Radio Astronomy

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Example Questions

Okay, so we will complete eight and nine. I already did a lot of explanation for which solution we get also. And I think this particular few things now we do will also help you to understand. So I hope this works and this will cover the essential part. So let's start with a couple of examples. And I think probably I will just wait for your reaction.

And if not, I will just give you a quick answer. We have already done these things, but we are just as we discussed that we will move back and forth and try to cover again different concepts as we are coming close to the end semester exam. And we'll do more revisions as promised. So I think that's what we're doing.

So first example is two radio antennas located along the Earth's equator are separated by separated by 30 meters. The baseline is 30 meters. And the lambda is given as six centimeter. And our angle H naught is given as one hour. So what is the time delay of a wave front survival at the more distance antenna, relative to the closer antenna? Now we know that that time delay between two antenna is given by what? V of sine theta over C.

We just we had this discussion earlier. So that's the thing. So you have V sine theta over C. If you just put in the angles, you will get, so you get the time as seconds. Our angle is one hour.

So the source is then 50 degrees. So our angle is our angle is equal to zero when the source transits from the zenith. And it is 15 degrees. So our angle is one hour. It corresponds to 15 degrees.

Why? Because 24 hours covers 360 degrees. So if you do that, you'll find one hour equal to 15 degrees. Okay. So you can compute the corresponding time delay. What about the second question? The second question was that what is the phase difference between two parts of the same wave front? So you have a time delay.

You convert it to a phase and that is easy. So this is what it's given. Okay. Again, look into the, when is the second eight with the assignment due? So the tau is equal to B sine of theta over C and two delta pi is two pi new tau. So you can calculate that.

This comes out to be 58.8 to pi radian. Okay. So remember that it takes, so full phase, if you have cosine of n pi, again repeats the function, right? N is an integer. So it has a time period repeats.

So here we have 0.8 to pi as additional. So that's the phase difference we have. Okay.

Let's do it. 58 pi is same. Okay. So 0.8 to pi radius is the difference. Any issue with this? Okay.

Hearing none, we'll go to the next one. Example two, imagine the source in the previous example, located at an hour angle of plus one hour or 15 degree to the west, 15 degree to the west of the baseline is a point source with a flux density of three gens. So based on the expression we have, we can use this as R is equal to F new cosine of two pi B over lambda sine of omega E t. What calibrated value is measured by the two antennas acting as a multiplicative interferometer. Then according to the same equation, what are the amplitudes and phase of the fringe? Okay.

So this is the correlation coming out from the interferometer, multiplying interferometer. So we have determined the fringe phase to be 0.82 radians, the previous example. So the response of the interferometer is three cosine of 0.

8 to pi radians. So then the Jansky value comes down because you have a fringe function. So that's what it looks like. As a single measured value, this should be seen straight. The information we can get from this interferometer does not come with a single measurement. We need to measure the output for a range of times to detect the oscillations of the fringes.

Although we could not infer the answer to this question from this single measurement from setup of the earlier example and our calculation in the previous example one, the part one of this example, we know that the amplitude is three Jansky, which is already given, and this is 0.8 radians. So that is, we can then calculate what is the instantaneous value of the flux. Now example number three, the multiplicative interferometric observation in example two, how long must we wait to measure one full oscillation of the fringes? Again, it is the same thing. It talks about the fringe spacing.

So you have a sinusoidal function. So any integer multiple of pi, it will come to the full period. And so you can calculate from this. You can work with this as well. Example number four, radio galaxy Cygnus A contains two especially bright points of radio emission separated by about 0.

71 half minute. Again you set out to build your own interferometer at a wavelength of six centimeters and decide to use the bright galaxy to test it out over what range of baselines should you make observations of Cygnus A to be able to measure the angular separation between these points. So very clearly you have two hotspots of the Cygnus and they are separated by an angular separation of 0.71 arc minute and the wavelength is given that is six centimeters. So what do you have? You have a baseline, right? So you have a resolution theta is given by lambda over d max. So you know lambda is six centimeter.

You know the theta, which is 0.71. So you have to have baselines, which so yeah, so your baseline has to be higher, larger than the value corresponding to this. So you put in all the values.

You convert 0.71 arc minute radian you get this particular value 2 to the power minus four. So you put that then baseline comes up with two meters. So in order to resolve the source, you need to have a baseline length greater than 90 meters. So this ideally should be, B should be greater than equal to 190 meters because if it is less than 90 meters, then we will not be able to do that. Feel free to ask questions if you have any.

Example five in an observation of a pair of point sources using an array of antennas, the visibility amplitude is seen to oscillate with the spacing period equal to that of the expected. That is equal to one over the belief separation angle of the other sources. However, the amplitude never goes to zero oscillating between two and four chance key. Why? What does it mean in terms of the sources? So you can use visibility function VAB as f a squared by f b squared by f two sources. And you can kind of the expression like this.

So maximum occurs when the cosine part will be maximum equal to first one. So four gens key is equal to this and f a plus f b, which is simple and the most minimum occurs when it is negative. So it goes from, when you have two sources, it goes from f a plus one, f a plus f b to f a minus f b. So if you have two equations, you have two unknowns, so you can solve for the individual flux density of the sources. Next example with an astronomical reception system, diverse elements collectively influence the comprehensive noise temperature.

This system encompasses an antenna characterized by a noise temperature of 80 Kelvin. So T n of antenna is equal to 80 Kelvin. It operates in an environment with a physical temperature, ambient temperature is given by 500 Kelvin. Demonstrates the efficiency.

So eta is 91%. Additionally, there exists a transmission line functioning at 400 Kelvin boasting an efficiency. So a transmission line is at 400 Kelvin and an efficiency of 90%. Okay. The receiver on the other hand comprises of initial three stages, each marked by a noise temperature of, so initially three stages. So receiver for first one is 100 Kelvin and gain is 20 dB.

The central query pertains to the calculation of full system temperature within this particular configuration. Okay. Let's see how people work it. So there is an antenna which is having a noise temperature of 80 Kelvin.

Ambient temperature is 500 Kelvin. Efficiency is 91%. It is connected to the receiver via a transmission line. So an antenna then connected by a wire or transmission line to a receiver. So transmission line is given by, its ambient temperature is 400 Kelvin and efficiency is 90%.

So let us see what is going to do. So yeah. So for a receiver noise temperature, which has three different stages, each one of them, yes, that was a nice reading. So receiver on the other hand, composed of initial three stages, each marked by a noise temperature of this. So this is true for one, two, three, and gain also is true for one, two, and three. So then we have done this problem before that we have the, how could I do it, TR is nothing but TR one plus TR of two multiplied by the gain of one. Then TR of three divided by the gain of one multiplied by the gain of two.

Okay. So that's where you finally come out for a total receiver noise temperature given by 201.01 Kelvin and system temperature then if you put it about there by all the efficiency parameters, it finally comes out to be 264.

96 Kelvin. Okay. Questions here? Let's move on. So the particular next example, in a particular observation, we have Umax minus Vmax. Am I audible for you all? Prashanth? G.

B. Raghuram Krishna? Yes, sir. Yeah. Thanks. Thanks. So in a particular observation we have Umax and Vmax is given by 50 kilo lambda. So they are corresponding to the Vmax, right? That goes to Umax. That is given by 50 kilo lambda.

So in terms of the lambda, it can be three lambda. Okay. So what is the lambda value? So FWHM is nine half minute. So the FOV is half of unit is nine half minute. And if you desire to just meet the Nyquist sampling rate with no over sampling and to image the full primary beam of the antennas, what should be the pixel and the map sizes be? Okay.

Yeah. So essentially we just discussed this. So you have the, when we do the imaging, we have the UV. Okay. So we have the visibility in UV grid. Okay. Since the maximum baseline is already given, so that's the extent of the UV values.

Okay. So for each pixel, you kind of consider that to be of one over twice of this UV values, because that's the inverse in the theta. So you're looking towards in the, in the theta domain. So that is kind of from, from proportional to the inverse of the U max, right? It is U and theta are the Fourier convicts. Remember V, which is defined by UV coordinates is nothing but a Fourier transform of I, which is in the theta X theta Y you can say, or LM, whatever you want to write in angular coordinates, assuming that the A is equal to one.

Okay. So A is equal to one, but A has a finite size, even by the half power beam width. So we will only consider that portion of the sky, which is coming from within this nine hour minute section of the sky around that point of center. So okay, let's go back to further. So U max, we know that is given by 50 kilo lambda, right? You know, lambda is, has it been defined? Yeah. So 50 kilo lambda, so 5, 50,000 and twice of that, because you are doing a Nyquist.

So twice of that, that is given by two arc seconds. So delta X or delta Y or theta is given by two arc seconds. Now to image the primary beam, you have a primary beam size of nine arc minute or nine times 60 arc seconds. So with each of the cells given by two arc seconds, you have to simply divide this by two arc second. So you have a nine times 30, that is equal to 270 itself.

Okay. So what happens is that, let me just a little bit take one more slide to explain this particular thing. It's maybe a very big, yeah, so we can just go ahead. So yeah, so we have, we stopped in that thing, this is that V u v is nothing but a Fourier transform of A l m times I l m. Now, if I just take it out, assuming this is equal to one, however, we only concentrate on that part of the sky, which is within the half power beam width.

Okay. But within half power beam width also we assume that the response of the primary beam to be equal to one or flat, just a simplification example, simplifying assumption, sorry, just to make life easier. So you have to only image the part of the sky, which falls within the half power beam width and that's it, you're going to do more than that. Given that you have now only V u v, which is a Fourier transform, two dimensional Fourier transform of sky, I l m, very simplistic assumptions. Now for a Fourier transform to take place, you know, there is something called a fast Fourier transform, which is only applicable if you can grid this data. So what you do is simply you grid the data in a regular Cartesian grid.

So you have different values for the grid. So certainly it starts from the center, which is u equal to zero and v equal to zero, and it extends to the maximum value of u max and v max. Okay. And that is given very clearly from the maximum baseline of the interferometer.

There is no doubt about that. That part is very clear. Now, given that v, just take a Fourier transform that to give to the sky plane, giving the i l m. So because it's the fast Fourier transform, so total number of elements, so suppose this is N u times N v number of grid points, that will also be equal to N l times N m. So this thing would be equal to that. So N u times N v would be equal to N l times N m. In fact, this values in both axis should be equal, equal to the squared grid in some sense.

So the maximum baseline corresponds to what? The minimum in the angular separation, right? That's resolution. So there's an inverse. So theta or l m is proportional to one over u. The maximum of u max is corresponding to the minimum value of the theta.

Okay. So what did we get? We had the u max, we calculated the theta min, the resolution. This is the resolution. We considered the Nyquist sampling, so we made it twice of that. And then with that, we know what is the extent of the beam is. So that was equal to nine arc minute and theta min was equal to two arc seconds.

So we calculated total number of theta. We needed it's 270 pixels. That's the easiest way to understand this one. Questions? We're running very close to the time. We want to finish by eight, just give me another five minutes. We will wrap it up. So next example, we have imagined that we wish to use an array with baselines from 10 meter to 30 meter to explore an area of the sky for new sources.

If you want to be able to detect sources within the field of view of 10 arc minutes on a site, what constraint does this pose on the bandwidth of the observation? I would say watch the detailed video, which will be coming up tomorrow for this. This will also be uploaded there. Example nine, we want to detect an unresolved source observing at 22.

2 gigahertz. A previous 22 gigahertz survey with a sensitivity of 0.3 Janske did not detect this source. We will use an array of three antennas, all with system temperature of 50 Kelvin,

effective apertures of 30.4 meter squared, observing a single polarization with a bandwidth of 50 megahertz, assuming correlation efficiency of 0.8. If we fail to detect the source, we would at least like to have the upper limit on its flux density using five times the RMS noise as a conservative upper limit or a non-detection.

What is the minimum amount of time that we should observe the source? Best question on number one, question number two says, what is the minimum flux density that will be detected in the cross correlations for an integration time of two minutes? So this deals with something called sensitivity. We discussed a part of it. We didn't go deeper. So sensitivity for an interferometer is given by this where your system temperature is same as your single dish system temperature. But instead of now for single dish, we had only thesis over delta nu times delta T that was given a sensitivity limit.

Here we have for interferometry, we have more number of samples. So that's multiplied by any 10 minus one, which gives you the rate of effect of the total number of base lines. And there will be some constants, which is given by C. So if you insert all the values, so you basically have 0.03 Jansky per synthesized beam to match the minimum SNR. If you put on top of that, then you finally get it to observe the particular source for delta T equal to 119 seconds.

Don't worry too much about all these things. It's up there to make the thing complete. This won't be, it's a bit more complex and we will not make it a part of the system or find ourselves. Even if we do, it will be very simple. Don't worry if you don't understand too much.

We will try to explain it more in this week's lecture. This will be available soon. Okay. So yeah, so the minimum flux density to go by this manner, a predictable is 0.0518 Jansky. Example 10, we obtain visibility data for a particular astronomical object.

A 3D plot of the visibility amplitude is shown in figure A. The amplitudes are shown in the gray scale. U is plotted on the horizontal scale, etc. So this is the visibility data in the figure A is shown here. And figure B shows the amplitude versus the Euler distance. What can be inferred about from this particular source? Now you have seen a lot of this 2D Fourier transform pairs, so by looking at this particular plot, what can you make out? That's all we're asking.

The source is separated by 6.3 arc minute. Sources are, it looks like a pair of sources. Visibility amplitude oscillates with a period of 550 inverse radian. This must compare to a pair of sources separated by 1 over 550 radian. With a peak amplitude of 6 Jansky and minimum amplitude about to be 0, then this gives you the minimum separation of 6.

3 arc minute. And two sources of similar, exactly same flux density of 3 Jansky. Because it's 3 plus 3, we're at the peak. You have seen that example, fA plus fB, when the cosine function peaks, and when it goes to negative, it is fA minus fB. So in this particular case, it is 3 plus 3, 2, 3 minus 3, that matches, so it is 0 in the minimum, and 6 Jansky at the maximum. So that's the two sources equal surface brightness of 3 Jansky, separated by a 6.

3 arc minute. So you can infer a lot from just the visibility plots, that's what I'm saying. Let's look at this 11th example. Yeah, so another visibility plot, a second astronomical object is observed. It's 3D visibility plot shows an amplitude dictating with UV distance in all directions. A 2D plot of the amplitude versus UV distance, which is given by this one, it shows about the structure. So it's like an extended source, a Gaussian source, which has a, yeah, source size is roughly a Gaussian width of WHM of 1 over 450.

And that gives size to 7.6 arc minute. How do we get this 450? It's kind of from where it goes to, from 8-ish to about 4. So it's kind of there in the middle of 300 and 600, so that is 450. So it goes from its peak value of 8 to its half power value of 4.

So that's how it is calculated. So every WHM is about 1 over this, so you get about 7.6 arc minute. And the peak flux is about 8 Janske. There are two, the visibility function for various brightness distribution models, solid lines, amplitude and dash lines are the phases.

This is what it is given and you can take a look. So we have done it also. So Gaussian, displaced Gaussian shape extended. So you can just look at the individual Fourier pairs. It's very self-explanatory.

If you look at it, you will understand. We have done it in the previous discussion as well. You just go through it and when it's available on the portal. Example 13, we learned a new source reported to have an angular size of approximately 3 arc minute with a brighter central core component of about 5 arc seconds. To obtain a reasonable map of this source at 1.35 centimetre wavelength, what baseline lengths should we use? So again, the same thing, similar expression.

So the baselines correspond to about five third of an arc second. The angular size of the source is given by this radian and the core is approximately 8.07 to the power minus seven. So that's how it needs to resolve the particular source. So the UV distance is then at the of this.

That corresponds to 15 metre of length at the observing frequency or wavelength of 1.35 centimetres and approximately 1700 metres or 16.70 metre to resolve the core. This is for resolving the core. So 15 metre for resolving the, yeah, brighter central core component of about 5 arc seconds and 3 arc minutes basically the angular size. So as the core is much, much smaller and compact, you need a longer baseline to resolve that. That's about 1.35 centimetre is about 16.70 metres or 1.67 kilometre is the baseline and 50 metre is for the just resolving source. Okay, thank you. That's the end of things close to what we wanted to discuss and share with you. So I can stop here and.