Radio Astronomy

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Radio Interferometery and Aperture Synthesis

Good evening everyone. This is a live session. We thought this is a good place to discuss a lot of things. So I think what we are doing is we will discuss a couple of more conceptual things of radiation. Last week we covered this radio interferometry at great length. A few things are still remaining.

So we will cover that before we proceed. Now we will also break up this particular today's live session. It will go up two hours. But first hour is to go through some more details of the lecture.

Everything will be captured and put up in your course sheet. And it will be available. The slides also will be available. There was a little bit of a mishap that we missed the first few. We couldn't post or it was not up to date till week seven.

Week eight and week nine will be updated today. And so in case there is an issue about the assignment of week eight, please let us know. We will take it up in the second half. So first we will have this discussion about the concept. After an hour or so, the TA will come in and show you exactly where all the materials are available and one by one and go through that.

Then again we will come back after that, the second half, next hour. And pick up some problems. But in between you can keep posting and let us know if you have any particular problem with the week eight assignments. We can go through that. As we discussed that we will have the live sessions every week from now on till the end of this course that we can get in contact with all the students and just bring in all the issues etc.

So let's begin with that and let's see if there is a question. Okay so Ashu are you there? Yes sir. So we can also, maybe in the next, when we start, after we finish this after maybe you can discuss a little bit about the forum questions and any issues they have with the week eight assignments. Okay so just to capture again, we showed that this is the, yeah we have the single dish and we know that for a single dish, for a single dish the size of the dish, capital D is basically the main thing. So lambda over capital D is kind of a measure of something called the resolution.

We have done that. As the wavelength becomes higher and higher, the size becomes impractical. So we started something shifted to something called radiotextile metering where now the single

dish size can be replaced by the dimension of the array or the maximum baseline which is covered. So this is Bmax, then the resolution of this array is given by disproportionate lambda over Bmax and that's how we avoid the impractical design. We can make much smaller number of dishes, many of them and the distance between the largest distribution of this kind of proxies for that dish.

So we can see that the distribution of this kind of dish is very distant between the largest distribution of this kind of proxies for that lambda over P for a single dish. And the other part is the effective area which we can take care of by summing up the area of all the individual dishes. So area of each dish, if it is a less effective area, the total number of antennas is 10 and this becomes a good area. This is with GMRT we have discussed before. So how this works, we have done it a couple of times before but just for the sake of it and as we go multiple times things become more effective.

So if you have two antennas you multiply that here first the remote, you multiply so as we consider a radio source coming from infinity or distance far away field, if it is moving in this direction to the both the antennas then it will appear faster in antenna 2 than antenna 1 and the additional path length converts directly to additional time which is given by P dot S over C and so this all has been done before. One thing is certain that as the earth spins this source gradually moves and appears in this direction. So at a given another time the rays will be coming first to antenna 1 and then to antenna 2 and then that relates to the compressive of antenna number 2. So essentially all this depends on the direction from where you're receiving the emission. So we did this the entire derivation we have E1 dot E2 the output from each antennas we correlate them or simply multiply them then do an average on time where the faster varying term goes to zero all this has been done before and you retain the slower varying term or which is function of tau or this delay and that is linked with the baseline and the lambda and that is finally linked with the earth's emission.

Okay so that was um derived earlier if you're looking at a single baseline or single pair of antenna then it gives rise to something with fringe the frequency of this is given by this thing in the bracket and that's for one. Now uh sorry just one more thing I think we have done it already. It also depends on this quantity D. So you can say as the baseline becomes larger and larger longer and longer this becomes very fast varying term the frequency increases and it becomes faster term. So for single spacing you have this kind of a fringe.

For multiple spacing you will have superposition of multiple out there. So again the Vmax in this case for the maximum baseline is the resolution and that compares with the resolution for a single station. Okay questions if you have any you can you can stop me and ask it immediately. This is a session to interact with you guys so I think feel free to interrupt whenever you can. You can just unmute yourself and ask a question whenever you feel it is going a bit faster or something which is not quite clear.

Anything so far? Okay so yeah so for different sources at different spacing you see different fringe. So we can just go ahead. Okay so I just took a bit of a different treatment. We introduced

something called Avancitor-Zernike theorem. What is it? Now you what you have is that for assume any astrophysical phenomena occurs at a position of r on the sky and it causes a time variable electric field.

Since the source where this electric field originates is in the far field we assume a convenient location over a celestial sphere of radius r centered on the telescope where it gets recorded. Okay so you have at a distance far away distance there is a some astrophysical phenomena occurred and it created some time varying electric field. Now since it's operating distance of r at r position of the sky you assume a sphere so that and this telescope at a distance of r where this particular event often something like modeling the concept of the celestial sphere. Similarly so assuming that the space within the celestial sphere is empty so there is nothing between that astrophysical phenomena and the telescope. Then you can write at the for the coordinate system r which is telescope coordinate system you can write the electric field which is measured the electric field which is recorded is linked to the electric field which is which occurs in the sky and they are linked by this equation but this is nothing but a propagator don't worry about it too much and just like this how the far away electric field transforms into the electric field which is measured.

We have done something similar early okay where did we do by the way am I coming in fine? Yes sir. So basically we have done this when we decided when we discussed the relationship between far field electric field pattern electric field pattern and the aperture distribution and we figured out that these two are linked by a Fourier transform. Go back to your part two the distribution yeah these two are linked by a Fourier transform and here we have shown that for different kind of aperture rectangular aperture circular aperture etc. And so the same thing similar thing also is linked over here so you have the field which is recorded on the telescope is just nothing but the electric field in the path part field in the sky propagated out okay this is the expression of that. Now if I just go ahead so this is the radio radio interferometer measures not a single field but the product of the two recordings from two different antennas so that gives rise to something called a visibility okay which is nothing but a product of two electric fields measured by two antennas one and two okay this is called the visibility function which is kind of something like a spatial coherence function and directly related to the average point in flux which measures the intensity of the electric flow of an electric field okay and this gives you the proportionality constant half epsilon 0 so if I do that and I express this and I put this expression over here I can see that this becomes quite cumbersome so visibility is a coefficient coherence function is linked to the integral of this epsilon capital R1 and eta sorry R1 and eta R2 and the two protocols over here.

Now we can assume radiation from an astronomical source is not specially coherent so this expression is equal to 0 if R1 is not equal to R if they're not coherent then this correlation will become 0 so R1 has to be equal to R2 so if I replace that in the previous expression of this I finally get an expression which is equal to this so they originate from the same point then only they become coherent okay coherent. So if I do that math finally after a few steps you essentially give rise to an expression like this where this is the visibility function v is the integral of a and i and e to the power minus 2 pi so and so so if you just do the math it will be supplied for you don't worry too much then you can see that slowly you're seeing that this expression over here is

nothing but if I can write v mu is nothing but a Fourier transform of a mu i mu that's what it says where a mu i mu is a definition of redefining this particular eta and mu R squared this function is redefined in terms of a and i. Now a and i have a significance what is i? i is the original sky brightness if you remember our first week or second week of lecture we defined something called the specific intensity so this i is nothing but linked to the specific intensity on the sky plane okay in the astrophysical sources and a is nothing but the primary beam pattern which is receiving this entire information so v and is just nothing but a Fourier transform a mu and i. You can also look into the detailed lecture which also refers this one so that's essentially what we do have okay now let's redefine the coordinate system on the sky in terms of something called direction cosines l m and n so l m and n has a relationship where l square is equal to m square plus n square is nothing but equal to one okay so two of them are independent one is depending on the other two the value so the two-dimensional image of the sky obtained from an interferometer is projection of the intensity distribution on the sky onto a tangent plane and s naught is equal to n equal to zero n equal to zero n equal to one which is the center of the field okay and so if you define that we are coming to the geometry then finally this d omega becomes can be expressed in terms of d l d m over one minus l square minus n squared squared of that you don't have to worry about this exact this derivations etc just remember the final final expression that the v and the product of a and i are Fourier transform pair okay and this this coordinate system on the sky represented by this direction s is broken down uh is derived in terms of l m n which are the direction cosines that's the easier way okay so what we do is so if i start doing that then we essentially define the coordinate system on the ground or telescope plane in terms of u v and w now remember one thing that this this direction s depends highly on the it changes position right and the rotation of the the rotation of the earth kind of influences uh the projection of this baseline so finally uh all what we measure is in terms of a projected baseline towards the direction of the source so the source is rising in the east with the same interferometric baseline we will have some kind of a projection compared to the source is setting towards the rest then the prediction will be changed so essentially for a same baseline fixed on the earth it sees the sky in a different way based on how the earth protects okay so that has to be somehow accounted in then we define this coordinate system for the interferometry now that purpose that forces us not to just stick to the the coordinate system of the the antennas on the ground which is expressed by x y and z we have shown this earlier so last week as well but x ij y ij and z ij is nothing but the x y z uh x i minus x j y i minus y j and z i minus z j corresponding to the 22 antenna they are given by y and j so that is fixed however the the hour angle and the declination versus center of the field is defined by h naught and delta naught while delta naught does not change with time as the earth rotates the h naught do change because h naught is what h naught is nothing but the ra naught minus lst and lst do change as a function of local time so as the source rises in the east and sits in the west because of the rotation of the earth the delta naught is fixed but h naught changes so for a given fixed baseline you have multiple u v and w for each okay and that's what we expressed in terms of the that's what the effect of the earth solution synthesis which we discussed last time okay so and also it depends on the lambda that's a pretty neat uh derivation so yeah so we can escape this right so uh if you if you agree with that then what we derived earlier the v r1 r2 is given in terms of the Fourier transform of a and i the product of a and i no ideal case you can assume that a mu as a function of reflection is unity okay and then this expression becomes much easy and it just becomes the Fourier transform of the sky however we know for a real antenna a

is not equal to one for any direction and it changes because it has a radiation pattern which is kind of looks like this so as a function of theta and phi the radiation pattern changes so a lm is essentially has to be measured so if you put all of them together and the actual values of an actual expression of ln so we can recast the this expression on the top into this huge cumbersome expression in the middle which is true for a three-dimensional sky where there is no assumption about how much is the extent of the field of view as well as it takes care of the curvature of the earth and the curvature of the celestial sphere for both of them it is taken care of and this is the total full expression exploring where the visibility or the spatial coherence function is nothing but a Fourier transform of the product of a the family beam pattern and i which is the sky brightness that's the expression now that becomes this is not a Fourier transform sorry but it becomes a Fourier transform if we do a simple approximation of a small field so the third dimension of w and n becomes negligible and this expression reduces to just two-dimensional Fourier transform where v is a function of u and v no longer w that can be ignored and a it is a i a and i the product of both e to the power minus two pi i u l and v m okay and d l d m okay this is nothing but a 2d Fourier transform equation and also known as the grand sector zerike equation it's a very powerful expression and the and the main equation for radio interferometry where we understand this is the correlation function which is the output of the spatial coherence function which is the output of the two interferometer and that is linked with the product of the primary beam and the sky so that gives rise to uh if i do go a bit go a bit i can like so so if i can write the visibility v i just drop everything else it's nothing but a Fourier transform of a the primary beam pattern multiplied with i the sky now the objective of the entire process is to understand how does the sky looks like so to make an image of this i so that is easy because now that becomes uh inverse Fourier transform of uh the v right if i know i if i know v i can do an inverse Fourier transform and get the okay and you if you know your antenna very well you can have a prior knowledge of the i which you can then divide from the image and you get the final value of i that's the very over simplistic uh they also presented but that is what to show is that the simple image which was coming for a single dish that you just do have a single dish remember that you have a single dish antenna and you were pointing at different parts of the sky you made an image by doing a dastro scan thank you so if you had a single dish telescope you have a single primary beam so what you did was you have pointed in one portion then pointed in the next portion and the next and the next and you made this kind of a scat last scan of the entire sky like point right and for each values you get something and then you know that supposedly there's a effort to source at the back which is like this so you can sample this and get the image that's what a single pixel camera represented by the single dish telescope like green light telescope for example okay here uh things are a bit different here you have a bunch of antenna in an interferometric array and you are getting visibility which is a function of frequency is a Fourier transform of a which is the primary beam pattern of any antenna times this type so if you have a priori knowledge from electromagnetics about this function then you can this is your observable so what you observe or measure from sorry this is your measurement this is your measurement and this is come from electromagnetic theory if you know your antenna very well designed very well and this is finally your sky which you are trying to image so based on this is not a single value because you have multiple baselines so if you have an antenna you have an antenna minus one the number of baselines each baseline has different distance so based on that you have different values of u and v so finally you get that and then you uh do the processing to find the image of i yes akash you

can go ahead unmute yourself akash you can raise your hand okay maybe we can move on and we can come back we'll be sexy okay so let's move on so one more thing we have to discuss is that as we showed this is again the same picture of the of the vla and what we're showing over here is that for a shorter spacing you have you have the fringe which is slowly varying and for a longer spacing the fringe varies much faster because the frequency of the fringe omega is proportional to the baseline so as the baseline increases the free omega increases and so that is pretty obvious obvious all right so you have different kind of fringes by looking at the fringes you can see you can compare and mention whether one is coming from a shorter baseline than the other one okay this is a video that is the animation which we have created and done in i think earlier phd student created this and we have to go through it slowly so yeah okay i got akash's question so he's asking the question that the question is about the a nu can i say it is the effective temperature or is it the fov essentially it is the fov okay it is the pattern the part field radiation pattern which we know we can characterize it in terms of the so the part field pattern you consider the half power beam width and that is the kind of overview so i think you can consider that to be a fov that is that's fine okay so coming to this expression let me let me start from let me go back and forth yeah so see it we will keep it running it goes in the loops don't worry if you miss one thing so on the left hand side on the top we have taken the sky or the i that is your i so this is your i nu l m okay just a point source that's it that's your model i we're simulating it okay and this is your antenna layout this is your n antenna and also we are showing how they are laid out we are varying the number of the antennas from two two antennas to 256 okay that's the 512. 512 will have some some significance we can avoid that right now it's not important for this but it is close to some real antenna design eventually so the as the animation progresses this n and value goes from two two increases to 512 very well the the topmost panel on the right hand side is the uv coverage what is that now we know that our values u v w is some kind of a matrix rotation of x y and c we have done this earlier right so this is what the uv w coordinates are for you know just a snapshot so you're looking at one particular direction given by this sky position over here that's your location so you know which direction to look so this defines your s right and you know your direction so you know x y z so just simply let the just a snapshot just a snapshot so you're looking for each antenna each configuration you're just taking a snapshot so you have the uvw values so you can then do the the visibility is a Fourier transform of i simply so we took the visibility measurement by handling the uvw coordinates and just in this Fourier transformer to get the the i but see the difference so as the channels goes from two to a huge number you see that the recovered image is finally coming close to what you actually see on the sky okay so what is that going on so when you start from two you have only one fringe then becomes four so let's look at it again now start so two then three then four then five and so and so so when first two the number of baselines were one when the number of antenna became three the number of baseline became three then four it increases further and so as so as the number of baselines increase the total number of fringe superposition of that also increases and so you are able to recover the original standard elliptical Gaussian which is the closest you can come to a point source okay so that's the Fourier imaging thing we'll post it again so we can keep taking let's go forward so the visibility function and the sky brightness distribution are nothing but a Fourier pair so your visibility v is nothing but a Fourier pair of the sky brightness i here we are assuming just for the simplicity that a is equal to one okay it's normalized completely very ideal case it is not real so for a point source the as a flat function uh this so this is brightness this is the

brightness i and it's the Fourier transform of the c the feasibility and we assume that a is equal to one okay so if the sky brightness is a point source just the brightness in the center then it becomes a dc signal as we know but Fourier transform we have done it earlier there are two sources over here with um so then it's basically it's a sinusoidal function two sources far away it gives a bigger function because now the distance is much more the frequency is increased and if you have multiple sources with multiple different brightness and we have a composite function which gives you like this i have various different kind of sources like an extended source looks like this a Gaussian kind of a thing and you have different kinds of functions on the physical distance so yeah so you have two different sources extended sources uh with the distance that gives you a function like this and so and so so you can have various different kind of pictures which gives you so you have a double source separated by a distance what is this if you separate two sources this looks like similar we'll come to that so essentially this is all uh nothing but as Fourier here the visibility given by this and the sky brightness here we write it in full glory again it would be 2d Fourier transform okay which assume assumption is that the sky is it's coming from a very small region of the sky sky okay that's that's doable so a few more things in this in this fashion so we we started from this expression we have the uv distribution what we can do is that you you understand that uh so the what happens is that if you have a fixed number of antennas there are a fixed number of baselines so it it doesn't have a complete coverage in the uv distribution it has discrete coverage so in order to absorb that discrete coverage in the continuous integral what we define is something called a sampling function now sampling function is very simple wherever there is a uv value the sampling function becomes one and wherever there is no uv value it becomes zero so it's zero or one okay it's just a mathematical way to absorb a discrete function into a continuous function defining another function which is like a mask you have a value at some positions you don't have a value at some positions because because this um your finally the expression of your yeah for expression for your uv w are these are fixed okay you don't have all possible values given a set of x y c we have multiple number of uvw but still not all values okay because it only can produce some set of uvw for all possible h noughts okay not all so it doesn't even though it is the kind of a projection but still it doesn't sample all possible values in uvw play so we still have some limited holes as you see for this one even with 512 antennas which is quite a large number of values you still see a lot of empty space in the uv uv space so uv coverage is still not complete hence to account for that we have a sampling function which gives zero when there is no value so if there's a gap the sampling function s is equal to zero and if there is a value like for this particular position you have a value so for this or this s is equal to one okay then we can write still this one in terms of integral so ideally we would have loved the coverage in the uv plane to be continuous and complete but because it is not so we don't have an ideal response for the sky brightness what happens is even a fixed compact source appears like a sinc function and we'll talk about it because of this missing coverage in the uv plane because we are not having a field aperture we're having discrete antennas which can only fill certain amount of the uv plane and not complete okay so that gives you something called a dirty or incomplete coverage and so it is linked in terms of a convolution so original sky convolved with something called the psf or point spread function it's called point spread function it will give rise to the first impression of the sky so if you know the the beam of this point spread function you can recover the storage in the sky okay what this says is the original image original measurement is not ideal we have talked about now the measurement as we do the real measurement for the interferometer

the original measurements are have some kind of a gain variations because we are finally getting measurements from two measurements from two different remember that we have a simple receiver chain system for a single antenna radiometer and we have gain variations for the electronics so this is the same thing we are not having any different kind of system instead of having one system we are having multiple systems previously we have only one in an system we have an antenna and we had some kind of an lna to amplify right you have a down converter a mixer to down convert and then again a lpf kind of a thing to cut off and then finally a spectrum meter etc and okay now every antenna has a similar system so the electronics involves which causes variation in the gain in frequency and time so ultimately that will affect the visibility which is measured the observed visibility is nothing but some kind of a gain for each of the antennas times the true visibility so that is taken care of in terms of something we call as calibration please watch the video original activity for this bandwidth thing they are expressed in details over there UV coverage we have already discussed and so we will skip this for this particular session just to show you that if you're looking for a PLA like telescope is that the UV coverage and for a particular source if you're looking at source for 3c147 with a declination of 50 with an antenna or just looking from VLA for VLA its latitude is 34 degree north okay Yeah, it's 34 degree north.

And so this is the different, for different hour angle, you can see different snapshots of UV coverages and that will change. So UV coverage is a tricky business and it changes as we look at different sources. So that was the source at the declination of, declination of 50 degree and with VLA being at 34 degree, almost looking at towards the zenith, little bit off. But now if you're looking with the same array, for a delta equal to 90 degree sources, like north pole, then you'll see something like this. If you're looking at 60, you're looking like that, minus 30, almost towards the south, this you see like something weird happened.

This is because of due to the location of the array on the surface of the earth, you can have different angle projection towards different parts of the sky. Okay, so I stop here and put it over to Harsha. For discussion of the material, we will take a, coming up to Campanal and again pick up certain topics from the problems which were covered in this particular. So what do you Harsha? Harsha, yes. And...