluster medium whatever I mean the space any space which can be even in vacuum as well as filled up with the plasma or other materials. They this radiation can suffer absorption and also emission from the medium itself.



So how do we take care of that while understanding the incident radiation which we detect and finally the actual radiation which actually comes in. So from the observed radiation we have to decipher the actual radiation and that will only happen if we know about the intervening medium and the different characteristics of the intervening medium and sometimes we know the background source well to use this principle to actually characterize the intervening medium. Sometimes we know the intervening medium well to be able to know this to decouple this effect of the intervening medium and to understand what the actual radiation is you can use it in both ways. Alright so it will transfer itself refers to the process by which electromagnetic radiation interacts with and propagate through a medium okay. Intermittent radiation can be absorbed scattered emitted or transmitted as it interacts with the particles and molecules present in the medium it travels through okay.

The behavior of the radiation depends on factors such as properties of the medium density composition the wavelength of the radiation and the temperature of the medium. So in astrophysics the radio transfer is fundamentally understanding how light from stars and celestial bodies travel through space interacts with the interstellar dust and gas. It helps an astronomers to infer properties of the stars and galaxies and their and other astronomical bodies. So the different kinds of emission thermal emission with blackbody radiation we have already discussed there are non thermal emission like which are non blackbody in nature and a process by which a material or object emits determine radiations visible light infrared radiation or radio waves. Emission can occur due to thermal variation particle interaction and particle field interaction emission can broadly classified into two categories thermal emission as we said the blackbody mostly it happens due to thermal energy of a material or object and non thermal because of different astrophysical processes it often associated with high energy phenomena synchrotron radiation and so and so.

So essentially non thermal processes are also there but whatever be the process of emission is they can change so typically we know that the lower frequencies the non thermal radiation of synchrotron is the most dominant and as we go to higher frequencies say about 3 gigahertz or so thermal radiation mostly of dust takes over and then finally turns over and near above x-rays. So we will come to the actual emission processes more in more rigorously towards the end of this course. So right now you just understand that there is an emission which can definitely happen from the stars itself and also from the intervening medium and so the actual emission from the stars or other cosmic sources they can travel and so they their incident on intervening medium and it can happen from the beginning and finally goes through the processes of absorption and emission and then comes up from the other side of the medium after being affected by that. So how do we go about defining all this so emission is defined by the emission coefficient J new which is kind of if we have the intensity of the radiation so the rate of change of that is given by this emission coefficient okay and the transfer equation in totality can be written as di nu ds is equal to minus Kappa I nu plus J nu where J is the emission term coefficient and K is the emission term coefficient and K is the absorption coefficient as you understand. So if system is in full thermodynamic equilibrium at temperature T then di nu ds is equal to 0 and I nu is given by this spectral radiance function B nu which is function of T.

In any infinitesimal volume ($ds\,d\sigma$) of thickness ds and cross section $d\sigma$, the probability per unit time that an isotropic source will emit a photon into the solid angle $d\Omega$ is directly proportional to the volume and solid angle:

$$P_{em} \propto ds\, d\sigma\, d\Omega.$$

The emission coefficient j_ν is defined so that:

$$j_\nu \equiv \frac{dI_\nu}{ds}$$

the equation of radiative transfer:

$$\frac{dI_\nu}{ds} = -\kappa I_\nu + j_\nu.$$

So rearranging the radial transfer equation it is we get that di nu ds is equal to 0 is given by Kappa beta nu plus J nu which is then rearranging that you get the spectral radiance function as a ratio of J over Kappa this is also called Kirchhoff's law. So absorption a process by which a material or a medium absorbs the radiation incident upon it. When radiation interacts with the material some of its energy is absorbed by the material leading to an increase in internal energy of absorbing medium. It is closely related to the materials properties and wavelength of the incident radiation. Different materials have specific absorptions characteristics and absorption behavior can vary significantly across the electromagnetic spectrum.

Have significant implication in various fields including astronomy, atmospheric sciences, remote sensing and spectroscopy. Applications of our absorption lines and bands in spectra are important tools for identifying chemical composition of the celestial bodies. Objects for example in stellar spectra dark absorption lines are caused by absorption of specific wavelength of light by element present in the stars exterior layer. So if we want to define the absorption coefficient then it is given by dp ds

$$\kappa \equiv \frac{dP}{ds}$$

or the infinitesimal probability of a photon being absorbed in a thin slab of thickness ds is given by Kappa. So for a absorption only case this Di nu over I is given by minus Kappa ds.

$$\frac{dI_\nu}{I_\nu} = -\kappa\, ds \quad \text{(absorption only).}$$

So that gives you something like I nu if you go from the s in to s out so if you come in to the medium you have yeah if you come in you are coming in the incident ray it is s in and when you are going out it is the s out. So the effect of the intervening medium is from between s in and s out. So then this integration should also be within s in and s out okay. So that defines the intensity at s out minus intensity at s in logarithm of both of that is equal to minus this integral Kappa s ds where s varies from s in to s out.

$$\int_{s_{\text{in}}}^{s_{\text{out}}} \frac{dI_\nu}{I_\nu} = -\int_{s_{\text{in}}}^{s_{\text{out}}} \kappa\,(s')\;\; ds' = \ln I_\nu \Big|_{s_{\text{in}}}^{s_{\text{out}}},$$

$$\ln[I_\nu\,(s_{\text{out}})] - \ln[I_\nu\,(s_{\text{in}})] = -\int_{s_{\text{in}}}^{s_{\text{out}}} \kappa\,(s')\;\; ds',$$

So if you rearrange that you finally get that your I nu at the final end when it comes to the intensity comes out of the medium over I nu when it was coming incident is given by exponential of minus Tau where Tau is given by this integral of Kappa s times ds which is also referred to as the optical depth of the medium.

$$\frac{I_\nu\,(s_{\text{out}})}{I_\nu\,(s_{\text{in}})} = \exp\left[-\int_{s_{\text{in}}}^{s_{\text{out}}} \kappa\,(s')\;\; ds'\right].$$

So that is the definition so basically your incident and the outgoing intensity is affected by the optical depth which is the density kind of makes sense. So if it is optically thick nothing almost comes out of the other end and is optically thin then nothing happens to the incident radiation both are equal. So the next thing is we can talk about the thermodynamic equilibrium at which we are making all these assumptions of dI nu ds equal to 0. It is a state in which thermodynamic systems is balanced and stable with no net exchange of macroscopic properties such as temperature pressure and chemical potential between its different parts. Thermal equilibrium refers to a state in which two or more objects or systems are at the same temperature and no heat transfer occurs between them when they are in contact.

In this state the net flow of heat between the objects is 0 because they have reached a balance distribution of thermal energy. If a medium is not in thermal equilibrium with its surroundings due to the rate of energy absorption and emission is not balanced leading to changes in the medium's temperature. Of course if you have a thermodynamic equilibrium or thermal equilibrium then there is no heat exchanges with the surrounding okay. So it is the equilibrium that is what. If not then there will be some exchanges happening one way or the other.

If there is complete equilibrium of the radiation which is surrounding the brightness distribution is described by the Planck function which depends only on the thermodynamic temperature of the surrounding. So we are reiterating that the Planck's law is the blackbody radiation is valid in thermodynamic equilibrium if there is still some exchange going on that is not the value. So often we define something local thermodynamic equilibrium that it may be true locally within the object and its immediate surrounding may not be valid for the global situation. So for an LTE or local thermodynamic equilibrium the following assumptions should hold uniform properties that is the properties of interest such as temperature pressure and chemical composition are assumed to be nearly constant throughout the local region. No macroscopic flow there is no macroscopic flow of matter energy within the local region.

Any transfer process like heat conduction diffusion occurs at much smaller scale and time frame compared to the region of interest. So at the large scale there is nothing happening all happening within very small boundaries. Rapid interaction that interaction and collision time scales of particles within the region are much shorter than time scales over which the properties of interest are changing. Equilibrium reactions chemical reactions in local region are assumed to be in equilibrium meaning the forward and the reverse reactions occur at the same rate. Full thermodynamic equilibrium will be realized only when very special circumstances in a black enclosure or say in stellar interiors.

Often Kirchhoff's law is applicable independent of the material and as is the case for complete thermodynamic equilibrium. So Kirchhoff's law gives that the e nu epsilon nu which actually J nu earlier over kappa nu is equal to the spectral radiance function. In general I will I nu which is specific intensity will be little different than B nu. If we define the optical depth as in the figure we have just defined it earlier also. So that is that the kappa nu ds from s naught to s or s in to s out and that is given by this particular quantity.

$$\frac{dI_\nu}{ds} = 0, \qquad I_\nu = B_\nu(T) = \varepsilon_\nu / \kappa_\nu$$

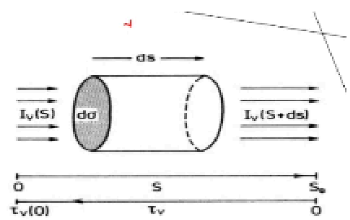
$$\boxed{B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}}$$

So you can write down the equation of radiative transfer in terms of now tau nu. So instead of writing it in terms of s you are writing in terms of tau nu. So how to do it? You have di nu ds is equal to minus kappa i nu plus epsilon nu or it was J nu earlier. So you can just take the divide the both sides like minus kappa nu and you get on the left hand side minus one over kappa nu di ds which is nothing but di d tau nu. You get on the right hand side i nu from the first term and the second term is actually spectral radiance or B nu.

$$\boxed{\frac{\varepsilon_\nu}{\kappa_\nu} = B_\nu(T)} \Longleftarrow \text{Kirchhoff's Law}$$

If we define the optical depth $d\tau_\nu$ (as shown in figure) by

$$d\tau_\nu = -\kappa_\nu \, ds \Longrightarrow \tau_\nu(s) = \int_{s_0}^{s} \kappa_\nu(s)\, ds,$$



then the equation of transfer $\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \varepsilon_\nu$ can be written as

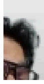
$$-\frac{1}{\kappa_\nu}\frac{dI_\nu}{ds} = \frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu(T)$$

The solution of the above equation can be obtained by multiplying by $\exp(-\tau_\nu)$ and then integrating τ_ν by parts:

$$\int_0^{\tau_\nu(s)} e^{-\tau}\frac{dI_\nu}{d\tau}d\tau = I_\nu e^{-\tau}\Big|_0^{\tau_\nu(s)} + \int_0^{\tau_\nu(s)} I_\nu e^{-\tau}d\tau = \int_0^{\tau_\nu(s)} (I_\nu - B_\nu)e^{-\tau}d\tau$$

Which gives

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)} + \int_0^{\tau_\nu(s)} B_\nu(T(\tau))e^{-\tau}d\tau$$



Okay so if you take you know take the integration from both sides you finally end up having this i nu at s is equal to i nu 0 e to the power minus tau nu plus this integral of B nu times e to the power minus tau. Okay so yeah so this if you can if you can finally compute this you finally get that this full radiative transfer equation is i nu s is equal to i nu 0 at the incident times e to the power minus tau nu s plus B nu t into 1 minus e to the power minus tau. Okay so for large optical depth that is for tau nu 0 goes to infinity i nu is basically the B nu.

Due to the definition $d\tau_\nu = -\kappa_\nu\, ds$

And from figure s and τ increase in opposite direction.

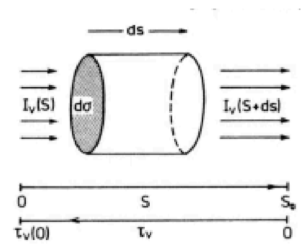
If the medium is isothermal, that is, if T(τ)=T(s)=T =const.

Then $I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)} + \int_0^{\tau_\nu(s)} B_\nu(T(\tau))e^{-\tau}d\tau$

can be computed explicitly resulting in

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)} + B_\nu(T)\left(1 - e^{-\tau_\nu(s)}\right)$$

For a large optical depth, that is for $\tau_\nu(0) \rightarrow \infty$, $I_\nu \rightarrow B_\nu(T)$



The observed brightness I_ν for the optically thick case is equal to the Planck black-body brightness distribution independent of the material. If the intensity is to be compared with the result obtained in the absence of an intervening medium, $I_\nu(0)$, we have

$$\Delta I_\nu(s) = I_\nu(s) - I_\nu(0) = (B_\nu(T) - I_\nu(0))(1 - e^{-\tau})$$

Okay so it is just the emission from the intervening medium it is not a nothing from the source actually travels to that because everything gets absorbed. The observed brightness i nu for optically thick case is equal to the plank blackbody brightness distribution independent of the material if the intensity is to be compared with the resultant obtained in the absence of an intervening medium i nu we have this expression.

RADIATIVE TRANSFER IN TERMS OF BRIGHTNESS TEMPERATURE

the equation of radiative transfer: $$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \varepsilon_\nu$$

1. Emission only: $\kappa_\nu = 0$

$$\frac{dI_\nu}{ds} = \varepsilon_\nu, \qquad I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} \varepsilon_\nu(s)\,ds.$$

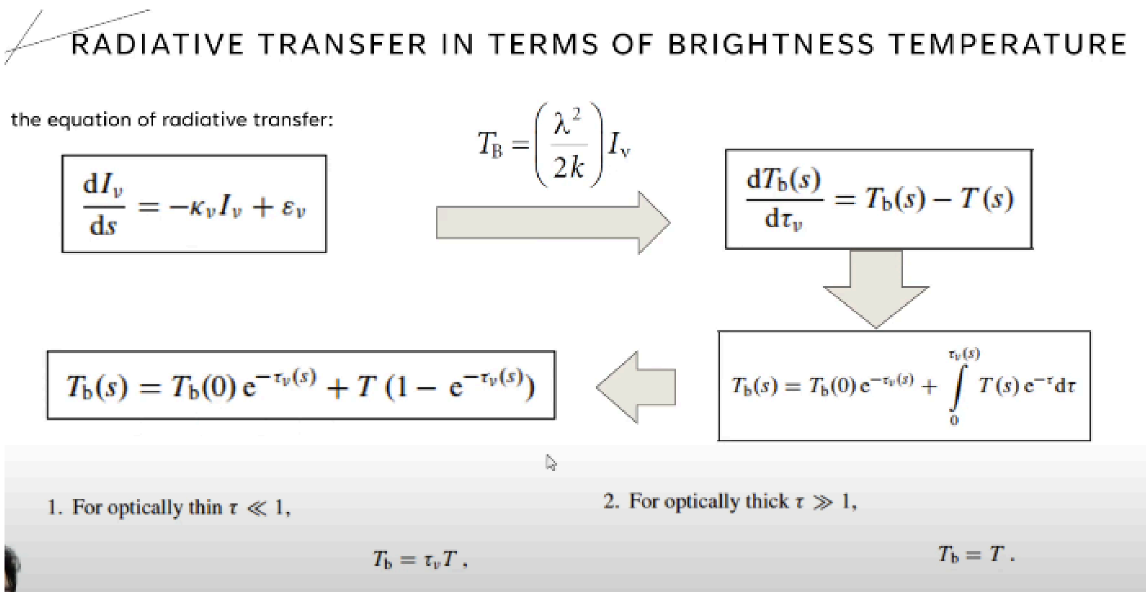
2. Absorption only: $\varepsilon_\nu = 0$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu,$$

$$I_\nu(s) = I_\nu(s_0)\exp\left\{-\int_{s_0}^{s} \kappa_\nu(s)\,ds\right\}.$$

$$T_B = \left(\frac{\lambda^2}{2k}\right)I_\nu$$

$$\frac{dT_b(s)}{d\tau_\nu} = T_b(s) - T(s)$$



So now we do have the optically thick case and optical thin case also we will discuss so the radiative transfer equation finally is of this nature di ds which s is the line of sight propagation is minus kappa nu i nu plus epsilon nu where epsilon nu actually is the emissivity. So if only absorption emission only kappa is equal to 0 so it goes to this level and for absorption only then it takes the value of e to the power minus tau where tau is given by this integral of kappa times ds. Now if I replace now in the Rayleigh-Jin limit if we replace the brightness temperature with the specific intensity with the brightness temperature then we end up having the radio transfer equation as this tb delta dt d tau is equal to t tbs minus t s. If we do that finally arrive in this radiative transferring expression which basically replaces the i with the t essentially so you have the same tau which keeps carrying on and you finally end up in this transfer equation where tb which is the brightness temperature when the radiation comes out of the intervening medium is equal to the incident radiation brightness temperature multiplied by the e to the power minus optical depth and then t of the medium times 1 minus e to the power tau nu. Okay for optically thin case tb results into tau nu times capital T and optically thick case it is just the small the t of the medium.

Great so I think we have tried to squeeze in mostly the discussion with regarding the blackbody radiation, Rayleigh-Jin limit and now the radiative transfer. It's a bit maybe faster so we'll see with a couple of example questions how to make it more explanatory. So let's look at some of the example questions.

A star emits most of its radiation at a wavelength of 450 nanometer. Use Wien's displacement law to calculate the temperature of the star.

1. A star emits most of its radiation at a wavelength of 450 nm (1 nm = 10^{-9} m). Use Wien's displacement law to calculate the temperature of the star.

So the wavelength we know the lambda max equals to b over t where b is the Wien's constant and so if we know the maximum wavelength is 450 nanometer then given the value of b we can calculate the value of the capital T as 6444.3.

ANS: Wien's displacement law relates the peak wavelength (λ_{max}) at which a blackbody emits most of its radiation to its temperature (T):

$$\lambda_{max} = b/T$$

Given the value of the peak wavelength:

$$\lambda_{max} = 450\, nm = 450 \times 10^{-9}\, m$$

We can rearrange the equation to solve for the temperature T

$$T = (2.898 \times 10^{-3})/(450 \times 10^{-9})$$

$$T = 6444.3\, K$$

Our next question is a metal surface at temperature of 5000 Kelvin emits thermal radiation at a frequency of 1 terahertz. Terahertz is 10 to the power 12 Hertz. Calculate the spectral radiance b nu according to the Rayleigh-Jin's law.

2. A metal surface at a temperature of 5000 K emits thermal radiation at a frequency of 1 THz (1 THz = 10^{12} Hz). Calculate the spectral radiance (B_v) according to Rayleigh-Jeans law.

So if we write it down the spectral radiance can be equated in terms of this temperature of the blackbody and also the frequency. So if you put all those values over here along with the proportionally constant you finally get the value of the b as 92 times the power minus 4 watts per meter squared plus radian per Hertz.

ANS: Sol: According to the Rayleigh-Jeans law, the spectral radiance(B_v) of a blackbody is proportional to the temperature (T) and the (v^2). The constant of proportionality is ($2k/C^2$)

We can calculate the spectral radiance (B_v) using the Rayleigh-Jeans law:

$$B_v = \frac{2\,(1 \times 10^2)^2 \times 1.38 \times 10^{-23} \times 5000}{(3 \times 10^8)^2}$$

$$B_v = 92 \times 10^{-4}\ W\, m^{-2} sr^{-1} Hz^{-1}$$

The next question we have a star has a temperature of 10,000 Kelvin. At what wavelength does this peak spectral radiance occurs according to Planck's law.

3. A star has a temperature of 10,000 K. At what wavelength does the peak spectral radiance occur, according to Planck's law?

Now according to Planck's law the peak spectral radiance occurs at a wavelength alpha peak when the exponential term e to the power hc over lambda kt becomes equal to 1.

So lambda peak is equal to this and given by this quantity of 4.5 to the power minus 7 meters. So basically you follow the Wien's law.

ANS: According to Planck's law, the peak spectral radiance occurs at the wavelength λpeak when the exponential term $e^{(hc/\lambda kT)}$ becomes equal to 1.

We can calculate the peak wavelength (λpeak) using the equation:

$$\lambda_{peak} = hc/kT$$

$$\lambda_{peak} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 10000}$$

$$\lambda peak \approx 4.55 \times 10^{-7}\, m$$

An object with the surface area of 3 times 10 to the power minus 4 meter square is observed to have a total power radiated of 5 times 10 to the power minus 3 watt at a certain temperature. Calculate the effective temperature T of the object along with according to Stefan Boltzmann law.

4. An object with a surface area of 3.0×10^{-4} m^2 is observed to have a total power radiated (P_{total}) of 5.0×10^{-3} watts at a certain temperature. Calculate the effect tive temperature (T) of the object according to the Stefan-Boltzmann law?

So the surface area is given total power is given so we calculate the effective temperature.

So we know that for Stefan Boltzmann law the temperature is given by P total so P total is sigma a 10 to the power 4. So T is given by 1 4th square power of the this ratio of P total over sigma a. So that is sigma is Stefan's constant okay. So that gives you the temperature to be 131 Kelvin approximately.

We can rearrange the Stefan-Boltzmann law to solve for T

$$T = \left(\frac{P_{total}}{\sigma A}\right)^{1/4}$$

$$T = \left(\frac{5 \times 10^{-3}}{5.67 \times 10^{-8} \times 3 \times 10^{-4}}\right)^{0.25}$$

$$T \approx 130.94\, K$$

So that this is where we kind of stop now. We have gone through several different examples of we have visited one example where you have used the Wien's displacement law then the Relegence limit then again the Planck's law and also the Stefan Boltzmann law from there. We will take up some more examples in the following lecture and we will also put it up if you are confused. This brings us the close at close with the lecture number 3 for this particular week. I hope you liked it if you have any questions you can also write back to us and we will try to answer them. So good I think we will see you for the next lecture. Thank you for joining.