

## **Radio Astronomy**

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### **Single Dish Observations**

Hello, welcome to the second lecture of week number 5 of radio astronomy. So, we will continue our discussions with single dish observations. In the last lecture, we have covered some few essentials of radio telescopes like the reflector antenna design, different types of reflector antenna design and why a reflector antenna is the most common chosen antenna design for radio telescope because it is a larger collecting area which produces a sharp beam or less beam with a lesser HVW or half power beam width and various others. We discussed also the different reflectors, different feed locations, Casagrand, offset Casagrand, prime focus, etc. We also established a very crucial relationship between aperture electric field distribution and far field electric field distribution which is by Fourier transform. We also noted that the far field power pattern is a Fourier transform of the autocorrelation function of the near field or the aperture electric field distribution.

These are the takeaways from the first lecture which we will carry forward and keep discussing more about the radio telescopes, a single dish which is the most common radio telescope to be used and the effect of design and observing strategies etc. So the first telescope which we would like to concentrate is one of the largest radio telescope single dish. It is 100 meters in diameter so from here to here it is about 100 meters and it is in Green Bank, West Virginia in the United States of America and this can point to any direction of the sky given the limitation of the elevation. The mechanical design allows it to only go down to some particular low elevation angles but apart from that it can point it has elevation encoder over here so gear and azimuth over here.

It can move this way and point in elevation in this way. It is kind of offset Cassegrain offset design because it is offset parabolic reflector so it is kind of it is not Cassegrain so it is not having a reflector which is coming back to the surface of the dish for the antennas but it is interesting thing is that the rays comes like this hits and then gets reflected and gets to the prime focus point. On the prime focus point it gets collected and then there are several feeds which are kept over here which collects the actual signal and finally sends it down through this rails over here and then finally down and to the receivers. So this is the more or less the design so let us see. Yes so these are the different structures so

number one is the primary reflector surface which is given by this one.

Number two is the reflector support structure which is below it. Secondary reflector is number four which is over here. Okay this is 100 meter in aperture that is all good. Number five it reflects to the counter weight which is over here. Number six is active some control link the shape of the surface is there.

Then the elevation bearing number eight so number eight reflects to the elevation bearing so that is kind of what points from here. Azimuth tracks is number 12 so which we discussed before already here and this is from the NREO single dish school lecture notes which we have received. So yeah that is kind of completes the schematic of this particular dish amazing dish 100 meter in aperture and points to very high precision in accuracy. Let us zoom in to this particular segment so if you do that you can see that there is this sub reflector over here instead of sending this back to the center of the dish it sends it to this particular assembly so it can point so over here all the different feeds are kept. So you can see some of the pictures over here like those dish so the again the geometry is the the the rays are coming hitting the dish getting up to here reflected and then coming down here so that is the that the more or less the path which is followed okay.

So once it comes and hits this sub reflector from the dish it then sends it down here to be collected over here. Okay and different frequencies are catered by this particular dish that will come later when we discuss about science later half of this this course but you remember this particular part. This is one of the structures of a radio telescope it is single dish 100 meter in diameter the largest fully steerable dish built so far the other 300 meter which is the Arecibo was fixed on the ground in the 500 meter also the FSD fast is also fixed on the ground so they are not fully steerable. The different dishes have different design this is another kind of design where you have the sub reflector the assembly of the all the elements of the receivers are here there are counterweights different types of assembly in the pedestal I've seen here also there's azimuth bearing which is not the like in in green man telescope the azimuth bearing was the real similar like the VLBA but in this case the azimuth bearing is a separate thing the pedestal is fixed they separate azimuth bearing over here and then the elevation is over here in this particular case elevation bearing is here and azimuth bearing is somewhere here as the more solid dish and so this is this quadrupole structures are different again for different dishes here this is structure is more or less coming down to the center near the center of the dish here it mostly comes up to the edge so different dishes have different structures and there is no unique design in that case depending on what kind of science you want to do good so a couple of things to keep in mind when we understand we said we have gone to the reflector design because it has more collecting area that means it will give to more

sensitivity however there are forces acting against it there are several inaccuracies which builds up because of the simply the reflector surface now depending on the wavelength you're observing the accuracy of the surface of the dish is very important okay like we have seen in GMRT the the design is like a mesh okay of wire where is in VLA the design is like absolutely fully covered it is fully solid and that depends on the highest frequency of observation if your wavelength goes to so little in in size for the highest frequency then you cannot build this kind of mesh design for GMRT the benefit is that the the advantage is that we have the highest frequency in L band or 1.4 gigahertz hence you can leave with the mesh design but really it goes up to 50 gigahertz so it's very difficult to maintain it so you have to build a solid surface now even when you build a solid surface what will be the accuracy of the surface to maintain a perfect parabola so there if there are deviations then those deviations gets reflected into the error in the signal so we say that the signals come gets reflected by the deal by the reflector and gets cut during the focus but supposedly it doesn't hit a perfect parabolic reflector then it will gather some errors okay so here is a easiest way to understand that so supposedly your best fitted parabola has a some error in the surface which creates an additional path length of epsilon okay for that the wave to travel additional distance of twice epsilon coming and going and so the error in the phase due to that is given by Delta which is given by  $2\pi$  by  $\lambda$  times twice epsilon so now you can see the Delta the phase change is inversely proportional to the lambda so as the lambda rises this phase change becomes negligible so for lower frequency it is lower for higher frequency it is higher and epsilon is the deviation okay so and that causes this relative power gain or power loss for this surface accuracy okay now so a rough estimate shows that the surface errors must be an order of magnitude smaller than those the shortest usable wavelength so it should be an order of magnitude smaller than the shortest usable wavelength so supposedly the our for GMRT the highest the largest wave the smallest wavelength is 21 centimeter that basically means that you can you can assume an inaccuracy of two centimeters up to that okay similarly supposedly for Green Bank telescope hundred meter in diameter the frequency can go up to high as high as hundred gigahertz or lambda minimum as three millimeter if that is the case then Sigma cannot exceed so another estimate is that the minimum Sigma should be lambda minimum over 16.



## Reflector Accuracy Requirements

- Where the actual reflector surface deviates from the best-fit paraboloid by a distance  $\epsilon$  the path length of the reflected wave will be in error by almost  $2\epsilon$  and the phase error  $\delta$  (radians) of the reflected wave

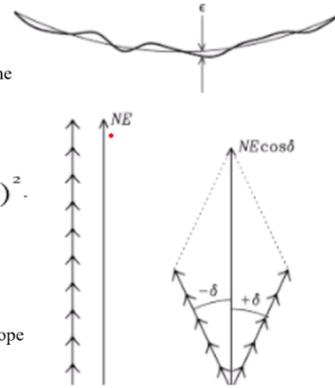
$$\delta \approx \frac{2\pi}{\lambda} (2\epsilon) = \frac{4\pi\epsilon}{\lambda}$$

- So, the relative power gain is  $\frac{G(\delta)}{G(0)} \approx \left[ \frac{E(\delta)}{E(0)} \right]^2 \approx 1 - \delta^2 \approx 1 - \left( \frac{4\pi\epsilon}{\lambda} \right)^2$ .

- This rough estimate shows that the surface errors must be an order-of-magnitude smaller than the shortest usable wavelength, a severe requirement indeed.

- A traditional rule-of-thumb for the shortest wavelength  $\lambda_{\min}$  at which a radio telescope works reasonably well is

$$\sigma \approx \frac{\lambda_{\min}}{16}$$



Credits: Essentials of Radio Astronomy, By Condon and Ransom



The 100-m diameter GBT is intended to operate at frequencies as high as  $\nu \approx 100$  GHz, or  $\lambda_{\min} \approx 3$  mm. To meet this specification, the RMS deviation from a perfect paraboloid must not exceed  $\sigma \approx 3 \text{ mm} / 16 \approx 200 \mu\text{m}$ ,

okay so roughly okay it's it's a bit of a back of the envelope calculation so trust us that is roughly the estimate you're looking for so Sigma cannot exceed three millimeter by 16 so 200 micrometer that's quite a huge accuracy 200 micrometer accuracy on the dish is quite large a requirement so that's why they have this active control of the shape which can put into the shape for the Green Bank telescope but anything else like there are Alma telescope operates at very high frequency same kind of accuracy is also required for for that okay so the next error which also arises from the single dish measurements is due to the pointing accuracy or inaccuracy so for a real telescope the the perfect accurate pointing is not known because this is a large structures you know you have see you see that there is this the entire feed is sitting on the center prime focus so he has the antenna rotates moves in different directions as a function of elevation angle this these things can move a little bit only slight movement is required to produce a huge error in the face why face because any movement from the ideal position of that parabolic reflector and the focus will create an additional path length to travel like same as the case for the surface accuracy any additional path length to be traveled by the signal to me to get recorded in the receiver is corresponding to gives rise to additional phase which will create an error okay and so that is the basic concept now so pointing accuracy is inbuilt you know inaccuracy sorry is inbuilt because you have a large structures and they keep moving and pointing in different directions and so little bit of a change in the relative positions of the feed and the reflector is quite possible not unknown okay even if you keep the for casagrand antenna if you keep all the antennas all the receiving things at the center of the dish and below it still the sub reflector itself can move okay so in a sense I mean yes if you want to reduce pointing accuracy in accuracy then probably Casagrand has a better chance of doing that than the prime focus the prime focus you're putting the entire

weight of the thing over there which has a which can create larger pointing errors so yes but pointing errors also is grows as a function of the half power beam width because if your beam is quite large then the pointing error can be you know can you can leave with a little bit higher pointing in a in accuracy or errors but if your power power beam width is narrow and if you're going to higher and higher frequency then this pointing lesser pointing in accuracy also can be result in a bigger error etc so let's say if you have a tracking error 1d tracking error in each coordinates like azimuth and elevation because you have to add two gears azimuth and elevation so suppose you have a tracking error you have an error because of the the motors are not working perfectly they're not encoded properly and so they have been moving slowly or they're bit behind they have some errors so that can be rest of tracking errors so that resultant tracking error for each direction is coming given by this which will result in a RMS error of this Rho square averaged so the on source gain can be given in terms of this G by G naught so ultimately the Omega s for that for the entire tracking error can be summed up as Omega s over s equal to Z over 1 plus twice C square root of that where Z is given by 4 ln 2 sigma 2 by theta power beam width square now let's say we can calculate the largest tracking error in arc second compatible with making flux density measurements with 5% RMS error using GBT 100 meter telescope at a frequency of 33 gigahertz so for the frequency 33 gigahertz first thing to be calculated is the half power beam width that comes out to be 23 arc seconds with 23 arc seconds



**NPTEL**

## Pointing-Accuracy Requirements

Credits: Essentials of Radio Astronomy, By Gordon and Barvain

Real radio telescopes don't have perfectly accurate pointing. Small errors in tracking a target source reduce the gain in the source direction and contribute to the uncertainty in flux-density measurements of compact sources. Tracking errors are just as important as surface errors in limiting performance of large radio telescopes.

- In terms of the beamwidth between half-power points  $\theta_{\text{HPBW}}$ , the relative gain at a point offset by angle  $\rho$  from the beam axis is
 
$$\frac{G}{G_0} = \exp \left[ -4 \ln 2 \left( \frac{\rho}{\theta_{\text{HPBW}}} \right)^2 \right]$$
- If the 1-D tracking error in each coordinate (e.g., azimuth or elevation angle) has a Gaussian distribution with rms  $\sigma_1$ , the tracking error  $\rho$  in two dimensions has a Rayleigh distribution
 
$$P(\rho) = \frac{\rho}{\sigma_1^2} \exp \left( -\frac{\rho^2}{2\sigma_1^2} \right)$$
- The mean squared tracking error is
 
$$\langle \rho^2 \rangle = \int_0^\infty \rho^2 P(\rho) d\rho = 2\sigma_1^2$$
- The rms value of the two-dimensional tracking error is  $\sigma_2 = 2^{1/2}\sigma_1$ , so small tracking errors reduce the average on-source gain by the factor
 
$$\langle G/G_0 \rangle = \left[ 1 + 4 \ln 2 \left( \frac{\sigma_2}{\theta_{\text{HPBW}}} \right)^2 \right]^{-1}$$
- The fluctuating on-source gain caused by tracking errors contributes a fractional uncertainty
 
$$\frac{\sigma_S}{S} = \frac{z}{(1 + 2z)^{1/2}}, \quad z \equiv 4 \ln 2 \left( \frac{\sigma_2}{\theta_{\text{HPBW}}} \right)^2$$

We can calculate the largest tracking error in arcsec compatible with making flux-density measurements with 5% rms errors using the GBT 100-m telescope at  $\nu = 33$  GHz.



$$\theta_{\text{HPBW}} \approx \frac{1.2\lambda}{D} = \frac{1.2 \cdot 0.0091 \text{ m}}{100 \text{ m}} \approx 1.09 \times 10^{-4} \text{ rad} \approx 23 \text{ arcsec.}$$

- Thus, the total tracking error must be smaller than  $\sigma_2 = 0.14 \times 23'' = 3.2''$  in azimuth angle and  $\sigma_1 \approx 2^{-1/2}\sigma_2 \approx 2.2''$  in elevation angle.
- Thermal changes by 1°C in the steel GBT support structure could produce a  $\approx 2''$  pointing shift.

we then go back and calculate this term called Z and that then gives you the Sigma 2 Sigma 1 from here so Sigma 2 from here and Sigma 2 then goes to this point 1 4 times 23 arc seconds which is 3.2 arc seconds given the relationship between Sigma 2 and

Sigma 1 given by Sigma 2 is square root of 2 Sigma 1 we can derive the Sigma 1 in this case which is about 2.2 arc seconds in elevation angle so yeah that is that is quite quite a small error giving rise to 5% RMS error in flux also should be noted is that just by thermal expansion of the steel supports for GBT that can one degree Celsius variation can lead to two percent to arc second pointing shift in the structures so overall I mean that basically says if you want to have an accurate measurement you need to have a pointing calibration done so you need to focus on a known source see how off that is from the center because source have a structure and so does the far field power pattern and so they have to be deconvolved out and understand that is it is it pointing at the right place at the right time or if it is not then correct for the model from the observation and then you reduce this error because if you are observing at a higher frequency and with a large structure like GBT pointing calibration is necessary same thing prevails for other telescopes like for very high frequency observations like ALMA or even with JVLA it's required to pointing calibration is very essential for GMRT also it is done but not as regularly throughout the observing cycle observing schedule as for the higher frequency observations because we are dealing with a little bit larger beam and so it's the inaccuracies are tolerable another important term to note this is not an error but it is a contributed to the something called a system temperature and this is the SEFD or the system equivalent flux density so that is given by  $2 \eta' k T_{sys}$  over  $A_{eff}$  and that the terms are quite known  $A_{eff}$  is effective area of the instrument in this case the radio telescope and this is the flux density to be detected SEFD this is the system temperature sorry and  $\eta'$  is the overall system efficiency which is a sum over all the inefficiencies built in so in other words as for a source of flux density  $S$  to be detectable the SEFD must be reduced by square root of bandwidth times the time stamps and so the  $S/N$  is over  $N$  signal to noise ratio is given by flux density

$$SEFD = \frac{2 \eta' k T_{sys}}{A_{eff}}$$

$$S/N = \frac{S \sqrt{Bt}}{SEFD}$$

of the source multiplied by the square root of bandwidth times time over SEFD okay this is one thing to be remembered about that you have to make the SEFD reduce that we increase the signal to noise the noise due to reception of radiation from the ground is smaller it is possible at shorter wavelength the secondary focus is preferred when you have a smaller higher frequency or smaller wavelength instead of a prime focus having a secondary mirror and a Cassegrain secondary reflector and a Cassegrain kind of a setup is

preferred because the noise due to the reception of radiation from the ground is smaller in the prime focus case your antennas are looking towards the ground so there can be spillover which we mentioned earlier in a diagram but for a Cassegrain setup it is looking up so chances of the ground leaking in is less it is possible to accommodate more receivers for large multi-beam systems field of view that is the region of the sky that can be accurately measured is larger that's not always the case but it is possible for to correction to correct for large-scale deformation of primary reflectors by deforming the sub reflector appropriately so there are a little bit of advantages and so as the frequency goes to higher and higher the wavelength goes smaller so that these inaccuracies of pointing the surface everything gets built up very fast so you can have a prime focus feed for lower frequency like 1.4 gigahertz or even over 2 to 3 gigahertz but as you go to higher and higher frequency 50 gigahertz 100 gigahertz having a secondary reflector is a better design than having a prime focus feed that's what essentially we meant single dish radio observations basically basic consideration for any observations made by a single telescope generally refer a single dish observation that's the definition the simplest type of observation you can make is to measure the power of the radiation coming from a source and entering in the antenna as we discussed earlier that the convolution of the beam primary beam with the source is what the resultant brightness which we observe from the telescope an equation that relates the power collected by the telescope to the flux density  $f_{\nu}$  I must note that we we keep changing  $f_{\nu}$   $s_{\nu}$  etc and apologies for that so here  $f_{\nu}$  determines the the flux density of the source okay so the  $P$  is equal to  $f_{\nu}$  times  $A_{\text{eff}}$   $\Delta\nu$  so  $f_{\nu}$  is the flux density

$$P = F_{\nu} A_{\text{eff}} \Delta\nu$$

of the source a effective is the area of the telescope and  $\Delta\nu$  is the bandwidth over which detection occurs observation strategy is pretty simple point to discover the source note the power measured by the so receiver and use above equation to calculate the flux density okay so that's how simple this is but there are lots of details like the transferring of power to a physical unit of flux density and request also some calibration calibrations are done with blank sky with internal loads etc etc and that gives rise to the real measurement of the of the power and translating that to the flux density but in broader perspective this is a very simple observation you point the telescope to the sky to the source we want to measure the power the flux density of and you simply observe it now as you know the source is not going to be static in the in the in the sky so source keeps on moving so depending on how long you need to expose to the source depends on how bright the source is and what is your SCFD of the telescope supposedly you do not have enough sensitivity then you have to keep observing for a longer time that means you may have to keep tracking the source on the sky for a longer time so for some sources you may just give a snapshot just look at the source and you are done for some sources you

may have to keep tracking for one one day maybe you have to keep tracking for the next day and the day after depending on how faint the source is you need more integration time to to resolve to to get the signal to noise ratio higher okay so that's the thing but nevertheless the just pointing to the source and tracking the source is all what is required now tracking the source is all what is done by that azimuth and elevation encoders which we discussed earlier for a single digital let's go and same for the interferometer so let's discuss about the basic measurement strategies so the first one which comes automatically is the strategy of switched observations select a pin yeah so the process to determine the power emitted by an astronomical source involves subtracting the contribution of the receiver noise okay so say we have these two positions on and off now we will come multiple times on this on and off and there are different you know meaning of that on and off okay so this time the on and off means in the on position the telescope is looking towards the sky so the the voltage which is recorded by the telescope is some multiplicative terms of the gain mostly the bandwidth and the antenna temperature which is a proxy for the sky temperature and the system temperature of the antenna of the telescope that's the on position the off position is on the other hand it is purely looking towards the receiver only and not looking connected to the sky so why these are done because we have a system temperature which is also not very well calibrated now if you want to really understand the the antenna temperature from these measurements we have to remove the system temperature from the equation so we need two equations because there are two unknowns  $T_A$  and  $T_{sys}$  there are many unknowns but I think we have some handle on the other two unknowns of  $\lambda \times G$  sorry  $\alpha \times G$  where  $\alpha$  is the responsivity of the detector and  $G$  is a dimensionless total receiver gain  $G$  is the Boltzmann so  $\Delta \nu$  is the bandwidth which is given so if you know the  $V_{on}$  and  $V_{off}$  voltage you can determine the  $T_A$  from there.

$$\begin{aligned}
 V_{on} &= \alpha G k \Delta \nu (T_A + T_{sys}) \\
 V_{off} &= \alpha G k \Delta \nu T_{sys} \quad \longrightarrow \quad V_{on} - V_{off} = \alpha G k \Delta \nu T_A
 \end{aligned}$$

okay that's a basic measurement of a single dish observation determining of the system temperature we determine the system temperature by different methods you can have a noise diode in the system which has a definitive calibrated signal or voltage and measurement and off when inactive are obtained so you have a calibrated signal from the noise diode given when in the on position and off position you have a just the  $T_{sys}$  so if you do that you can calculate the  $T_{sys}$  in terms of this often cal position also okay this is further you know derivation of the system temperature itself so moment you know system temperature you can actually deal with only one on position for the calculation and just put in plug that this is in here in here and you can get the value of the  $V_{on}$  sorry that the  $T_A$  okay then measurement of antenna temperature that's a third way so we have

two system so we have on and off so that this essentially gives you the V on and V off take the ratio of that to get to calibrate the gains you know the system temperature anyway from the Tcal so you can take care of the T antenna from this V on and V off measurements that's another another way to to look at things

$$\rightarrow \frac{V_{\text{cal}}}{V_{\text{off}}} = \frac{T_{\text{cal}} + T_{\text{sys}}}{T_{\text{sys}}} \quad \longrightarrow \quad T_{\text{sys}} = \frac{V_{\text{off}}}{V_{\text{cal}} - V_{\text{off}}} T_{\text{cal}}$$

so just to summarize again because it may look confusing so in the first one we assume that the gain the gain is very well known so you just get rid of the system temperature by this having two measurements because there are two unknowns this thesis and ta okay so you can do this on and off measurement when on is to the sky and off is towards the load and you can do the measurement next is you can also have an independent handle on the system temperature by connecting to a calibrator source internal calibrator source or noise diode and then you can measure measure the thesis from there if you measure the thesis from there then you can go back here and put it and also measure the V on and ta separately another thing is you can do a ratio of V on and V off so what happens is it cancels out the the the gains and what remains is it just retains the ta thesis and T ta and thesis as a function of on and off now if you already know thesis from the T calibration measurement then you can put it there and you can get a TA clearly okay that is also possibility very often another thing happens is that you point towards the source but the source may not be only source which is filling this entire your beam so you may have to point on and off to different direction so on meaning when the source is in your beam and off position is a standard position where there is no source okay and you keep on nodding between these two position to get a reference signal only for your source and subtract the background from the source so various such thing will happen and we will discuss this more when we discuss science done by single dish telescopes later on okay single dish telescopes typically you are just collecting the entire power collected by a single telescope and it's like a single pixel camera right so you don't have the image basically shows one pixel or one power value for the entire sky which is inside the beam now but a single pixel also you can use to map the sky how do you do it you keep on pointing different position at different time supposedly you have this background source a supernova remnant and you can keep on just you can keep on pointing at different location in a raster scan mode and you keep doing this on the fly mapping technique basically this is how the sardinian radio telescope operates and makes an image and we have just borrowed it from there so in a sense if you do this kind of cross scans in in in pointing fashion so for every point you get a value of the amplitude rate and so you can put them up in a in a stack them up in an image form and you can get the image from the even from a single dish observation but for images of course the interferometers are

much better and we will talk about it later so uncertainty in the measurement temperature of course there is uncertainty lots of uncertainties are there and like when you first we have shown the radiometer expression in the in the last week's lecture where we if you take the first round your your your variation across the spectrum is looks very very coarse but if you keep on taking a to get intricate over multiple multiple runs your your this noise reduces same exactly what you are seeing over here the first one is without averaging second one is with averaging and you can see look at this area and you can see the noise have reduced it was so large before it has gone down significantly okay however the signal itself is is more strengthened so signal to noise ratio have improved okay the best place to go back for the antenna temperature errors is going back to the radiometer equation where you have the error on the TA is linked with the system temperature and it can be reduced by the more number of times of observations and the bandwidth of observations antenna beam is very important

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$$\sigma(T_A) = \frac{T_{\text{sys}}}{\sqrt{\Delta f \Delta \nu}}$$

because it again it comes down to the thing that your full width half maxima is defining your field of view and that's your main beam but field of view defines what is your source if it is within your field of view then that cannot be resolved by this by this particular thing there by single dish but supposedly a source is extended like that and you are having your individual pointings as something like pointing are like this like this so your single single pointing field of view is like this so then you can get multiple such beams and you can actually image and resolve this particular source but supposedly your your entire pointing or entire field of view is like this then you probably will get a single measurement for that and cannot be able to resolve the source so then field of view basically is a very important topic quantity for designing an experiment with a single dish so whether you can image or not that a particular source depends on the field of view okay beam pattern we have already discussed in details the beam pattern has a for for our aperture beam pattern goes to sine squared a sinc square function the primary beam pattern for a uniformly illuminated thing and this is also with for discussing over here what some few things follow so this is already what we have just let me change the color yeah so this is the this is already via some expression we have gotten from the previous lecture where we define the primary beam the five-field power pattern of the antenna

$$P_{\text{bm}}(\theta) = \frac{\sin^2 \left[ \left( \frac{\pi D}{\lambda} \right) (\sin \theta) \right]}{\left[ \left( \frac{\pi D}{\lambda} \right) (\sin \theta) \right]^2}$$

which has a uniform aperture illumination and from there you can integrate and you can get the beam solid angle or antenna solid angle from there because your P beam

$$\Omega_A = \int P_{\text{bm}}(\theta, \phi) d\Omega \quad \text{where:} \quad \Omega_A = 2\pi \text{ sr.}$$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

$$\Omega_A = \frac{\lambda^2}{A_{\text{eff}}}$$

is known so you can define that and get the beam solid angle from there so if you get that then the beam solid angle also from the antenna theorem is related to the lambda squared over a effective that is it depends on mostly on the the wavelength of observation as well so if you know the the main beam is linked to this expression of the theta the power collected by the main beam is linked with the measurement of the theta full width half maxima or H p w they are the same and so the solid angle of the main beam can be gathered from the measurement of the theta f w h m or theta H v p w by using this particular formula

$$P_{\text{main}}(\theta) = \exp\left[-4\ln 2\left(\frac{\theta}{\theta_{\text{FWHM}}}\right)^2\right]$$

$$\Omega_{\text{main}} = \frac{\pi}{4\ln 2} \theta_{\text{FWHM}}^2$$

we have already mentioned that for we have mentioned and showed that for the aperture the theta f w h m is proportional to lambda over D or 1.02 times lambda over D,

$$\theta_{\text{FWHM}} = \frac{1.02\lambda}{D} \quad (\text{rad})$$

for particular aperture illuminations uniform illumination is not necessarily desirable as we discussed before also because it can result into large speed of world losses so we mentioned in the in this previous lecture about uniform illumination pattern and that's a very simplistic and it gives you a good handle on all the concepts but in practically uniform illumination is not very desirable because as illumination goes till the edge of the telescope that the dish so it is chances of spillovers is pretty high a tapered distribution is more desirable that's why we showed there the second case after the uniform illumination a tapered illumination pattern same here H tapered illumination is more desirable and that's a better case than the uniform illumination now let's take our case case study consider observing at a wavelength of 21 centimeter with a radio telescope of diameter 2 meter as in this small telescope in his tech the angular resolution of this observation is 1.15 lambda over D so this is the angular resolution for that so that comes out to be 6.9 degree that's the HP BW so it can differentiate between two sources which are 7 degree apart so this is but this is almost 14 times the angular diameter of a moon of the moon so if there are where two radio sources separated by say eight times the moon's diameter in a simple observation with this telescope at this wavelength you will not be able to determine that radio emission was coming from two distinct sources just a case study so if you have a two meter class telescope then it will not be able to separate out even if they're separated by eight times the diameter of the moon distance it should be clear that this antenna at this particular wavelength cannot resolve any structure of the moon and even determine its angular size so yeah that is the limitation so so certain telescopes are meant to do something and not made to do others so that's what all it means now let us consider a short radio wavelength say for example one centimeter and a large telescope of diameter of 100 meter like the green-bent telescope the angular resolution for such an observation is about 1.15 times this ratio which comes out to be 24 arc seconds

What is the best resolution obtainable with a single radio telescope?

- Let us consider a short radio wavelength say  $\lambda = 1$  cm and a large radio telescope let us use a diameter of 100 m (the diameter of the Green Bank Telescope).
- The angular resolution for such an observation is  $\sim 1.15 (0.01 \text{ m}/100 \text{ m}) \sim 0.000115$  radians  $\sim 0.0066^\circ \sim 24$  arcsec.
- The same telescope, operating at 21 cm, has an angular resolution 21 times poorer; however, if it could operate at the upper end of the radio window,  $\sim 1$  mm, then the resolution would be 10 times better.

so that's the HP BW of this green-bent telescope at one centimeter wavelength so the same telescope operating at 21 centimeter will 21 times poorer than this resolution so in some sense that what we're hinting is that even at the largest telescope fully steerable 100 meter class also having 24 arc second resolution at a one centimeter wavelength so at a lower wavelength higher wavelength sorry at 21 centimeter it will have 21 times poorer it's almost 400 arc seconds or 440 which is much higher almost like six arc minutes kind of a thing we are seeing so even though telescope beam becomes pencil beam like single dish may not be the most desirable you know telescope type to observe or having the highest resolution because at some point the for certain wavelengths the lower the higher wavelength part the size becomes impractical and quite costly to build and to maintain there is something called main beam efficiency for say for example regard related to the pointing accuracy the main beam efficiency

$$\eta_{mb} = \frac{\Omega_{main}}{\Omega_A}$$

$$\eta_A = \frac{A_{eff}}{A_{geom}}$$

is also very important so that the power collected by the whole telescope versus the power collected by the main beam itself and if that this ratio of the power collected by the main beam over the power to control power character by the entire antenna is is higher that means the directivity is higher that means the sensitivity of the particular telescope is higher towards that particular direction so if you if you do the math if you keep following because we have been discussing this multiple times you can see that this sigma main is less than equal to  $0.97 \lambda^2$  by a effective is is what we get if we have 10 DB H taper illumination so the efficiency parameter for this main beam can be defined as this as I said the sigma of the Omega of this sorry Omega Omega main which is the solid angle of the main beam and over the solid angle of the entire antenna so that is the the main beam efficiency it's also linked with the aperture efficient there's another thing called aperture efficiency and that is linked with the effective area over the geometric area okay so that's the two things and so if you plug in some numbers for this main beam and the solid angle and the antenna efficiency aperture efficiency we get that the ratio of the main beam efficiency and the aperture efficiency is about 1.2 so now we know that the power collected by a single source with the flux density of  $f_{nu}$  and a

bandwidth of Delta nu and effective area of a effective is given by P in addition to that we know that the main beam can have its own field of view so that when we desire when we calculate the power collected by the antenna we convolve the intensity of the source by the beam pattern of the antenna and it's a convolution and so actually Delta P becomes this if we integrate over the whole sky then we for different directions then we finally end up having the power pattern is is given by this for at least to the integration over the sky for polarization also if you consider my polarizations then you have a half factor coming in front we'll talk about pollution later when we talk we will again discuss it further and so finally you can use the the power and temperature relationship to get the effective antenna temperature in terms of the power and so P is k T a times Delta nu and so Delta nu goes away and T a becomes a 50 over 2k times that integration

$$P = \left\{ \int_{\text{whole sky}} I_\nu(\theta, \phi) P_{\text{bm}}(\theta - \theta_0, \phi - \phi_0) d\Omega \right\} A_{\text{eff}} \Delta\nu$$

$$T_A = \frac{A_{\text{eff}}}{2k} \left\{ \int_{\text{whole sky}} I_\nu(\theta, \phi) P_{\text{bm}}(\theta - \theta_0, \phi - \phi_0) d\Omega \right\}$$

$$F_\nu = \frac{2k}{A_{\text{eff}}} T_A$$

$$\Omega_{\text{main}} = \frac{\pi}{4 \ln 2} \theta_{\text{FWHM}}^2$$

$$\langle I_\nu \rangle = \frac{2k}{A_{\text{eff}} \Omega_{\text{main}}} T_A \xrightarrow{\text{By Substituting the value of } \Omega_{\text{main}}} \langle I_\nu \rangle = \frac{2k}{\lambda^2 \eta_{\text{mb}}} T_A$$

observing resolved versus unresolved sources so if as I said they've the half-power beam width is is smaller than the distance between the two sources then you can resolve the two sources or resolve even the single source and and see the structure of that particular source if it's an extended one if the main beam size is larger than the structure of the source then it will be as a unresolved source so yeah so I think finally we get the factor of a F nu in terms of antenna temperature and if the antenna temperature is for a entire source it will integrate over the entire source for a resolved source and we collect the information of the antenna temperature from there once we do that we can we can derive the flux density of the source in terms of the antenna temperature by using this formula the calibration factor is often expressed in units of Kelvin's per Jansky

$$G = \text{DPFU} = \frac{A_{\text{eff}}}{2k}$$

$$\sigma(F_\nu) = \frac{2k}{A_{\text{eff}}} \left[ 2 \frac{T_{\text{sys}}}{\sqrt{\Delta t_{\text{obs}} \Delta \nu}} \right] \quad F_\nu(\text{min}) = \text{SNR} \left( \frac{4k}{A_{\text{eff}}} \frac{T_{\text{sys}}}{\sqrt{\Delta t_{\text{obs}} \Delta \nu}} \right)$$

$$\text{SEFD} = \frac{2k}{A_{\text{eff}}} T_{\text{sys}} \quad F_\nu(\text{min}) = \text{SNR} \left[ 2 \frac{\text{SEFD}}{\sqrt{\Delta t_{\text{obs}} \Delta \nu}} \right]$$

and is sometimes called degrees per flux unit or DPFU so DPFU is nothing but this proportionality constant if you can say between the antenna temperature and the flux density of the source which is basically the effective area of the antenna over twice Kelvin Boltzmann constant for resolved source the situation is a bit complicated because that is for the unresolved source standard temperature and the flux density are written by that for a resolved source there has to be a half take into account the main beam solid angle as well as a source solid angle okay both has to be taken into account so the finally the I knew the the source comes out in terms of the antenna temperature and the main beam efficiency lambda square etcetera given by this for a just to extend for different cases which arises during the practical observations for a source which fills the sky any form source you have this finally the antenna temperature which is given by  $T_a$  and multiplied is equal to the effective area over lambda square times sigma  $\Omega_a$  so then total solid angle is considered because that's the of the antenna and  $T_B$  which is the the brightness temperature of the source so that is the another case there are several cases where brightness temperature versus the internet temperature beam dilution so if a source for an unresolved source there is an interplay between the source structure and the antenna solid angle and these two interplays and that has to be corrected properly from the measurement in order to understand the actual source temperature for that particular part of the source in terms of a temperature okay so that's how we get so the  $T_a$  this decrease in  $T_a$  relative to  $T_B$  is referred to as the beam dilution because the source solid angle over the antenna solid angle if the source solid angle is is lower than the antenna solid angle then the brightness temperature of the source may appear to be larger than actually the measured antenna temperature okay in addition for spectral line observation so if you're not observing not looking only for the amplitude or the flux density of the source but also looking for spectral response so suppose you have a atomic transition line like for the hydrogen hyperfine transition from 1s to 2s at 1.4 gigahertz that's a specific line at a given frequency so you want to observe those lines spectral line observations then the spectrometer has to be configured and you have the bandwidth of observation is important the central frequency has to be tuned because you want to keep the location of the spectral line or you know which you propose to target has to be in the center of the band and spectral resolution has to be such that you resolve the entire spectral line into several such channels we will come to this later on also when we discuss about spectral line observations both with the interferometry as well as single dish so but it's needs to be discussed here for completeness because yes if you are looking at a spectral line then the number of channels spectral channel has to be sufficient to to the re construct the spectral line the center of the spectral line of the band

should be configured such that the expected spectral line should show up in the center of the band because sometimes there are other most of the time other things has to be done like continuum subtraction etc which requires you to have sufficient channels on either side of the band on both side of the spectral line to be subtracted to get the line with more significance these are just a few more recipe for getting a good spectral line observations.

Optane radio images of course you have a single pixel camera with a single telescope and you can do the raster scan as we discussed there are other methods to increase this area of the coverage and having multiple pixels so if you can think of something like having a instead of a single feed you have a array of feeds at the focus it's also called a focal plane array we will be discussing it in in weeks to follow more and but that that's like if you have a three by three focal plane array that means you have a three nine pixel camera kind of at the focus and so that gives you more more field of view and more resolution on the on within that field of view. There are single dish observations have relatively small number of antenna temperature measurements often just one measurement per pointing direction then imaging using single receiver telescope involves pointing observations moving and repeating move the telescope to a new position and then again repeat the observation process covering the desired sky area converting antenna temperature so convert measure measured internal temperatures at all positions into a comprehensive radio source map. These are all already discussed and there are imaging challenges of course you can do raster scan and make an image but that image won't be as as good as the image which you will get from a radio interferometer and the reasons are obvious but we will definitely cover both the aspects of imaging when we discuss the interferometers and the advantages and disadvantages of each techniques. So you have antenna temperature which is kind of again is a measure of the source intensity convolved with the beam pattern far field beam pattern and that's the final measurement of the internal temperature different given the different size of the of the primary beam pattern given different size of the sources you can have different effects of the convolution between the source structure and the beam in this particular case we haven't determined the size of the source itself but given the the thing and different beam the result can be different for different telescopes. Yeah just elaboration of that so we can keep on doing it in in detail for different shape of the beam for different shape of the source itself you can do that and finally to deconvolve the beam from the measurement to be able to understand the source temperature and source structure clearly.

To calibrate the telescope we have already pointed out in the beginning of this lecture that a lot of issues happen because of pointing so pointing correction has to be done you need to have to choose a particular pointing source to observe that and correct from it

details are given over here. Calibration of the gain is necessary to get the gain out from the measurement of the antenna temperature effective area also has to be measured and the gain curve as a function of frequency elevation or different antennas have different as our telescopes have different responses or the gains as a function of elevation frequency temperature ambient temperature it changes so you need to understand the gain as a function of all these different parameters before you can correct from the ongoing observations. Again measurement of the beam pattern and the main beam efficiency is also very vital and those are has to be known a priori to be able to get a sensitive observations and also this radiometer equation at the end we can we have revisited it multiple times so you need to know the system temperature quite well you can calibrate it as we have shown earlier and you can derive this this RMS noise or sensitivity measure for the given telescope. If you know the sensitivity of telescope you can derive the minimum flux density which can be measured by your telescope in terms of the SCFD and that's also we have discussed earlier and so this is just for the completeness. The different types of polarization also or the astronomical sources and different telescope beam have their own polarization characteristics so they have to be calibrated out different hardware's different feeds have different effects all those can be done and so we come to the end of another lecture and this is the last lecture of this particular week we discussed in depth how a single dissol observation works we tried to cover the essentials of how the calibration is done between the so that we can get an accurate measurement of the source properties like the flux density the measurement of the of the source size the structures and how to deconvolve out the primary beam characteristics of the antenna telescope itself to get the accurate measurement of the source in the sky.

However I think things will become more clear when we take an observation into account and explain this more so that will come to the next half of the after the interferometry is done we will talk about the science with the telescopes there we will take a particular use case and discuss that to make these concepts more clear so some of these are a little bit repetitive some of these concepts are here for completeness so if you are feeling overwhelmed by all of this information don't worry we will come back and use it and show it how it is used for actual observations in later part of this in this course. Again this is a reference list we do prepare our notes and the slides based on existing literature and we try to reference them when they are used on that particular slide but if we have missed I apologize for that this is a disclaimer for that and so I hope you like the lecture and thanks for joining see you in the next lecture next week thank you bye