Radio Astronomy

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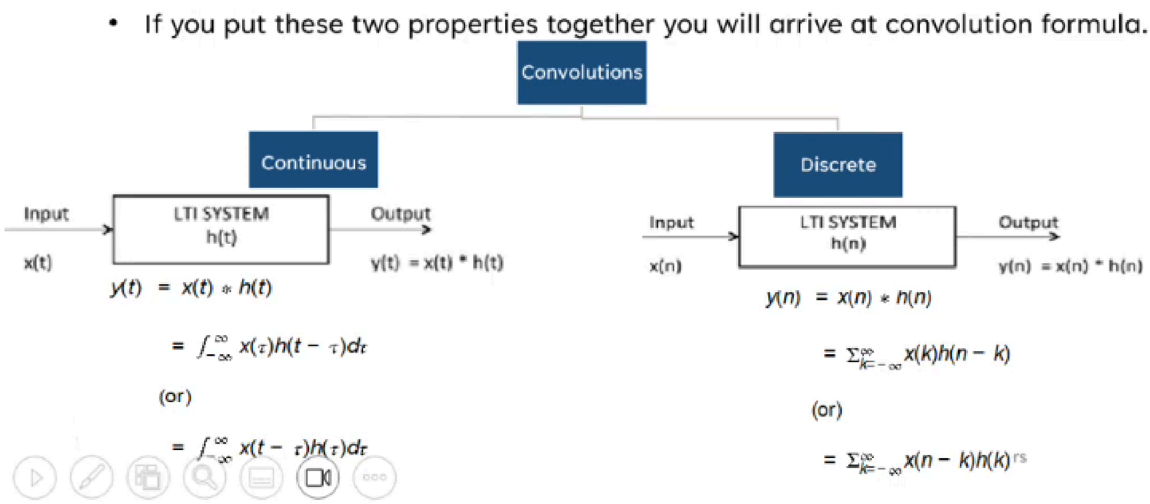
Lec-14

Signal Proceeding and Receivers - Part 02

Hello, welcome to the second lecture of week 4 of radio astronomy. We are continuing our discussion regarding signal processing and receivers. So, we have covered like a few things about basic signals and properties, what is the system, Fourier series and transform continuous in discrete forms. We will teach today about Nyquist sampling and aliasing problem and different radiometer basic radiometer design and finally end with heterodrome receivers. In last lecture, we established that understanding of the signal processing techniques and understanding of a system is very important in order to detect a radio signal from the sky. We also discussed that any periodic signals can be represented by Fourier series and Fourier transform.

These are essential tools which we will require in subject a bit more. Now even to begin with observation study, we need to detect the source. So, next few lectures, we will try to understand the theory behind such system which mostly is LTI or linear time invariant systems. So, our next concept we want to discuss is convolution and correlation.

If you look into a linear time invariant system or LTI system, it is a system that produces output signal from any input signals subjected to the constraints of linearity and time invariance. If a system is LTI, then it provides predictability of the output signal and makes it easy to characterize system and remove the effect of the system from the signal. Exactly or approximately is applicable to many important physical systems. The response y t of the system to an arbitrary input signal x t can be found literally using convolution y t equals to x times h t where h is the systems impulse response and the star represents the convolution and not multiplication. Now the question comes why convolution and what is convolution? The response of an LTI system is convolution because it follows linearity and time invariance.

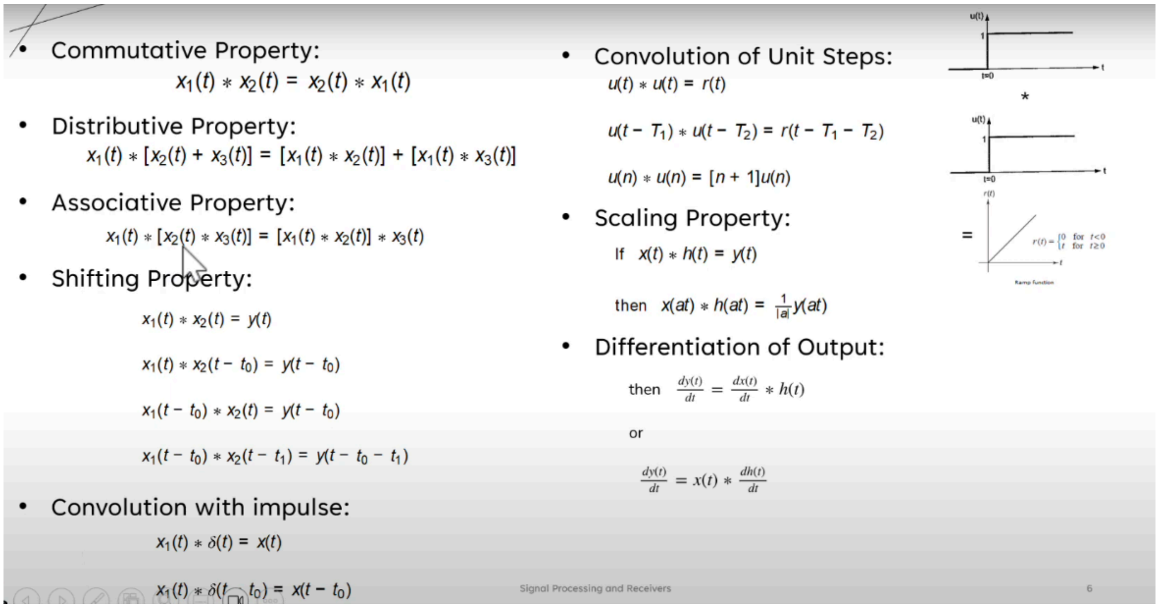


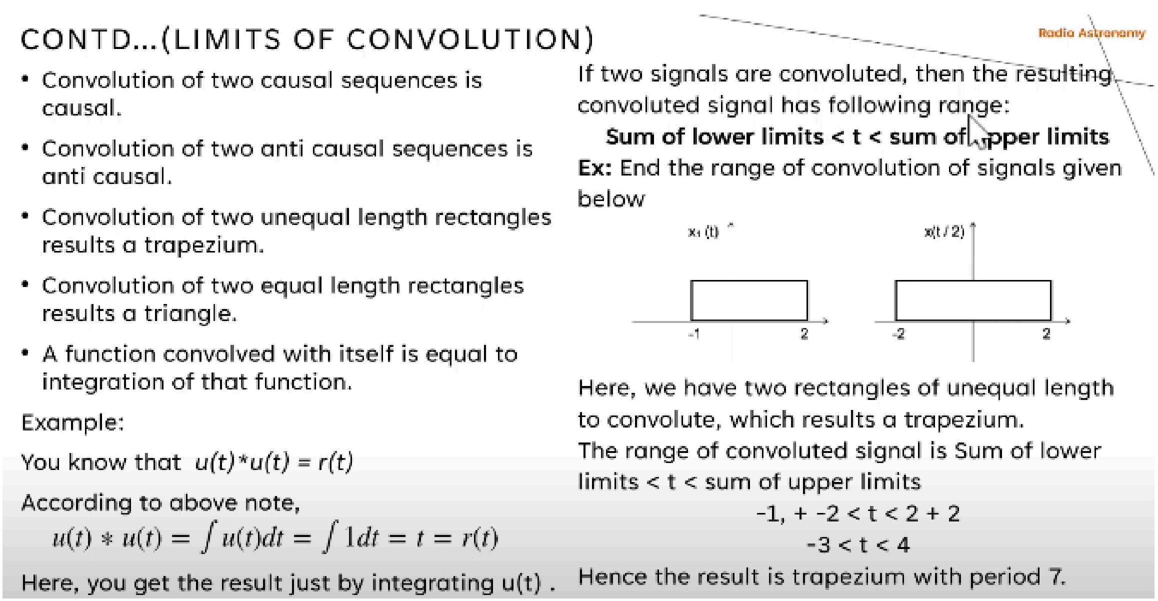
If you put these two property together, you will arrive at a convolution formula. So, a convolution can be two types continuous and discrete just like previous thing. You have an input signal and you have an output signal where we say the h is the effect of the system and so the output is basically nothing but x convolved with h. It can be like something like you have a dish, you have an antenna and you are measuring the sky. So, you have a sky which is coming in through your antenna.

So, you have a antenna have a far field radiation pattern or the beam and so the what signal which comes is convolved with the beam. So, signal of the sky convolved with the beam is what we record finally. So, that is one way you can understand the convolution. Convolution have different properties. Commutative property where you have x 1 signal convolved with x 2 can also be similarly equal to x 2 convolved with x 1.

Convolution of unitary unit steps can also be done. So, distributive property x 1 convolved with x 2 plus x 3 is x 1 convolved with x 2 plus x 1 convolved with x 3. Associative property you have the in particular order of association can be interchanged. Shifting property x 1 convolved with x 2 is y, x 1 convolved with x 2 t minus t naught is y of t minus t naught. Similarly, x 1 t minus t naught is also y t minus t naught.

If you have both x 1 and x 2 are shifted by t naught and t 1 respectively then y shifted by t naught plus t 1. Convolution with impulse x 1 can be convolved with the delta function and so x 1 convolved with the delta function of t minus t naught is gives you x of t minus t naught. Limits of convolution if two signals are convoluted then the resulting convoluted signal has the following range. Sum of lower limits to sum of the upper limits that t extends from the sum of the lower limits to sum of the upper limits. So, that is the one of the properties.





Here we have two rectangles of unequal length to convolve with which results in a trapezium. The range of the convoluted signal is sum of the lower limits to sum of the upper limits. So, we have minus 1 to 2 and minus 2 to 2. The resultant limits become minus 3 to 4. The area under the convoluted signal is given by A y is equal to A x times A h where A x is the area under the input signal and A h under the impulse response.

The area under convoluted signal is given by $A_y = A_x A_h$

Where A_x = area under input signal; A_h = area under impulse response; A_y = area under output signal.

Proof: $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Integrating on both sides we get

$$\int y(t)dt = \int \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau dt$$

$$= \int x(\tau)d\tau \int_{-\infty}^{\infty} h(t-\tau)dt$$

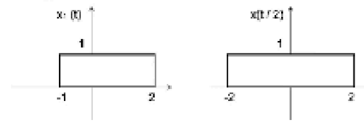
We know that area of any signal is the integration of that signal itself.

$$A_y = A_x A_h$$

DC component of any signal is given by

DC component = area of the signal/period of the signal

Ex: what is the dc component of the resultant convoluted signal given below?



Here area of $x_1(t)$ = length × breadth = 1 × 3 =3

area of $x_2(t)$ = length × breadth = 1 × 4 = 4

area of convoluted signal = area of $x_1(t)$ × area of $x_2(t)$ = 3 × 4 = 12

Duration of the convoluted signal = sum of lower limits < t < sum of upper limits = -1 + -2 < t < 2+2 = -3 < t < 4

Period=7

Dc component = 12/7

So, the area of the final output signal or convoluted signal is given by the product of the input signal and the impulse response both and it can be proven. So, you can prove that y t is given by x times h and integrate the both sides you get y t dt. So, that gives you the A y and here if you integrate you have one integral for tau and one for the t. So, one for the t is given by A h and for the tau is given by the A x. To calculate discrete linear convolution, convolute two sequences x n which is A B C a discrete sequence and h n is a sequence of E and F and G.

Let us see how to calculate discrete convolution:

1. **To calculate discrete linear convolution:**

Convolute two sequences x[n] = {a,b,c} & h[n] = [e,f,g]

Convoluted output = [ea, eb+fa, ec+fb+ga, fc+gb, gc]

Note: if any two sequences have m, n number of samples respectively, then the resulting convoluted sequence will have [m+n-1] samples.

	a	b	c
e	ea	eb	ec
f	fa	fb	fc
g	ga	gb	gc

Example: Convolute two sequences

x[n] = {1,2,3} & h[n] = {-1,2,2}

Convoluted output

y[n] = [-1, -2+2, -3+4+2, 6+4, 6] = [-1, 0, 3, 10, 6]

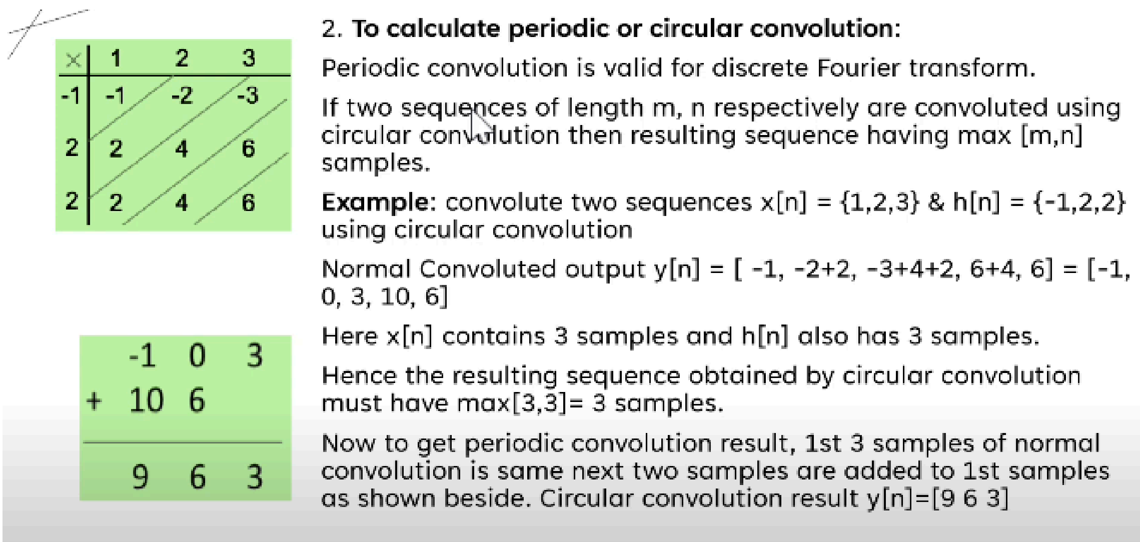
Here x[n] contains 3 samples and h[n] also has 3 samples.

so the resulting sequence having 3+3-1 = 5 samples.

×	1	2	3
-1	-1	-2	-3
2	2	4	6
2	2	4	6

So, the convoluted output will be E A times E B plus F A, E C plus F B plus G A, F C plus G B plus G C. If any two sequences have m and n m n numbers of samples respectively then the resulting convoluted sequence will have m plus n minus 1 samples. Convolutive sequence x n 1 2 3 and h n minus 1 2 2. Convoluted output will be given as following the above formula you basically get minus 1 0 3 10 and 6. So, x n contains three samples and h n contains also three samples.

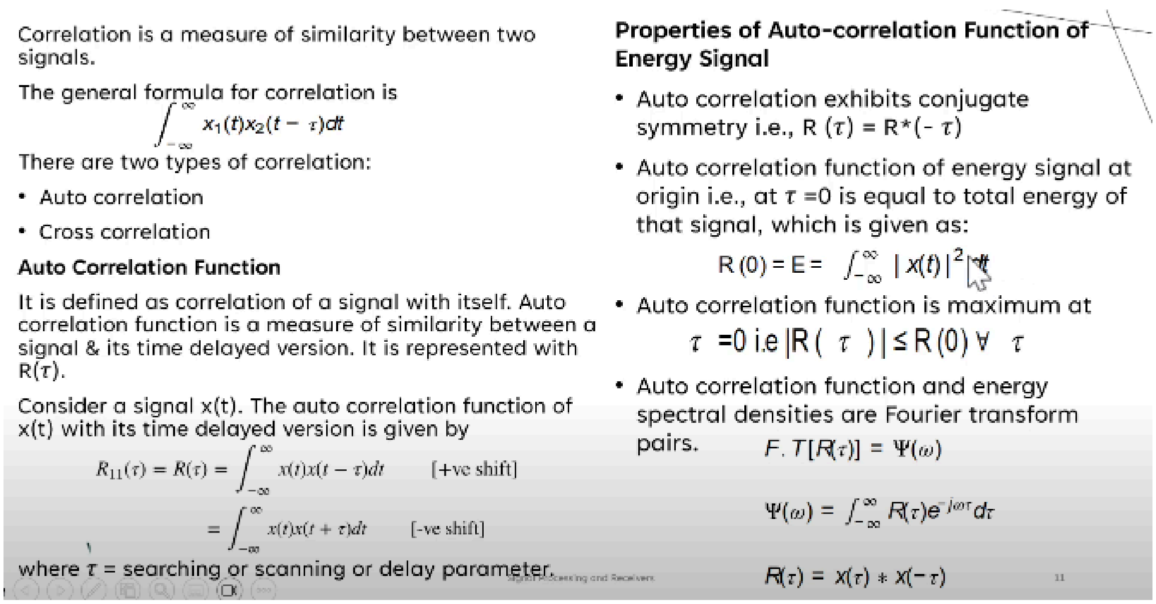
So, according to the rule the final convoluted sequence should be of five samples which is the case. To calculate periodic or circular convolution. Periodic convolution is valid for discrete Fourier transform. If two sequences of length m n respectively are convoluted using circular convolution then resulting sequence having max m n samples. Convolutive sequence x n 1 2 3 and h n minus 1 2 2 using circular convolution.



Normal convoluted output will be given by minus 1 0 3 10 and 6 following the rule as we discussed in the last slide. Here x n contains three samples and h n also contains three samples. So, the final must have the maximum of three and three so three samples. Now to get a periodic convolution result first three samples of the normal convolution is same next two samples are added to the first samples as shown below beside. So, minus 1 0 3 and 10 and 6.

So, if basically for circular convolution you return the first three and simply add the fourth one to the first one and fifth one to the second one in this process. So, now to get periodic convolution result you basically do this the following thing and so you finally have the periodic convolution result become 9 6 and 3 with a length of 3 which is the case as per the rule. Correlation and autocorrelation. Correlation is a measure of similarity between two signals. General formula is given by x 1 t integral of minus infinity to plus infinity of x 1 t and x 2 t minus tau dt.

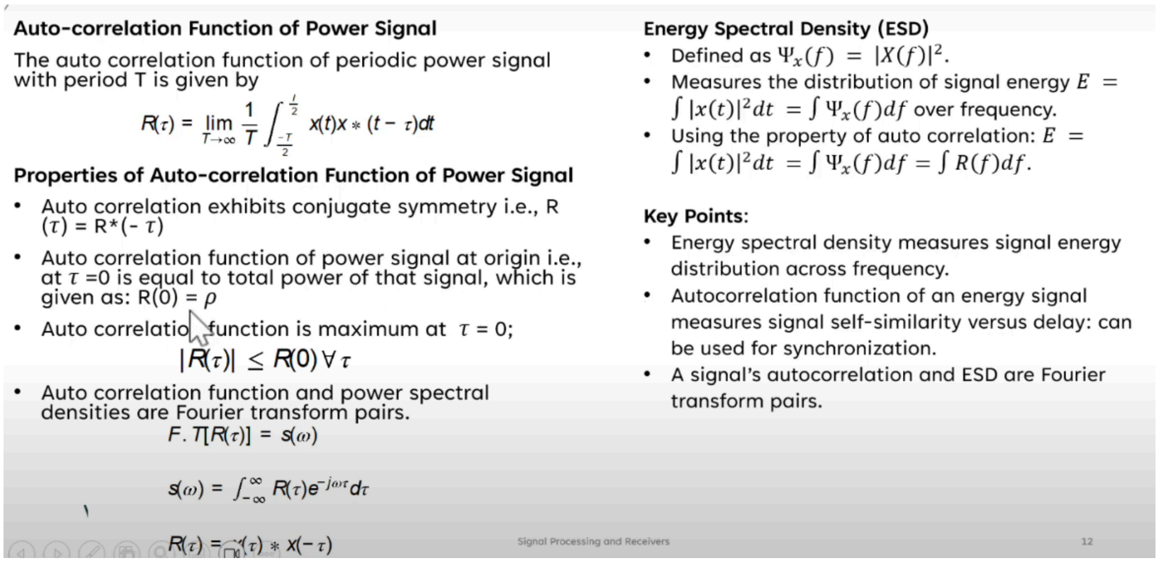
There are two types of correlation autocorrelation and cross correlation.



Of course autocorrelation is defined as a correlation of a signal with itself. Autocorrelation function is a measure of the similarity between a signal and its time delayed version. It is represented by r tau. r of tau of 1 1 that is the same signal is given by this x t into x t minus tau which is the positive shift and x t and x t plus tau which is the negative shift.

Tau is searching or scanning for delay parameter if any. Properties of autocorrelation function of energy signal. Autocorrelation of r tau is equal to r star minus tau. Autocorrelation function of energy signal at the origin at t equal tau equal to 0 is equal to total energy of the signal itself which is given by r 0 is e minus infinity plus infinity integral of x t whole square modulus square dt.

Yes. The autocorrelation function of a power signal is given by limit tau t tends to infinity 1 over t minus t by 2 to t by 2 and x t then convolved with x t minus tau dt. Sorry about this type. The property of autocorrelation function of the power signal. Autocorrelation exhibits conjugate symmetry r at tau is r conjugate at minus tau. Autocorrelation function of power signal at the origin at tau equal to 0 is equal to total power of that particular signal which is given by r 0 is equal to rho.



Energy spectral density E s d is defined as the x of f modulus square. This measures distribution of the signal energy over frequency. Using property of autocorrelation e x t modulus x t square dt is equal to psi of x f d f is equal to r f d f. So major points to learn from this particular slide is energy spectral density measures signal energy distribution across the frequency. Autocorrelation function of an energy signal measures signal self-similarity versus delay can be used for synchronization.

A signal's autocorrelation and E s d are Fourier transform pairs. Cross-correlation function next is the measure of similarity between two different kinds of signal. Two different signals not kinds of signals sorry. Consider two signals x 1 t and x 2 t the cross-correlation of these two signals gives you r 1 2 tau is given by r 1 2 tau is equal to integral of minus infinity plus infinity x 1 t into x 2 t minus tau dt that is positive shift and x 1 t plus tau into x 2 t that is a negative shift. If signals are complex then this second one will be should be the conjugate okay.

Properties of cross-correlation function of energy and power signals autocorrelation exhibits conjugate symmetry. Cross-correlation is not commutative like convolution. Cross-correlation function corresponds to multiplication of spectrums of one of the signal one signal to the complex conjugate spectrum of spectrum of another signal. So r 1 2 is basically x 1 omega if we transform x 1 omega times x 2 conjugate omega. This is called correlation theorem also known as Wiener-Kinchin theorem.

The cross correlation of these two signals $R_{12}(\tau)$ is given by

$$R_{12}(\tau) = \int^{\infty} x_1(t)x_2(t-\tau)\,dt \quad \text{[+ve shift]}$$

$$= \int_{-\infty}^{\infty} x_1(t+\tau)x_2(t)\,dt \quad \text{[-ve shift]}$$

If signals are complex, then

$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)x_2^*(t-\tau)\,dt \quad \text{[+ve shift]} \quad = \int_{-\infty}^{\infty} x_1(t+\tau)x_2^*(t)\,dt \quad \text{[-ve shift]}$$

$$R_{21}(\tau) = \int_{-\infty}^{\infty} x_2(t)x_1^*(t-\tau)\,dt \quad \text{[+ve shift]} \quad = \int_{-\infty}^{\infty} x_2(t+\tau)x_1^*(t)\,dt \quad \text{[-ve shift]}$$

We will be using this quite well in the weeks to come. So the cross-correlation function is nothing but the Fourier transform of a the multiplication of the spectrum also called the power spectrum of the both the signals Fourier transforms x 1 omega and x 2 conjugate of x 2 omega. Let us see an example to make things a little bit easier. An LTI systems impulse response is h t given by 2 times delta t minus 1 and 3 times delta t minus 2 where delta t is a direct delta function. If the input to the system x t is an exponential function e to the power minus t for all values of t greater than equal to 0 find the output y t at t equal to 2.

So the output y t is obtained by convolving the input x t with the impulse response of course. So y t is x t convolved with h t and that is given by x tau h tau t minus tau d t. x t is nothing but e to the power minus t unit function because it exists only in the t greater than 0 so this is represented easily by the unit function. So h t is also here so then y t becomes this value. Substituting t equal to 2 you get y 2 is equal to 4.73.

An LTI system's impulse response is $h(t) = 2 \cdot \delta(t-1) + 3 \cdot \delta(t-2)$, where $\delta(t)$ is the Dirac delta function. If the input to the system is $x(t) = e^{-t} \quad \forall\, t \geq 0$, find the output $y(t)$ at $t = 2$ [Hint: convolution].

Answer: The answer lies between 4.7 to 4.8.

The output $y(t)$ is obtained by convolving the input $x(t)$ with the impulse response $h(t)$:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\,d\tau$$

Given that $x(t) = e^{-t}u(t)$ and $h(t) = 2 \cdot \delta(t-1) + 3 \cdot \delta(t-2)$, the integral becomes:

$$y(t) = 2 \cdot e^{-(t-1)}u(t-1) + 3 \cdot e^{-(t-2)}u(t-2)$$

Substituting $t = 2$ gives:

$$y(2) = 2 \cdot e^{-(2-1)} + 3 \cdot e^{-(2-2)} = 2e^{-1} + 3 \approx 4.73$$

Okay so that should be clear. So again I'm just going through. h is given the impulse function is response is given. The input signal is also mentioned which is x t e to the power minus t is only valid for t greater than equal to 0. So we replace that condition with a unit function unit step function and then h t is there so if we just replace them you have a delta function. So delta function property is that it is it is it responds is only to that particular period so t equal to 1 it will respond no nowhere else.

So you have y t as all these things. Finally you put the y t value equal to 2 and you get y of 2 given by 4.73. That's your output signal at t equal to 2. Let's start our attention to something called sampling. Sampling is very important in real systems real receiver systems.

Sampling is a process which helps in conversion of an analog signal into a digital signal. This is important because data transmission in the form of digital signal offers various advantages like high efficiency, fast speed, low cost, low interference, low distortion, high security etc etc etc. In context to radio astronomy provides us a way to store and analyze the signal. Hence sampling is essential to improve the quality and transmission ability of signals over the communication channel. Sampling is performed using an electronic device slash component called analog to digital converter or ADC.

This is a device which either samples a signal first and quantize it into fixed level of voltages or quantizes a signal first into fixed level of voltages and then discretizes it. ADC helps in converting continuous signal digital into digital format such that it can be stored or digitally processed with help of computational devices like APGs, PCs etc. Now the GPU also is used. A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f is greater than or equal to the twice of the highest frequency of the component message signal.

This is a very powerful theorem. What it says is we are discretizing the signal. Okay. Now is it possible to discrete, what should be the rate of discretization? At what frequency should we do it? This sampling theorem or Shannon Nyquist sampling theorem says that this rate of the sampling should be greater than twice the highest frequency available in the signal itself. Let us consider what it is. So let us consider a continuous time signal X of t.

Let us say the spectrum of X t has a bandwidth of f sub m hertz. That means spectrum of X t is 0 for omega greater than omega m. So omega greater than omega m, its response is 0. So the highest frequency is, so yeah, bandwidth of fm right. Then the sample input signal X of t can be obtained by multiplying X t with an impulse train delta t of period t.

Then output will be a discrete signal called sample signal and is represented by Y t in the diagrams. So you have a constant signal, a continuous signal X t and now you are trying to discretize it. So you have impulse train of delta t coming from the other end and then you multiply them and you finally get Y t from the other side. So here you can observe that the sample signal takes period of the impulse. The process of sampling can be explained by the following mathematical expression.

Y t is the sample signal is X t times delta t. Trigonometric Fourier series representation of t is given by delta t is equal to a 0 summation over n equal to 1 to infinity a n cosine n omega s t plus b n sine n omega s t. This gives delta t is equal to 1 over 1 over t sub s and then summation over n 2 over t s cosine n omega s t plus 0. So you finally get the a 0 value as 1 over t s a n is given by 2 over t and b n s are all 0. If you substitute delta t then finally we get Y t is equal to given by this particular series 1 over t s X times t X of t plus 2 cosine omega s t X t 2 cosine 2 omega s t X t and so on. Fourier transform of that expression gives us omega and Y omega Y omega is 1 over t s X of omega X omega minus omega X X omega plus omega s X omega minus 2 omega s and X omega plus 2 omega and so on and so forth.

$$\delta(t) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos n\omega_s t + b_n \sin n\omega_s t)$$

Where

$$a_0 = \frac{1}{T_s}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)dt = \frac{1}{T_s}\delta(0) = \frac{1}{T_s}$$

$$a_n = \frac{2}{T_s}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\cos n\omega_s\, dt = \frac{2}{T_2}\delta(0)\cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\sin n\omega_s t\, dt = \frac{2}{T_s}\delta(0)\sin n\omega_s 0 = 0$$

Gives

$$\therefore\ \delta(t) = \frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t + 0)$$

So it can be given by this particular summation series. So to reconstruct Y t you must recover input signal spectrum X omega from the sampled signal spectrum of Y omega which is possible when there is no overlapping between the cycles of Y omega. So Nyquist rate for a band of frequency is the minimum sampling rate Nyquist is the minimum sampling rate at which signal can be converted into samples and can be recovered with back without distortion. So we are trying to discretize a signal.

It is a continuous signal. We want to discretize it and the discretization has to be done following a rate by which we can recover the entire signal and do not lose any information. That is the idea. However we are discretizing. Okay so discretizing meaning we are we are putting some missing some time stamps in the signal. But we have to make sure ensure that this loss or of those time stamps do not cause any loss of information in the final signal.

And if we do the sampling in a particular rate or above it then we can recover the signal without any loss of information. That is what this Nyquist channel Nyquist sampling theorem is all about. So Nyquist rate is f s greater than twice of f m where f m is the maximum signal. So Nyquist interval is one over that. In case of a band pass signal or signal with a frequency bandwidth the spectrum of the band pass signal x omega equal to zero for the frequency outside the range of f 1 f 2 where it is the upper and the lower limits.

Okay. Now if the frequency f 1 is greater than zero as per Nyquist rate and there is no aliasing effect when f s is greater than twice f 2. This leads to couple of issues. Sampling rate is large in proportion with f 2 and sample signal spectrum has spectral gaps. To overcome this Nyquist rate theorem was modified to the following. The input signal x t can be converted into samples and can be recovered back without distortion when the sampling frequency f s is less than twice the maximum frequency but greater than the twice the bandwidth of the signal itself.

So where B w is the bandwidth of the signal. This is valid and it is possible to up convert and down convert frequencies. This is the change or modification in the original Nyquist rate theorem for a band limited signal. Sampling in DFT analogy earlier it was noted the sampling is necessary for DFT such that the signal DFT is discrete Fourier transform such that the signal can be reconstituted from its DFT. Sampling theorem says the complete reconstruction is possible if the sampling frequency is greater than equal to twice the bandwidth. Similarly if we take n samples per period of the continuous frequency periodic signal x f with period 1 we expect the spectrum can be reconstructed if n is greater than the duration of the time signal x of n.

Recall sampling operation is equivalent to multiplication by a train of impulses. Hence the sampling at intervals of 1 over n effectively forms a new spectrum given by x tilde f of this. The impulse train in discrete form can be written as summation over delta n minus r of n r n. The convolution theorem shows that inverse discrete time Fourier transform x tilde n of the sample spectrum x tilde f is the convolution of the original signal x n by a periodic train of unit samples. So x tilde n is nothing but a convolution of the original signal x n with this impulse train.

The relationship between x n and x tilde n shows the signal x tilde n is a periodic with period of n. It is said that to be a time aliased version of x n by analogy with the frequency aliasing formula. Thus if the sampling is improper then aliasing will be present in both frequency as well as time domain. So if sampling is not done properly in time domain or in frequency domain there will be an effect called aliasing which will be present in both frequency and the time domain as well. So take away message is that if you want to discretize the signal why do you discretize it? To convert it from the analog to the digital form which makes life easier.

We can transport the signal quickly. We can store the signal properly etc etc without loss of the signal without other interferences etc etc. However when you're doing a continuous signal to a discretized signal you should also make sure that it is sampled in a particular date that is given by the Shannon Nyquist Simply Theorem. For a continuous signal there is no problem but for a band limited signal which is a start and the end frequency so it has a bandwidth B w there the sampling rate becomes lesser than the highest frequency present in the band but greater than the bandwidth of the signal itself twice the bandwidth signal itself. This is all which we have learned.

If you do not sample at that rate you face aliasing. So examples which of the following is correct statement about sampling theorem? Nyquist rate causes aliasing. Sampling theorem for signal with a bandwidth of f s less than equal to twice B w. Aliasing can be avoided by following the sampling theorem and none of the above.

So c is the true. Aliasing can be avoided by following sampling theorem.

1. Which of the following is the correct statement about Sampling theorem.
   a) Nyquist rate causes aliasing
   b) Sampling theorem for signal with a bandwidth is $f_s \leq 2BW$.
   c) Aliasing is can avoided be following sampling theorem
   d) None of the above

Answer: (c)

2. If you have a signal bandwidth of 100MHz the choose the correct sampling rate out of the following
   a) 150MHz
   b) 220MHz
   c) 180MHz
   d) 100MHz

Answer: b) 220MHz as the Sampling theorem for signal with a bandwidth is $f_s \geq 2BW$ and 220MHz is the most correct in all the option.

Second question, if you have a signal bandwidth of 100 megahertz choose the correct sampling rate out of the following. So the signal bandwidth is 100 megahertz choose the correct sampling rate 150 megahertz 220 megahertz 180 megahertz or 100 megahertz. So it should be at least twice the bandwidth so it should be greater than equal to 200 megahertz. Out of all the possible options only B satisfies that condition so answer is B.

So now we shift from the sampling to something called noise. We have introduced noise a little bit let's keep discussing further in this particular lecture also. So far we have learned about signal. But in order to detect a signal it has to be significantly higher than the noise in the system. Noise in the system can appear in multiple ways.

That's what we are going to discuss now. So unless we characterize the noise and be aware of all possible noise or interferences present in the system or in the surrounding it will be difficult to make a proper detection of the signal. Particularly the signals if they're so weak like the radio astronomy signals. So noise is an unwanted signal maybe intentional or random. There are different types of noise thermal noise, short noise, partition noise, flicker noise, burst noise, transit time noise etc. Most of the above stated noise are no dominant at higher frequencies except thermal noise.

Characterizing and quantifying thermal noise is the most important for those who are working in radio astronomy instrumentation. So here on we will discuss about different types of noise but keeping a focus on the thermal noise itself. Short noise in electronic circuits consists of random fluctuations of DC current which is due to electric current being the flow of discrete charges or electrons. Partition noise where current divides between two or more paths noise occurs as a result of random fluctuations that occur during this division. For this reason a transistor will have more noise than the combined short noise from its two gain junctions.

Flicker noise also known as 1 over F noise very important characteristic noise we'll talk about it later we'll talk about ionosphere wave propagation etc. But it's a very important thing to note particularly regarding the electronic components and calibration of the gains of the electronic components 1 over F noise or flicker noise or even called sometimes pink noise. The signal or process with a frequency spectrum that falls off steadily into higher frequencies with a pink spectrum. It occurs in almost all electronic devices and results from a variety of effects.

So white noise and the pink noise have two different characteristics. White noise typically is can be defined by a Gaussian random process where you have a zero mean Gaussian or a Gaussian with a fixed mean mu at mu 0 and and general deviation variance of sigma square. Pink noise is a particular variety where the mu is not fixed. It varies with time so there is no real mean for a long time if you if you track the signal for a long time. So that's one of the one of the particular effects of flicker noise. Next is burst noise consists of sudden step like transition between two or more discrete voltage or current levels random and unpredictable at times can be removed easily by averaging of signal.

The white noise also if a noise has a characteristic of white noise that basically means the fluctuation against zero mean and if the mean is zero or the mean is the signal itself without a positive DC bias then averaging over multiple times reduces the noise particularly. Transit time noise if the time taken by electrons to travel from emitter to collector in a transistor becomes comparable to the period of the signal being amplified that is at frequencies above VHF and beyond the transit time effect takes place and noise input impedance of transistor decreases. From the frequency at which this effect becomes significant it increases with frequency and quickly dominates other sources of noise. Now how do we characterize this thermal noise? So let's go back a little bit and talk start from the place where we left the discussion of the black body and the Rayleigh-Jens law. Let's consider a resistor passive electronic component that absorbs electrical power and converts that power into heat simply joule heating.

As for the black body laws the temperature T greater than zero kelvin generates some electromagnetic radiation aka electron electrical noise. The frequency spectrum of this noise depends only on the temperature of an ideal resistor and is independent of the material of the resistor itself. The derivation of the electrical power per unit bandwidth P nu generated by a current in the resistor. So at low frequency h nu very very less than kT and the Rayleigh-Jens approximation is accurate. Recall Rayleigh-Jens derivation of B nu starts with a large cube of side of length greater than lambda containing standing waves of thermal radiation.

Consider two identical resistors at temperature T connected by a lossless transmission line as a pair of parallel wires of length a much larger than the longest wavelength of interest. Standing waves on the line must satisfy a equals to n lambda over 2 where n is 1 2 3 4 and so on. Where lambda is the wavelength electrical signals do not travel at exactly the speed of light on a transmission line but at some slightly lower velocity than that. Okay so the setup is like this there is two resistors connected by two wires there's lossless transmission line okay and that is kept like in this particular diagram. For a which is the length of the transmission line is very very larger than the lambda of the highest wavelength possible.

- At low radio frequencies, $h\nu \ll kT$ and the Rayleigh–Jeans approximation is accurate. Recall that the Rayleigh–Jeans derivation of B_ν starts with a large cube of side length a $\gg \lambda$ containing standing waves of thermal radiation.
- Consider two identical resistors at temperature T connected by a lossless transmission line (e.g., a pair of parallel wires) of length a much larger than the longest wavelength of interest. Standing waves on the line must satisfy

$$a = \frac{n\lambda}{2}, \qquad n = 1,2,3,4, \dots\dots\dots$$

- where λ is the wavelength. Electrical signals do not travel at exactly the speed of light on a transmission line, but at some slightly lower velocity $v < c$ so $v = v/\lambda$ and $n = 2a/v$

The number of modes per unit frequency is given by n nu is equal to twice a over frequency. The classical Boltzmann law says that each mode has an average average energy of kT in the equilibrium so the average energy per unit frequency e nu in the transmission line is given by e nu is equal to n nu k times T that gives you 2 a kT over nu. This energy takes a time T which is a over velocity to flow from one end of the transmission line to the other. So the classical power energy per unit time per unit frequency flowing on the transmission line is given by p nu of energy over delta T which is the time it takes that is given by twice kT however p nu is also independent of the velocity. So here the p nu is given by 2 kT if the noise power has a bandwidth of B then the total power sorry the bandwidth over which the noise is measured is called B then the total power of the noise power density is becomes equals to p is equals to twice kT times B.

Radio Astronomy

THERMAL NOISE ....

- For a > λ, the number of modes per unit frequency is $N_v = 2a/v$
- The classical Boltzmann law says that each mode has average energy ⟨E⟩ = kT in equilibrium, so the average energy per unit frequency Ev in the transmission line is

$$Ev = N_v kT = \frac{2akT}{v}$$

- This energy takes a time t = a/v to flow from one end of the transmission line to the other, so the classical power (energy per unit time) per unit frequency flowing on the transmission line is

$$P_v = \frac{E}{\Delta t} = 2kT$$

- However, it is to be noted P_v is independent of velocity v.
- Now, here we will slightly deter from the last time and move on to define noise power as

$$P_v = 2kT$$

- and noise power density as

$$P = 2kTB$$

- Where, B is the bandwidth over which the noise is measured.

So noise power is determined by only and only the temperature okay that gives rise to this thermal noise effectively and the bandwidth of B. In radio astronomy we are interested in learning about structure of signal from the sky for which we use radio antenna essentially antenna temperature you remember that is given in terms of this the flux density. So you look at you remember that lecture where we considered S is k times delta T a where delta T a is nothing but the change in antenna temperature when the source of interest was within the field of view minus the antenna temperature when the source of interest was not in the field of view. So it's the background subtracted. So the k times k is the Boltzmann constant delta T a is the differential antenna temperature off source and on source on source minus off source over the effective area.

So that gives you delta T a S is the flux density of the source itself. Now correspondingly the effective antenna temperature is also given over here omega a over omega S where omega S is the angular size of the source and T S is that source temperature sorry it's not antenna temperature and A is the effective collecting area of the antenna itself and omega is the antenna beam width okay.

$$S = \frac{k\Delta T_A}{A_e} \text{ and } T_s = \frac{\Omega_A}{\Omega_s}\Delta T_A$$

So now if antenna is getting a change in temperature then that will show as a deflection in the received signal. This deflection does not come purely from only the sky. So we are definitely looking for signals from the sky but in the path the signal itself collects a lot of corruptions due to the wave propagation through the atmosphere etc. In the neighborhood the antenna is also collecting lot of interfering signals etc from the neighborhood okay which are propagating in the similar same bandwidth as the signal itself.

So if you are looking for a signal that is a signal that is a signal that is a signal that is if you are observing a reflection in the voltage or the power received then we can define a quantity called signal to noise ratio which will quantify how much signal strength the signal strength when compared to the background noise. So signal over the noise is what we have to maximize. The SNR higher the SNR the more confident we will be about detection of the signal okay. The most significant the signal detection will be. So we have seen that the signal is already weak and because of the propagation through medium it becomes weakens further.

Not only it weakens but also it gathers other noises, other interferences which makes the signal to noise even further reduced. So effort should be taken to improve or increase the signal to noise ratio. So when we detect the signal the signal will undergo different steps like amplification, filtering etc. This post processing of the signal might contribute also to the noise. Thus there is a fear that it might reduce signal to noise to such a level that signal will not be recognized.

For this reason a quantity called as noise figure was defined which will provide the idea about how much noise is being contributed by the system itself. So the signal is coming from the sky. It gets detected by the antenna and then pushed through the other front end and back end system of electronics which amplifies, filters the thing, mixes signal as squalor detector is there etc etc. A lot of other electronics is there. While passing through these different stages of electronics the signal also suffers attenuation, suffers a lot of other things like contribution of the noise in the signal increases because of system itself.

So the desirable thing is the system itself should be at a lower temperature, lower noise temperature. Sometimes that is ensured by putting in the system inside the cryogenic devices because thermal noise the major thing is the system noise itself and the thermal temperature is maintained at a lower ambient temperature then the system noise contribution can be lowered. Other thing is that we choose by default low noise electronic devices like low noise amplifiers. So in radio astronomy low noise amplifier is the by default term.

The RF amplifier is used only after the low noise amplifier is initially put in place. So noise figure of each individual component of the electronics matters a lot. So designing a proper adequate receiver requires the specification of individual components also has to be checked. Okay so one such thing is noise figure which we discussed before also and we are again discussing now.

So amplifier of course is a term that amplifies a signal. Amplification is required. Suppose our antenna is very far away in the field and we are bringing the signal back to our location of where the computers are placed through coax cable or any other cables or even OFC optical fiber cable. There will be some attenuation because of the length. So the before sending the signal through the cable the typical thing is to amplify the signal. So amplification itself comes with a cost that there could be some noise induced by the electronic component itself.

So first thing is choosing the low noise amplifier is required. Second thing is choosing a cascade of such amplifiers to give it a more higher gain. Okay so we will talk about it now. So say Xt is the input signal to this amplifier and Yt is output signal and the signal to noise when the beginning is SNR i and leaving the output is SNR o or output and this particular amplifier is a LTI system having gain g and noise figure of N sub a. So that the noise figure noise Na sorry and the noise figure of this simple system is SNR i over SNR o that comes down finally to 1 plus Na over g times Ni where Ni is the noise in present in the input signal itself.

So, Here we can define a figure of merit:
Noise Figure (F) as

$$F = \frac{SNR_i}{SNR_o} = \frac{\frac{S_i}{N_i}}{\frac{GS_i}{GN_i + N_A}} = 1 + \frac{N_A}{GN_i}$$

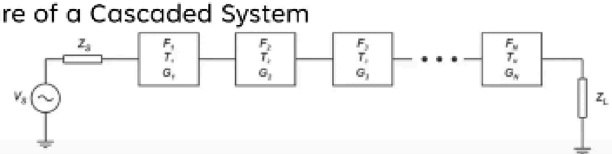
In terms of noise temperature noise power is defined as N is proportional to T. Considering that the environment temperature is T0 if there is a detection it this will cause the noise temperature to increase in the system then the noise figure of the system can be defined as f which is given by noise temperature at the output over at the input. So T0 plus Te is noise temperature at the output and T0 is the one at the input. So Te is the device contribution to the noise temperature overall. So 1 plus Te over T0 is the total noise figure of the system itself.

As I said a cascade of amplifiers are required sometimes to provide a higher gain. So for a cascaded such system cascade system the noise figure kind of varies like this it is not just simple sum but following this kind of a thing you can understand that the total noise figure of this system where you have a cascaded components of 1, 2, 3 etc up to N there which has a noise figure of f1, T1, g1, f2, T2, g2 etc etc. So f i, T i and g i are the three different components of this individual cascaded sub components then the total noise factor is equal to f1 plus f2 minus 1 g1 plus f3 minus 1 g1 g2 and like this. So for the nth component is f n minus 1 g1 g2 up to g n. Similarly the cascaded system temperature equivalent noise temperature is given by T1 plus T2 over g1 plus T3 over g1 g2 and then therefore the nth component T n over g1 g2 up to g n minus 1.

- In the terms of Noise Temperature
  - Noise power is $N \propto T$,
  - Considering that the environment temperature is T_0,
  - If there is a detection it will cause the noise temperature to increase in the system. Then the Noise figure of the system can be defined as

$$F = \frac{Noise\ temperature\ at\ output}{Noise\ temperature\ at\ input} = \frac{T_0 + T_e}{T_0} = 1 + \frac{T_e}{T_0}$$

  - Where, T_e is device related noise temperature.
- Noise Temperature of a Cascaded System



  - Where F_i is the noise factor; T_i is the equivalent noise temperature and G_i is the available power gain of the i^{th} stage. Now, the the noise factor of the cascaded system is:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 \ldots G_{N-1}} \qquad T_{cas} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots + \frac{T_N}{G_1 G_2 \ldots G_{N-1}}$$

This is very important to know while designing a proper radio receiver. Let us take a look at the noise temperature since we are talking about the noise temperature take a look at the noise temperature or system temperature in addition with the antenna itself. So system temperature refers to the effective temperature of the entire system including the antenna and associated components that receive or transmit determatic signals. It is a critical factor to determining the sensitivity of the and the signal to noise ratio of a system. See sensitivity of a system is very important because we need to understand we have to make a build a system which is sensitive to a particular signal. If the signal is already faint the system has to be super sensitive and supposedly the let's let's take an example like if i am looking at the sun for that the telescope which i will be building and electronics which are required will be of much much relaxed specification then supposedly i am looking at a distant star or a galaxy which is millions and millions of light years away.

Sun is brighter and nearby whereas this particular faint galaxy is far away and so i have to build a much more sensitive system. So that this budget of the system temperature is very important to note in order to make the system more sensitive and build a correct system for a correct signal to be detected. Not every single every telescope is built for every possible signal. So it the system temperature represents the equivalent temperature of an ideal noise source that would produce the same amount of noise power as the entire system. Contributing temperatures come from contributing components to the system temperature antenna itself feed lines receiver transmitter electronics etc.

A fair culmination of the system temperature can be written as Tc is equal to Ta which is antenna temperature times TAP the antenna physical temperature multiplied by 1 over epsilon this e1 which is the antenna efficiency minus 1 then TLP which is the line physical temperature multiplied by 1 by e2 minus 1 which is the line efficiency and Tr over e2 which is the Tr is the receiver noise temperature. There are plenty of other terms like the atmosphere creates one kind of noise there are different noises for the surface if there is a system if the antenna is made up of reflector dishes in the surface accuracy itself creates another another kind of contribution to this thing various efficiency parameters etc. This is just the one of the thing we will see more detailed expressions in the coming lectures. This gives rise to 180 Kelvin.

$$T_{\text{sys}} = T_A + T_{AP}\left(\frac{1}{\varepsilon_1} - 1\right) + T_{LP}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{1}{\varepsilon_2} T_R$$

So the next thing to discuss about is the minimum detectable temperature because we are talking about sensitivity. So what is the minimum temperature that our system can detect that also gives a sense of what is the sensitivity of the given system. So let us say define that it was delta T mean of a receiving system equal to the rms noise of the system itself. So that is given by delta T mean is given by k prime Tc over delta f times T that is also called delta T rms. This is a radiometer equation which we will again see in the next week mostly that how this is derived from the scratch. But what it says is that the minimum detectable temperature is a function of the system temperature which is given in the previous one more detailed can be also shared in the coming weeks.

$$\Delta T_{\min} = \frac{k' T_{\text{sys}}}{\sqrt{\Delta f t}} = \Delta T_{\text{rms}}$$

$$\boxed{\frac{S}{N} = \frac{\Delta T_A}{\Delta T_{\min}}}$$

The essence will be the same but there will be more components added to this system temperature explicitly. So system temperature is one of the reason it is in the numerator. So as system temperature rises the delta T mean goes down also rises. The minimum detectable temperature rises that means sensitivity deteriorates.

So thesis has to be low to increase sensitivity. It is further lowered by delta f which is the bandwidth of the receiver. So if the receiver bandwidth increases the sensitivity increases because we are getting more samples of signals remember. So more the sample the all the fluctuating components of the noise they will reduce and only the mean component of the signal will get amplified. And T is the time. So the delta T mean remember has to be low for sensitivity to rise.

So a system with delta T mean lower is considered to be higher in sensitivity. If it is high then it is not that sensitive because we can detect only the signals the faintest signals which are lesser which are above this delta T mean is only which we can detect by the system. So delta T mean has to be lower that means system temperature has to be lower has to be lower because that is in the numerator and delta f has can be has to be larger to reduce delta T mean and T time of observation also has to be larger to reduce this delta T mean which makes sense because time samples and frequency samples will reduce the noise at least the fluctuating components of the noise which are just basically white noise components. Right. So in this case the signal to noise ratio is then it is by N delta T A over delta T mean very vital.

We will come back to this when we talk about single dish observing strategies and more. So before we proceed further let us take a couple of examples. A receiving system has an antenna with total noise temperature of 50 kelvin. A physical temperature of 300 kelvin and an efficiency of 99 percent. A transmission line at physical temperature of 300 kelvin and an efficiency of 90 percent and a receiver with the first three stages of all of 80 kelvin noise temperature and 13 db gain.

Find the system temperature. So system temperature is nothing but the receive okay firstly compute the receiver temperature. So it's a cascaded noise temperature formula we have used so there are three stages so total at tempera the temperature of the receiver is T1 plus T2 over G1 plus T3 over G1 G2. We have done it earlier in this yeah in this particular slide we did the same so we use that same formula and come back with that so all three are 80 and all the gains are twin 13 db.

So 13 db gain is approximately linear scale is about 20. So 80 plus 80 by 20 plus 80 by 400 is equal to 84.2 kelvin you can try. Now the total system temperature is nothing but Tc's given by so this is with Tc's by antenna temperature plus ambient temperature physical temperature time 1 over e epsilon ep 99 percent minus 1 then T of line temperature is given by this Tlp is given by 300 again and 1 over lp is e epsilon lp is 90 percent and then tr. So we essentially follow this this expression over here this is Tc's there's a typo this is from Tc's and this is the antenna which is 50 degree 50 kelvin Tep is 300 kelvin 1 over 0.99 minus 1 then 300 times 1 over 0.9 minus 1 and then 84.2 divided by 0.9 this gives rise to 180 kelvin.

Question 1: A receiving system has an antenna with a total noise temperature of 50 K, a physical temperature of 300K and an efficiency of 99 percent, a transmission line at a physical temperature of 300 K and an efficiency of 90 percent, and a receiver with the first 3 stages all of 80 K noise temperature and 13 dB gain. Find the system temperature.

Solution:

The receiver noise temperature can be calculated using cascaded noise temperature formula [Gain = 13dB = 20 (appx.)]

$$T_R = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} = 80 + \frac{80}{20} + \frac{80}{20^2} \approx 84.2K$$

Then the total system temperature will be:

$$T_R = T_A + T_{AP}\left(\frac{1}{\epsilon_{AP}} - 1\right) + T_{LP}\left(\frac{1}{\epsilon_{LP}} - 1\right) + \frac{1}{\epsilon_{LP}} T_R$$
$$= 50 + 300\left(\frac{1}{0.99} - 1\right) + 300\left(\frac{1}{0.9} - 1\right) + \frac{1}{0.9} \times 84.2 \approx 180K$$

So our next example is finding the noise power of a cascaded system output assume that the source noise temperature is Ts equal to 150 kelvin. Also suppose that the cascade noise factor gain and bandwidth are respectively 1.8 db 6 db and 10 megahertz find the available noise power at the output of the cascade. So going by the previous um slides if we go back to that slide where we discussed the cascade given by this and also the f noise figure is 1 plus Te over T0.

Question 2: Finding the Noise Power of a Cascaded System Output Assume that the source noise temperature is Ts = 150 K. Also, suppose that the cascade noise factor, gain, and bandwidth are respectively F_{cas} = 1.8 dB, G = 6 dB, and B = 10 MHz. Find the available noise power at the output of the cascade.

Solution:

To find the noise temperature of the cascade:

$$T_{cas} = (F_{cas} - 1)T_0 = (1.8 - 1) \times 290 = 232\ K$$

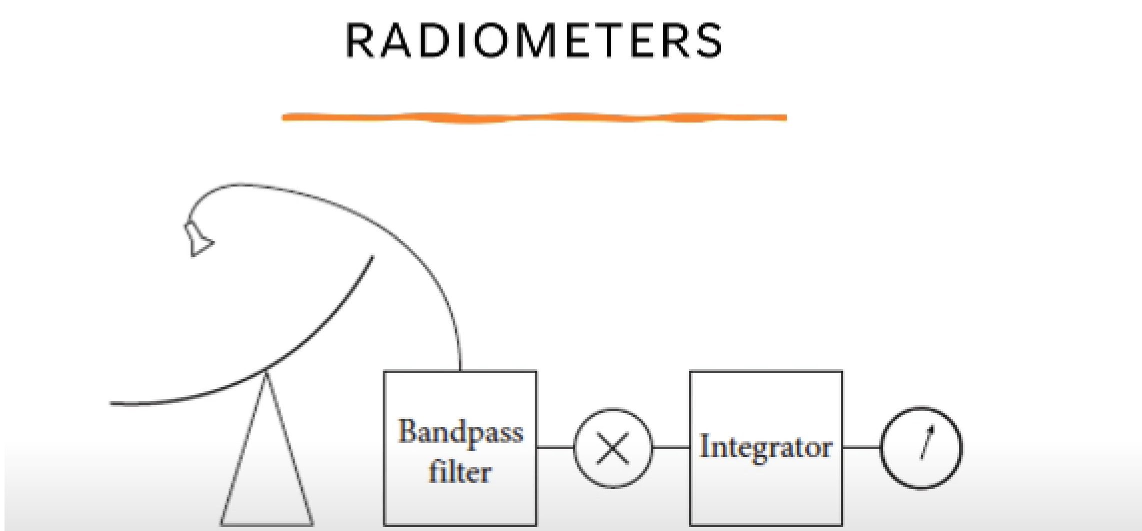
Therefore, the noise temperature of the overall system is $T_{sys} = T_s + T_{cas}$ = 150 + 232 = 382 K.

Finally, we have:

$$\begin{aligned} N_o &= k(T_s + T_{cas})BG \\ &= 1.38 \times 10^{-23} \times 382 \times 10 \times 10^6 \times 10^{0.6} \\ &= 2.099 \times 10^{-13}\ W = -96.8\ dBm \end{aligned}$$

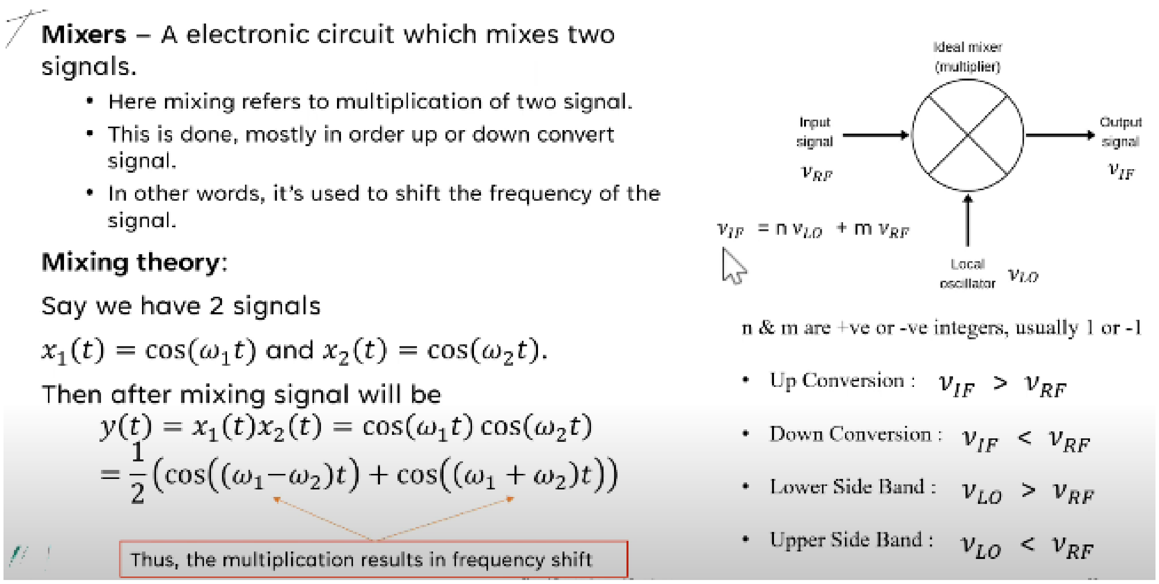
Let's see if we can solve this. So T cascade is given by f cascade minus 1 times T0 that is 232 kelvin. The noise temperature of the overall system Tc's is given by Ts Tc that is 382 kelvin. So then we have the noise power is K times total temperature times the bandwidth. Okay so that will give rise to 1.38 this is the kelvin this is the Boltzmann constant plus 382 plus you have a gain of 6 db so that you convert into the linear scale and also you have bandwidth of 10 megahertz.

So 10 megahertz is 10 to 10 to the power 6 this is the bandwidth and 10 to the power 0.06 is the is the gain so total kama's out to be minus 96.8 db m that's the answer. So uh we come to the last segment of our um of our discussion talking about it's all good so we discussed about different techniques to do signal processing signal analysis different signals different systems Fourier transform Fourier series convolution autocorrelation cross correlation we spoke about Nyquist sampling theorem various um you know tools as well as the noise discussion of noise sensitivity etc.



Now let us combine them together and see how a basic radiometer basic block of a radio telescope looks like so it's something looks like something similar an antenna with a reflector to begin with it may be antenna without a reflector also then a bandpass filter because we do not want to see uh outside our band because there's been a lot of more spurious signals coming from the outside the band then an integrator which integrates the signal which kind of lowers the noise level and increases the signal level so SNR gets increases. So there are different bands we will come that later also but VHF band the high frequency is 3 megahertz to 30 megahertz the VHF for very high frequency goes from 30 megahertz to 300 megahertz UHF band or ultra high frequency band is 300 megahertz to thousand megahertz L band goes from 1 gigahertz to 2 gigahertz S2 to 4 uh C band is 4 to 8 X 8 to 12 K U band 12 to 18 K 18 to 27 K A or K above is 27 gigahertz to 40 gigahertz then V band 40 to 75 and W band 75 to 110 gigahertz there are higher bands also but this is what we will be concentrating on mostly I mean below four to five gigahertz.

So different components which are used different kinds of RF connectors will be used because we have wires adapters attenuators RF amplifiers high gain amplifiers low noise amplifiers mixers and filters that compose comprises this entire radiometers. RF connectors specific type of removable mechanical coupling which is attached to the an RF cable there are several types of RF connectors differentiate based on frequency of operation and size. N-type connectors works up to 11 gigahertz big in size and is for rugged use SMA type connectors sub miniature version A works up to 26.5 gigahertz to 2.92 mm connectors which is similar to SMA connector can go up to 40 gigahertz. RF adapters it's a mechanical interconnect which connects the or links different types of connectors N-type male to female adapters so if you have to join different cables these are the adapters which is required. Attenuators it is equivalent to resistors however there is thoroughly characterized such that it provides a particular amount of power loss across it for a fixed band of frequency so very very accurate it is used for isolating different components such that the operations are smooth RF amplifier or high gain amplifier it's a basically high gain amplifier the the signal low noise amplifier as it signifies if the noise is kept low typically the after the end of the after soon after the collection of the thing from the antenna there is a low noise amplifier because you want to amplify the signal without adding much noise to the uh signal chain another set of the components are extremely useful it are called filters as the name suggests it filters out the signal based on the frequency first one is low pass filter it is only allows something signals below a particular frequency to to pass and anything above that should be stopped high pass filter is just the reverse of the low pass filter it passes allows the higher frequencies to pass the lower frequency are completely uh chucked out a band pass filter is as name suggested it allows the operations bit within the minimum and the maximum frequency as it is designed for so it allows that pass band to pass and then and restricts the other other frequencies outside the band uh to to transfer the reverse of the band pass is the band reject or a notch filter which basically only rejects the signals between that frequency minimum to maximum to stop suppose you have a very strong line a gsm signal line at 900 megahertz or so you build a notch filter so that your telescope can operate in the same frequency without getting affected by that particular signal mixers um they have a very very strong role when we talk about superherosine receivers they mixes two signals so uh um RF signal comes in and IF signal goes out of this it's RF is radio frequency it's an intermediate frequency and what happens is mixing with the LO signal or local oscillator okay so if the IF signal is higher than the RF that's up conversion down conversion is a reverse one lower sideband is the when LO is less than greater than the RF frequency and when LO is lower then it's called the upper sideband so it mixes the two frequencies and causes a frequency shift at the end so often we do a down conversion to because if the frequencies are higher we want to bring it down conversion to bring it to the lower passband baseband signal a typical digital scope looks like this a telescope then with the front end electronics this is typically near the telescope itself then a long wire connection or ofc connecting to some back end area where you kept another set of electronics receiver back end and then the digital back end at the end these are made by ln's mostly and with some filter combinations similarly at the back end is also



some several other filter combinations the and some ln's RF amplifier a local oscillator for the mixer and the low pass filter at the end so as we know that the antenna temperature is uh temperature is given by p nu uh which is the power per unit frequency over kelvin uh which is also linked with the effective and effective area times the source flux over 2k now so for a simple system like this signal is collected it's it's uh it's collected over a band limited region so it's collected over the central frequency is nu RF then the bandwidth is delta nu so it is calculated between nu RF minus delta nu by 2 to nu RF plus delta nu by 2 so a typical the radiometer consists of four stages a low loss band pass filter a square or detector whose output voltage is square proportional to the square of its input voltages so it squares the voltages in the input and a signal integrator that smooths the rapidly the fluctuation detector component as a integration to time integration part mostly and a volt meter to detect the signal finally so the integration you can see this is the sample after n 50 samples you can see it's so much of a spread that the noise power is still high and if you say if you smooth over 200 samples you see that the fluctuations are going down and so you're critically defining a mean signal more rapidly so sigma t which is the uncertainty in the signal is given by this ts the system temperature over this delta nu by tau but also some gain variations is the full expression we will take a little bit deeper look at the next week the different kind of radiometers this is a simple one the first stage compared with the differential one where the two different feeds and you keep switching back and forth between the two it reduces a lot of this gain errors so you can see the gain error coming from this simple radiometer is kind of dropped when you actually use a differential radiometer more more again on this in the next week finally coming to something very important called super heterodyne receivers where we actually the rf there's a there's a provision of the we do not have to operate the entire signal chain at the simple radiometer we have to operate the entire signal chain at the particular receiving frequency where the antenna is designed but in this particular case we have frequency where the antenna is designed but in this particular case we can in the second case of super heterodyne receivers we can design our yellow frequency such that the remaining part of the chain can operate in a single baseband frequency no matter what kind of a receiver we can put in here it has various advantages some advantages are following that as we know we cannot build a single system which covers a wide band because our scientific interests are varied we do not want to restrict ourselves to a particular wavelength so we need to build separate receivers at this at this stage of the antenna to observe separate things however for all of those the back end can be similar extreme back end because of this down conversion it comes down to the same baseband frequency and some of the systems are same for different other bands to be used that's one advantage there are several others which we will mention in the second next slide so let us understand what is heterodyne means basically okay this is like another design of the same suppose example we have rf signal at five gigahertz with a bandwidth of 100 megahertz now mixing signal nu1 and nu2 will result into a heterodyne frequency of nu1 plus minus nu2 if nu1 is received uh frequency or nu rf with a bandwidth of b which we are mixing with a signal with frequency nu2 or nu l o local oscillator then the shifted frequency will look like nu rf plus minus b by 2 minus nu l o and nu rf plus minus b by 2 plus nu l o minus nu l o and plus nu l o this means heterodyning a bandwidth signal doesn't change its characteristics but it just shifts the frequency up and down so typically we take the lower value we don't allow the higher value because typically a super heterodyne receiver is followed by a high low pass filter actually the different stages there's a rf gain stage and rf line filtering stage rf gain stage amplifies signal power to a sufficient level such that the faithful mixing can be performed generally there are lower noise amplifier is kept first to make overall noise field system to be low or minimum rf line filtering in between rf filtering line a bpf or a band pass filter or a combination of lpf and i hbf are used for a band selectivity it is important as amplifiers are not always narrow band so they might amplify the signals out of our band of interest so filtering will allow us to remove unwanted frequency mixture mixer and l o so l o is one of the most vital part of the super heterodyne receiver generally l o's are near pure sine and cosine wave such that it is it will have a single frequency in the spectrum impure l o have harmonics resulting in various impurities if line filter now in case of a receiver which we are interested in in interesting the right after mixing an lpf is used to cut off the higher frequencies so our idea is to down convert if you have to up convert we will have a high pass filter which will allow the higher frequency to go through that is new rf plus l o and and top edge of the lower frequency but in this particular case we're interested in the down pass filter which will interest in the down conversion so we use a low pass filter which will allow the new rf plus new minus new l o to pass but stop this higher frequency to pass i f amplifier then after that band pass some attenuation will be um anyway introduced so you need another i f amplifier after at the end to amplify the signal to retain its characteristics uh so that completes the definition the discussion of the super heterodyne receiver again the

- The basic formula for a link/RF/Power budget is given by

$$P_r = P_t - L + G \qquad (all\ quantities\ are\ in\ dB\ Scale)$$

advantage is you can keep some part of the signal chain constant even if you use different kind of front end for collecting different signals at different wavelength time there are other advantages of of doing down common signal to a lower frequency uh that we'll discuss later sometimes up conversion is also required it depends on the nature of the surrounding if you have too much of interference in a particular baseband you can also up convert and then down convert finally so different combination of up converter and down converter can be used in a signal chain as well it is not uncommon uh the last topic of today's lecture is the rf budget and power budget link budget etc so we have to make a budget of how much is the power loss uh attenuation the signal will suffer from its very first tapping in the antenna till it finally gets recorded individually okay and that that budget is very very important for example i just have you just use the example to demonstrate that a radio antenna receives the power of minus 100 dbm which is then amplified by an lna having a gain of 20 db then it is transported to the base station at 400 meters using an rf cable with attenuation losses of 0.5 0.25 dbm db per meter losses assuming the receiver requires a power of minus 40 dbm for faithful detection then calculate the required gain so the receiver requires certain input power to do its job properly there is minus 40 dbm and the antenna receives the power of minus 100 dbm so the best possible the possibility it would be maximum of 60 db loss so the gain uh required by the required uh bx then by the above formula we have minus 40 dbm is equal to minus 100 dbm minus 0.25 into 400 db plus 220 plus x db so if you do all this thing x comes out to be 140 db additional amplification required at the antenna state before it is getting transported over the key

Example: A radio antenna receives a power of -100dBm, which is then amplified by an LNA having gain of 20dB, then it is transported to base station at 400m using an RF cable with 0.25dB/m losses. Assuming the receiver requires power of -40dBm for faithful detection then calculate the required gain.

Solution: Let the gain required be x. Then by using above formula we have

$$-40dBm = -100dBm - (0.25 * 400)dB + (20 + x)dB$$
$$-40dBm = -180dBm + xdB$$

$\therefore$The gain required will be $x = 140dB$

okay so um we have come to the end of this uh this week's lecture uh again a disclaimer we have not generated all the material by yourself we have referred to several uh uh books lectures available and so we have referencing them here and acknowledging that contribution thanks for joining um see you next week in the class thanks