Radio Astronomy

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Lec-04

RadioI Astronomy - Tutorial 01

Hello everyone, I am Hashim Naasthanti and I will be your TA for this radio astronomy course. I am currently pursuing my PhD under the supervision of Prof. Agirug Datta, who is also the course instructor of this radio astronomy course. Now let's begin with discussing what we are going to learn from this tutorial session. So in this tutorial session, we will start with knowing, means like discussing about how to approach a MCQ type question. Also how to approach a numerical answer type question.

I hope most of you might know how to do that. However, we will still revisit it and shortly discuss like how to solve this type of questions. And after that we will see some model questions regarding this first week of assignment. And also with this we will also study some concepts like image charges and method of images.

Also, we will see how this electromagnetism goes into radiation and how the antenna radiates. And we will take an example Hertz Dipole or Infinitesimal Dipole to see how the mathematical equations looks like for the antenna radiation. Also after that we will do some MATLAB simulations also to visualize how the electromagnetic fields look like. So let's begin with reviewing how to solve an MCQ type question. Basically there is only two methods to solve MCQ type question.

The first is you know the entire concept as well as the correct formula to solve for the answer or the next method is method of elimination wherein you don't have the entire knowledge of these formulas and the subject matter but you know the relations between the quantities as well as some of the concepts from which you can build on to solve the answer. Now let us consider this example over here. You don't have to take any intention because if this is not you studied in the first week of your lecture or the first previous two lectures, this problem will be again discussed in the third week of the lecture, means the similar kind of problem which is related to antennas. Let's kind of review this question to understand how to solve the MCQ type question. Now this question says that earth sustains an angle with the sun which is of solid angle 4 into 10 to the power minus 5 steradian.

For your information the solid angle is the ratio of the area subtended by the cone this small a over the radius square. And also the question says that there is an antenna you are using to study the sun which has a narrow beam which covers the sun exactly and the question is asking that find the directivity of the antenna. Directivity is essentially the design metric for the antenna which you will study in the third week of this course. Now if you go by method one, you will know the entire concept and you will know the formula that directivity is equal to 4 pi by the solid angle. So the correct answer will be C because 4 will get cancelled and your answer will be pi into 10 to the power of 5.

Q1: The solid angle subtended by the sun as viewed from the earth is $\Omega = 4\times10^{-5}$ steradian. A microwave antenna designed to be used for studying the microwave radiation from the sun has a very narrow beam whose equivalent solid angle is approximately equal to that subtended by the sun. What is the approximate directivity, D?

A) 10^5 B) 10^6 C) $\pi\times10^5$ D) $\pi\times10^6$

Now if you go by the second method, you won't have the formula remember but you will know kind of the concept like directivity will be inversely proportional to the solid angle. So the most probable answer will have the order of 10 to the power 5. So there is C and A which are the most probable answer. Now since you know that the directivity is inversely proportional, you have to see the word proportionality. So proportionality means there should be a proportionality constant.

So that means C is most probably the correct answer because you can see the pi as a proportionality constant there. So this is how you go to solve MCQ type question. Now let's move on to reviewing how to solve a numerical answer type question. On the contrary to the MCQ type question where you can solve by having partial knowledge, here you have to know the entire knowledge about the question or the subject matter. So let us take this as an example.

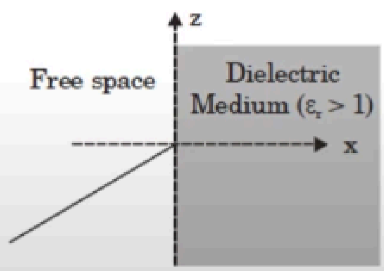
You might have studied this in this week itself. So it will be easier for you to relate. So this question is taken from set 1 of the GATE EC 2017 paper. So now let's see the question what it is. So the question says given an electromagnetic wave which is traveling in a free space having electric field as shown here, you have to calculate the relative permittivity of the dielectric medium if there is no reflected wave.

For solving a Numerical Answer Type questions you will need both concept and the necessary formula.

Example (a question from set-1 GATE ECE 2017): A uniform plane wave traveling in free space and having the electric field

$$\bar{E} = \left(\sqrt{2}\hat{a}_x - \hat{a}_z\right)\cos\left[6\sqrt{3}\pi \times 10^8 t - 2\pi\left(x + \sqrt{2}z\right)\right] \text{V/m}$$

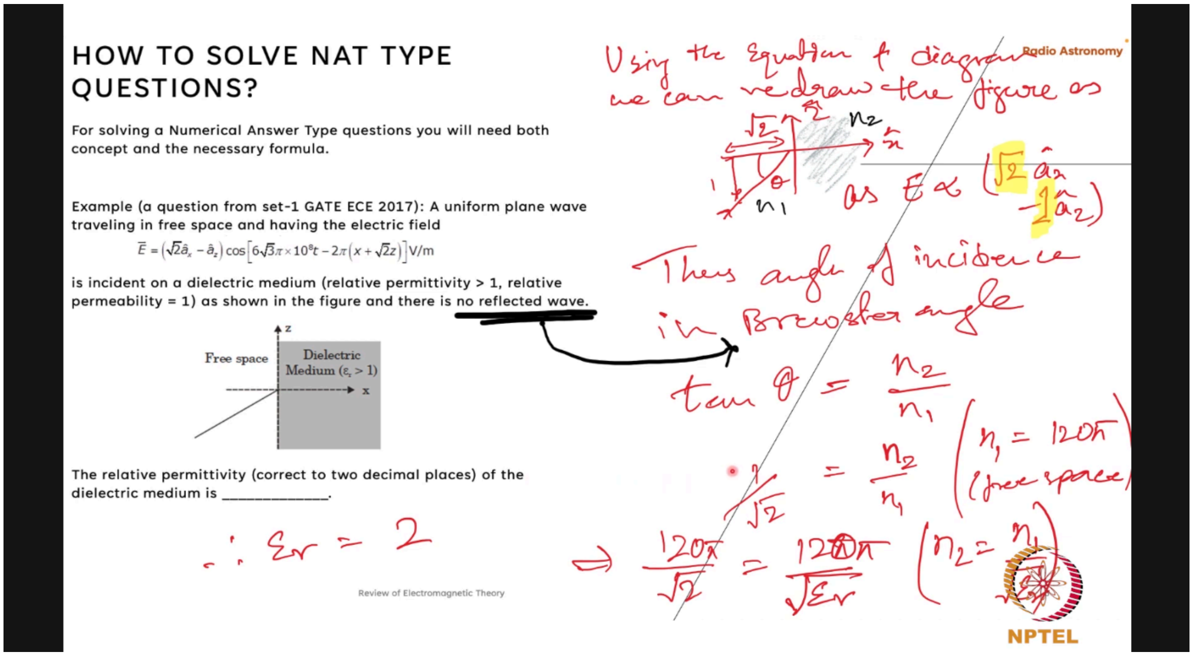
is incident on a dielectric medium (relative permittivity > 1, relative permeability = 1) as shown in the figure and there is no reflected wave.



The relative permittivity (correct to two decimal places) of the dielectric medium is ______________.

So here the trick is there is no reflected wave. So let's see how to solve this. So as I said here the trick is there is no reflected wave. So this means they are talking about the relative angle. Now looking at the equation we can build an geometry, how the magnetic field, sorry electric field is in the space, oriented in the space.

So here you can see there is along x you have root 2 and along z you have minus 1 which is towards the negative side. And also you know that the free space has the relative permittivity of 1 whereas the other medium will have some value which will be greater than 1. Now since the condition is the Brewster's angle so what you will have is tan of theta is equal to n2 by n1 where the n1 and n2 are the reflection coefficient or the impedance of the media. So if you solve you will get dielectric constant of 2. So here you will get 1 by root 2 tan and n1 is equal to 120 by pi so you will get er is equal to this value.



How? Because the medium, if you know the equation for the impedance of the medium it is equal to 1 over root epsilon naught and mu naught mu r and relative permittivity means it is directly proportional to that. If you, that has been already covered. So you will know all these formulas so you will arrive at the er is equal to or the relative permittivity is equal to 2.

Now let us have a look at another nat type question.

The electric field component of a plane wave traveling in a lossless dielectric medium is given by

$$\vec{E}(z,t) = \hat{a}_y 2cos(10^8 t - z/\sqrt{2})$$

V/m. The wavelength (in m) for the wave is _______.

So in this what is given is electric field of a plane wave which is travelling through a lossless dielectric medium. So there are no losses in the dielectric medium. This is the main key to answering this question. And you have to find the wavelength of the plane wave. So remember what is the property of the lossless dielectric medium. A hint is you can take a lossless dielectric medium as a field space and from there you can start calculating the quantity.

So I will give you 10 seconds here and after that I will, we will proceed to see what is the correct answer. So let us see what is the correct answer. If you got answer somewhere in range of 8.5 to 9.1 then your answer is absolutely correct.

Given that:

$$\vec{E}(z,t) = \hat{a}_y 2cos(10^8 t - z/\sqrt{2})$$

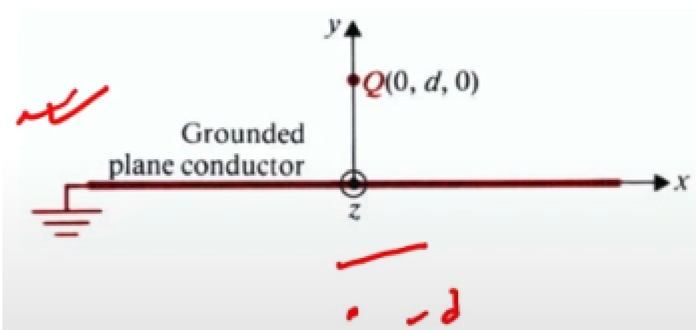
V/m Comparing with the general form

$$\vec{E}(z,t) = \hat{a}_y Acos(\omega t - \beta z$$

we get phase constant $\beta = \frac{1}{\sqrt{2}}$ rad/m Also, we know that β is generally in the form of $2\pi/\lambda$ Thus, $\beta = \frac{2\pi}{\lambda} = \frac{1}{\sqrt{2}}$ or, $\lambda = 2\sqrt{2}\pi$ m Therefore, $\lambda = 8.89$ m

Why is that so? Because the general form of the electromagnetic wave is this where Bz is the wave number which is represented by 2 pi by lambda also known as the phase constant. Now what we know comparing this with this is that beta is equals to 2 pi by lambda which is equals to 1 by root 2. So lambda comes out to be 2 root 2 pi meter. Therefore lambda is 8.89 meter.

So the sample questions which I discussed earlier is a similar kind of question you will get for your assignment. Now let us move on to another exciting topic which is image charge and method of images. Now coming to the point what is an image charge. Say you have a ground plane. Let us consider this diagram over here where you have a ground conducting plane and these are charged directly above it.

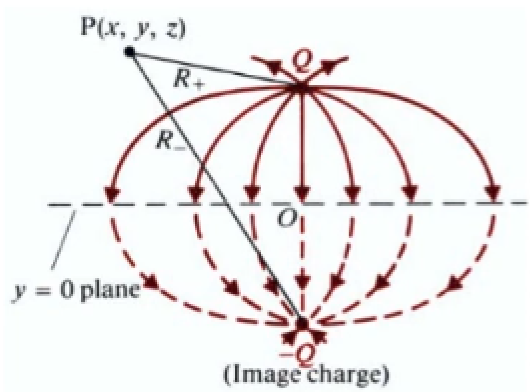


So there is an image charge created due to the what method of images directly below the ground plane equidistant from the conductor. Like say if this is D your other charge will be created at minus D. Now let us see what is method of images. So by definition method of images replaces the original boundary condition by a straight image charge in lieu of a formal solution of Poisson's or Laplace equation so that the original problem is greatly simplified.

This is the definition. What does this does is we can use this to solve for potential or the force at a particular charge or potential at a particular point by using this method. So this method is applicable if and only if it satisfies these two conditions. The first condition is it should satisfy Poisson's or Laplace equation. That means either del dot E should be equals to your rho or the proportional to rho or E dot E is equals to zero where E dot E you can replace it by del square V which is proportional to rho or del square. So in Laplace square V equals to zero.

So this is Laplace and this is Poisson. The second thing it should satisfy the given boundary condition and the simplest solution should be taken. So basically this method is for simple boundary conditions. Means like here we have a grain plane, here we can have a ground plane, here too we can have here too where you can define boundary conditions in a very simplistic manner. If you have some kind of boundary condition like this, this where it is difficult to define a boundary then this method most probably won't work or it will be very difficult to implement.

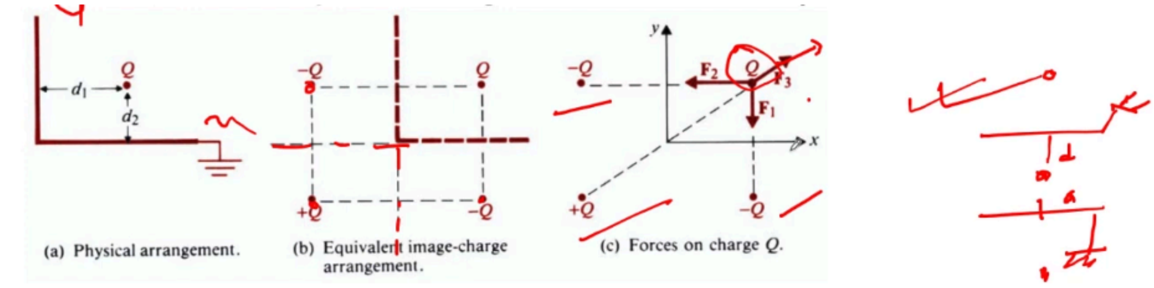
Now let's see what happens if we go by the book.



So if we go by the book, if you want to calculate the potential at x, y, z like this at point P, so you will get the cumbersome equation where you have to integrate this charge enclosed over the r1 where r1 is the equivalent distance from the Q or the location of the observer from the origin. Now if you use method of images what will happen due to this charge there will be an opposite charge created below the surface having minus Q and you can calculate the potential as this where plus r is the distance from or the vector relative vector from Q and r minus is relative vector from Q minus. So this is how the method of images simplifies a problem. Also if you take some numeric value for this location you will find that these two values will be same.

This is for you to do by your hand. Take some random values of Q as well as the observer location of x, y, z as 1, 2, 3 some point and see and evaluate the function this and this and see you will arrive at the same answer.

Now I also said that the method of images is also valid for forces. So if you see here, you have a charged Q at a distance from a ground conductor, grounded conductor at d1 from the bottom and from the vertical side, d1 from the vertical side and d2 from the bottom. So let us say that this is y and this is x for simplicity.



(a) Physical arrangement. (b) Equivalent image-charge arrangement. (c) Forces on charge Q.

So due to the ground conductor in x what will happen you will get a minus Q charge here and due to vertical you will get a minus Q charge here. Now to balance it out what will happen since this is also grounded and this is also grounded along x and along y you will get an image charge plus Q here. So then the effective force on Q will be simply this force simply the effective force by minus Q minus Q and plus Q which is what F1, F2 and F3. F3 is outward because it is positive Q and positive Q will generate a repulsive force on Q whereas minus Q will generate an attractive force. So the equivalent force is F equal to F1 plus F2 plus F3 which you can evaluate as this and the final force will come out to be as shown here.

By using method of images we get $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

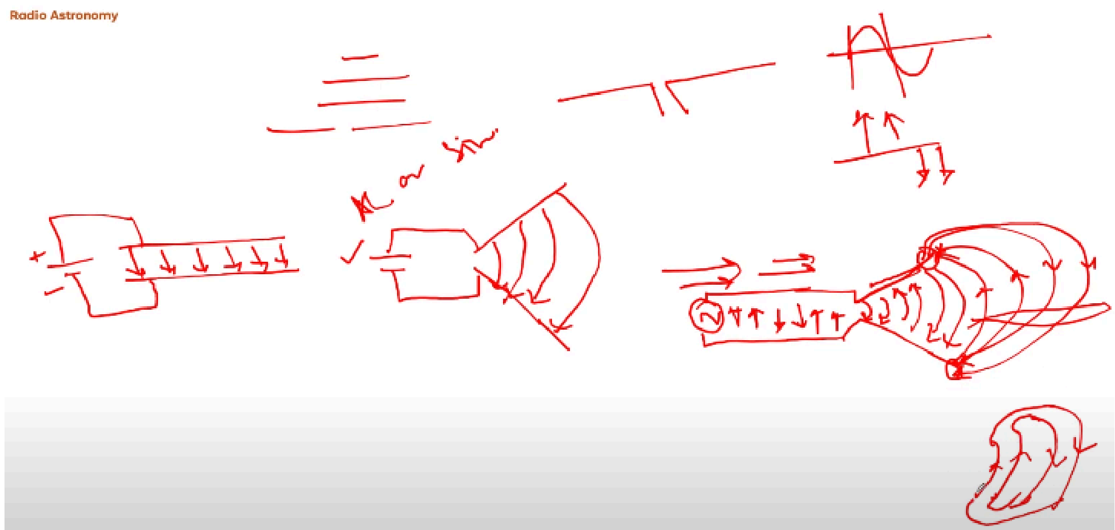
$$\mathbf{F}_1 = -\mathbf{a}_y \frac{Q^2}{4\pi\epsilon_0 (2d_2)^2} \qquad \mathbf{F}_2 = -\mathbf{a}_x \frac{Q^2}{4\pi\epsilon_0 (2d_1)^2} \qquad \mathbf{F}_3 = \frac{Q^2}{4\pi\epsilon_0 [(2d_1)^2 + (2d_2)^2]^{3/2}} (\mathbf{a}_x 2d_1 + \mathbf{a}_y 2d_2)$$

$$\mathbf{F} = \frac{Q^2}{16\pi\epsilon_0} \left\{ \mathbf{a}_x \left[\frac{d_1}{(d_1^2 + d_2^2)^{3/2}} - \frac{1}{d_1^2} \right] + \mathbf{a}_y \left[\frac{d_2}{(d_1^2 + d_2^2)^{3/2}} - \frac{1}{d_2^2} \right] \right\}$$

So you can take any system of charges for example you can take like there is a two ground plane conductor like this, two grounded conductors and there is a charge here. In this case you can also calculate how the force is acting upon this charge. So what will happen here you will get Q by 2 and here you will get minus Q by 2 minus Q by 2 both side so that this is balanced. So this is how you calculate if the distance is say both same d then this will be Q by 2 and this also will be minus Q by 2. So this is how you calculate force by using method of images for a simplistic boundary conditions, instead of solving the cumbersome integral equations.

Let us move toward another exciting topic of the day which is antenna radiation. So in this week you have learnt about how the time varying fields leads into radiation or plane waves something like that. But how to radiate using a transducer which can radiate the electromagnetic wave which can convert electrical energy into electromagnetic wave. So you might have seen in your early childhood days or you might have seen in pictures also some antennas like this, which is like this, this, some Yagi-Uda antenna or some this is called Yagi-Uda or some kind of log periodic antenna and this is there is a simple dipole antenna like this which is connected from here.

So now here we will try to develop an analogy how the antenna starts radiating. You do not have to take tension because this will also be covered in week 3 but let us create a kind of knowledge not knowledge we create some kind of pictorial representation how the radiation might happen. A crude analogy to say as we cannot see any electromagnetic wave. So we need to have some kind of pictorial representation how the electromagnetic wave starts radiating. So let us consider first we will start from the electrostatics and move towards electrodynamics things like how the radiation will start happening.



So let us consider two parallel plate capacitors which is connected to say for example a DC source. Then how will be your field look like. So this is positive terminal, this is negative terminal. So your field will be from here to here.

The field lines will come here to here. Now what will happen if you stretch out the capacitor. Let the same DC source be connected. So what will happen? The fields lines will be stretched out like this. Understood? Now let us consider this source replaced by a alternating source or sine wave. Now if you see a sine wave like this so what is here? One at first half you will get up field and the second half you will get down field.

So let us proceed with this analogy. So you have a sine wave here. So this will be connected to your what this kind of stretched capacitor. So to represent this first half up first half down what we will do we will put arrow here. So the first half coming down up down up.

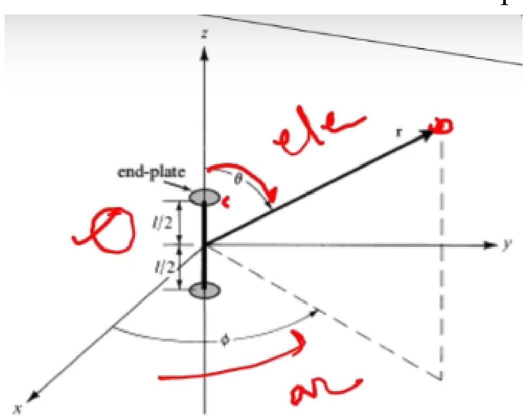
It will come like this. It will travel like this. So you will get a field here and the reverse field then to here. Then again here. So this how you will get. Now what will happen? This generator will push sine wave this way.

So the fields will be pushed in the same direction over and over. So what you will see the fields are bulging out like this. It will bulge out and kind of it won't intersect because field lines never intersect. So for example the drawing.

So this is how the field lines will stretch. Now if you see it is hanging at a very narrow space. What will happen as it pushes forward and forward this field lines will break. Break not break leave this conductor. Kind of leave this conductor and become free.

Like say this I will write this four set of line. So like this two fields will be like this and two fields will be like this. So what will happen since there is two up field two down field it will try to reconnect with each other and make a loop. As soon as makes a loop so if electric fields curls around then what will happen it will generate a magnetic field and magnetic field in turn will generate a electric field which will then lead this to a propagation. This is how antenna will start radiating. So now let's see how the equations turn out to be for this thing.

So we will take infinitesimal dipole or the Hertz dipole as an example here.



(a) Infinitesimal dipole

Now if you see what we will have is a dipole of length L, L which is very very less than the wavelength. So above the origin you have L by 2 below the origin you have L by 2. So you are calculating field at r at distance r from it where theta is the elevation and phi is the azimuth angle. Now what so since it's infinitesimal dipole or a very short length of wire you can consider current to be constant here.

So you can take IZ as AZ I0 because it's oriented around Z and from this what we can do we can calculate a vector potential at point r using the vector potential formula. Now from there what we can do we can replace your IZ and we will arrive at this equation, Thus we can write Vector potential as

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \mathbf{I}_e(x',y',z')\frac{e^{-jkR}}{R}\,dl'$$

where, $\mathbf{I}_e(x',y',z') = \hat{\mathbf{a}}_z I_0$

Thus, $R = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} = \sqrt{x^2+y^2+z^2}$

$= r = \text{constant}$

Therefore, dl' = dz'. Thus we can rewrite the vector potential as

$$\boxed{\mathbf{A}(x,y,z) = \hat{\mathbf{a}}_z \frac{\mu I_0}{4\pi r} e^{-jkr}\int_{-l/2}^{+l/2} dz' = \hat{\mathbf{a}}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}}$$

After that if we go into spherical coordinate system, spherical coordinate system then we will have r theta and r variation of A which is vector potential at ar and a theta.

$$A_r = A_z \cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$A_\phi = 0$$

That means if we have like infinitesimal dipole at distance r, so the vector potential is varying along the elevation theta and with respect to r. So there is a variation with respect to elevation and with respect to r.

Now we know that the vector potential A is related with what magnetic field B by a curl relation. So what we will have, we will have an H field in phi direction.

$$\mathbf{H} = \hat{\mathbf{a}}_\phi \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

So if you remember correctly you have current in I direction, I am sorry in Z direction, I in Z direction. So if you apply a right hand thumb rule you will find that the magnetic field should curl along like this and from the equation also you find it is along azimuthal plane that is phi is this one azimuthal plane. So this shows that the current, the direction of the current follows the magnetic field and if you follow the Maxwell's equation and apply this formula for current j equals to zero because you are calculating at distance r from the conductor so there is no current density at that location then you will get two field values, electric field values at r, e r and e theta, radial as well as elevation value.

Thus from above equation and maxwell's equation we have (J=0 as current is constant spatially).

$$H_r = H_\theta = 0$$
$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$
$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$
$$E_\phi = 0$$

So if you see H and e field are what orthogonal to each other which it should be for a transverse electromagnetic wave.

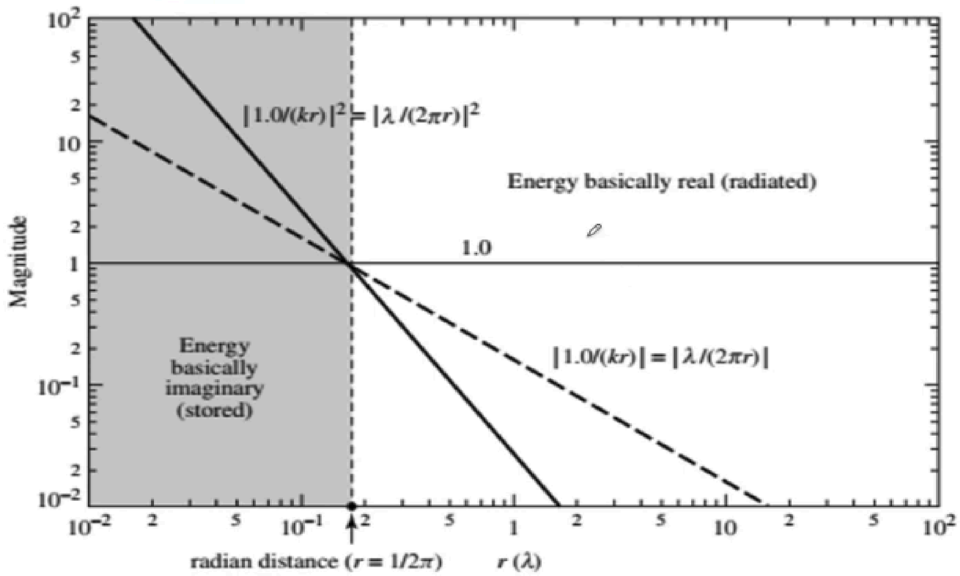
Now let us examine these equations. So if we see there are k r and k r square terms here. That means it has a distance dependency and this will vanish at as far as you, means if you go very far away from the source or the infinitesimal dipole.

So this due to this we can categorize fields in three parts. First part will be near field. Second will be personal region or intermediate field. And the third one will be farrenhopper or far field. So there is three fields you can differentiate it with. So if this k r is very less than one, so this value will be very high.

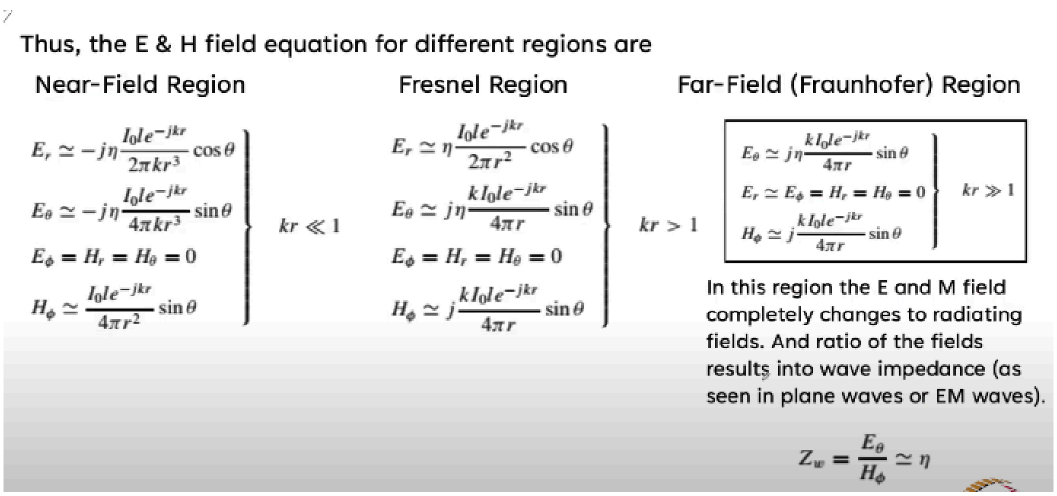
$$H_r = H_\theta = 0$$
$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$
$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$
$$E_\phi = 0$$

So you will have a distance dependency. So that means your field will be capacitive in nature. If you see the capacitor analogy, you will find that if you go near the field, you will find the field values. So that says that you are in the capacitive region. And you can also draw this chart which is taken from, you can read in more detail in Antaran theory by Balaanis, Image Credits: Antenna Theory (By C. Balanis)



However this you can also create by using these terms here,



So if your k r value is very less than lambda or r is very less than lambda by 2 pi, so what will happen is your field is in near field region. If that is greater than lambda, k r is greater than lambda or r is lambda by 2, then it is in intermediate region or personal region. So these two fields mean the antenna is still not radiating. It is not completely converting the electrical energy into electromagnetic energy. When you go very far beyond the lambda, greater than lambda, you will enter the far field region where this term will diminish so much that it will be almost equals to 1.

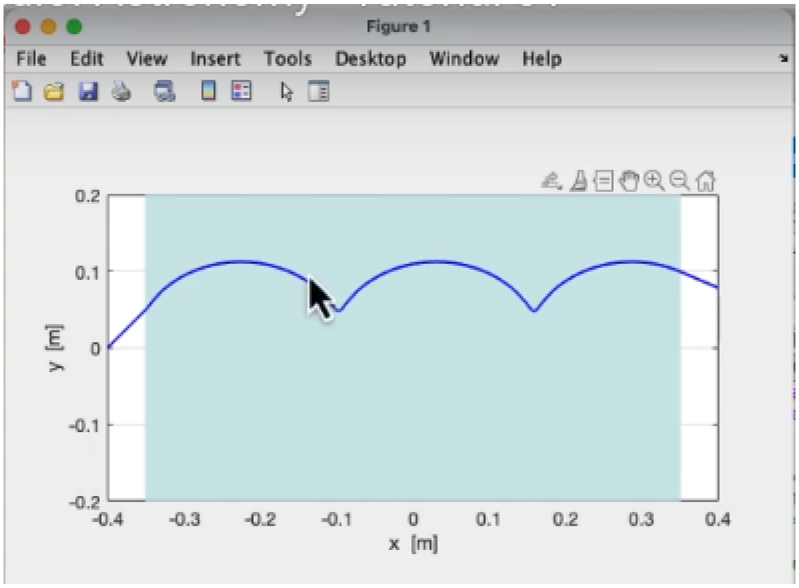
In that region you can say that the total electrical energy has been converted into electromagnetic energy. Not total or some part, there will be losses. So but in ideal conditions everything will be converted. So this is what the equations for radiation looks like.

So to summarize what we have? We have near field region where k r is 1. Why k r is 1? So k is equal to 2 pi by lambda. k is equals to 2 pi by lambda. So you remember this, that is why this relation is coming. So very less than this, then it will come under near field region. If it is greater than 1, then it is personal region and if it is much much greater than 1, then it will come under Poornhofer region where the antenna will start radiating and there will be no radial term, there will be no radial dependency.

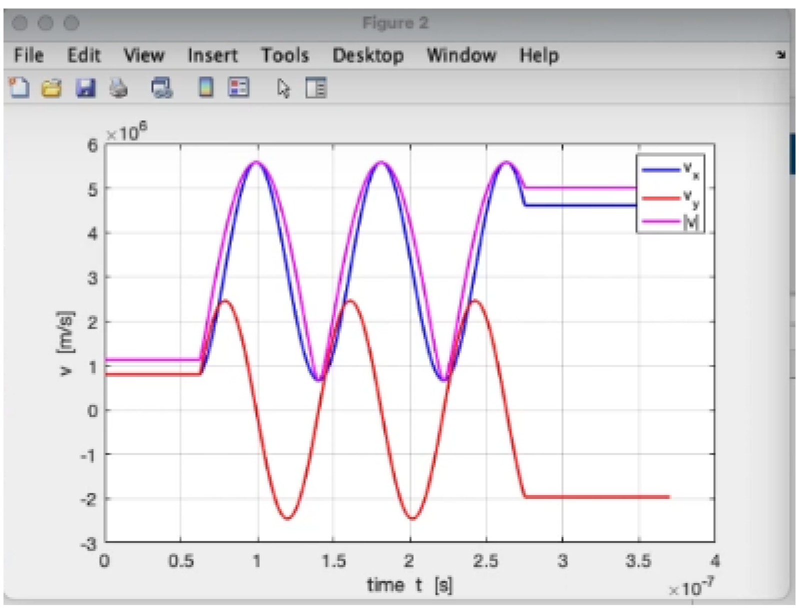
And also a quick bit that there is something called eta quantity here which is nothing but the impedance of the media which is calculated by E theta by h phi which is for space phase is equals to 120 pi. So this you need to remember because this comes in most of the comparative exams very frequently. So now let us move towards another exciting part of this lecture series, lecture session which is MATLAB simulation. So what we will try to do? We will see some MATLAB simulations used to enhance our understanding about electromagnetism.

Now let us have a look at our first example MATLAB code. So do not have to worry about having MATLAB or not. I already had this code with me, so I am trying to demonstrate and we will also revisit sometime in future with using Python. So now this first example is about a charged particle in an time varying electric and magnetic field. You have a charged particle which is moving at a velocity in a time varying electric and magnetic field. So in this you can see we have defined the number of time steps, in number of how many time steps we have to evaluate, the element of the field and proton and everything, the definition is here and the rest of things are the equations turn into like program format to evaluate and find at each and every time step what is happening and we are plotting that.

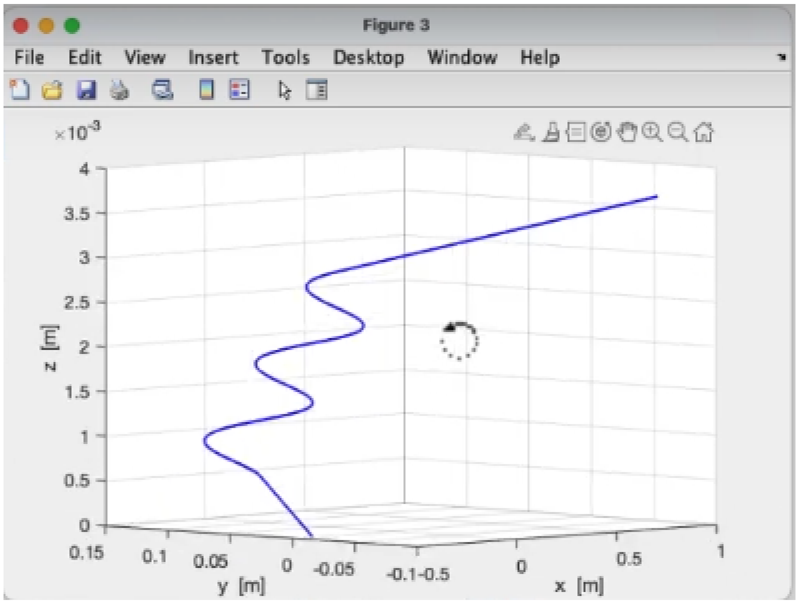
So when you run this code, what you will get? You will get a graph with x and y coordinates, how the particle is moving in two dimensions.



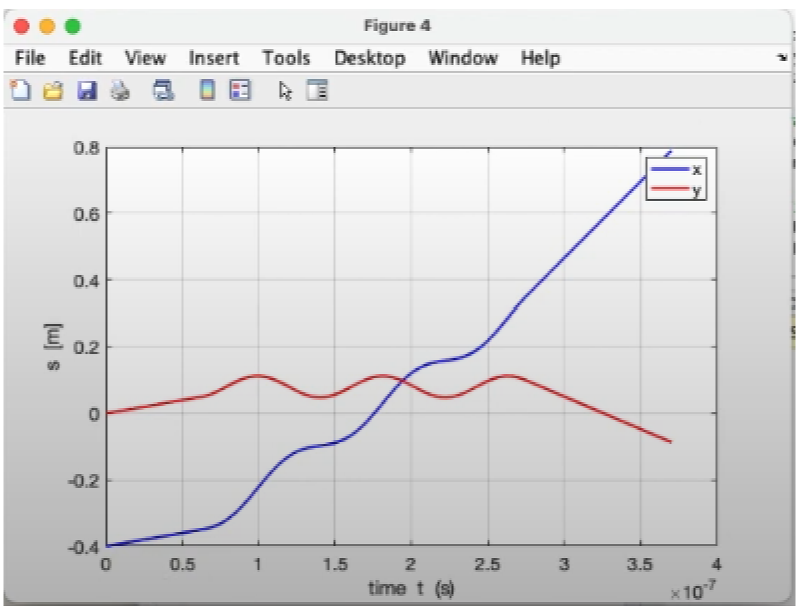
Then you will get a figure showing the velocity with respect to time in vx and vy and its magnitude also.



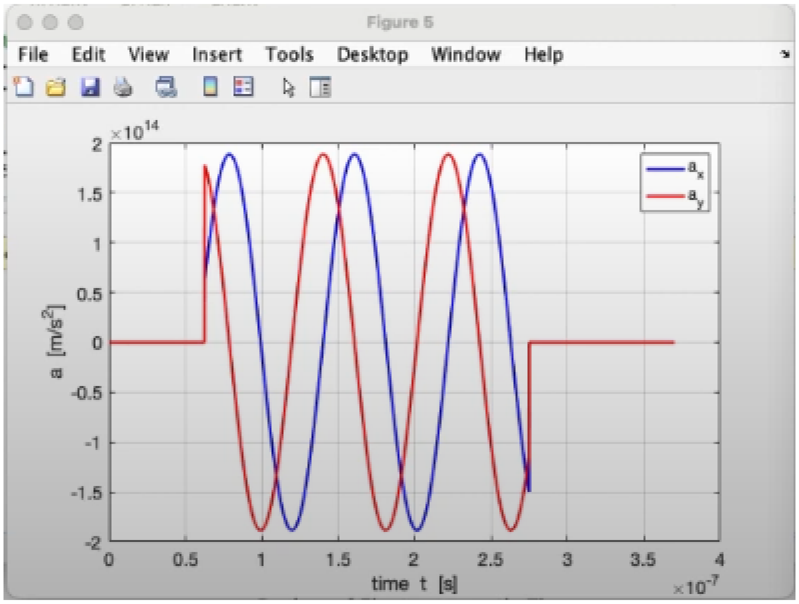
Now in the third figure what you will get? You will get a 3D plot of how the particle is moving in three dimensions.



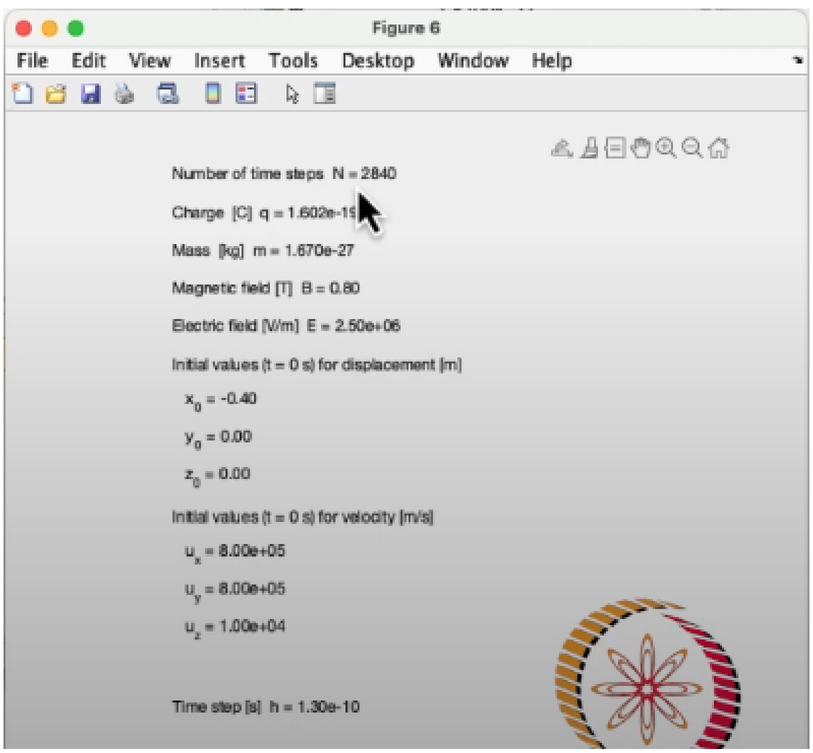
So you can see it is moving in a sinusoidal way. Now another thing you will get how much the particle is displaced, quantifying that displacement of the particle, how much particle has been displaced from its location.



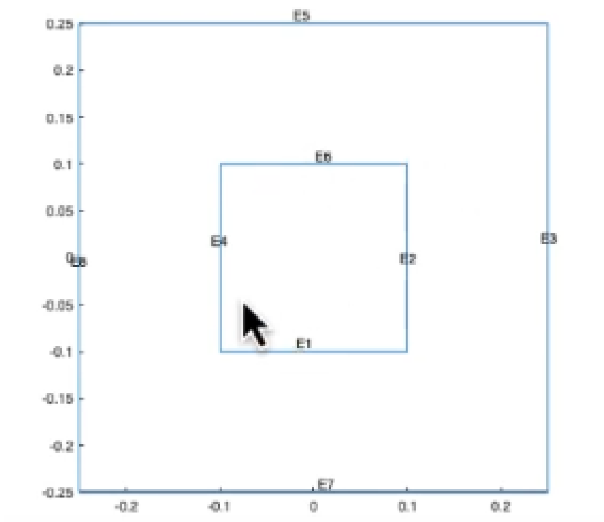
Then one more thing which you will get is the acceleration of the particle, how much particle is being accelerated in the space,



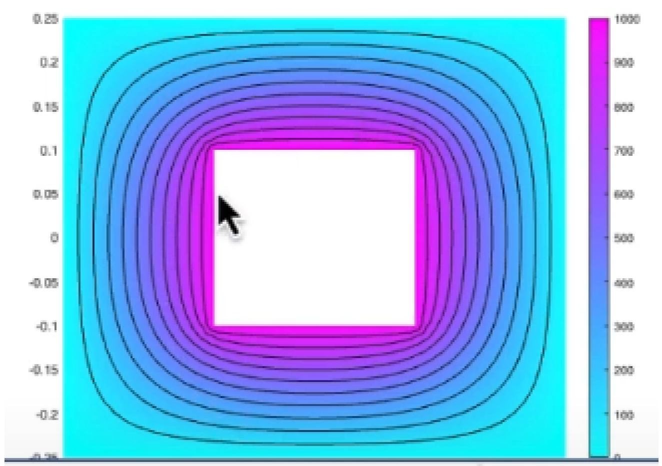
and then you also you will get one window showing initial and final initial velocities, charge, the magnitude of the charge, mass and etc.,



And the another example is by MATLAB itself which shows kind of a boundary problem, wherein you will find a rectangular box inside a bigger rectangular box and the inside rectangular box will have a higher voltage than the outside one.



Then what will happen, what you will think is the field lines will be created like this and this and this. If you plot and contour using that you will find that the field is more intense towards means near the high potential value square than the ground,



So if you think intuitively that if you have a very high potential box what will happen? The near that box you find very high means the strength will be high and it will dissipate as you increase the distance from it.

So which is the basic Coulomb's force law also. Now also you can imagine in this way you have a kind of a water jet. If you go towards the water jet, towards the water inlet means hose, towards the hose you will feel more and more pressure than you are at the outside. So this is a way to create an intuition how the magnetic field and electric field will act as. So this marks the end of this particular lecture. But before saying thank you

I would like to summarize what we have learnt in this lecture.

So what we have seen is how to solve, effectively solve MCQ type question, then how to approach Nat type questions or numerical answer type questions or then we went to see some model questions. After that we discussed an interesting topic related to image charge and method of images as well as we saw how the antenna, created an analogy how an antenna radiates as well as gone through the equations how the simplistic equations turns into a radiation equation or the simple vector potential gives rise to radiation. And different fields, means different fields, near field, intermediate field and far field of a antenna, different regions of an antenna. So with this also we covered some two of the MATLAB simulations where we saw how a charged particle moves in space with a time varying electric and magnetic field. Also we saw how the electric field varies is when if you are going away from the means high potential region. With this thank you.