Radio Astronomy

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Radio Interferometers

Hello and welcome to this second lecture of week 7. We will be starting on a very interesting topic of radio interferometers. Before we go into that we would like to a little bit motivate the reason for bringing up interferometers. Now remember this particular telescope it is the Green Bank telescope GBT. It is in West Virginia and the speciality is that this entire dish has a 100 meter diameter. So 100 meter in diameter.

This is in West Virginia in United States of America. And this is one of the largest dish which is fully steerable. You can see this we already discussed earlier in the lectures. You can rotate in azimuth through the mortars over here and in elevation with the mortars over here.

Ok. So that is the two rotation. You can move in any direction. There was this entire feed assembly which we discussed earlier. So rays do come in from the distance sources.

A parallel rays gets reflected and gets to this secondary reflector or even the feed. Sometimes there is a feed over here and sometimes it goes to the secondary reflector and then it gets reflected and there is lot of other receivers over here by which it gets absorbed and collected. Ok. Now we also have known that there is an interesting thing about the resolution. You can able to resolve two sources on the sky if they are proportional to lambda over d where d is the dimension of the dish.

So if the lambda rises the d also has to rise to balance the lambda over d ratio and to keep the resolution to be similar. The other thing which the large dishes does is the effective area, geometric area is kind of proportional also to this d square. Right. So that also gets done with this. But remember one thing, if you go to a fairly lower and lower frequency or higher and higher wavelength then the dish design becomes quite impractical to go further in that direction.

For example, say we use something called a lambda is equal to 1 centimeter. Now for that if I am making a dish of 100 meters to preserve the same resolution at lambda equal to say 1 meter or that is 1 meter or 300 megahertz the same dish, sorry this is the dish, same diameter has to be grown up to how many zeros are there. This becomes 10 kilometer. So 100 centimeter is one, yeah. So 100 times 100.

So same thing needed is you have to go up to 10,000 meter or 10 kilometers that becomes quite impractical to build it. So what we introduce here is something called radio interferometers. We refer to GMRT, these individual dishes are also quite magnificent and they are also state of the art. There is a 45 meter diameter dish and we have 30 of them, 30 dishes across. But now the lambda, the resolution is no longer determined by lambda by d.

Instead it is by lambda over B max. What is that? There is a maximum baseline between any two antenna inside the array. Roughly they say this direction or this, that is the size of the array or the maximum baseline. So now if I do the same math and calculate that now instead of to preserve the same resolution I do not have to make a single dish which is 10 kilometers. I just have to make an array which is spread over of 10 kilometer because the resolution element is now proportional to lambda over B max and no longer lambda over d for an interferometer.

For an absolute relief because suddenly the pressure is taken off from making a single dish of very large size. Instead we replace with n number of small dishes of a reasonable size diameter and that is your effective area. Right? So that can be easily worked out. So instead of creating a dish which is of 100 times the size of the GBT we can replace them by hundreds of dishes with much lesser size or more so. That is a possibility.

So or even one dish of GBT size 100 meters we can replace them by multiple 6 meter or 10 meter plus dishes of much smaller size and we can compare and get the same equivalent effective area at much lesser cost. So why interferometry? Because now the resolution which is the ability to differentiate between two sources on the sky is no longer lambda by d but it is lambda by d max which is the maximum baseline between any two antennas in the array. And the collecting area, what is the collecting area becomes? It effectively becomes your total number of dishes so 30 and then total area of each dishes which is d squared over 4. So effective collecting area becomes like that proportionally. Without counting in the constants of proportionality or the efficiency parameters etc etc.

Alright? So let us see what is this fascinating thing all about. So since the radio star or radio source is at infinity, okay, or very very large way away, so even if the two antennas are looking at the same source in the near field they will be having parallel rays because what happens is this star emanates spherical waves at a very far away but the moment it goes to the far field they become parallel rays. Okay? How? That is true. That can be easily understood. Supposedly you have a source, you have one arc, right? So if you just draw this, this is a small r, so s which is the circumference that is equal to nothing but r times theta.

So s is equal to r times theta. Alright? So when theta, when r becomes infinity, infinitely large, the radius of curvature, okay, the distance between infinity and large, this thing gets transformed into something like parallel rays. Okay? So that can be easily proven. So what we will use, we will use that same property of this and assume that what were spherical waves at the near field to the star or the radio source becomes parallel waves at the far. So even though these two antennas are looking at the same source, they are effectively getting parallel waves in this near the telescope.

So instead of now having a single dish, we have multiple dishes and we combine the signals collected from each of the dish or the antennas. They are simply reflecting antenna as we did before and they are separated by some baseline of p. So either they can be additive or multiplicative. For today's lecture, we will just restrict ourselves to multiplicative interferometers, but there is also additive interferometer and multiplicative. These are the two types of interferometers.

And there are many more. Okay? So let us see. So the radiation is coming from the far field, they are parallel to each other. So what is the problem? So when the star has emitted the spherical waves, they are coming and at the far field to the stars or near the two antennas, they are like parallel to each other. So if I sample the same wave front, which is coming from the star, there will be a little bit of a delay between the two antennas, particularly, right? If they are coming from this direction, as we mentioned.

So this arrives, if the star is located here and coming in this direction, it will arrive antenna 2 earlier than antenna 1. So the path delay between these two is delta s, which can be again by simple geometry, can be broken down in terms of this theta, which is the direction to the normal and the B, which is the baseline or the distance between the two antennas. We colloquially say radio astronomers tell this term as a baseline. Okay? So that's the difference. So that can be easily done.

Let us see how it is done. So, no, sorry, before that one more concept. So this is the additional path difference. This is the path delay. Okay? And we have seen that path delay is also due to the time delay. So t g, tau g is B dot s times c or B sine theta times c, you can easily do that.

This is the time, geometric time delay between the two. Okay? And so that can easily be responded into a time delay into a phase. So one of the antenna e 1 is, say it is having in this character e 2 should be having a cosine of 2 pi nu t, the other one should have an additional delay of delta phi. And then they can be, they're multiplying. So they are, we are multiplying both of them.

This is the response from antenna 1 and this is from antenna 2. Okay? So in order to sample the same wave front, they have to be delayed. One of them have to be delayed by this delta phi, which is the thing, but just the function of the theta, which is the direction of the viewing and the distance between the two antenna. Okay? So we just multiply the two electric fields and we just use this cosine 2 pi nu t and cosine 2 pi nu t plus tau. If I represent that and we get these two terms.

Now, if I see that you can easily derive from here to there by using cosine rule and from cosine rule, we have used this. And now you can see that this particular term has two terms. One of them has a time and another one is independent of this time. So if I now average it over time, these two function, I can easily say this term goes to zero.

It is a cosine function. It's a sinusoidal function, right? So if I average over large amount of time, then this averages down to zero. That's a very way of telling this. So you can prove that e1 dot e2 averaged over time can be represented by just simply e0 squared by 2 cosine of 2 pi nu tau. Now tau is nothing but B sine theta divided by C or nu times tau can be represented by B sine theta divided by lambda. So if I represent that, then that is there.

Now, if you represent theta and take into consideration the rotation of the earth, then omega e is nothing but the angular rotation of the plate of the earth. If you replace that into the previous equation, you derive into this particular fringe rate. Okay. So remember, we will have one example in the next lecture on this particular aspect. So what is this? This is nothing but e1 dot e2 average over time is given by this e0 squared divided by 2 cosine of 2 pi B over lambda sine of omega e times t.

Okay. The omega is nothing but the angular rotation rate of the earth. So two antennas looking at a one direction towards the radio star or radio source gets the spherically waves which are emanated from the star, but because it has traveled over large distance to infinity, they have transformed, spherical waves transform into plane waves. And so if we want to sample the same wave front, you have to put this delay compensation on any one of the antennas. And that gives rise to this beautiful function e1 dot e2 because we are multiplying an interferometer and it gives rise to this particular function which is the fringe function given by this rate which is proportional to the rate of the earth rotation. So if you multiply and draw it, this is a fringe.

This is like this, if you have seen the corrugated asbestos sheet, it is like basically like that. It goes up and down. This is the top and this is like there and this is the bottom that is in the dark and so on. It's a sinusoidal radiation.

Okay, that's the fringe. So this is a response of the interferometer for any two baselines. For example, this is the, let's use a lighter color so that we can, yeah, this is for any one. So that same thing will happen for any other. So for like for VLA, we have 27 antennas.

So any two can give rise to one fringe. So then 27 will give rise to how many? That is 27 into 26 divided by 2. How did we do that? It is 27 into 27 minus 1 divided by 2. That will give rise to this. So how much is this coming to? This is precisely 27 into 13.

That gives rise to 351. So for all this combination, we will get 351 fringes for a given time and we will superpose them to reconstruct what we have for the sky. We will talk about it later in the next coming, very exciting. Just to do a similar math for GMRT. Now GMRT has how many dishes? It has 30 dishes. So for GMRT, total number of such fringes for a given time will be 30 into 29 divided by 2 in the same manner.

So you have something like 15 multiplied by 29. Okay. So how much will that be? If you can calculate that? 29 times 15 is given by 435. Okay. Those many fringes will come for a given baseline, for total number of baselines here.

Okay. So I think we have come close to this. This is a very short lecture just introducing this entire thing. And of course we have mostly followed, taken the visuals and some of these equations from this particular book. But also we will be following other two books in the upcoming weeks. So stay tuned for more lectures. In the next lecture we will be dealing with little bit more examples.

So we are moving little bit slowly to make you able to catch up with the concept and then we will be done. Okay. See you in the next lecture. Thank you for joining us.