Radio Astronomy

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Lec-15

Radio Telescopes

Hello and welcome to this week 5th of the lectures on radio astronomy. This is the first lecture of week 5th and we will be starting our discussion about radio telescopes. In the past we have discussed about different kinds of antenna. So we continue in that direction for this particular lecture as well. If you remember we discussed about different wire antennas to begin with like simple dipole antenna. We even showed derivations of the dipole antenna's far field radiation pattern.

We also defined different zones of radiation starting from the near field reactive field zone to intermediate Fresnel zone and also to the far field or the front of the zone. Simple antennas are like simple wire antennas a dipole a monopole a Yagi Uda to begin with here in the Yagi Uda was very popular antenna for the TV sets before the dish TV or similar came across and so you can see the three directors and this is the one is the feed over here and the last one is the reflector. Okay this is still used for certain purposes like the ST radar facilities they uses the similar Yagi Uda antennas. Now we also mentioned that earlier that in order to because a single dipole is restricted to the particular wavelength so the bandwidth of the antenna is remains restricted.

To increase the bandwidth of the antenna we sometime resort to designing of this log periodic dipole array and that basically has different such dipoles at different wavelengths corresponding to and the entire different lengths correspond to different wavelengths. So the largest one is the shortest frequency that's the lowest frequency and the smallest one corresponds to the highest frequency that defines kind of band within which LPDAs work. Very popular antennas and a version of LPDAs is also used as a part of the SKA1 flow design we will see later on. So aperture antenna is the next kind first was used by the major discovery of the CMB cosmic microwave background radiation was done using one such aperture antenna where Arno Penzias and Robert Wilson when they were trying to observe a radio source they found that they are receiving a similar kind of noise coming from every direction not just a particular direction and that's how they kind of hinted in the discovery of CMB microwave background radiation which kind of gave an evidence for the big bang theory. The other types of different types of aperture antennas includes horn antennas which present transition or a matching section from the guided mode inside the waveguide to the unguided free space mode outside the waveguide.

So it's kind of a transition between the kind of what precedes is a transmission line and what comes out is the expanded one where you open up the angle and the wave starts going into free space. So the E-plane sectoral horn is a type of horn where the flare is in only one direction E-plane and H-plane similarly is the sectoral horn where the flare is in only around the H-plane whereas the pyramidal horn typically used flares in both E and H-plane which forms like this one.

For any horn antenna the HPBW or the beamwidth and Gain can be approximated by

$$\theta_H = \frac{67}{a_{1\lambda}}, \qquad \theta_E = \frac{56}{b_{1\lambda}}, \qquad and \qquad G_0 = e_{cd}\left(\frac{4\pi}{\lambda^2}a_1 b_1\right)$$

We have a particular another variance of this pyramidal horn where you can use the conical horn where instead of this rectangular flaring you can have a conical flaring and that defines a conical horn. In relation to the radio astronomy instead of focusing on how the antenna will radiate and work we are more interested to know about how the power pattern looks like or what is the half power beam width to resolve the sources. What it means is that basically we are definitely interested in the knowledge of the antenna but mostly what defines is the half power beam width which defines our field of view and the power pattern which defines the gain of the antenna in the receiving mode in the far field.

So they are very important and further discussions will follow in that direction. The other antennas which we discussed earlier also is planar antenna. I am just covering it again for completeness and there are mostly the micro-steep or patch antennas which are very popular used in different applications like mobile communications etc. In radio astronomy also it is becoming a bit popular nowadays particularly in respect to the space-based radio astronomical applications but for the ground typically we use more like the horn antennas, the conical antennas etc. are more used than the planar antennas.

So all those have been discussed before and what we haven't discussed earlier is a type of antenna which is very popular in radio astronomy is a reflector antenna. Now before we start discussing the reflector antenna one thing we have to understand is that an ideal radio telescope should have a large collecting area because ultimately we are detecting very very faint sources from the sky. So unless the collecting area is large the sensitivity of telescope won't be sufficient to detect the faint signal. Remember we discussed about the signal to noise ratio how the signal has to overcome the barrier of the noise and the SNR has to be much much higher than one in order to detect the source with significant confidence. So that is the entire requirement from the typical radio telescope design.

So one way to do that is kind of starting with collecting area. So far what we discussed the horn antennas, the wire antennas like dipoles they're active antennas. Here what we use is simply those kind of antennas but we put it on top of a passive reflector. Okay and what happens is that we have a larger collecting area that is typically kind of either in the receiving mode they will be coming from the sky hitting the surface of the reflector and then reaching the central region where the feed is kept or in the transmit mode we can assume that there is a centrally placed element which radiates and that will be this reflector will produce a directivity like much like the effect the director has. So in this context we can go back and relate us to this effect of the Yagi Uda where you have a reflector at the end a single reflector just assume that the reflector antennas are in for the reflector antennas this reflector is expanded to two dimensions sometimes to three dimensions also we will we will check out those.

$$G_f(\phi) = 2\sqrt{\frac{R_{11} + R_L}{R_{11} + R_L - R_{12}}}\,|sin(S_r\, cos\varphi)| \quad where, S_r = 2\pi S/\lambda$$

Okay and what it provides is much better directivity and for transmitting antenna and also a larger collecting area for a receiving antenna. So there are different types of reflector antennas one is a basic these are the basic design of of all the known telescopes like GMRT the very large array in the US, CHIME in Canada, LOFAR, FAST, SKA you keep on naming each of the SKA mid uses a reflector antenna SKA low uses different kinds of an antenna which will come later LOFAR also has a bit different reflector aperture antenna kind of a design it's not quite same as a reflector antenna so we'll come to that as well later. Meerkat ASCAP on the other hand has designs which matches with reflector antennas. So these are the most popular type of antennas used in radio astronomy and communication purposes it provides high gain directivity and narrow bandwidth and high major to minor lobe ratios but that basically means a high directivity in the sense. We will be majorly discussing about the sheet reflector then the corner reflectors and mostly we'll be using the parabolic reflectors.

So a sheet reflector it's just just having a like a what it says if you just expand that director for the Yagi Uda and expand it into a two-dimensional sheet that's what this is effectively is. Not very well used in case of radio astronomy but there are some early antennas were based on this but not recently. Then there are corner reflectors very well used in remote sensing purposes but in this case also the not very well used for the radio astronomical purposes. The most popular ones are the parabolic paraboloidal reflector antennas and that has a very interesting reason. The reason is that for a parabola all the rays which are if supposedly you place a radiating antenna at the set at the at the focus of the parabola then it either it sends out the rays out it reflects from the dish and then becomes parallel.

So all the rays which are coming from the focus reflects on the dish and then they start becoming parallel and then become plane waves and moves out in the free space. Hence we can now consider the reverse case where in the receiving mode this antenna will be receiving the rays from the faraway sources since the source is at faraway position they will be sending spherical waves but by the time it reaches the antenna they have become more or less parallel okay and we can we can prove this later on and so if the parallel rays are coming from the from the source in the sky they're hitting the antenna and then they're reflected by the parabolic reflector and then they come and meet at the at the focus itself and that's the property of the parabolic dish which allows it to happen. So if you keep the the receiving feed at the at the focus of the parabolic dish then it will receive all the rays which are coming and hitting on the surface of the parabola. So in that case the collecting area from over which it is collecting the all the rays increases further than its own this area whatever okay so supposedly we have used the same kind of horn antenna so horn antenna has a certain dimensions so if I just put it on the focus instead of just putting it looking at the sky the collecting area increases by large okay so that's what we have discussed over here that way if you follow the geometry you can easily understand that any any waves emanating from the the feed at the at the focus will will finally become send out parallel waves to the sky to the free space and in the receiving mode the the same the parallel waves coming from the sky at infinity comes and hits the parabolic reflector and then comes and reflects back to the focus point and it basically helps that parabola has a single focus so all the rays comes and meets there. So the paraboloid has the property of directing or collimating the radiation from the focus into a beam of para beam parallel to the axis in the transmit mode and the reverse mode it collimates the entire parallel waves coming from the sky and it reflects and on the parabolic dish and then comes and meets at the focus itself.

We elaborate the case of the receiving mode for the parabolic reflector antenna so here you can see that there is this the the diagram over here shows that the plane waves are coming from the from the sky and since the source is at distance also they can be assumed to be at infinity so even though the source is emanating spherical waves by the time it is coming and reaching the neighborhood of the antenna kept at infinity the waves become parallel to each other or plane waves. Now the idea is that if you have a parabola then the say for the on axis point which contains the the focal point and the vertex the rays will come like this hits the reflector and then finally coming back to the focal point okay. For that particular case the total path length is F plus H for on axis on axis reflection so you can let me try it again to explain it better so you can basically it just comes through the axis and again reflects back okay I'm just trying to demark it so and meets so the total length is F plus H so H to begin with if it is coming from the point of

H at a height of H okay and then it is coming and meeting that that goes to the height of F so F plus H is the on axis total path length. Now the same thing can happen for off axis where the waves comes hits the reflector and goes and meets at the at the focus let me try to draw it again so for the off axis case it is coming reflecting and going and meeting toward the focus yeah that's better and this is given by this right hand side and then if they both have to be equal then it finally gives you the relationship of Z equal to R squared over 2 4 over 4 F which is the equation of a parabola.

$$(f+h) = \sqrt{r^2 + (f-z)^2} + (h-z) \quad \Rightarrow r^2 = 4fz \quad \Rightarrow \boxed{z = \frac{r^2}{4f}}$$

So the another important aspect of parabolic tiches is the F by D ratio and that it in terms controls the gain of the antenna the collecting area the half power beam width or the half power beam width of the far field radiation pattern which also is equivalent to the field of view of the antenna itself they all depend on this F by D ratio so this is very important for design of a parabolic dish.

So now we will start discussing about the radiation fields of the field antennas particularly with the reflector antenna. So what appears is that the near field aperture field distribution is like a Fourier transform with the far field radiation pattern and that's what we will going to discuss and establish. So consider a continuous current sheet or field distribution over an aperture assuming a current our field perpendicular to the to the page in the y direction that is uniform with respect to y the electric field at a distance R from the elemental aperture dx dy is given by de is equal to minus j omega d ay where ay is the vector potential given as a function of the jy which is the current density.

$$dE = -j\omega\, dA_y = -\frac{j\omega\mu}{4\pi r}\frac{E(x)}{Z}e^{-j\beta r}\, dx\, dy$$

Okay so the electric field de is given in terms of this electric aperture electric field distribution exy the intrinsic impedance of the medium capital Z omega which is the angular frequency 2 pi times frequency and mu is the permeability of the medium. Okay so the infinitesimal electric field distance R is given by minus j omega delta d ay which is the a is the vector potential.

Okay and jy is the current density. So that is given in now in terms of the ex and z and e to the power minus j beta R. Okay that's the infinitesimal electric field at a distance R because of the current density in the aperture. So now if you if we do that then we expressed again for an aperture with a uniform dimension y1 perpendicular to the slide and with the field distribution over the aperture a function of only x the electric field as a function of phi at large distance from the aperture R greater than greater than a can be defined as e of phi so electric field has only the phi dependency which kind of can be expressed in terms of this aperture electric field distribution ex by this relationship. Now if we take a closer look at this expression okay we can easily use substitute the following x lambda is equal to x over lambda and then we can express e of sine theta phi is given as e of x lambda minus infinity to plus infinity.

$$dE = -j\omega \, d\mathcal{A}_y = -\frac{j\omega\mu}{4\pi r}\frac{E(x)}{Z}\, e^{-j\beta r}\, dx\, dy$$

$$E(\phi) = \frac{-j\omega\mu y_1 e^{-j\beta r_0}}{4\pi r_0 Z}\int_{-a/2}^{+a/2} E(x)\, e^{j\beta x \sin\phi}\, dx$$

So, using the concept of FT we can rewrite this as

$$E(\sin\phi) = \int_{-\infty}^{\infty} E(x_\lambda)\, e^{j2\pi x_\lambda \sin\phi}\, dx_\lambda$$

Here $x_\lambda = x/\lambda$.

$$E(x_\lambda) = \int_{-\infty}^{\infty} E(\sin\phi)\, e^{-j2\pi x_\lambda \sin\phi}\, d(\sin\phi)$$

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$$E(x_\lambda) = \int_{-\infty}^{\infty} E(\sin\phi)\, e^{-j2\pi x_\lambda \sin\phi}\, d(\sin\phi)$$

Now we can express minus infinity to plus infinity but the electric field the aperture electric field is only going to be nonzero at from minus a by 2 to plus a by 2 but however we can definitely expand this to minus infinity to plus infinity because the rest of the integral beyond minus and minus a by 2 to plus a by 2 is 0. So we can write this expression to give the following expression and they are nothing but just the Fourier transform pairs. So we can write the forward transform e of sine theta phi given in terms of e of x lambda which is the aperture so this is the aperture electric field distribution and we can gain the far field electric field pattern like this and the reverse. So what appears is that that the e of phi and the e of x are basically nothing but Fourier conjugate Fourier pair. The electric field distribution or e sine theta phi or angular spectrum refers to the angular distribution of plane waves and sine phi is less than equal to 1 the field distribution represents radiative power and if sine phi is greater than 1 then it represents reactive or stored power.

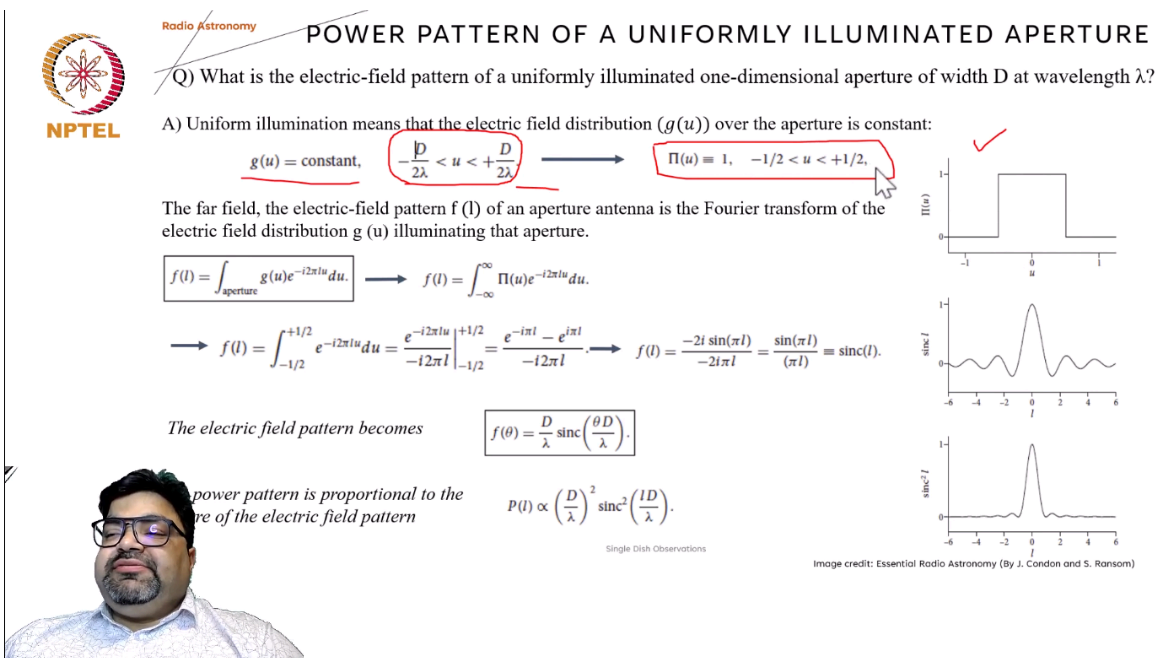
$$E(\phi) = \int_{-a_\lambda/2}^{+a_\lambda/2} E(x_\lambda)\, e^{j2\pi x_\lambda \sin\phi}\, dx_\lambda$$

$$E_1(\phi) + E_2(\phi) + \cdots = \int_{-a_\lambda/2}^{+a_\lambda/2} E_1(x_\lambda)\, e^{j2\pi x_\lambda \sin\phi}\, dx_\lambda + \int_{-a_\lambda/2}^{+a_\lambda/2} E_2(x_\lambda)\, e^{j2\pi x_\lambda \sin\phi}\, dx_\lambda + \cdots$$

So finally we express this as a Fourier integral and we can utilize this in later explanations to understand how does different aperture distributions give rise to different far field radiation patterns. So to start this with the first example is what is the electric field pattern of a uniformly illuminated one-dimensional aperture of width D at a wavelength lambda. So just to elaborate it further so we have a aperture electric field distribution given by G of u which is given over here which is constant it is a constant electric field distribution between the aperture area which is minus D by 2 lambda to plus D by 2 lambda. So it is basically given by something like this. To represent it better we can do the unit unit function which is a step function so it is 0 outside this particular area of minus half to plus half that's equal to 1 and outside the this particular area of minus half to plus half it is 0.

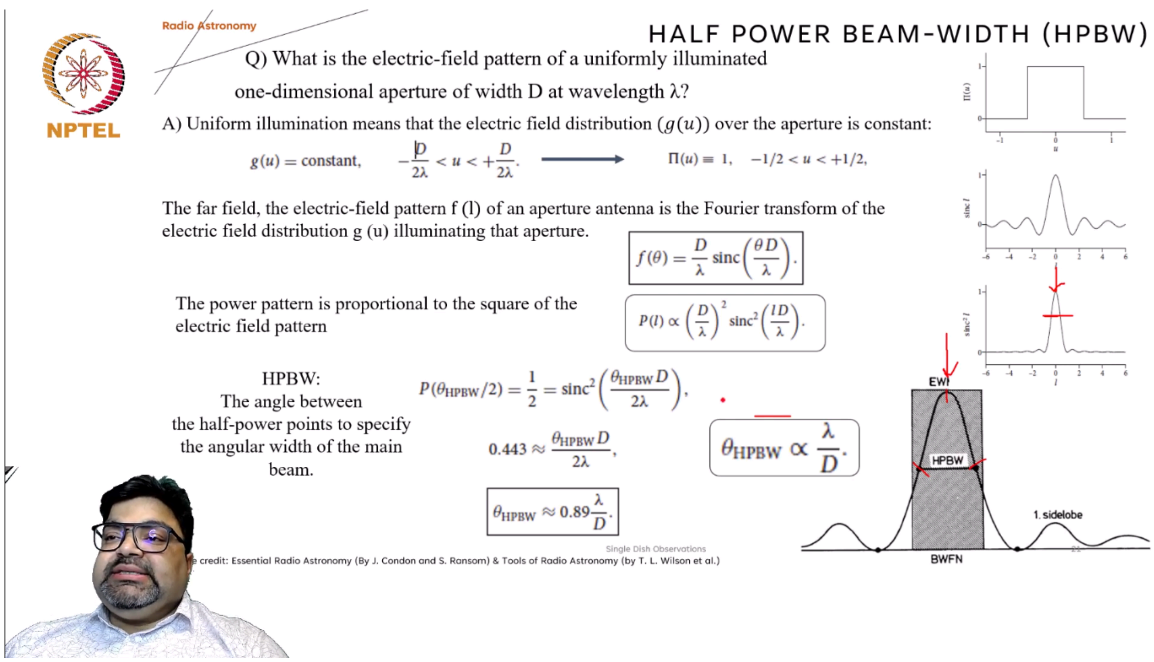
Okay so it's a unitary function. Hence what we can write is we can write the aperture electric field pattern in terms of this unitary field unitary function. So f is the far field electric field far field electric distribution and the G u is the near field or the aperture electric field distribution. And so you can rewrite G u in terms of this unitary function and that transforms the f L is equal to minus infinity to plus infinity this unitary functions times e to the power minus i 2 pi L u. Okay of course there are some scaling involved but we assume that the constant basically is unit.

So if I do this Fourier transform finally you will end up being that the f of L is equal to a sinc function which is nothing by sine pi L over pi L that is given by this function. So electric field pattern at then becomes at the far field becomes f of theta is d over lambda sinc of theta d over lambda. The power pattern is just the square of that so that becomes proportional to sinc square of L d over lambda and is shown by this graph over here. Okay so we do it again once more. So our unitary function so our distribution which is constant across the aperture we take the constant value to be 1 an aperture to be minus d by 2 lambda 2 plus d by 2 lambda.



Then the far field pattern is equal to just the Fourier transform of this. If I do the math then we get a sinc function corresponding to the far field pattern and if I do the square of the far field then that gives you the power pattern in the far field that is sinc square. So a constant electric field distribution inside the aperture gives rise to sinc function for the electric field in the far field and gives rise to a sinc square function for the power pattern. Let's see how it goes for the what we can do with that. So when we have done this now next thing what we can do is we have this far field pattern this sinc square function so that is the power pattern in the far field.

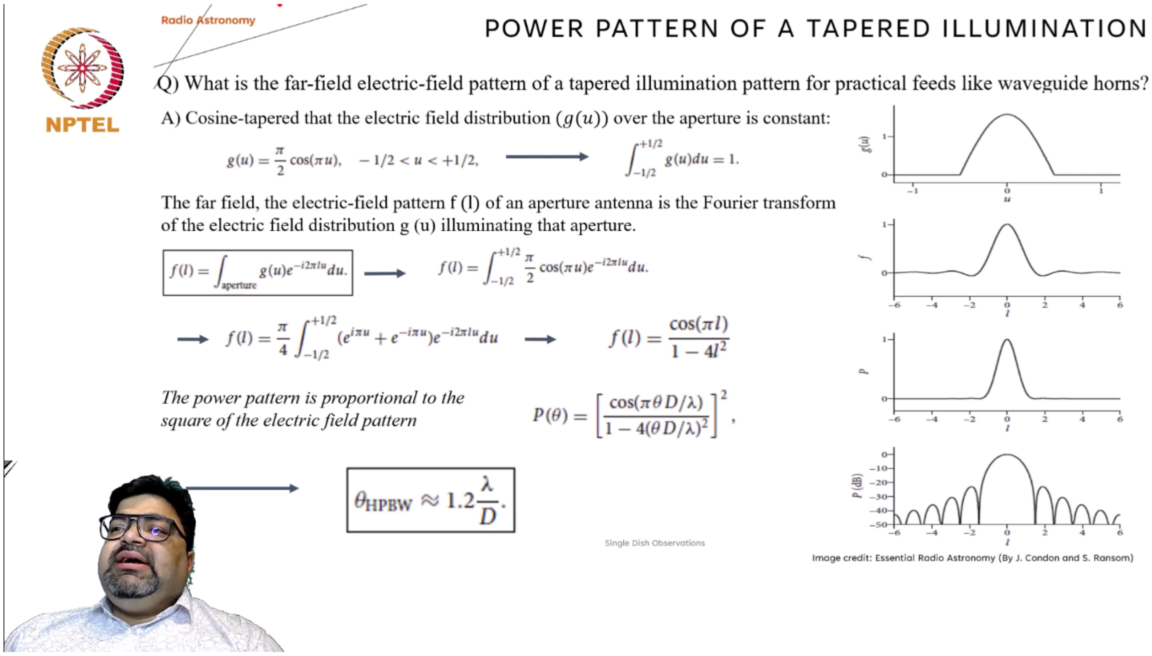
Now we know that in the in the bore side so at the center of that power pattern we have the power to be maximum. And if the power comes down to the power maximum by 2 that is the points where we have the points where we have something called a half power beam width. So let's consider that maximum power and consider the power coming down to the half of that maximum value at the two end. So the angular separation between these two points is known as half power beam width or that stands as a proxy for the field of view of the radio telescope antennas. So we do the math we determine what is the p of half power at each side and we evaluate that so that gives us the theta half power beam width to be equal to 0.89 times lambda by D.



So for the sake of completeness we can say that this is like proportional to lambda over D. So lambda over D is a very powerful measurement of the field of view. They may have some prefixes depending on the exact aperture distribution electric field distribution but the nevertheless it depends on lambda and D. So hence we can conclude that for a radio telescope the field of view increases with increase in lambda or decreases with increase in the diameter. So if I want to make my primary beam narrower then the aperture should be increased for a given value of the wavelength.

This is a very important concept which we'll be using more in the upcoming weeks. So going to the next example, typically the power the electric field distribution in the aperture is not exactly constant. So a very closer one to the reality can be a tapered illumination. So what happens is you assume that the electric field is distribution in the aperture is like a cosine function okay.

So it's like a little bit tapered. So it is a maximum at the center and goes down a little bit towards the edge. The far field electric field pattern going by the previous Fourier transform relationship then is equal to the this integral of this which comes down to be the finally the f L the far field electric field pattern is cosine pi L minus 1 by divided by 1 minus 4 L square. If I take the square of that we get the far field radiation pattern power pattern which is cosine square function minus 1 minus 4 pi square d square over lambda square and whole square of that. If I do the same expression like how to calculate the maximum power so this is the electric field pattern in the in the aperture this is in the far field this is the power pattern in the far field and since we don't see much variation in the in the side lobes what we do is we change from the linear unit down to the log unit so in dB scale. So in dB scale now you can see the side lobes more prominently this is the main lobe is the first side lobe second side lobe third side lobe and so on okay.



So the distance between the two half power points is given by this 1.2 lambda over D for a tapered illumination pattern.

Now let us move to more realistic case if you are looking only in the one-dimensional case which is not real so for two-dimensional case now you can have the far field radiation pattern and denoted by this f of LM where LM are general direction cosines or angular coordinates and you express the near field aperture in the dimensions in terms of u and v which are just like in terms of length in physical length but in terms of scale by the wavelength it's a very standard practice so like it is essentially u is basically nothing but dx over lambda and v is nothing but dy over lambda where dy and dx are the two aperture dimensions in x and y directions. So then the the far field is basically proportional to the sinc function along x and along y just following the one dimensional equivalence and then the power pattern is given by this sinc square L dx over lambda and sinc square M dy over lambda.

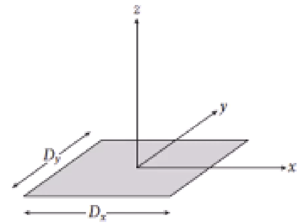
Radio Astronomy

POWER PATTERN OF A 2-D APERTURE

The electric field pattern of a two-dimensional aperture is the two-dimensional Fourier transform of the aperture field illumination

$$f(l, m) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(u, v)e^{-i2\pi(lu+mv)}du\,dv,$$

For the Uniformly Illuminated Rectangular Aperture



The electric field pattern becomes →
$$f(l, m) \propto \operatorname{sinc}\left(\frac{lD_x}{\lambda}\right)\operatorname{sinc}\left(\frac{mD_y}{\lambda}\right).$$

The power pattern is proportional to the square of the electric field pattern

$$\longrightarrow \quad P_n(l, m) = \operatorname{sinc}^2\left(\frac{lD_x}{\lambda}\right)\operatorname{sinc}^2\left(\frac{mD_y}{\lambda}\right).$$

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FOR A CIRCULAR APERTURE (explore the circular symmetry and express in polar coordinates)

Since the integral representation of the Bessel function of order zero is

$$J_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{iz\cos\varphi}\,d\varphi$$

$$f(u) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} g(\rho)\,e^{-2\pi i u\rho\cos\varphi}\rho\,d\rho\,d\varphi .$$

The electric field pattern becomes →

$$f(u) = \int_0^{\infty} g(\rho)\,J_0(2\pi u\rho)\rho\,d\rho$$

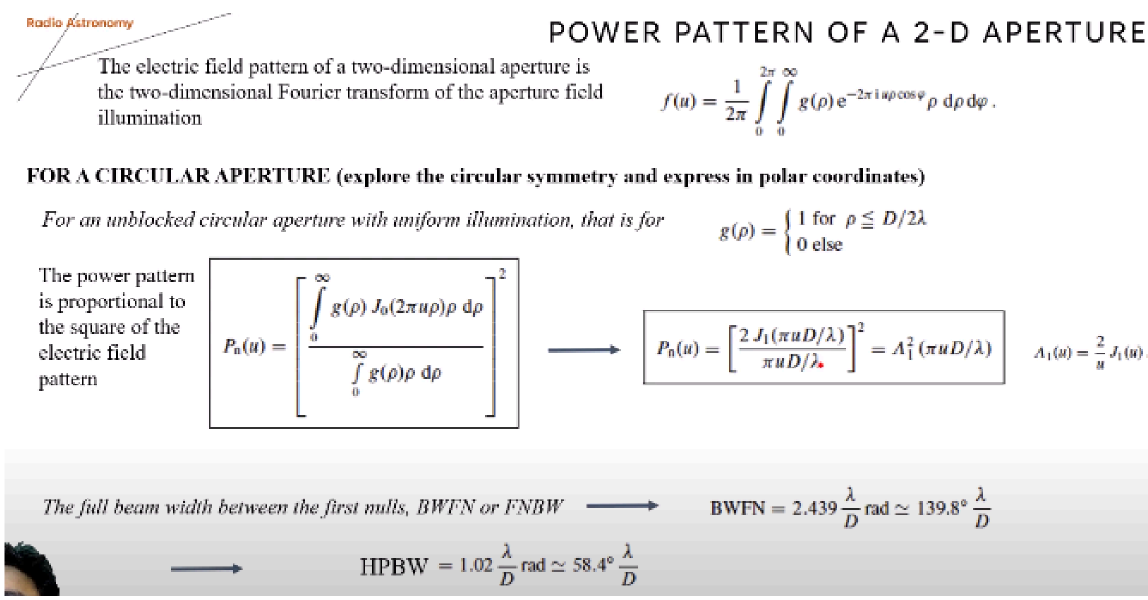
The power pattern is proportional to the square of the electric field pattern →

$$P_n(u) = \left[\frac{\int_0^{\infty} g(\rho)\,J_0(2\pi u\rho)\rho\,d\rho}{\int_0^{\infty} g(\rho)\rho\,d\rho}\right]^2$$

It's a very generalized form. Let's look at for a particular case of circular aperture. So for a circular aperture we essentially have the the aperture distribution function sorry the far field electric field pattern is related to the to the aperture illumination pattern by with respect to this. Here we have used the the cylindrical symmetry of the system and the electric fields hence becomes it's double integral 0 to 2 pi and 0 to infinity and in that case for 0 to 2 pi part of this e to the power i z cosine of phi we identify that to be the Bessel function of 0th order. So we substitute that to this we get the far field electric field pattern given by this expression where J naught is nothing but the Bessel function of 0th order. If we follow the similar math then if I have the square of this we get the power pattern in the far field and that is given by this expression whole square of that.

If we keep keep doing the math we finally end up in the power pattern is given by 2 J 1 pi ud over lambda by pi ud over lambda whole square and that is given by the first Bessel function of the order 1. This lambda 1 is nothing but this scaling scaled function.

So for this particular power pattern the half power beam width comes out to be 58.4 degree times lambda over D. So lambda over D is still there and that is the pre factors just keep on changing depending on what kind of aperture illumination function we are using.



Now we have skipped a lot of steps and those steps are important but it also very cumbersome so what we like you to do is focus on the final result. For example when we started with the first case a constant electric field distribution that gave rise to a sine sinc function in the far field electric field pattern that also gave rise to a power pattern of sinc square. A tapered illumination pattern which is given by a cosine function gives rise to another cosine square form in the far field power pattern. For circular aperture in two dimension simple you know distribution of electric field in the aperture gives rise to a Bessel function in the power pattern and finally we have the concept of half power beam width which is nothing but a proportional to the lambda over D where lambda is the wavelength of operation of radiation or observation and D is the dimension of the telescope or the reflector. So utilizing the nature of the Fourier transform we can relate several different antenna electric field aperture distribution to the their far field electric field pattern.

It not only helps to just to understand the functional form but also it also gives the half power beam width for each and every form of illumination pattern. So depending on if we have a particular scientific observation in mind that we have to have the field of view up to some level we can design our telescope and aperture distribution in a way that it reflects the right amount of field of view. To come to couple of such issues like the side lobe levels often are a big botheration. We typically want to see some part of the sky with a high sensitivity that is true but we do not want that there are spurious sources getting picked up by the side lobes because we were not paying attention to those while doing the data processing. So one of the requirement for having a idealized beam or far field radiation pattern is having the lowest side lobes possible.

So that one can check different functions. So here we have for the completeness we have said the uniform function leads to a sinc square function in the far field, sinc function in the far field for this, a triangular function also have a similar pattern, a cosine function gives rise to this kind of function, cosine square gives rise to almost a sinc square, a Gaussian gives rise to a Gaussian, inverse taper goes to this alternate negative and positive side lobes and edge also goes gives this kind of a pattern. Inverse taper just to summarize less amplitude at the center than the edge yields a smaller beam width but larger minor lobes than for the uniform distribution. That's one interesting thing to note. Here we also give some more such examples of different aperture field distributions, specific cases of tapering and etc for rectangular and circular aperture for just completeness. Now in the next one we come to a very important you know relationship.

We knew that there is a relationship of Fourier transform between the aperture electric field distribution and the far field electric distribution. We also have done the squaring of the electric field in the far field to get the power pattern in the far field. So in this case if we just put things together it will not be very difficult to understand that the Fourier transform of the antenna power pattern is proportional to the complex autocorrelation function of the aperture distribution. So e x lambda minus x naught is a displaced aperture distribution and e x lambda is the original distribution at center at zero. The correlation function of autocorrelation function of these two is proportional to the some power pattern.

$$\overline{P}(x_{\lambda_0}) \propto \int_{-\infty}^{\infty} E(x_\lambda - x_{\lambda_0})\, E^*(x_\lambda)\, dx_\lambda$$

$$S(\phi_0) = \int_{-\infty}^{\infty} B(\phi)\, \bar{P}_n(\phi_0 - \phi)\, d\phi$$

$$\overline{S}(x_\lambda) = \overline{B}(x_\lambda)\,\overline{P}(x_{\lambda_0})$$

Here the bar represents FT

Okay that is the Fourier transform of the antenna power pattern. Okay so if I take the individual electric fields and we Fourier transform them we get the far field pattern and the squaring the far field pattern we get the power pattern of the antenna. So the power pattern if we Fourier transform it back or inverse Fourier transform we get p x lambda zero. Okay hence it is easy to understand then the Fourier transform of the antenna radiation pattern is proportion to the autocorrelation function of the aperture distribution or aperture electric field distribution. Okay that's a good valid relationship and we will be using it later.

This is first discovered by Booker and Klema. So another concept in this line is an antenna response to a moving source. So let's say the s theta s phi zero is the observed power distribution and b phi is a true source brightness distribution and p n phi is a mirror image of the normalized power pattern. So the s which is the observed power distribution will be equal to minus infinity to plus infinity b phi which is a true source distribution times the antenna power pattern. So it's the convolution of the antenna power pattern and the source brightness which is the final observed power pattern which is kind of obvious. So you have a primary beam of the antenna and you see the source coming and going out of the beam.

Inverse of this quantity will provide us an approximate angle (or beamwidth for the aperture)

$$\phi_c = \frac{1}{a_\lambda} \; radians$$

This is because a_λ is aperture in terms of wavelength ($a_\lambda = D/\lambda$) and we know that $\theta = 1.22\lambda/D \approx \lambda/D$. It follows that this (cutoff) angle ϕ_c is equal to 1/2 the Beamwidth between First Nulls (BWFN) for a uniform aperture distribution (ϕ_c = BWFN/2), and is 12 percent greater than the beamwidth at half-power (ϕ_c = 1.12 HPBW).

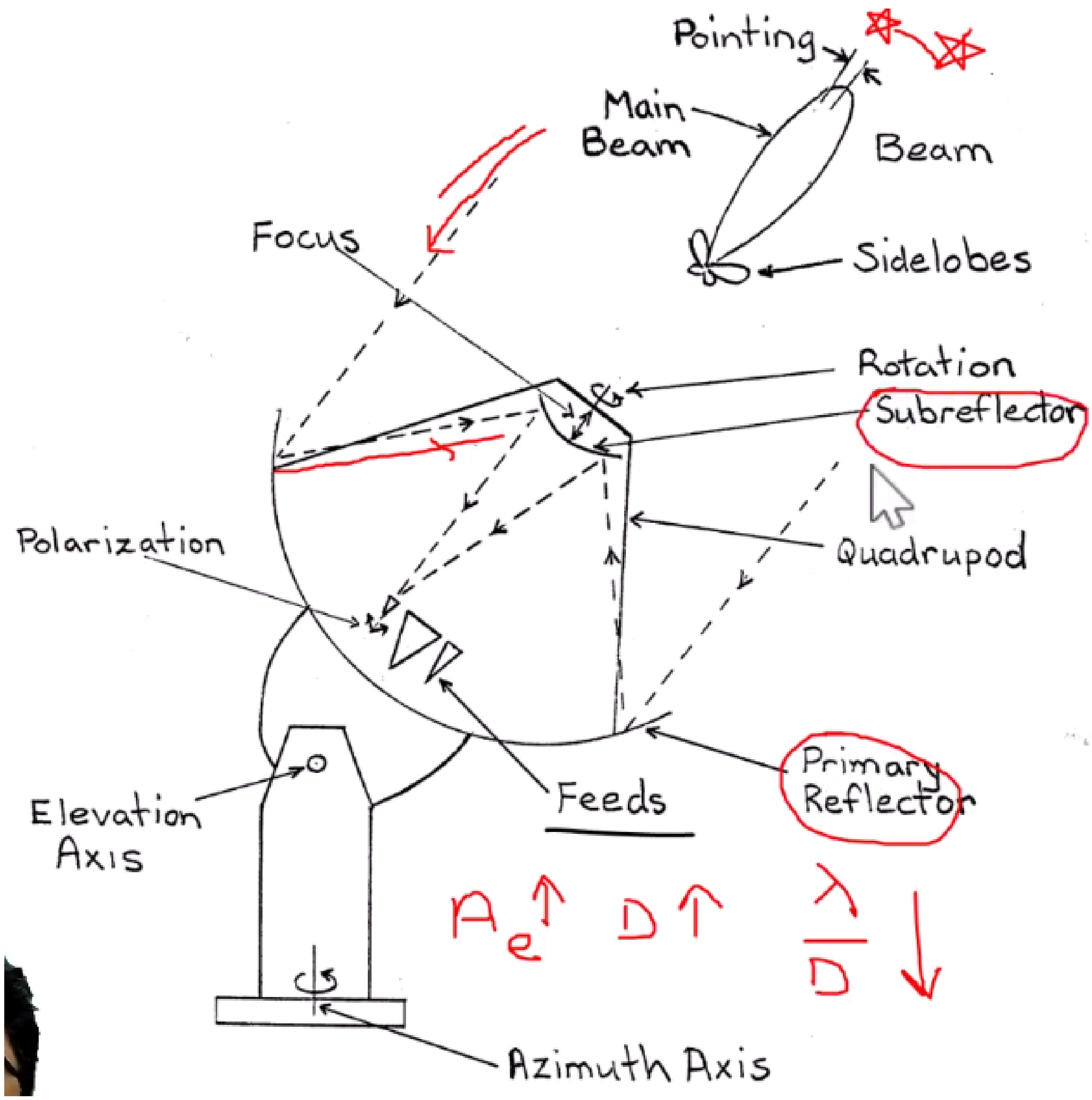
Now I will we will not be able to see or determine the exact source distribution. Okay but we will see the source brightness convolved with the antenna power pattern as our observed or inferred source distribution. So there will be a convolution with the primary beam pattern of the antenna which we are using to observe the source. Okay that will be there.

So that's what we wanted to mention it over here. So if we have a narrow beam then we will be may be able to see different you know the we can resolve the source if the source is is finite in size. If the beam is very large then we may infer our extended source to be a point source or compact source because it is larger than the size of the source itself. We also have it depends on we have various quantities like the half power beamwidth is one factor which we discussed before. We also have something called full width sorry the beam width at first null BWFN or FNBW and that also is another measure of the field of view and so that can be characterized from the this final antenna power pattern. If we get the power pattern we have the way to understand where the power pattern goes from maximum to half of the maximum value and so you can basically just measure the distance between the two half power points and that will give you the half power beamwidth.

$$E(\phi) = \frac{2\lambda}{\pi D} \frac{J_1[(\pi D/\lambda)\sin\phi]}{\sin\phi}$$

Same if you want to calculate the distance between the two null points you have to put the value of the power pattern to be equal to zero and you will get two points and that's how you calculate. We in fact did some examples in our previous weeks when we discussed about the both the first null beam width FNBW over here it says BWFN and the half power beamwidth. So it's just putting them in context how do you calculate from a given normalized field pattern which is given for that two-dimensional circular aperture the this is the closest to the actual reflector antennas how do we calculate the BWFN that is you can you're basically showing that. So having done that discussion about the relationship between the aperture distribution and the far field pattern and the far field power pattern of the antenna it's important to look into couple of basic structures of the radio telescope. So the most popular as we said is a reflector dish which gives more directivity because of the larger D you have a smaller lambda by D for a given lambda so that the half power beam width becomes narrower so you are able to resolve different sources in the around line of sight.

Supposedly there is a star over here I'm just trying to make one and a star over here it's a better one so let's just say the star over here I will be only be able to differentiate between the two stars if my half my primary beam is smaller than the angular distance between these two stars. So to get more resolution I need to lower the primary the half power beam width and so the larger D as D goes increases lambda by D for a given lambda goes down and so that gives you higher resolution. That is one also as D goes up the effective area also goes up so that helps in the correcting area to go large so hence the the effectively the reflector antennas are actually quite helpful for radio astronomical purposes. But apart from just the dish and the size of it there are a few other important things so we will see in the next following slides also. So first of all it has to move in different direction so typically it is a azimuth axis so it is the azimuth rotation and it has elevation axis so elevation rotation along that.


Pointing
Main Beam
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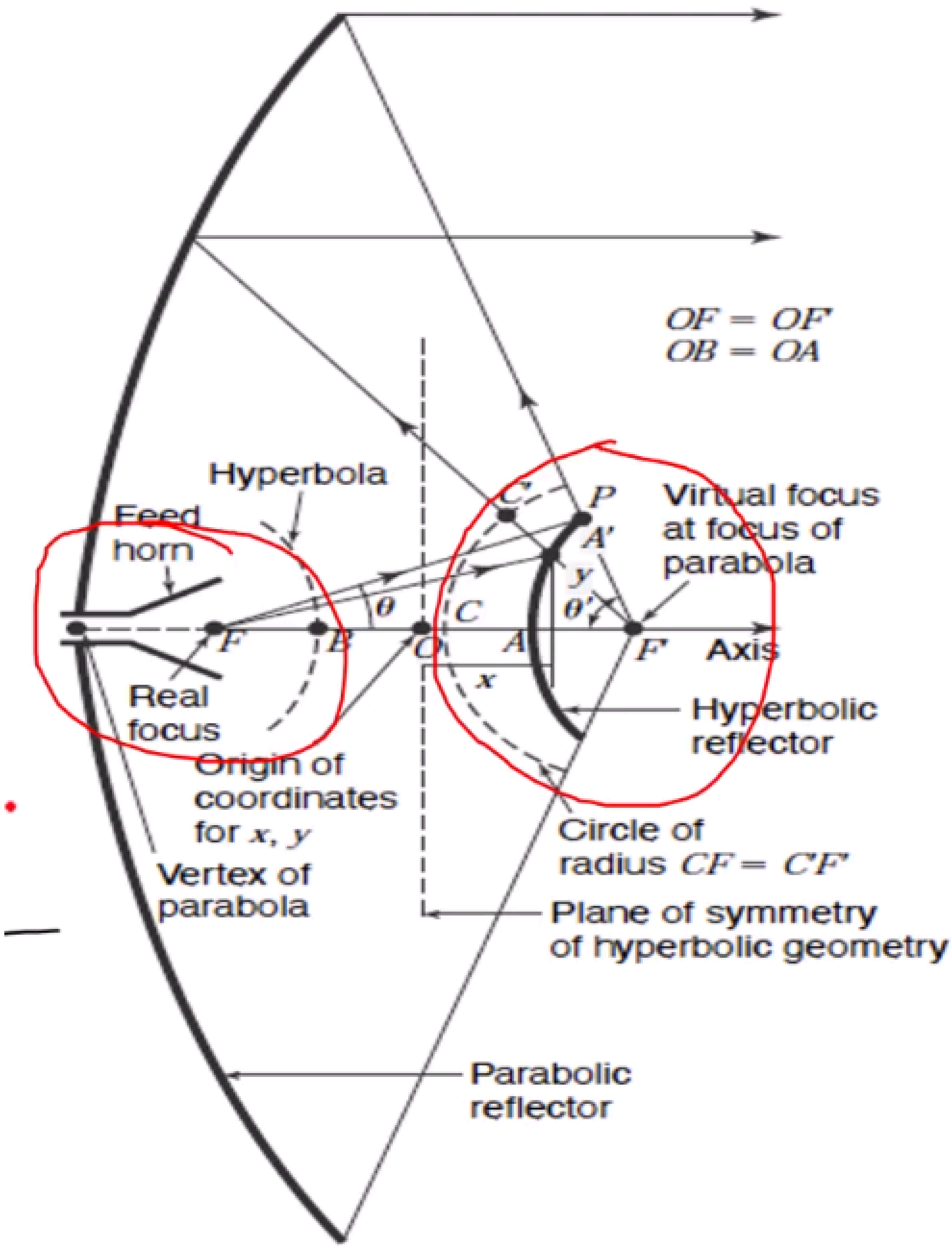
Focus
Rotation
Subreflector
Quadrupod
Polarization
Feeds
Primary Reflector
Elevation Axis
Azimuth Axis



Typically most of the antennas are able to point in different parts of the sky barring very low elevation sources because of physical constraints of the antenna itself. This is a diagram of a Cassegrain antenna and we will show you what Cassegrain means in a bit so but the dish is parabolic so the as the as the rays from the sky are coming they are hitting the surface of the dish and they are getting reflected back and to the focus there is another secondary reflector which then focuses it down to the feeds or the antennas which are then situated on the central part of the dish as a whole. So this is the primary reflector the dish and this is the secondary reflector sub reflector which is at the focus okay. There are more things to that there is a as we said there is a elevation axis so there is an elevation encoder okay in that case to which I asked it to move in a in separate in in particular way to track a particular source on the sky. Similarly there is a azimuth encoder for because this is having a movement in the base of it and it moves like that.

There's a quadrupole which supports this this focal sub reflector and the focus and there is other different structures foundations of all are radio telescope is a engineering marvel by itself mechanical electrical all combined. Yeah this is a dish from a VLBA array the very large baseline array in US operated by NRIO. This is the Alma dish and you can see that there are several compared to the VLA or VLBA dishes that the surface is further very smooth and we'll come to that there is a different kind of this quadrupole structure which which supports the center sub reflector and then the all the necessary antenna everything are in located here in the this color is not very good let me just try some of the color yeah so this is located no yeah this is located here near the center hole so all the antennas kept inside one of the reason is that as you go to higher and higher frequency the temperature of the ambient temperature to maintain for the receivers are very low so that more or less they are cryo-cooled so this cryo cooling requires a lot of space and that is only can be done inside the dish and not somewhere outside that's one of the reason why this is design is like that way and Alma operates at a at a quite higher frequency hundred gigahertz plus this is a similar design of another reflector dish a different kind it's called mere cat it operates bats near a 1 gigahertz about 800 megahertz up to few gigahertz similarly another antenna is called ASCAP in Western Australia this is in South Africa the America and they to have different dishes this is a quadrupole design but this is something like offset cassette and design where the reflectors are somewhere here we will show you one in the next detailed design aspects of this kind of dish in the next lecture yes coming to a simple log periodic we discussed before yeah this is also not an obsolete design radio telescopes do use it particularly for SKA low or this kind of arrays we use several such log periodic or variants of that this is the inverted V this is a low for telescope at high band they have this kind of biconical design but in in to in 2d only then there's this is a antenna for the design for the SKA one low and this is called Scala for this is a very unique design but it is ultimately the operating principles are similar to like the long bit of the dipole antennas this is a low for low band array it's a inverted V kind of a shape and this is the MWA tiles operating somewhere in 100 megahertz to 200 megahertz this design of the MWA are more or less similar and this what different executions so different characteristics in addition to those different kinds of designs different kinds of reflector antennas we also have the mount like the optical telescopes mount is also very important so you have equatorial mount and that is they have some advantages and disadvantages and they have alt as azimuth elevation mount like we showed before we haven't shown you a picture of our equatorial mount yet but as we start interferometry we will show you some pictures the different types of reflector antenna also they can parabolic cylinders the conventional paraboloid reflector fed by a point source typically a waveguide horn it generates a pencil beam by rotating the parabola about its axis there can be cylindrical reflectors it's used very commonly in fact the OT telescope in India is a cylindrical reflector and GMRT is a parabolic reflector so even in India you have different versions for a parabolic reflector a very simple we have already seen the design we have seen for Cassegrain so let's let's look it for a different thing so parabolic reflectors are where the feed itself is kept at the the focus of the parabola and so the antennas rays of rays of it comes and then the reflects and they meet at the focal plane very simple there is no sub reflector okay of course there is a problem there are of course pros and cons in every every design if we have the feed over here which is quite large then there is a significant blockage creates because of that okay that's a parabolic reflector the the main design which we will follow for this is for GMRT which has this is its rotation around azimuth around this place and the elevation rotation happens in this area encoder this is a dish which is unique by its design these are the quadruports over here and this is the the feed assembly in the focus so very simple as it looks like but very sophisticated indeed and we'll come to that later on the addition to parabolic reflector you have a spherical reflector and one of the important telescope called fast in China you know adopts this design and so they have a fixed telescope at the at a location and there is a movable central feed which moves from one end to the other and it also the dish also deforms a little bit to give it a instantaneous 300 meter diameter kind of aperture and it can focus at a particular point in the in the sky but it cannot focus I cannot point at any location of the sky because of its limitation the dish is fixed on the ground nevertheless it follows this spherical reflector design the con which we showed already the Cassegrin where you have a have a sub reflector let's change the color of the pointer you have a sub reflector in the focus and you have the main antenna feed designs are kept in the center part of the of the dish this is a Cassegrin and that is operate operational for several antenna designs like the VLA the VLBA antennas the Alma antennas we showed and so and so so this is the VLA antenna and you can see the central sub reflector the at the center you can see on the dish there are all these antennas sticking out that is the different feeds for different wavelengths okay there are feeds for L band feeds for the higher frequencies and going up to I mean 50 gigahertz the VLA has feeds so as you said it's not they cannot be a single antenna which covers the entire electromagnetic spectrum even within the radio range so you have to build several different receivers and feeds to be able to do that what happens is that this sub reflector actually actually moves a little bit hmm and it focuses on and it sends a signal the collected rays into though that particular feed which is operational for that particular time suppose you are doing a L band observation then it will focus on the L band you're doing something above L band then it will focus on the particular that particular antenna and so on like the prime focus feeds this also Casa Grande also creates a little bit of blockage however the the good part of this is that if you are using the higher frequency receivers then it can be cryo cooled and in the space below the dish so just to summarize there are different kinds of feed locations the Casa Grande we have already discussed the Gregory which is like you have a sub reflector but it is inverted parabola not in this way so that is different and so it comes out and goes into the central part there is nice meet where you have a sub reflector but then it comes through and the receiver is kept somewhere else not at the center of the dish and that is called nice with offset Casa Grande when you think that this is you're using only one part of the dish of the Casa Grande and you are not using the other part that gives rise to the offset Casa Grande antenna this is same for meerkat and for Green Bank telescope we will discuss it later so prime focus just to summarize once very quickly the prime focus is for GMRT Casa Grande is for VLA and Allen telescope array offset Casa Grande is is one thing where the sub reflector actually moves and can point little bit here and there to point to different parts of the central part and you can keep multiple not only single feet but multiple feeds over here so that it can just move the sub reflector and point nice with we have already discussed there's also something called beam waveguide and they are a little bit more complicated and dual offset or offset Casa Grande for the Allen telescope array and for the Green Bank telescopes the Casa Grande focus here is for the Australian telescope compact array these are the different telescopes for that particular feed locations so we just wanted to have a summary that we have all the very great telescopes they have different collecting areas we have the first which is quite the largest single dish and the cutting area basically just a measure of the D square of the diameter square of the particular antenna but then you end up having interferometers where you have n number of such antennas so n D square is the kind of collecting area how it scales so in this fashion we are getting up from the telescopes which we know today to something like SKA which is how going to have a square kilometer of collecting area and that's the goal and that's the aim in future thanks for listening we will be having our next lecture on the remaining part of the radio telescope designs and see you in the next class thanks one last thing before ending as we say every time that all of the lecture notes have been created from existing materials of different books so we try to refer them wherever they are relevant in that particular slide but in case we are missing them please Apple I apologize and so we have a list of references printed at the end of each lecture which basically shows from where we are basically you know borrowing the some of the contents like some of the diagrams equations etc so thanks for your understanding see you in the next class thank you bye