# Radio Astronomy

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**Examples**

Okay. So can you see the slides? Thumbs up if you do. Great. Thanks a lot. So, the first problem we want to look into is, am I audible also? There was a problem from somebody that there was a bit of audio issue. Now is it okay? You are audible, sir.

Great. Thanks a lot. So, the first example we are concentrating on, it is a spotlight of beam, spotlight beam of intensity I mean, is equal to 5^10 to the power minus 8 hertz per second, per centimeter squared per hertz per selleion, at a frequency of 5^10 to the power 14 hertz, pointed at a cube of gas. On the opposite side of the cube, which responds only to light at 5^10 to the power 14 hertz measures the intensity of the light to 3^ 10 to the power minus 8 hertz per second, per centimeter squared per hertz per sr^-1. The spotlight is turned off and the intensity of the light emanating from the cube at the same frequency is then measured to be 2 * 10^-8, the following units. Okay, so there is a spotlight beam of intensity I knew given over here and a frequency of 5^10 to the power 14 hertz, that means, so it's pointed to a cube of gas on the opposite side of the cube, the spot light detector, which responds only to the same frequency measures the intensity of the light as 3. Okay, so it was initially at 5, it went to 3. The spotlight is now turned off and the intensity of the light emanating from the cube at the same frequency is measured to be 2. Okay, so what are the optical depth and source function of the cube of gas at this particular frequency? So that's the question.

So the classic question, we go back to the radiative transfer part, you have some kind of a cube, so this is not the right question. Yeah, so you have some sort of a cube. So I knew the most of my ability and you're shining some spotlight beam from this side, the I knew over here is given at s, it traverses s plus ds and comes out of there as I knew prime at s plus ds. So I knew is known, I knew prime is known, and then when everything is off, the particular gas in the cube is emitted and the intensity is also known. So the question asked is that what is the what are the optical depth and source function of the cube of gas at this particular frequency? That's number one.

Number two is if the cross section of the interaction between the photons and the gas particles is 5.7 to the power minus 18 centimeter square, the cube is 50 centimeter in length and uniform in density, what is the number density of the gas particles? Compare the source function of the gas to the intensity of the spotlight beam and contrast these two objects as sources of light. What does the source function tell you about the photon object? So it looks complicated, but let us go ahead and try to take it one by one. Okay, so we have two unknowns, the source function and the optical depth. The equation we have using this I knew, this part is already known, we have been using this particular expression before, when we did the transfer.

So when the spotlight is off, there is no incident intensity, so I knew not equal to zero. And so in that case, I knew not, I knew off is, is just going to be represented by the s new one minus tau. Okay, but it is on, then it is governed by this particular expression. So this is the absorption part, and this part is talking about the emission. So if I subtract the two equations, you suddenly get an expression with the I knew on, I knew off, and I knew zero minus tau. So if I do plug in all of those values, this is on one is, when it goes on, then it is saying, it is three, then when it goes off, it is two, and I knew not is basically the original one, which is equal to five. So if I do all those things, then finally I will get the value of tau is equal to one point, tau name as one point six one. And so if you solve the source function, you put in that into this particular expression, the value of tau into this expression, and you can figure out what the value of s new is, which is given by this. Okay, so that's the quick and easy part. Now, you have the absorption coefficient kappa new, which we discussed earlier. And that is equal to number density times sigma new. Okay. And what is sigma new that is given as five point seven, two to the power minus 18 centimetres square. Let's go back and figure out what this is. So if I go back to slides.

This is given as. Yeah, and the gas particles. The cross section is given by five point seven to the power minus 18. So then this absorption coefficient is the number density. Multiplied by the cross section, cross section is given. So you can. Then kind of know what to do. So if you know kappa new, then you already know sigma new. So you can be right. But that's what you know. Now tau new is already known. That is one point six one. We have time for the previous slides. So tau new is equal to kappa new times L because it's a cube. So we know. The sides of the cube. So if you put that in each side is centimetre. The length is centimetre also. So if you put that in the expression, I get from tau new. Is equal to sigma new times L and I know sigma. I know L and I know tau. So I can calculate for the value of N is that gives out to be five point six five. And for my three to the power 15 gas particles, five centimetres of the cube. OK, that's a good question to answer. Now let's go check the third one. The third one says, compare the source function of the gas, the intensity of the spotlight beam and contrast these two objects as sources of light.

What does the source function tell you about the full-bound object? The source function of the gas is smaller than the intensity of the spotlight. OK. This indicates that the gas is fundamentally a weaker light source at this frequency. The source function indicates the brightest the gas could get on its own. And this is less than the intensity produced by the spotlight.

So essentially you have the source function was given by s new. The original spotlight have this thing of I not new. So definitely this is five in the units. And this was the same unit, it was around two. So that made the source function of the gases weaker.

So it's a bit it looks a bit complicated, but then when you put in the knowledge you have from the transfer, then it becomes the only new part which we learned over here is that the kappa is a product of the absorption coefficient, the product of the number density of the gas and times the cross section. This is something which we learned. So now going forward, the second example, you have a positive and electron that undergoes simple harmonic motion in y direction. The y position is given by y is equal to y naught sine omega t. What is the time average power of the radiation produced by this oscillating electron? So the acceleration of the electron is a function of time taking the second derivative.

So d two by d two, the second order differential equation is equal to minus omega square y naught sine omega t. Now that is already given. So you have the formula of the D'Aumont formula, where p is given by two times q squared, so charge times a squared by one, this is squared. So if I substitute the equation we get dt is equal to ct squared over cc cubed minus omega squared y naught sine omega t. OK, so the acceleration part we are replacing by the d two by dt two. So you have p, the formula is p squared over pc squared. It was having a squared. A squared is nothing but time d two by d two squared. So you get the double derivative of y from here, which also we call y double prime for easier attention. So we put this over here and finally get the answer. OK, and the time averaged part, so this still has an omega t. So if you do this time averaging part, you finally get the value of p squared omega two to the power four by two squared, you get p squared by two squared. This is that formula. The only trick over here is to relate the acceleration with the double derivative of y. Example number three, an electric stove is turned off and the heating element or burner turns from red back to black. But the stove warning light indicates that the stove is still hot. An infrared sensor is aimed at the burner and the radiation at a frequency of 3.33 into 14 hertz emanating from the burner is determined to have an intensity of 1.4 into minus 15 hertz per second per hertz per centimeter squared per second. With that is so, then estimate temperature of the stove burner, that's one, and estimate the flux of the recommended variation limited by the stove burner.

So we have a frequency. That is 3.3. So frequency is 3.33 into 14 hertz. And the intensity is given by 1.46 times 10 to the power minus 25. The entire unit. I just couldn't. A lot of the terms are there. So we do compute temperature of the stove.
That is one. And total flux of the recommended variation of the stove burner. Let's go ahead. The stove burner is solid and opaque and so the Planck function is a good approximation to the intensity emitted. It is not no longer in the relative limit.
So we cannot use this. This difference is pretty high. It's close to 14 hertz. So h nu by kT for Boltzmann times T is less than one, does not fold anymore. So to use this thing, the relative limit does not fold.
So Rj limit does not fold in this case. OK. So to solve the temperature, we rearrange. So we have this Planck function, which is fine. We rearrange the above equation and we rewrite it.
And we put this thing. So you put the value of i nu, put the value of nu and the h and k are constant. C is also constant. So if you put all of those values in, finally you get the T. It's basically simple. It's a little bit more of a calculation. So this kind of thing comes when asked in an exam like scenario. First thing you have to understand is this frequency. Whether the frequency is low enough to apply the relative limit. That is the first question you should ask yourself. And this is clearly pretty high, in frequency of 14. So it is not near the lower frequency of the relative limit. And so the relative limit is not cold. So if you use the actual expression out of it, it's a bit tricky. Because the only thing you can concentrate on is the nu.
What the value of nu actually is. So if it is close to like a few pence of gigahertz, you can still get the approximation. But it's not at least much higher than that. 100 gigahertz or more, you can

find the blank slot. So there's no exact clear demarcation where it holds and doesn't. But you can take a few pence of gigahertz, it's still OK.
The algorithm goes to hundreds of gigahertz, then you can stop using the actual function. You can actually make the calculations and try to see which frequency exactly breaks down. But that's how the field for this particular function works. This is the burner, so we can still get the total flux emitted. So that is sigma 2 to the power 4 by sigma Boltzmann law.
And we can just simply put t over there. The entire example is very simple, provided you don't make a mistake in using the averages in this particular case. Once you do that, this is perfect. Next example, volume of ionized gas in interstellar space is known to emit thermal radiation at a frequency of 400 megahertz. Flux density of 200 Jenske is determined to come from an area of the sky with a solid angle of 2. If you have any questions, you can stop me and keep asking.
Use this data to infer a lower limit to the temperature of this gas. So you have the I mu, calculate the intensity from the flux density and solid angle. So you basically have f and you have the solid angle. So derive the I mu from there. And then since the Planck function is the upper limit to its intensity and substituting in the values, we find the gate.
The temperature should be greater than equal to 200. So use the Planck's function and put the values of I over there. Let's take the previous example and you find the upper bound. So lower bound of the chemical. Next example talks about cyclotron frequency. The strength of the magnetic field in a sunspot can be as large as 1000 Gauss.
What is cyclotron frequency of electrons in a sunspot? What is the radiation? Do this electrons emit? So the equation is omega c is equal to 3 perpendicular over r. If you use cyclotron, you get e b over m c. So omega c, if you just simply substitute the values, you finally will get that it goes to 1.76 10 to the power 10 radian per second.
The conversion of the relation is very confusing. The CGS units of magnetic field are close to the charge, so one process so and so. So just rearrange the particular. You express the Gauss in terms of the CGS units and do the calculation. So one Gauss in more fundamental units are gram per centimeter per second, ESU inverse. So if you do that, put that, you have these changes, you put it there, you find it there in radians per second.
The immediate radiation occurs at a frequency of mu c and omega c over 2 pi, which is the cyclotron frequency. So it's 2.8. And it's simply simple, you just have to use the right expression. The next example, we have a radio observation at a wavelength of 6 centimeter is the determination that a particular radio source has a solid angle of 7.

18, 10 to the power minus 6, is opaque and thermal and has a flux density of 250 channels. So it's wavelength is given as. As mu is equal to. So we've been given a lambda as 6 centimeter.So you can calculate the mean. From there. C over lambda. And it has a solid angle of 7.18. 10 centimeter to the power minus 6.
And the flux density. And the density of. The density of. 350. Yes. So what is the temperature of the radio source? So. As it is 6 centimeters, so 6 centimeter corresponds to how much? So frequency is given as C over.

Lambda. And the for a. 10 centimeters. So this is six. Several five figures.
Yeah. So frequency is about five. And. Still can approximate this. In terms of the religions. So you do that.Then we can use the brightness temperature expressions. This is TV. Six. OK, again,

if you judge.  So.  Next question was.  What is the intensity of the source at a wavelength of 2.7 centimeter?  And what is the.  The density of the source and it.  2.7.  So intensity of the source act.  2.7 centimeter is corresponding to.And there you can simply use this and get the final. Value.  Since you got the temperature, you can.  Put it back and then calculate the I.This time for a different frequency. So you.  Essentially use the same.  Same expression.  But here you have a temperature. The frequency and the flux.  The intensity both have changed. So you can just.  Just calculate the others.  So you know the frequency and the temperature.Second factor.  And.  And because we've given.  So it's angle.  OK, so in the first expression, the.  Knowing the flux density, we calculated.  The temperature, the brightness temperature.In the second one, the use of brightness temperature.  And we knew what is the.  What was the frequency at different frequency.  Evaluate the intensity and repeat that.  Once we got the intensity, then the flux density can be just a very much.  With the solubility.  Third, let's say the third expression.  Third question. An observation is made at another.  Radio source that is twice as hot as the first source.  What is the intensity as two point seven.  So the other source is twice as hot as the previous source.The previous source, the brightness temperature is sixty three point six.  So here you just simply multiply by two.  So if you if you go by that, you already have.You're going to keep it.  You already have the flux density.  At two point seven.For a source which is.  So for another source, which is our twice.  T.B. as its own temperature.  Then.  This expression has to be.  So it is calculated that the intensity was previously.Two point four one in the minus 14.  In this particular case, it is four point eight two.  Just twice or not. So intensity is not just just a scale.  So giving it from space.  In this.  But essentially.  OK, what is the brightness temperature of the source discussed in the previous example, the source of the lambda six centimetres.  The source is opaque with thermal radiation and has a temperature equal to sixty three point six.  So.  It's physical temperature and temperature.

Next question, next example, a spherical region containing only honest hydrogen gas.  Ten P.C. in diameter. So what is one percent?  You remember what is one percent?  Approximately three times centimetre.18 centimetres.  OK.  So.  That's one percent is observed to have a spectrum that fits new square at lower frequency and need to be for minus point one at higher frequencies.  The turnover frequency of a set five gigahertz.  At one point seven gigahertz, the measured intensity of radiation is I knew this is going to be this.  Assuming the gas is of uniform is uniform in density and temperature in temperature and density of the hydrogen gas.So we have.  The size of the.  The region, spherical region, it is for the.  Diameter is given as 10 percent.And you have two different parts of the spectrum.  There is.  At the lower frequencies.  This has.It goes down as a lot.  Spectrum.  So you have a inverted.  Something like this.  So if we have a log of.

New.  You have a new people.  You square.  As new rises.  This thing also rises.  And this is inverted as new. Rises this.  This is about.  The typically.  Is kind of proportional to.
The frequency.  You have.  So if I know.  That's just different.  We can.  Constant.  Zero.  Okay. So in the lower frequency.  This thing behaves in terms of.So how do we go from here to there? Now if you take log of both sides.The log of.  It's nothing but.  If you.  Minus.  Okay.  So.  That minus.Now, if I.  This.  So log of.  Is basically a straight line.  With the.  That's how you. Okay. So just starting again.  You can.  You can make a discount, but let us move on and.  See how. Again.  Okay.  So.  We can use the intensity at a frequency.  Well below the.  Where the emission is.  At 1.7 gigahertz Which is below the turnover.  The intensity is.  And it is given by the

relatives.So I could calculate.  We can calculate.  Because we know.  Okay.  7 gigahertz.  C is known.  So that brings us.  Temperature.  The brightness temperature.
10,000.  So the.  optical.  And if that is so that you can.  In terms of.  Temperature.
Okay.  And we finally.  What was the.  First.  So the original.  So.  So act.  The density of radiation is given by this.  As you mean.  In for temperature and density of the hydrogen gas.
To compute the temperature and density of the hydrogen gas.  So temperature.  To the density.  So.  So we have the.  Transfer.  Equation.  We have.  Optical depth is equal to one.  This is the.Function of emission measure.  And so the EM.  Is equal to.  The other constant.  And then.  Put the frequency.  So the temperature.  Six centimeters.  And the temperature.  To that.  The emission measure comes out nine point.  Seven.  Centimeter.  Six.  And.  So.  The density was 3000.  Centimeter.  In this.  Okay.  Next question.  There's a magnetic field.

Treading through the.  Of the Milky galaxy.  The magnitude is typically.  There is also an interstellar medium.  Of the space between the stars.  That contains.  And.  At what frequency do these.  Emit radiation.  Due to their interaction with the.  So we have seen this expression.  Is that.  One over two.  Times the main.  Okay.  Now, if I put all the.  The.  And C.  And finally get this.  Five point six.  It's an extremely low frequency.  Okay.  Okay.  Simply. So  Radiation.  We have.  We have discussed.  So it's basically one of the.  Radiation.  Strong intensity.  And.  The.  Polarized.  And first.  So we have a good description in the.  And the.  Part because interesting.  Was that the flux.  I would function of.  He has as a pocket.  Okay.  That part.  So.  That's density.  F mean.  Proportional.  Minus.  The new is the frequency where the flux is measured.  And I'll find.  Nothing but the spectrum.  Okay.  So what happens is supposedly you have.  A radio source.  You're.  You're.  Two different.  Typically.  Okay.  As I said, if you're plotting the log.  Log scale plot.  That you're talking about.  New.  Over.  New.  This will be basically be a straight line, but inverted.  Invert slope.  So as the.  Frequency.  The flux.  That's the.  So at a higher frequency.  You get much lower.  Than the.  So it's kind of lighter.  As you go.  Even truly what happens.  If you something.  Absorption.  Just discussing.  And.  Point.  So.  It's.  Frequency.  And.  The reasons.  However, for a simple observation.  Any.  The galaxy.  We look for this particular. Okay.  This particular feature.  The spectrum index.  He's a very important.  Show.  And this.  This.  Alpha has a lot to do with.  The basic nature.  Of the medium.  Which is emitting.  Simple radiation.  And so what we do typically do.  Because this is a.  Expression where the two unknowns.  Because you can write down.  The air.  As some.  Constant.  And.  And.  Right.  So if we just have one equation.  You have two unknowns.  If new.  So what you do is you have.  frequency of measurement.  We do.  Which is.  Not.  One.  And you have another frequency.  Say.  Which is.  Not.  Two minus alpha.  And then it did the ratio of the.  Then you see that.  Not.  You have observation of.  If.  One is observed.  And if you.  Observed.  You know all the.  New one and new.  So by doing this.  You can calculate.  Alpha.  Which is nothing but.  One over.  Two.  Divided by.  One.  That's.  A simple way.  Of deriving the spectrum.  And knowing the exact nature.  Now this alpha is not fixed.  It has a variation.  It changes.  So.  Listen to the.  To the.  Lecture notes.  Understand more.  The purpose of.  Solving.  Expressions and.  And problems.  This is.  Okay.  Next example.  Imagine a spherical.  With a uniform magnetic field.  Ten people minus two dollars.  Angular diameter of three million seconds.  A spectral.  Tux density of.  Janssen.  Up.  It is set the source time.  Source is diameter. Five times larger.  What would.  Tux density.  Have to be.  One.  So.  The.  We spoke about.  It happens because of some.

But to be a phenomenal. Synchrotron. But essentially the spectrum looks like this. It turns over at a frequency. You can say SSA.So the cause of this turnover is. Synchrotron. Self absorption. SSA. Stands for. Synchrotron. Self. Absorption. Yeah. And so that. It depends on. B. For point five. And I knew. Point four. So. If I know. The. Turnover frequency. And I know the intensity. At a particular frequency. I can actually. Which is. Which is. And I know the intensity. And we'll discuss this more in the next coming. Last two weeks. Of this. Of course. More details about this. Okay. So. Let us go back again and. Look at the question. What you asked. So a spherical. Source. Is given. Angular diameter. The milliamp seconds. Spectrum. T-flex density. Is given. For. P. Opening at one gigahertz. So you have. The kind of. Offering at one gigahertz. Also this. One. And I know the peak. That's value. And then. In particular case, you know, the B also. So you can put all of them together. And then. You asked the question that. The source diameter. Would have been five times larger. What would be. It's. T-flex density. For its spectral. Turnover. Still. One. Okay. So new. Is the same. You have the. T-flex density. Is. Is given by. I. As I knew. Times the. Omega. Solid angle. Which is nothing but I knew. Pi. Theta squared. So for the new source. It is five times larger. In diameter. So. Theta. Goes to. 25. People square. Okay. And so you can easily just. So. Even. Downward frequency constant with a five foot increase in. Theta. And. If new. Also increased by that. So everything remains the same. I knew. Is. Just. I knew also is the same. The same. Omega. Just changes to. 25 times. The flux has to balance. By. In order to keep the. Turnover frequency. The same. Okay. So that basically says that. That's. The new source. The five times. Larger diameter. Should have a flux. Is equal to. 25 times. That's of the previous. So if. Is this because. Is five times.The past. Which is the. Okay. That's again. Simple. The birth of a next example, the birth of a. Astro radio astronomy marked by the first detection of extra. Terrestrial radio signals. Was an observation of radio emission from a galaxy. From our galaxy by. 1932. This emission. We know is due to. radiation indicated.The plane of the galaxy. Is infused with. Particles. In a. Yeah. The spectrum of the. Has been determined. We have a spectral index of. As equal to. According to this data. What is the power law index? Of the energy distribution. Of the interstellar. So. Alpha, which is spectral index. Is one minus P over two.

The P is the power law index of the energy. Electron. Well, that comes from the simple. Radiation. I think for this particular. Purpose is. Better to know this. Up front. The theory. Is. Over the power In the lectures. For the purpose of. This. Problem. Remember that. The spectrum index. Is. Related to the. The power law. distribution. By. This particular. So P is one minus two. Alpha. So in. So here we're relating. So if we know. So. Coming. Just. This discussion. Coming from the previous. . So it's so the observations now. Getting some sighs. So if you know how difficult obligations are. Slowly slowly. of physics and science term of the quadratures. So last example for this particular discussion, if the spectrum of the galactic emission has been determined to have a spectral index of alpha is equal to minus 0.676, sorry, and when observed at a frequency of 90 megahertz, flux density of 10 milligrams was observed, then what will happen? What will be the approximate flux density at 120 megahertz? So exactly what we did, we have the flux now represented by S instead of F, but they were the same thing. So you know that this is the relationship and if you compute, if you know the flux density at both frequencies, sorry, flux density at one frequency, one of the frequencies, you know that the water frequency and know the value of alpha, then you can calculate the flux density of the second frequency, which is P of x. So a very common way to ask the question numericals for the simple combination. Okay, great. We can stop there for a little bit and thank you.