Radio Astronomy

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Lec-19

What Have we Learnt so Far? - A Review

Hello, welcome to this sixth week, lecture 2 of Radio Astronomy. In this particular week, we have been a little bit light. We started off with a particular demonstration or simulation of how antenna radiation patterns can be derived using a package called NEC. This particular lecture we are going to revisit on some sample problems on all the topics which have been covered so far in the past five weeks. We also held a live session and in that sense we gathered the reaction from the students and based on that we thought this is the best way to move forward. So we are going to revisit some of the concepts and try to address them using simple problems as before.

Maybe we have been a little bit light on doing some more problems. It may be beneficial for students to understand the concepts. So let's proceed. So the first problem which we are trying to tackle here is that of electromagnetic radiation is emitted from a distant object with a frequency of 1.

4 GHz. What is the energy of the photon and what is the wavelength? So we revisit the basic thing that energy E is given by h nu where nu is the frequency. And also you can visit that nu is nothing but C over lambda where lambda is the wavelength. So if you use these two concepts that the energy is h nu and nu is C over lambda we derive energy and h is the Planck's constant so that has a known value given over here. And we put the frequency so that is 1.

4 GHz so 1.4 10 to the power 9 GHz and that gives us 9.28 10 to the power minus 25. The next one is given the nu we have to understand the lambda so use the second formula and you just write this 0.214 meter.

Q 1: Electromagnetic radiation is emitted from a distant object with a frequency of 1.40GHz.

a. What is the energy of each photon?

b. What is the wavelength?

Ans.

a. To get the energy,

$$E = (6.626 * 10^{34}\ \text{J s})(1.40 * 10^{9}\ \text{Hz})$$
$$= 9.28 \times 10^{-25}\text{J}$$

b. For wavelength, we have

$$\lambda = \frac{3 * 10^{8}}{14 * 10^{9}} = \boxed{0.214\text{m}}$$

$E = h\nu$

Frequency

$\nu = c/\lambda$

Or we can say 21.4 cm. So it should be very easy for you to tackle. Let's go to the next one. Consider a radio source that is found to be elliptical.

So it is elliptical in nature. So in shape with a major axis of 0.3 degree and minus axis of 0.1 degree. What is the solid angle of the source? So here we are discussing about if a source is elliptical in nature and it is a major axis and minor axis defined then what will be the solid angle obtained by the source.

This is a very useful concept because we often have derivations with respect to the point sources but when it comes to the extended sources we end up with a travel. So the equation we use is that suppose the angular extent is theta then the solid angle is given by this particular expression which we have done earlier. So it comes out to be pi by 4 theta square. Here in this case for ellipse we have not theta is not same that's for circular source so we have theta major and theta minor. Now we have been given theta major and theta minor that is 0.

3 degree and 0.1 degree. Now what we do is that we convert the degree into radian that's what we have done here and once you put that into this expression you get finally that omega or the solid angle is given by 7.18 10 to the power minus 6 steradian. Again a very simple problem.

Q 2: Consider a radio source that is found to be elliptical in shape with a major axis of 0.3° and a minor axis of 0.1°. What is the solid angle of this radio source?

Ans: In this case, we use an equation

$$\boxed{\Omega = \pi\left(\frac{\theta}{2}\right)^2 = \left(\frac{\pi}{4}\right)\theta^2}$$

Since this source is elliptical instead of circular, we use the equation for the area of an ellipse, that is,

$$\Omega = \pi\left(\frac{\theta_{major}}{2}\right)\cdot\left(\frac{\theta_{minor}}{2}\right)$$

But we must first convert our angles to radians.
θ_{major}= 0.3° (π/180°)= 0.00524 radians
and θ_{minor} = 0.00175 radians.

$$\Omega = \pi\left(\frac{0.00524}{2}\right)\cdot\left(\frac{0.00175}{2}\right)$$

So then, $= 7.18\times10^{-6}\,\mathrm{sr}$

The third problem which we are encountering here is you use a radio telescope that has a collecting area of 1.2 meter square to observe a radio source at a frequency of 1420.4 megahertz and a bandwidth of 2 megahertz. You measure a detected power of 1.2 10 to the power minus 19 watts.

What is the flux density f mu of the source in Jansky's. Okay. So first of all one Jansky we can again revisit this so one Jansky given by J y is 10 to the power minus 26 watts per meter square per hertz steradian inverse. Okay. That is already known.

So this is one Jansky. Now let us define so f mu is the flux density that is dependent on the power over effective area over the bandwidth delta mu. Now bandwidth we already knew that is equal to what that is equal to 2 megahertz and the central frequency is given and the power is given. Collecting area also is given that is effective is equal to 1.

2 meter square. So we plug all of them together so the p is in watts p is also given by 1.2 into 10 to the power minus 19 watts. So we plug that in we plug all the other values. Finally it came out to be 5 10 to the power minus 26. We do this scaling of one Jansky.

Q 3: You use a radio telescope that has a collecting area of 1.20 m^2 to observe a radio source at a frequency of 1420.4MHz with a bandwidth of 2.00MHz. You measure a detected power = 1.20 × 10^{-19}W. What is the flux density, F_ν, of the source, in Janskys (Jy), at the observed frequency?

$1\ Jy = 10^{-26}\ W\,m^{-2}\,Hz^{-1}\,Sr^{-1}$

Ans: We know for flux density,

$$F_\nu = \frac{P}{A_{eff}\,\Delta\nu}$$

$\Delta\nu$

$$= \frac{1.20\times10^{-19}\ W}{1.20\ m^2 \times 2.00\times10^6\ Hz}$$

$$= 5.00\times10^{-26}\ W\ m^{-2}\ Hz^{-1} = 5.00\ Jy$$

So finally we get 5 Jansky. So finally the flux came out to be 5 Jansky in this case. It is a very standard problem. So we have an effective area of the telescope given. We also have the frequency of observation which does not come in this particular picture.

We have the bandwidth of observation 2 megahertz and then we have the detect power. Okay. And so we finally get 5 Jansky. Next question. We have a radio source at a distance of 20 mega per sec.

Now 1 per sec is already given over here is 3.09 times 10 to the power 16 meters. It is important to define a larger unit because the distance becomes too large in astronomical scales. So it's 20 mega per sec so you can define 20 mega per sec is equal to you can do the math by yourself. So 20 times 10 to the power 6 multiplied by 3.

09 into 10 to the power 16. So roughly this should come out to be around something like of the order of 6 times. This is 7, 13. So 10 to the power 23 kind of in meter scale.

Okay. That's the distance. It's observed by a telescope having a collecting area of 300 meters squared. That's given over here. The frequency is given by 22.2 gigahertz. So 10 to the power 9 hertz and the bandwidth of 250 kilohertz.

Determine have a flux density f nu is also given by 20 Jansky and is found to be a uniform circle with an angular diameter of 30 arc seconds. Okay. Now with that those information in the distance is given by 6 times 10 to the power 23 meter.

Q 4. A radio source at a distance of 20.0Mpc (1pc = 3.09×10^{16}m) is observed with a telescope, which has a collecting area of 300m^2, at a frequency of 22.2GHz, and with a bandwidth of 250 kHz. e source is determined to have a flux density, F_ν, at this frequency of 20.0Jy, and is found to be a uniform circle with an angular diameter of 30.0 arcsec.

a. What is the intensity, I_ν, of the radiation from this source at this wavelength?

b. What is the total power of the radiation detected by the telescope?

c. What is the luminosity of the source over the observed spectral range?

$A_c = 300\,m^2$

$1\,pc = 3.09 \times 10^{16}\,m$

$20\,Mpc = 20 \times 10^6 \times 3.09 \times 10^{16}$

$\cong 6 \times 10^{23}\,m$

With that information given we are supposed to figure out what is the specific intensity I nu. Total power of radiation detected by telescope and so we need to figure out the p, the I nu and the luminosity.

Okay. So let's see how we go about this. So first of all the source, substance and solid angle. So we know that is 30 arc seconds. So we convert that to the circular source. So theta major is equal to theta minor if 30 arc seconds and it has, so we do that all of them and then finally we get 1.

66 into minus 8 Cd. So intensity is 20 Jansky is the flux over the solid angle. So finally intensity is given by this particular expression 1.2 10 to the power minus 14 watts per meter square per hertz per steradian. Okay. Total power detected is f p is given by f nu times area of the telescope times delta nu.

So we put those in 20 Jansky times theta to the power 26 that takes it to Jansky 2 watts per meter square per hertz per steradian. Yeah, Jansky. So and then 30 meters, 300 meters square is the area and 250 kilohertz is the bandwidth. So that comes out to be the power is given by 1.5 to the power minus 17 watts.

Ans.

a. the source subtends a solid angle of

$$\Omega = \frac{\pi}{4}\left(30'' \frac{1^\circ}{3600''} \frac{\pi \text{ radians}}{180^\circ}\right)^2$$
$$= 1.66 \times 10^{-8} \text{ sr}$$

The intensity, then, is

$$I_\nu = \frac{20 \text{ Jy } (10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \text{Jy}^{-1})}{1.66 \times 10^{-8} \text{ sr}}$$
$$= 1.20 \times 10^{-14} \text{ W m}^{-2} \text{ Hz}^{-1} \text{sr}^{-1}$$

b. The total detected power equals the flux density multiplied by the bandwidth and the collecting area of the telescope

$$P = F_\nu A_{\text{telescope}} \Delta\nu$$

$$P = 20.0 \text{ Jy } (10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \text{Jy}^{-1}) \; (300 \text{ m}^2)(250 \times 10^3 \text{ Hz})$$
$$= 1.50 \times 10^{-17} \text{ W}$$

Okay. Simply a substitution. So the previous question when we did we had the p and the a effective and the bandwidth given we derived the flux. In this particular case we have angular not a compact source but it's extended source and we have been given the flux density the angular extent so we calculate the specific intensity from that and then from the flux density we also calculated the power. The last one is the luminosity. So we express the flux density multiplied by the bandwidth and the distance.

Okay. So distance is remember 20 mega per sec. So 20 10 to the power 6 times mega per sec. So 1 per sec is 3.09 times 10 to the power 16 meters. So that whole square is the luminosity the distance to that and so if you multiply this with times the bandwidth times the flux density you get the total luminosity in this particular band as 2.4 10 to the power 29 watts.

c. The luminosity of the source, over the observed spectral range, is given by the observed flux density times the bandwidth,

$$L = 20.0 \text{ Jy}(10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ Jy}^{-1}) \, 250 \times 10^3 \text{ Hz}$$
$$\times \left[4\pi(20 \times 10^6 \text{ pc} \times 3.09 \times 10^{16} \text{ m pc}^{-1})^2\right]$$
$$= 2.40 \times 10^{29} \text{ W}$$

Okay. Question number five. An alien civilization is at a distance of 7 per sec uses radio antenna of the diameter of 100 meters acting as a transmitter to beam a radio signal towards the earth. So now you are using a radio antenna as a transmitter. The civilization converts 5 times 10 to the power 6 joule of energy into this signal which is transmitted at a constant power of for power for 100 seconds centered at a frequency of 1 gigahertz and the bandwidth of 1 megahertz.

In a beam with a solid angle of 9.3 10 to the power minus 6 radian.

Q 5. An alien civilization at a distance of 7 pc (1pc = 3.09×10^{16}m) uses a radio antenna of diameter 100m, acting as a transmitter, to beam a radio signal toward Earth. The civilization converts 5.00×10^{6}J of energy into this signal, which is transmitted at a constant power for 100s, centered at a frequency of 1.000GHz with a bandwidth of 1.00MHz, and in a beam with a solid angle of 9.30×10^{-6}sr.

a. What is the luminosity of the beamed signal?

b. What is the intensity of the beamed signal?

$d = 7pc = 7 \times 3.09 \times 10^{16}$ m

$\nu = 1$ GHz $= 10^{9}$ Hz

$\Delta\nu = 1$ MHz $= 10^{6}$ Hz

$\Omega = 9.3 \times 10^{-6}$ sr

So what is the luminosity of the beam signal and what is the intensity of the beam signal. So what do we have we have a distance which is nothing but 7 parsec. So 7 multiplied by 3.

09 times 10 to the power 16 meters. Then we have total energy and then total time and the frequency is given by 1 gigahertz. So 10 to the power 9 Hertz and delta nu or the bandwidth is given by 1 megahertz or 10 to the power 6 Hertz and the beam solid angle is given by omega equals to 9.3 times 10 to the power minus 16 standard. Given those we need to calculate the luminosity and the intensity.

Let's see. So luminosity is nothing but the total energy over total time and that is given by 50 kilowatt. Okay total energy was 5 times 10 to the power 6 joules and it was beamed for 100 seconds and B is the intensity. So now you have the luminosity from there you know the area of the transmitting antenna. Yeah its diameter is 100 meter. So then you can calculate the area and the omega of the beam and delta nu.

So if you put that so I nu is L over area effective times omega times delta nu. Okay. Relating it again. So area times omega times delta nu. That is our and if I put all of them together so area already we know it is omega is 9.3 times 10 to the power minus 6. Delta nu is 1 megahertz because 6 Hertz and this is the area is 5 pi times 50 meter diameter is 100 meter so radius is 50 meter square that gives you 7.85 into the power 3 meter square and omega this is the solid angle is equal to this. Okay and this is the sorry the delta nu the bandwidth. Yeah. Okay so that gives you the final answer to be if you just plug it in so I nu the intensity is given by this expression 0.685 watt per Hertz per meter square per solarium.

a. $L = 5.00 \times 10^6$ J/100s = 5.00×10^4W = 50kW.

b. Here, we can use the equivalence of intensity and surface brightness. e intensity is the power emitted per area of the emitting surface per solid angle of the transmitted beam per bandwidth, that is,

$$I_\nu = \frac{5.00\times10^4 \text{ W}}{(A \text{ of transmitting antenna})\times(\Omega \text{ of beam})\times\Delta\nu}$$

We are given that the transmitting area, beam solid angle, and bandwidth are as follows: $A = \pi(50.0\text{m})^2 = 7.85 \times 10^3 \text{ m}^2$, $= 9.30 \times 10^{-6}$sr, and $D_\nu = 1.00 \times 10^6$Hz.

$$I_\nu = \frac{5.00\times10^4 \text{ W}}{(7.85\times10^3 \text{ m}^2)\times(9.30\times10^{-6} \text{ sr})\times1.00\times10^6 \text{ Hz}}$$

$$= 0.685 \text{ W Hz}^{-1} \text{ m}^{-2} \text{ sr}^{-1}$$

So in the previous problem we did the reverse one we have the we calculated the luminosity from the flux density and the delta nu and the distance. So if you have a distance so that you draw the sphere and so you multiply that with the entire area and you get the luminosity. In this case you have the distance the bandwidth the frequency the solid angle and you calculate the intensity and the luminosity. Luminosity was easy and intensity was right from the luminosity itself.

So let's go to the next one. An electric stove is turned off and heating and the heating element of the burner turns back to black but the stove warning light still indicates that the stove is hot. An infrared sensor is aimed at the burner and the radiation and the radiation at a frequency of 3.33 10 to the power 14 Hertz so it's quite high 3.33 10 to the power 14 Hertz was detected from the burner having an intensity of 1.46 10 to the power minus 28 watt per meter square per Hertz per solarium.

Estimate the temperature of the stove.

Q 6: An electric stove is turned off and the heating element (or burner) turns back to black, but the stove warning light still indicates that the stove is hot. An infrared sensor is aimed at the burner and the radiation at a frequency of 3.33×10^{14} Hz emanating from the burner is determined to have an intensity of 1.46×10^{-28} $WHz^{-1}m^{-2}sr^{-1}$. Estimate the temperature of the stove burner.

$\nu = 3.33 \times 10^{14}$ Hz

$I_\nu = 1.46 \times 10^{-28}$ $W m^{-2} Hz^{-1} sr^{-1}$

$T = ?$

That was the question. So this is a question from the black body radiation and we did a lot of some problems for the Rayleigh-Jin laws etc. So let us see. So given such a high value of the frequency 10 to the power 14 Hertz it is difficult to approximate by Rayleigh-Jin. So we consider the entire black body spectrum the Planck's law.

So we consider that the spectral radiance B nu function of temperature given by the expression 2 h nu h nu cube over c square 1 over x it is power h nu over kT minus 1. So this entire thing is coming out to be 1.46 so you have all the other values given h is a Planck's constant c is also a constant speed of light k is Boltzmann constant and nu is given nu is nothing but 10 to the power 3.33 nu is given as 3.33 into 10 to the power 14 Hertz and intensity is given 1.46 10 to the power minus 28. This entire thing yeah. So if you put all the other values together you will see now we can solve for kT or T and finally T comes out to be 322 Kelvin or 120 degree Fahrenheit.

Ans. The Planck function is given by , $B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$

$$\Rightarrow 1.46\times10^{-28}\ \mathrm{W\,Hz^{-1}\,m^{-2}\,sr^{-1}} = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$$

By substituting in the values for *h*, *c*, *k*, and *ν*, rearranging we get

$$\Rightarrow \exp\left(\frac{1.60\times10^4 K}{T}\right) - 1 = \frac{5.44\times10^{-7}}{1.46\times10^{-28}} = 3.73\times10^{21}$$

$$\boxed{\begin{aligned} T &= \frac{1.60\times10^4\ \mathrm{K}}{\ln(3.73\times10^{21})} \\ &= 322\ \mathrm{K}\ (\text{or } 120°\mathrm{F}) \end{aligned}}$$

$\nu = 3.33 \times 10^{14}$ Hz

$I_\nu = 1.46 \times 10^{-28}$

So the next example what we are looking at is in an article in an article about a visible light visible wavelengths spectral observation of a nearby galaxy the measured flux density centered at a wavelength of 4250 angstrom and with a band pass of 50 angstrom is reported to be 2.3 10 to the power minus 8 watt per meter square per angstrom. Meanwhile a radio observation of the same galaxy at 22 gigahertz with a bandwidth of 50 megahertz and yields a flux density of 42 Jens. So the two bands visible band and radio band the invisible band the wavelength is mentioned the band pass is mentioned and the flux density is mentioned in terms of watt per meter square per angstrom and radio the same thing is mentioned 22.2 gigahertz bandwidth of 50 megahertz and the flux density of 42 Jens. In which band does this galaxy have a large larger flux density compare the detected fluxes in the two bands? Very interesting question. We first note the conversion between f lambda and f mu is the same as between i nu and i lambda.

Q 7. In an article about a visible-wavelength spectral observation of a nearby galaxy, the measured flux density centered at a wavelength of 4250 Å, and with a bandpass of 50.0Å, is reported to be $2.30 \times 10^{-8} \mathrm{W m^{-2} Å^{-1}}$. Meanwhile, a radio observation of the same galaxy, at 22.2GHz with a bandwidth of 50.0MHz, yields a flux density of 42.0Jy.

a. In which band does this galaxy have a larger flux density?

b. Compare the detected fluxes in the two bands.

So f nu is lambda square over c times f lambda and is given by just the scaling just put those values and finally you get f nu as 1.38 uh to the point is 29 watts per meter square per hertz. And f lambda is given in terms of yeah in terms of f nu so that is in terms of angstrom. Okay so if you just convert each others you can get back to this so we converted the um the f lambda on the optical to f nu in the radio and we converted the f nu in the radio to f lambda.

That was the the exercise we did in the first place. This is for the radio in it was in reported in f nu so we converted that to f lambda and in the optical it was in f lambda and we converted that to f nu. The detected fluxes are given by multiplying the measured flux densities by the bandwidths. So um if we do that the f is multiplied by the bandwidth which is 50 in in the case of the radio uh with 4.2 to the minus 26 times 50 megahertz is given 2.1 to the minus 18 watt meter. Similarly if your uh watt per meter square sorry and if you're doing the for visible it's 2.38 into the power minus 8 uh so that is 1.15 10 to the minus 26 watt per meter square.

Ans.

a. We first note that the conversion between F_λ and F_ν is the same as between I_λ and I_ν:

$$\boxed{F_\nu = \frac{\lambda^2}{c} F_\lambda} = \frac{(4250\times10^{-10}\ \mathrm{m})^2}{3.00\times10^8\ \mathrm{m\,s^{-1}}} 2.30\times10^{-8}$$

$$= 1.38\times10^{-29}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$$ $F_\lambda \rightarrow F_\nu$

$$F_\lambda = \frac{c}{\lambda^2} F_\nu = \frac{\nu^2}{c} F_\nu = \frac{(2.22\times10^{10}\ \mathrm{Hz})^2}{3.00\times10^8\ \mathrm{m\,s^{-1}}} 4.20\times10^{-25}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$$

$$= 6.90\times10^{-23}\ \mathrm{W\,m^{-2}\,\AA^{-1}}$$ $F_\nu \rightarrow F_\lambda$

b. The detected fluxes are given by multiplying the measured flux densities by the bandwidths. For the radio, this is $F = 4.20 \times 10^{-26} \times 50.0 \times 10^6 = 2.10 \times 10^{-18}\ \mathrm{W\ m^{-2}}$

For the visible, $F = 2.3 * 10^{-8} \times 50 * 10^{-10} = 1.15 * 10^{-6}\ \mathrm{W\ m^{-2}}$

So you can clearly see that the density flux is much much brighter in case of optical than the radio. So the next question is about the Orion Nebula which is the H2 region containing hot ionized gas that emits thermal radiation.

At a frequency of 400 megahertz uh the Orion Nebula is found to have flux density of 220 over 200 janske sorry and comes from an area of the sky which have a solid angle of 2 times into the minus 5 steradian. We use this data to infer a lower limit for the temperature of the gas in this nebula. First of all there's a typo it should be 200 janske and so if it's 200 janske you have a steradian 2.2 minus 10 to the minus 5 steradian so I mu is given by 200 divided by the steradian and so we have done similar problem before so it comes out to be 10 to the power 19 watts per meter square hertz inverse steradian inverse. Now what we do is we can we know what is a Planck's law so I mu can be go to 2 h mu cube c square 1 over e to the power h mu k t minus 1.

Now that basically says that this for the lower limit of the temperature so everything will else we knew except temperature so if you substitute that in this expression all the other values h we know from before nu we know nu is around 400 megahertz c is a constant so everything else is known except the temperature Bose-Mann also is a constant so if you put substitute everything in just remember this value of I mu is less than equal to this Planck's equation given by that and so that gives you an upper limit a lower limit for the temperature of the gas which is 2040 kelvin.

Q 8: The Orion nebula is an HII region containing hot ionized gas that emits thermal radiation. At a frequency of 400 MHz, the Orion nebula is found to have a flux density of 200 Jy and comes from an area on the sky with solid angle of 2×10^{-5} sr. Use these data to infer a lower limit to the temperature of the gas in this nebula.

200 Jy

Ans: We first calculate the intensity from the flux density and angular size.

Or,

$$I_\nu = \frac{200\ \mathrm{Jy}\ (10^{-26}\ \mathrm{W\ m^{-2}\ Hz^{-1}\ Jy^{-1}})}{2\times 10^{-5}\ \mathrm{sr}} = 1\times 10^{19}\ \mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}$$

$$1\times 10^{-19}\ \mathrm{W\ Hz^{-1}\ m^{-2}\ sr^{-1}} \le 9.42\times 10^{-25}\ \mathrm{W\ Hz^{-1}\ m^{-2}\ sr^{-1}} \frac{1}{\exp(0.0192\mathrm{K}/T)-1}$$

$$\Longrightarrow \exp(0.0192\mathrm{K}/T)-1 \le 9.42\times 10^{-6}$$

which requires that $T \ge 2040$K.

Okay the next question is a radio observation at a wavelength of 6 centimeters yields determination that a particular radio source has a solid angle of 7.18 times 10 to the power minus 6 steradian is opaque and thermal and has a flux density of 350 janske what is temperature of the radio source what is the intensity of this source at 2.7 centimeter flux density is measured at 6 centimeter which is still in the realm of Rayleigh-J's approximations so t we just take f nu as 2 k t over lambda square times the steradian we just substitute all the values and get the value of t to be equals to is the brightness temperature to be goes to 63.6 kelvin and a wavelength of 2.7 centimeter the frequency becomes 11.1 gigahertz so the I mu is given by 2 k t over this thing so and then finally the lambda value is given the t value is given so if you substitute all of them then you get the I mu at this frequency to be equals to 2.41 to the minus 18 watts per meter squared per hertz per steradian

Q 9. A radio observation at a wavelength of 6.00cm yields the determination that a particular radio-source has a solid angle of 7.18×10^{-6}sr, is opaque and thermal, and has a flux density of 350Jy.

a. What is the temperature of the radio source?

b. What is the intensity of this source at 2.70cm?

Ans. a. Since the flux density is measured at 6.00cm, which is still in the realm where the Rayleigh–Jeans approximation works well

$$\boxed{F_\nu = \frac{2kT}{\lambda^2}\Omega}$$

$$T = \frac{3.50\times10^{-24}\ \mathrm{W\,m^{-2}\,Hz^{-1}}(0.060\ \mathrm{m})^2}{(2\times1.38\times10^{-23}\ \mathrm{J\,K^{-1}})7.18\times10^{-6}\ \mathrm{sr}}$$

$$\boxed{T_B = 63.6\ \mathrm{K}}$$

b. At a wavelength of 2.70 cm, or frequency of 11.1GHz, we find the intensity to be given by:

$$I_{11.1\mathrm{GHz}} = \frac{2\,(1.38\times10^{-23}\,\mathrm{J K^{-1}})\ 63.6\mathrm{K}}{(0.0270\,\mathrm{m})^2}$$

$$= 2.41\times10^{-18}\,\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$$

so that's was an easy problem just a simple substitution so if you are confused these are there more and more examples we are doing here for your benefit a spotlight the next question is a spotlight has an intensity of two times 10 to the power 11 watts per hertz per meter squared per steradian at a frequency of 5 10 to the power 14 hertz what is the brightness temperature this is a bit higher in frequency 10 to the power 14 is a bit higher frequency for Elegance approximation to hold so you have to write everything in terms of the Planck's law if you put it like that way there is a intensity and there is a frequency so all you need to is to solve for the brightness temperature and that's comes out to be 2100 kelvin

Q 10: A spotlight has an intensity of 2.00×10^{-11} $\mathrm{WHz^{-1}m^{-2}sr^{-1}}$ at a frequency of 5×10^{14}Hz. What is its brightness temperature?

Ans: The only measurement of the spotlight beam we are given is that at the frequency of 5.00×10^{14}Hz, which is far too high a frequency to use the Rayleigh–Jeans approximation. So, using Planck function we obtain;

$$2.00\times10^{-11}\,\mathrm{W\,Hz^{-1}\,m^{-2}\,sr^{-1}} = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT_B)-1}$$

RJX $\cdot\ \nu \sim 10^{14}$ Hz

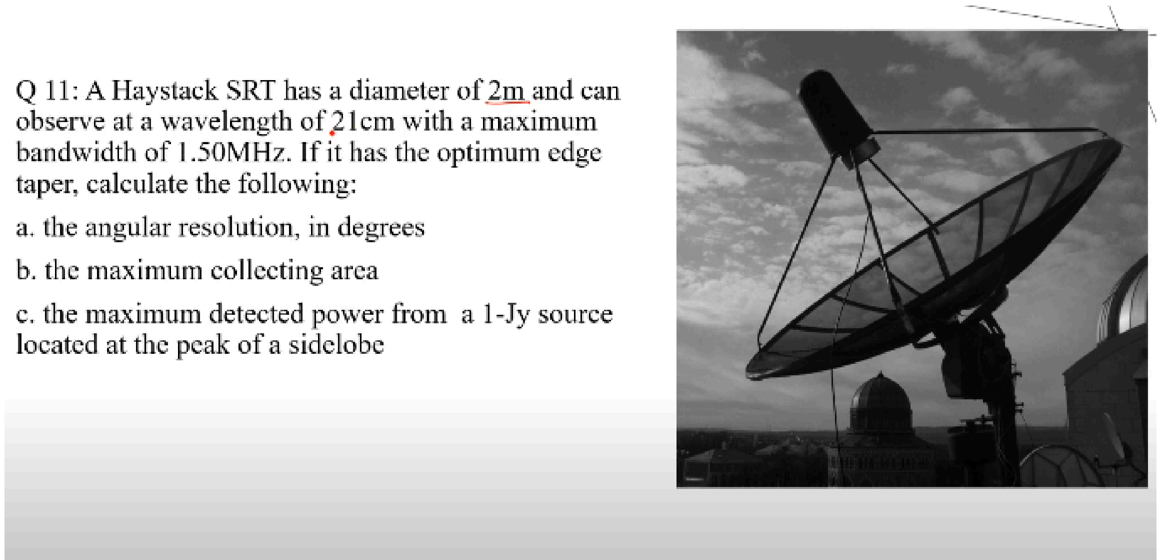
By substituting in the values for h, c, k and ν, we get

$$\exp\left(\frac{24000\,\mathrm{K}}{T_B}\right)-1 = \frac{1.84\times10^{-6}}{2.00\times10^{-11}} = 92000$$

The brightness temperature (T_B) is given by :

$$\boxed{T_B = \frac{24000\ \mathrm{K}}{\ln(92000)} = 2100\ \mathrm{K}}$$

so its relevance does not hold so you cannot have religions law do not hold at this frequency because frequency is to the power of almost to the power 14 hertz that's why so you put the entire thing into the normal planck's law and you get the answer our next example is



that of the haystack observatory um run by mit uh has a small radio telescope srt with a diameter of two meters and can observe it wavelet 21 centimeter or 1.4 gigahertz about with the maximum bandwidth of 1.5 megahertz if it has this is the srt if it has the optimum edge taper calculate the falling if it is optimal edge taper then you can calculate the following the angular resolution in degrees the maximum collecting area the maximum protected power from a one jenski source located at the peak of a side lobe so those are the three questions to be asked so resolution of the optimum h taper has theta full width half maximum or hpbw the same thing is 1.5 1 5 divided by the d uh as the lambda is there i believe yes the typo so if you put all of them together you get the fwhm is 6.92 degree which is also the half power beam width they're the same thing we are given the h taper but not any information about the physical blockage by the feed horn on receiver not about the imperfection in the reflector so maximum collecting area is a geometric area pi r squared multiplied by 0.82 due to the edge taper that's the edge taper we have then the maximum effective area is 0.82 times pi times one because it's yeah diameter is two meters so our radius is one meter so edge taper introduces an efficiency of 0.82 percent efficiency at the center of the beam the maximum edge so okay so for the effective area is becomes 0.82 times pi r squared r is is d by 2 d is 2 meters so r is 1 meter you put all the values in you get 2.5 meters squared 5 8 meters squared sorry third question is the center of the beam the maximum detected power due to one janske source in site low becomes uh so power is p one janske source is in the site low so and then this is times the effective area times the uh bandwidth 1.5 this is the delta nu this is the effective area this is the conversion janske to what meter squared dot dot this is um it's a one janske source so this is the source flux density and you have a um a reduction of 0.4 percent because you're now protecting in the site law so that's the reduction because it's not the main beam but it's in this in the cycle so if you're having a source coming in the side lobe then the flux density goes down further in this particular case it is 0.4 percent of this so finally the power is given by 1.55 to the minus 22 watts

Ans. a. The resolution with optimum edge taper is $\theta_{FWHM} = 1.15\,/\,D$;
therefore, we have FWHM = 1.15(0.21 m) / (2.00 m) = 0.121 radians = 6.92°

b. We are given the edge taper but not any information about the physical blockage by the feed horn and receiver, nor about the imperfections in the reflector, so the maximum collecting area is the geometric area, πR^2, multiplied by 0.82 due to the edge taper. We have then the maximum $A_{eff} = 0.82\,\pi(1) = 2.58\ m^2$

c. At the center of the beam, the maximum detected power due to the 1-Jy source in the sidelobe is:

$$P = 0.004 \times 1\ \mathrm{Jy}\left(10^{-26}\ \mathrm{W\ m^{-2}\ Hz^{-1}\,Jy^{-1}}\right) \times 2.58\ \mathrm{m^2}\ \left(1.5 \times 10^{6}\ \mathrm{Hz}\right)$$
$$= 1.55 \times 10^{-22}\ \mathrm{W}$$

so tricky thing over here is to figure out this factor which comes from of course the actual thing here we are doing this just to not make you um aware of the fact that if a if a source is not coming into the primary beam but coming through the side lobe so we have discussed this so if you cut the primary beam and then this is the null over here the same thing so if you draw that this is the center this is the p naught which is the maximum value if it comes to p naught by two that becomes this half power beam width and in the first side lobe so first side lobe the maximum value is much much lower than the p0 so in this case we are assuming this is normalized this is a 0.004 times p naught so whatever the value is okay next question is assume that the srt the small radio telescope which we discussed in the previous example is a prime focus telescope of two meter diameter and has an rms of the reflector irregularly approximately 0.7 centimeter and that total loss due to the blockage due to the feed and receiver feed legs is 12 percent of the collecting area assuming optimal feed taper again observation producing the meter with a bandwidth of 1.5 megahertz calculate the following the effective collecting area the a effective detected power grain observing a source one jensky

Q 12: Assume that the SRT discussed in previous example, which is a prime-focus telescope of 2-m diameter, has an *RMS* of the reflector irregularities of approximately 0.7cm, and that the total loss due to blockage due to the feed, receiver, and feed legs is 12% of the collecting area. Assuming optimal feed taper again and observations at 21cm with a bandwidth of 1.5MHz, calculate the following:

a. The effective collecting area

b. The detected power when observing a 1-Jy source at the center of the beam

at the center of the beam different from last time so you can assume that the reduction of the efficiency further you had 0.82 before you have include the other 12 percent so the blockage blockage reduces the factor of 0.88 so if you go by the expression you calculate the further reduction and so the effective area effective further reduces to 1.1 9.91 remember the last time it was 2.58 with just the tapering and now it is 1.91 it was in the previous example it was 2. so problem it was 2.58 in the previous example where there was no reduction of 12 so if further reduction of 12 happens it becomes a 1.91 meter square he did power is nothing but one jenski this is minus 26 minus 26 times 1.91 the effective area times the bandwidth and this gives you this thing total power is the total flux density one jenski is one so it is a the power is given by one jenski so one times ten to the power minus 26 times those what per meter square etc etc times the effective area 1.91 times the effective bandwidth which is 1.5 into 10 to the power six so that effectively finally comes out to be 2.86 times 10 to the power minus 21.

Ans. a. we can determine and include the reduction in effective collecting area including the other two factors. e blockage introduces a factor of 0.88. We calculate the reduction due to the surface irregularities using the Ruze equation,

$$\frac{A_\delta}{A_0} = e^{-[4\pi(0.7\text{ cm}/21\text{ cm})]^2} = 0.84$$

Our final estimate of the effective area of the SRT, then, is

$$0.88(0.84)(2.58\text{ m}^2) = 1.91\text{ m}^2 \quad A_{eff}$$

2.58 m

b. the detected power of a source at the center of the beam.

$P = 1 * 1^{26} * (1.91) * (1.5*10^6) = 2.86 * 10^{-20}$W

$P = 1\times10^{-26}(\ )\times1.91\times1.5\times10^6$

$= 2.86\times10^{-21}$

okay uh question number 13 uh consider a sequence of three amplifiers with the gains of g1 is given by 100 that is 20 db g2 is 20 that is 13 db g3 is 100 it is 23 db and noise temperatures are 20 kelvin 100 kelvin and 500 kelvin calculate total gain total noise temperature of the sequence total gain is multiplication total and i think so it is 4 10 to the power 5 that's the in linear unit in decibel it is just the addition of them so 56 db so gains are just 9 g1 plus g2 plus g3 in db of course and in linear scale this is a multiplication total noise temperature will be given by tn is 20 the first one that is tn1 plus tn2 is 100 divided by the gain of the first one so it is it is like this it is tn1 plus tn2 divided by the gain of the first one plus tn3 divided by the gain of the first multiplied by the gain of the second okay so like that so if you put that in it finally comes out to be 21.3 kelvin that is what we tn we discussed this in one of the examples previously also

Q 13: Consider a sequence of three amplifiers with gains of G_1 = 100 (20dB), G_2 = 20 (13dB), and G_3 = 200 (23dB) and noise temperatures $T_N^{\,1}$ = 20K, $T_N^{\,2}$ = 100K, $T_N^{\,3}$ = 500K. Calculate the total gain and total noise temperature of this amplifier sequence.

Ans: The total gain is given by $G_{total} = 100 \times 20 \times 200 = 4 \times 10^5$

Note that you can also find the total gain by summing the gains of the amplifiers in decibels, so G(dB) = 20 + 13 + 23 = 56dB. $G_1 + G_2 + G_3$ (dB)

For the total noise temperature, we find:

$T_N = 20 + 100/100 + 500/(100 \times 20) = 20 + 1 + 0.25 = \boxed{21.3\text{K} = T_N}$

$T_N^1 + T_N^2/G_1 + T_N^3/G_1 \times G_2$

Review 1 21

so now a radio telescope has a receiver the total noise temperature of 100 kelvin total gain of 10 to the power 8 this is 10 raised to 8 this is in superscript sorry and a bandwidth of 1 megahertz what is the total noise power detector so total noise the total noise power detector so total noise power detector will be the gain times the kelvin in sorry in boltzmann the k in boltzmann constant times the bandwidth times ta plus tn okay so um total gain is given or this is given total gain we have written g is 10 to the power 8 then the boltzmann then the bandwidth and then 100 which is the noise temperature okay because that this is goes to zero for just the noise and so this becomes 10 to the power minus seven 1.38 10 to the power minus seven watt this is the noise power

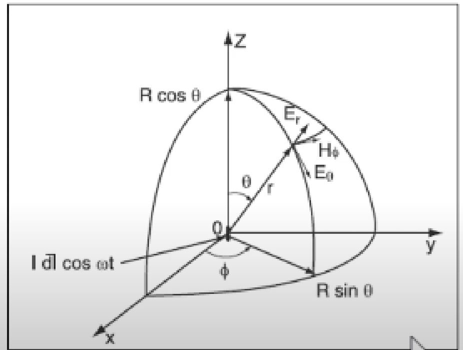
Q 14. A radio telescope has a receiver with a total noise temperature of 100K, a total gain of 1 × 108, and a bandwidth of 1MHz. What is the total noise power detected?

Ans: The power is given by $P = G\, k_B \Delta\nu\, (\overset{0}{\cancel{T_A}} + T_N)$.

$P = (1 \times 10^8) \times (1.38 \times 10^{-23}) \times (1 \times 10^6) \times (100) = \boxed{1.38 \times 10^{-7}\ \text{W} = P_N}$

or pne question number 15 the radiating element shown in this figure is of 10 meter length and carries a current of one ampere it radiates in theta to 30 degree direction in free space at frequency f given by three megahertz estimate the magnitudes of e and h at a point located at 100 kilometer from the point of origin this is regarding the radiation so at three megahertz lambda equal to 100 meter as long as the length of the element remains less than or equal to lambda by 10 the field components are given by the previously we have done so you can derive the field components from there and if you since only those field components which are inverse proportional to the r contributes so

Q 15. The radiating element shown in Fig is of 10 m length and carries a current of1 amp. It radiates in $\theta = 30°$ direction in free space at $f = 3$ MHz. Estimate the magnitudes of E and H at a point located at 100 km from the point of origination.



we have discussed this at very large distance r squared and r cubed terms vanishes or goes to zero so at very large l er will go to zero e theta has one one over r term this two will go to zero the term which survives is this one for h pi again the second term the first term in this case goes to zero and the second term survives okay these are two terms which survives as r tends to infinity so e has a theta component and h has a phi component in this particular case okay so what it happens is that if you write it down in this expression that is all there so finally if you now uh goes for the value of theta is equal to 30 degree so sine 30 is given and i is let's go back and check sorry the i the length is 10 meters current is one ampere theta is 30 degree and frequency is three megahertz okay so if you put the values of uh for dl um all these values if you put in you finally get the value of e given in terms of 45 to the 19 and h is given in terms of this put in the values you get the answer not very difficult the last of the problem today we're discussing is calculate the distance at which the electromagnetic wave will have the same magnitude for induction and radiation fields if its frequency is 10 megahertz

Ans: At 3 MHz, $\lambda = 100$ m. As long as the length of the element remains less than or equal to $\lambda/10$, expressions of field components:

$$E_r = \frac{2Idl\cos\theta}{4\pi\varepsilon}\left[\cancel{\frac{\cos\omega t'}{r^2 \upsilon}} + \cancel{\frac{\sin\omega t'}{\omega r^3}}\right]$$

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{r\upsilon^2} + \cancel{\frac{\cos\omega t'}{r^2\upsilon}} + \cancel{\frac{\sin\omega t'}{\omega r^3}}\right]$$

$$H_\phi = \frac{Idl\sin\theta}{4\pi}\left[\cancel{\frac{\cos\omega t'}{r^2}} - \frac{\omega\sin\omega t'}{r\upsilon}\right]$$

Since only those field components which are inversely proportional to r contribute to the radiation field, the relevant expressions can be rewritten as:

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{r\upsilon^2}\right]$$

$$H_\phi = \frac{Idl\sin\theta}{4\pi}\left[-\frac{\omega\sin\omega t'}{r\upsilon}\right]$$

Ans 14. cont…

Thus $E = E_\theta\, a_\theta$ and $H = H_\varphi\, a_\varphi$

The magnitudes of the remaining two terms can be written as below.

$$|E_\theta| = \frac{Idl\sin\theta}{4\pi\varepsilon} = |E| \quad \text{and} \quad |H_\phi| = \frac{Idl\sin\theta}{4\pi} = |H|$$

Putting the constant value, we will end up

$$|E| = \frac{Idl\sin\theta}{4\pi\varepsilon} = \frac{10\times 0.5}{4\pi\times 10^{-9}/36\pi} = \frac{45}{10^{-9}} = 45\times 10^{9}$$

$$|H| = \frac{Idl\sin\theta}{4\pi} = \frac{10\times 0.5}{4\pi} = 1.25/\pi$$

so the induction radiation fields have the same magnitude at a given r given by frequency over the circular frequency and that is uh so this is the c over the circular frequency so this is given by three ten to the power eight uh divided by two pi ten times ten to the power 16 is 10 megahertz so r is given by four point seven seven meters so r is the distance so r is the distance which is given by c over omega c over two pi new new is 10 megahertz c we know so if you put the value you get finally four point seven seven meters that's the answer

Q 16: Calculate the distance at which an electromagnetic wave will have the same magnitude for induction and radiation fields if its frequency is 10 MHz.

Ans: the induction and radiation fields will have the same magnitude at:

$$r = \frac{c}{\omega} = \frac{3 \times 10^8}{2\pi \times 10 \times 10^6} = 4.77\,\mathrm{m}$$

$$r = \frac{c}{\omega} = \frac{c}{2\pi\nu} = 4.77\,\mathrm{m}$$

so that brings us close to the this today's this session of additional problems i hope this helped a little bit uh to your for catching up to the material which has been covered so far and we will keep interacting in the in the live sessions and and try to come up with more solved problems which can help you in understanding the concepts better so thanks for joining thanks a lot hope this helps see you in the next class thank you