Radio Astronomy

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Lec-06

Radio Astronomy Fundamentals Part - 1

Hello, welcome everyone to this second lecture of week two for the radio astronomy course for the first time in NPTEL and we are quite excited about it. If you remember last week we actually went through first telling you introducing the entire course and then we went on revisiting some electromagnetic theory particularly Maxwell's equation and also the electromagnetic waves, how they propagate, the pointing vector and also a little bit of polarization. This week we started off talking about different important discoveries in radio astronomy and some very important milestones and development of radio astronomy across the world. This lecture we are going to concentrate on defining very specific fundamental quantities of electromagnetic radiation. We talk about the brightness, the intensity, specific intensity, flux density etc. We also talk about how when the waves propagate, radiation propagates through a medium, they suffer emission, absorption so there's a radio transfer process involved in that we talked about that.

We talk about how stars radiate, how galaxies, stars emit different forms of radiation, how they're very close to something called blackbody radiation, what are the different limits of blackbody radiation, talking about Wien's law, talking about Ellington's limit which is mostly applicable to radio astronomy and also Stefan Boltzmann law and ending with a concept very important to the astronomy called brightness temperature.

So this is again a similar slide as we have shown before multiple times. The entire electromagnetic spectrum is quite large than the radio band itself. What happens is most of the band higher in frequency, lower in wavelength than radio are actually absorbed, radiation is absorbed in the Earth's atmosphere and so we cannot observe those rays from the surface of the Earth.

That's why we have to send space telescopes to detect any radiation at those frequencies, the gamma rays, x-rays, ultraviolet, infrared most of it, we have to send space-based telescopes to observe them. Only a very minor fraction of the optical visible wavelength actually can be done from the ground. There also a lot of effects comes because of the

Earth's troposphere, the water vapor content of troposphere. That's why you will find most sensitive optical telescopes to be housed on a very top of the mountain at a very higher altitude. We are lucky in radio that we are we observe most of our band without any hassle from the atmosphere.

Only if you get to very lower frequencies as anosphere becomes a problem. If you go to a little bit higher frequencies mostly towards the microwave, the troposphere starts becoming a problem. But most of the band we can observe from the ground and that's the advantage. Now what we observe, this is a very latest and the greatest image from LOFAR telescope. LOFAR is a telescope which is located in Netherlands and expanding to other parts of Europe and this is an image made at 150 megahertz or 2 meter wavelength.

Now this is a huge part of the sky, definitely the image represents a 2D projection of a three-dimensional sky. We cannot see the distance towards from line of sight because we are only observing from one angle. So the extent is large, you cannot see much in the central image but you can zoom in zoomed in version of central image based on this kind of squares actually shown in the periphery. So you can see there are a lot of structures, very important stuff, radio galaxies, jets are being observed. So what do we observe? We see there is a spatial extent in two dimensions in X and Y you can say but we astronomers define it as right ascension and declination.

We will talk about that later on, in later weeks to come. And what else do you see apart from the two-dimensional extent? We see there is a third dimension, the color coding of the image. You see that there are places where you see higher values than the other places. That means the emission is not uniform across the entire spatial scale. Something is happening more vigorously and some places than the other.

Okay, there are much low level emissions extending over a larger places but very high level emission located concentrated at some locations particularly in these hot spots which we call. How do we make sense of this unless we actually understand, go beyond and understand some critical definition of the radiation? Okay, so let's start doing that. Astronomers study an astronomical source by measuring the strength of its radiation as a function of direction of the sky means the spatial location, frequency. We will come later, it's under spectroscopy plus other quantities time, polarization, etc. So depending on what you are doing you can either be just mapping the sky, so looking at the image.

You can see as a function of frequency there also things change. Supposedly you have some particular transition, atomic or molecular transition that will only be detected at a particular frequency. So depending on where you're located, how far away from us you are located, the source is located, it will appear in a particular frequency based on that. There are other things like time and polarization like so some features are visible only in a polarized manner, some emissions are polarized, some are not. Okay, time varying, there are things which are transients which are only visible in some time and not visible on the other.

Like pulsar, when it rotates sometimes you see for another time you do not see depending on the pulse period, the rotation period and the duration of the pulse you can either see it or miss it. So it is kind of a transient. Let us say there is a huge uproar of something called fast radio bursts, FRBs, also transients in radio. There are other transients you know in other part of the wavelength like gamma ray bursts. Okay, so these are really varying in time.

In a particular time you will see a huge amount of radiation emission and then suddenly it falls off for the others. So that is how it is quantified spatially, frequency in frequency domain, also in time and polarization. So there are four different domains. Clear and quantitative definitions are needed to describe the strength of the radiation, how it varies in direction, frequency, distance and distance between the source and the observer is required. Consider the simplest possible case of radiation traveling from a source through an empty space so there is no absorption, no scattering or emission along the path of an observer.

It is purely vacuum or free space. The angular size of the Sun depends on the distance between the Sun and the camera or the telescope but the number of photons falling on the detector per unit area, per unit time, per unit solid angle does not. The photo taken from near Venus would not be overexposed or one from the Mars will not be underexposed. Total number of solar photons from all directions reaching the camera per unit area, per unit time does decrease with increasing distance but that is because only that if you see the Sun emits, it emits in all direction. So if you go farther away you are only intercepting a sub part of the solid angle so that flux may decrease but not decrease because of the distance per se.

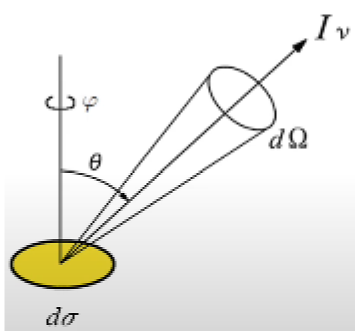
So we distinguish between the brightness or intensity of Sun's radiation which does not depend on the distance from apparent flux which does depend on the distance. Okay so these are the critical things which will come. It's pretty confusing actually to begin with. Something depends on distance, something doesn't. What are we doing? So let us go forward and understand a little bit more of the entire formalism so that will make it much more clear.

So we define total brightness is the contribution of photons of all frequencies. So if we sum over all the frequency of emissions, so in different frequency there are different emissions as we see that the entire electromagnetic spectrum is full of such radiations. So here also even with the radio band we have to take into account different frequency ranges. Only then we can get the entire information. The brightness per unit frequency is called specific intensity.

So brightness is total brightness is total contribution of all photons emitted in all possible frequencies. Brightness per unit frequency range is called specific intensity, also called spectral intensity, also called spectral brightness depending on who writes the book or who is writing that. So the notation is I sub new where the subscript new is used to indicate per unit frequency range. So that's the definition. Let's go a little bit further and try to understand.

We'll come back and again keep explaining more more lucidly. So I new is the specific intensity. Now let us see how it does. So you have a simple cartoon that there is d sigma which is the infinitesimal area of the surface. And we have a direction I in the upwards I new and it is making a solid angle of d omega.

$$dP = I_\nu \cos\theta \, d\Omega \, d\sigma \, d\nu$$



So the infinitesimal power intercepted by the surface d sigma is given by this dp is I new which is specific intensity of the source cosine of theta d omega d sigma d new. Where dp is the infinitesimal power in watts, d sigma is the infinitesimal area of the detector in in meter squared. Then bandwidth is in Hertz and theta is the angle between normal to d sigma and the direction to d omega. So that is the thing and I new is the brightness or specific intensity in watts per meter squared per Hertz per steradian. So when we define this it's a very interesting one anecdote.

In radio just to make sense of how the radio signals from the cosmic sources are there compared to any of the cell phone emissions or anything like that. So the definition of the unit of this intensity is in Jansky and that one Jansky is 10 to the power minus 26 watts per meter squared per Hertz per steradian etc. So you understand that in typically the cell phones or even in any gadget it's merely what units are really where they operate even higher and this is 10 to the power minus 26 watts. So about 23 orders of magnitude lower in in the emission in the strength to the signal. So if you that's why if you have any electric gadgets most of the telescopes are indeed in quiet zone no transmission from any other things otherwise you kill the signal.

Okay, carrying on intensity brightness and specific intensity. So intensity often termed as brightness in astronomy it's essentially measurement of the electromagnetic radiation. This measure has the property that it does not decline with distance defined in such a way that it remains unaffected by the spread of the radiation also. This measurement is the quantity of the energy of the electromagnetic radiation traveling from a finite size portion of the source to a finite size portion of the recipient or the detector. Example of small circular region of the Sun as seen in the sky that covers a specific angular diameter of the sky then the EMR or recommended radiation from that portion of the Sun striking at a specific plot of the ground has a particular intensity and if the Sun were closer or somewhat further the intensity from that angular diameter would remain unchanged.

Some way to explain this this particular phenomena that if if it the definition of intensity such a way that no matter with a disclosure or further that the the intensity remains the same. So we have the previous definition we have this dp the power intercepted by the detector for unit solid angle for unit surface area of the detector per unit frequency range that is there and cosine theta is just the alignment projection to that direct line of sight. So we can just change the you know rearrange the variables and find that I knew the specific intensity can be given in terms of the power over those many terms. If we integrate the power and I think so this is basically the power over the area of the detector over the solid angle over the bandwidth times the cosine of theta which is the angle of angle which it makes to the line of sight. So what is that okay so again we are writing the same expression we are defining now the flux density the next important quantity intensity is defined remember it doesn't depend on the on the distance.

Now what about flux flux is defined in a way that we have the same expression as before the infinitesimal power intercepted by the detector and now we are just substituting this d sigma d nu on the right hand left hand side and you get I knew cosine theta d omega. If you integrate if you have an extended source then you integrate over the entire solid angle and you get total total integrated intensity through that solid angle towards that line of sight and you get something called something called flux density. So s nu is basically integral of I knew cosine theta d omega. Okay now this is the flux density all right so that is the integration over the entire solid angle okay. So and then if you want to integrate over the entire frequency range then that is also possible that total flux is s nu which was per unit frequency times delta nu integral of that.

So that is the flux density.

$$dP = I_\nu cos\theta d\Omega d\sigma d\nu$$

$$or, \quad \frac{dP}{d\sigma d\nu} = I_\nu cos\theta d\Omega$$

so, integrating over the solid angle subtended by the source yields

Flux density, $S_\nu = \int_{Source} I_\nu(\theta, \phi) cos\theta d\Omega \approx \int_{Source} I_\nu(\theta, \phi) d\Omega$

Flux or total Flux S from a source is the integral over frequency of flux density

$$S = \int_0^\infty S_\nu d\nu$$

So now will flux density depends on the distance intensity was not but will flux density depends on the distance that's a good question think about it we will come back and answer it. So then we define something called specific luminosity which is total power per unit bandwidth radiated by a source at a frequency of nu it's m case unit are what watts per Hertz. Okay so the area of a sphere of radius d is 4 pi d square so L nu is linked with the flux density by L nu equal to 4 pi d square s nu.

$$L_\nu = 4\pi d^2 S_\nu$$

Note that some radio sources emit beam quasars for example in this case this equation cannot be used to calculate total spectral luminosity of a beam quasar with only flux density measurement made towards just one direction. So if it just towards one direction then these measurements are wrong only if there is a source which is you know emitting isotropically then this kind of treatment is valid.

So that is that goes for that so we're assuming the solid angle measuring over the entire solid angle or per solid angle the the emission is the same in every per solid angle direction. So if a particular source is has a beaming direction so we basically has a specific emission towards one direction over the other from the source point of view then this treatment doesn't work this assumes that the source is emitting equally in all possible direction that's why this per solid angle concept comes when you talk of the detector. Same thing is not true for the beam quasars which is which have a very narrow angle of emission. It's an intrinsic so luminosity is an intrinsic property of the source and is independent of the distance. So luminosity is not dependent on the distance but s the definition is such that it is dependent on the distance because s is L over 4 pi d square.

So s depends on the distance. Similarly if we can tell from here also that s was depending on the distance because as the distance goes further the solid angle decreases and so the is I knew is independent of the distance so s will depend on the distance because of the solid angle dependency. Alright total luminosity L of a source is defined as the integral over all frequencies of the spectral luminosity.

$$L = \int_0^{\infty} L_{\nu} d\nu$$

So spectral luminosity is per unit Hertz. So total luminosity or volumetric luminosity is basically integral of the specific luminosities for every given frequency intervals and so you define over the entire range of frequencies. So you also note that for the black body luminosity we will come later in the next lecture about Stefan Boltzmann law in more detail but here we just you know relating that L is related to the T the temperature of the black body as sigma a t to the power 4.

$$L = \sigma A T^4$$

Sigma is the Stefan's constant. We talk it and talk about it more later. For stars which are lying on the main sequence that's a particular class of stars. Luminosity is also related to the mass appropriately. So L and M the mass and the luminosity and has a relationship.

So L over L solar. So as we're defining things in astronomical scales the mass of the cannot be anymore in kgs or gram. It has to be in terms of the mass of a star of the Sun. So M is over M Sun and L is over L Sun. So it's just a kind of a unit we remember it easier. So L over L Sun is M over M Sun to the power 3.

$$\frac{L}{L_{\odot}} = \left(\frac{M}{M_{\odot}}\right)^{3.5}$$

5 for main sequence stars only. This is just a must light ratio. So supposedly you have

information about mass you can infer the luminosity you have and do do vice versa. So you have a higher luminosity you have a higher mass and vice versa. So these scalings are very powerful tools which keeps coming back and forth to the our astronomical literature because it makes us easy to interpret the results. These are little bit hand wavy sometimes mostly derived from correlation of data like we observe have observed this at this particular frame.

So it is a correlation and but these are very powerful tool in interpreting results from different observations. So very easily you can read it now from from something called intensity which you are observing to the mass okay of the object and this this thing's works very well. Let's take a few example to sort out these issues any issues regarding this this confusing definitions of specific intensity flux density and luminosity. Okay so the first one,

1. Which of the following statement is true?

   a) Specific intensity is directly proportional to the frequency
   b) Intensity is inversely proportional to frequency
   c) Total Flux Density is independent of frequency
   d) Specific intensity is equal to the mean intensity

we are just trying which of the following statement is true specific intensity is directly proportional to the frequency intensity is inversely proportional to the frequency total flux density is independent of frequency and specific intensity is equal to the mean intensity. So the answer to this question is total flux density is independent of the frequency because we know total flux is integration of s new over d new okay. So that is the true answer.

Let's try one more specific intensity is also known as source brightness mean intensity brightness temperature none of the above,

2. Specific intensity is also known as
   a) Source brightness
   b) Mean intensity
   c) Brightness Temperature
   d) None of the above

and now specific intensity also called source brightness by the definition. Now some problems calculate the power received by the detector of area of 50 square meter receives mean intensity of 0.005 meter watts per meter square when the source is at zenith.

3. Calculate the power received by the detector of area 50 sq. meters receives mean intensity of 0.005 watts per sq. meter when the source is at the zenith. ________

Ans. Given that

Mean Intensity of the Source, I_m = 0.005 Wm^{-2};
Area of the detector, A = 50 m^2

Thus, the power received,

$$P = 4\pi I_m A cos\theta \ \approx 3.14\ W.$$

As the source is at zenith – 0°.

So this is the same classic example where we relate the power with the specific intensity multiplied by the area of the detector by the solid angle by the bandwidth. So mean intensity of the source or specific intensity is 0.005 watt per meter square the area of the detector is a is 550 meter square. So the power received is nothing but 4 pi steradian times the IM times the area times cosine of theta remember cosine of theta is the theta is 0 right because it was at zenith source was at zenith so cos theta is equal to 1 so it's 4 pi times IM times a which should give you 3.14 watts.

Next example what is the source brightness if the power received by a circular detector of radius 5 meter is 70 watts when the source is at zenith and subtends a solid angle of 15 steradian over a bandwidth of 100 megahertz okay.

4. What is the source brightness if the power received by a circular detector of radius 5m is 70 watts when the source is at zenith and subtends a solid angle of 15 steradian over the bandwidth of 100 MHz. _______

So similar problem little bit different in the framing so here we have given the power the radius of the detector circuit vector so area to be calculated solid angle is given 15 steradian and the bandwidth is given of 100 megahertz.

So P is 70 watt steradian solid angle is 15 steradian and area is a is pi r square since it's circular so pi 5 square which is 78.53 meter square hence the source brightness or the specific intensity is given P over a sigma cosine of theta which is 0.059 watt per meter square per steradian okay.

Ans. Given that

P = 70 W; Ω = 15;

Area of the detector, $A = \pi r^2 = \pi(5)^2 = 78.53\, m^2$

Thus,

the Source brightness, $I_0 = \frac{P}{A\Omega cos\theta} = 0.059\, Wm^{-2}sr^{-1}$

Next problem is coherent measure radiation from a source at a distance of 2.3 kilo parsec has a flux density of 103 Jansky over a frequency band of 1 kilohertz if it is isotropic what is the power radiated.

5. Coherent (maser) radiation from a source at a distance of 2.3kpc has a flux density of 103 Jy over a frequency band of 1 kHz. If it is isotropic, what is the power radiated?

So we have given the distance of the source the flux density is given and a frequency band is given it is told it is isotropic so what is the radiated power. So for an isotropic emitter we know that the L the luminosity is dependent on the flux density by 4 pi D square. So here the distance is given as 2.3 kilo parsec 1 kilo parsec is 10 to the power 3 per sec and 1 parsec is 3.

09 10 to the power 16 meters. So if you put everything in there it is 7 into 10 to the power 19 meter is the distance comes out to be. The total flux is specific the flux density times the bandwidth so 103 Jansky times 1001 kilohertz so that is there and then one Jansky is 10 to the power minus 26 watts per Hertz per meter square so that is what it's given so it is 10 to the power minus 20 watts per meter square. So L the power radiated isotropically is given by 6 10 to the power 20 watts.

ANS. For an isotropic emitter we know that the total power radiated can be written as

$$L = 4\pi D^2 S$$

So, all we need to do is convert our distance to meters (7×10^{19} m)

and understand that the total flux density

$S = S_\nu \Delta\nu = 103\, Jy \times 10^3\, Hz \times 10^{-26}\, WHz^{-1}m^2 = 10^{-20}\, Wm^{-2}$.

Then,

$$L = 4\pi(6.9 \times 10^{19})^2 10^{-20} = 6 \times 10^{20}\, W$$

The last example and I put a large one a radio source at a distance of 20 mega parsec 1 mega parsec is 10 to the power 6 parsec and 1 parsec is 3.09 10 to the power 16 meter is observed by a detector having area of 300 meter square at a frequency of 2 gigahertz with a bandwidth of 250 kilohertz.

The source is determined to have a flux density f new at this frequency of 10 Jansky and is found to be of a uniform circle with an angular diameter of 30 arc seconds. So what is the intensity of the radiation from this source at this wavelength? What is the total power of the radiation detected by the detector? What is the luminosity of the source over the observed spectral range assuming that the radiation is isotropic?

6. A radio source at a distance of 20.0 Mpc (1 Mpc = 10^6 pc and 1 pc = 3.09×10^{16} m) is observed detector having area of 300 m^2, at a frequency of 22.2 GHz, and with a bandwidth of 250 kHz. The source is determined to have a flux density, F_ν, at this frequency of 10.0 Jy, and is found to be a uniform circle with an angular diameter of 30.0 arcsec.

a. What is the intensity, I_ν, of the radiation from this source at this wavelength?

b. What is the total power of the radiation detected by the Detector?

c. What is the luminosity of the source over the observed spectral range (assuming that the radiation is isotropic)?

So calculate I new power and luminosity. So let's take the first one the average flux density is divided by the is the flux is divided by the solid angle of the source and since the source is found to be an uniform circle the intensity equals the average intensity. We first need to find the solid angle since we are only given the angular diameter using the relation as below. So since it is 30 is the diameter so theta by 2 is radius so we figure out the solid angle and that comes out to be 1.

66 10 to the power minus 8 steradian. The intensity then is just simply the flux density 10 Jansky converted to the watts per meter squared per hertz per steradian into yeah that unit and then divided by the total solid angle which is 1.66 divided by multiplied by 10 to the power minus 8 steradian. If you put the numbers it comes out to be 6 10 to the power minus 15 watts per meter squared per Hertz per steradian that's the I.

Ans. The average intensity is the flux density divided by the solid angle of the source, and since the source is found to be a uniform circle, the intensity equals the average intensity. We first need to find the solid angle, since we are only given the angular diameter. Using relation

$$\Omega = \pi\left(\frac{\theta}{2}\right)^2 = \left(\frac{\pi}{4}\right)\theta^2$$

$$\text{Or, } \Omega = \frac{\pi}{4}\left(30''\frac{1^\circ}{3600''}\frac{\pi\, radians}{180^\circ}\right)^2 = 1.66 \times 10^{-8}\, sr$$

a. The intensity, then

$$I_\nu = \frac{10\, Jy\, (10^{-26} W m^{-2} Hz^{-1} Jy^{-1})}{1.66 \times 10^{-8}\, sr} \approx 6.0 \times 10^{-15} W m^{-2} Hz^{-1} sr^{-1}$$

Okay number two the second question was total power of radiation detected by the detector. So the first part is I new remains the same the P new is nothing but the flux this density multiplied by the area multiplied by the bandwidth and that comes out to be 7.5 into the power minus 18 watts that's the power.

b. The total detected power equals the flux density multiplied by the bandwidth and the area of the detector

$$P_\nu = S_\nu A \Delta\nu = 10(10^{-26})(300)(250 \times 10^3) = 7.5 \times 10^{-18}\ W$$

We have a third question which is what is the luminosity of the source observed over the observed spectral range assuming that the radiation is isotropic. So s new the flux density is total flux over delta new and also s is luminosity over 4 pi D square. So given that we can relate the specific flux sorry luminosity over the entire range to be equal to the flux density times the bandwidth times the 4 pi of distance square. If you put all the values distances remember it's 20 mega per sec 20 times 10 to the power 6 and 3.09 into 10 to the power 16 meter. So if you put all the numbers it comes out to be 2.4 10 to the power 29 watts.

c. The luminosity of the source, over the observed spectral range, is given by the observed flux density times the bandwidth, and then

$$S_\nu = \frac{S}{\Delta\nu}, \qquad and \qquad S = \frac{L}{4\pi d^2}$$

Then,

$$L = S_\nu \Delta\nu 4\pi d^2$$

Or,

$$L = 10(10^{-26})(250 \times 10^3)(4\pi)(20 \times 10^6 \times 3.09 \times 10^{16})^2$$

Therefore,

$$L = 2.4 \times 10^{29}\ W$$

So this brings us to the close of another another lecture I hope that this specific problems which we tackled in the in the end actually helped you a little bit understand more and and de-entangle the issues between flux density intensity and luminosity but we will discuss this more all through the lecture the series and hope it helps. Thanks for joining and see you on the next lecture. Thank you very much.