# Radio Astronomy

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## Fundamental of Antenna Theory - Part 03

Hello, welcome to this fourth lecture of week 3 of radio astronomy. We are continuing our discussions on fundamentals of antenna theory. We have done a few details of the regarding antenna in our previous class. We have done radio antenna, importance of antenna by applications, antenna fundamentals, how does radiation happen particularly in context to Hertz dipole, how to characterize an antenna, we have understood what is radiation pattern, what is a gain, directivity, radiation impedances, effective area, etc. We also will today in this particular class we will focus on duality theorem, reciprocity theorem and what is antenna temperature. We will discuss about some wire antennas and aperture antennas but mostly the discussion on the reflector and the antenna array we will postpone to until week number 5, okay, where we talk about single dish we will talk about that.

So that is the plan, let us proceed. So reviewing the antenna in the past two lectures, past three lectures actually, we have discussed about different parameters of antenna, how the antennas are characterized, this is already done. However, radio astronomers does not operate antenna in the active mode that means that we are not operating in the transmit mode. We do operate the antenna in a receiving mode mostly.

So there are few concepts still need to be tackled before we actually introduce that part. In radio astronomy mostly the antenna is used in passive mode or receiving mode. In other words, we are interested in observing what signals are coming from the stars, galaxies, other planets, etc. We are not transmitting anything. So in this context how we will will the antenna behave in a passive mode and how will we receive and observe a source.

In this lecture we discuss about this and try to arrive at an answer conceptually as well as mathematically. So first one is duality theorem. What is the duality theorem? When two equations that describe the behavior of two different variables are of the same mathematical form, their solutions will also be identical. The variables in the two equations that occupy the identical positions are known as dual quantities and a solution of one can be formed by a systematic interchange of symbols to the other. The table in the right shows a dual equations of dual quantities of electric and magnetic sources.

By using duality in an abstract manner you can explain motion of magnetic charges given rise to magnetic currents when compared to their dual quantities of moving electric charges creating electric currents. So there are two different things of electric sources. So J not equal to 0 but M equal to 0. For magnetic sources M not equal to 0 but J equal to 0 and you have this set of equation. This is required to set the stage for the next one.

| Electric Sources ($\mathbf{J} \neq 0, \mathbf{M} = 0$) | Magnetic Sources ($\mathbf{J} = 0, \mathbf{M} \neq 0$) |
|---|---|
| $\nabla \times \mathbf{E}_A = -j\omega\mu\mathbf{H}_A$ | $\nabla \times \mathbf{H}_F = j\omega\epsilon\mathbf{E}_F$ |
| $\nabla \times \mathbf{H}_A = \mathbf{J} + j\omega\epsilon E_A$ | $-\nabla \times \mathbf{E}_F = \mathbf{M} + j\omega\mu\mathbf{H}_F$ |
| $\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}$ | $\nabla^2\mathbf{F} + k^2\mathbf{F} = -\epsilon\mathbf{M}$ |
| $\mathbf{A} = \frac{\mu}{4\pi}\iiint_V \mathbf{J}\frac{e^{-jkR}}{R}dv'$ | $\mathbf{F} = \frac{\epsilon}{4\pi}\iiint_V \mathbf{M}\frac{e^{-jkR}}{R}dv'$ |
| $\mathbf{H}_A = \frac{1}{\mu}\nabla \times \mathbf{A}$ | $\mathbf{E}_F = -\frac{1}{\epsilon}\nabla \times \mathbf{F}$ |
| $\mathbf{E}_A = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{A})$ | $\mathbf{H}_F = -j\omega\mathbf{F} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{F})$ |

It's reciprocity theorem. So what does it do? It is, so reciprocity theorem states the mutual relationship between the behavior of the sources and responses. According to this theorem in any network composed of linear bilateral lumped elements if one places a constant current generator between the two nodes in any branch and places a voltage current meter between any other two nodes in another branch makes observation of the meter reading then interchanges the locations of the sources in the meter. The meter reading will be unchanged. A bit complicated.

How do we apply? We apply it in the sense of the transmitting and receiving antenna properties. We go a little bit more so that we understand a little bit more of the theorem. Lorentz reciprocity theorem states that in a if you have a E1 dot H2 minus E2 dot H cross H1 E1 cross H2 and E2 cross H1 and divergence of that then you have E1 J2 plus H2 dot M1 and E2 dot J1 minus H1 dot M2 where J1 M1 and J2 M2 are the two sets of sources which are allowed to radiate simultaneously individually taking the same medium and the same frequency and produce fields E1 H1 and E2 H2 respectively.

$$-\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2$$

Okay so taking the volume integral you can get on the two sides and for a source free J1 is equal to J2 equal to M1 equal to M2 equal to 0. In the region the above equation reduces to divergence of this expression equal to 0 and hence this is also equal to 0.

$$-\oiint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot ds' = \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2)\, dv'$$

- For a source-free (J1 = J2 = M1 = M2 = 0) region, the above equation reduce, respectively, to

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = 0 \quad \text{and} \quad \oiint (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot ds' = 0$$

These equations are special cases of Lorentz reciprocity theorem and must be satisfied in source free regions. Taking that putting in the context of transmitting and receiving antenna if you if you have trouble in understanding the previous one don't worry let's understand this particular context. Consider two antennas whose impedances are Z1 and Z2 they're complex and are separated by a linear isotropic medium. One antenna number one is used as a transmitter and that number two as a receiver. Okay the equivalent network of each antenna that is the internal impedances generator ZG is assumed to be conjugate of the impedance of antenna number one while the load impedance of ZL is conjugate of the impedance of this thing so that maximum transfer is possible those are all given okay.

$$P_2 = \frac{1}{2}\text{Re}[Z_2(V_g Y_{21})(V_g Y_{21})^*] = \frac{1}{2}R_2|V_g|^2|Y_{21}|^2$$

So after those things are done the power delivered by generator to enter a number one is given by this under maximum transfer theorem and if the transfer admittance of the combined network consisting of generator impedance and antennas and load impedance Y21 the current through the load VG Y21 is given by and the power delivered to the load is given by this taking the ratio of P2 and P1 we get 4 R1 R2 Y21 whole square.

- Taking ratio of $P_2$ and $P_1$ , we get

$$\frac{P_2}{P_1} = 4R_1R_2|Y_{21}|^2$$

- In a similar manner, we can show that when antenna #2 is transmitting and #1 is receiving, the power ratio of $P_1/P_2$ is given by

$$\frac{P_1}{P_2} = 4R_2R_1|Y_{12}|^2$$

$(Z_g = Z_1^* = R_1 - jX_1)$

$(Z_L = Z_2^* = R_2 - jX_2).$

The power delivered by the generator to antenna #1 is given by $P_1 = \frac{1}{2}\text{Re}[V_1 I_1^*] = \frac{1}{2}\text{Re}\left[\left(\frac{V_g Z_1}{Z_1 + Z_g}\right)\frac{V_g^*}{(Z_1 + Z_g)^*}\right] = \frac{|V_g|^2}{8R_1}$

Now what is Y21 it is for this one okay so number one is transmitting number two is receiving number one can be broken into the generator source and the impedance as well as antenna impedance you have a receiving and also the receiving impedance and the load impedance itself. So on the maximum power transfer theorem the P1 is already defined so what is the power transmitter is already defined now only the power which is delivered to the load is can be defined by this where Y21 is the load impedance. In a similar manner we can show that when antenna 2 is transmitting and one is receiving it is basically the same P1 by P2 is this under the conditions of reciprocity Y12 and Y21 are the same. Hence the power delivered in each direction is also the same that basically means that the whether I interchange the quantity of first antenna and the second antenna if the they are matched in any case with the antenna impedances the load or the source impedances on each side and antenna impedances are anyway matched because they are identical okay.

Hence this Y21 and Y12 the reverse order are basically same. Hence the P1 and P2 are also the same. So the reciprocity of the theorem for reciprocity for antenna radiation in order for reciprocity to be maintained and for the antennas to exhibit matching polarization between transmit and receive modes it is essential that their polarization characteristics including the direction of rotation aligned with each other. Let the antenna under test is number one which is here and with while a probe antenna number two is oriented to transmit or receive maximum radiation. The voltages and currents V1 I1 at terminals 1 1 over here and V2 I2 at terminals 2 2 over here are related by V1 Z11 I1 and Z12 I2 whereas V2 is Z21 I1 and Z22 I2 where Z11 and Z122 are self impedance of antenna 1 and antenna 2.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Z12 and Z21 are mutual impedances between antenna 1 and antenna 2 okay. If a current I1 is applied at the terminals 1 1 and voltage V2 is measured at the open I2 equal to 0 terminals of I antenna 2 then equal voltages V1 OC will be the measured at the open I1 equal to 0 terminals of antenna 1 provided the current I2 of antenna 2 is equal to I1. So if I1 is the current which is applied at terminals 1 1 so the 1 1 terminal you have I1 is the current going through is measured at the open of at the open I2 equal to 0 of terminals of antenna 2 okay and voltage V2 is measured at the at this particular stage I1 is the current flowing through antenna 1 and voltage V2 is measured in across terminals 2 2 okay and similarly then an equal voltage V1 OC will be measured at the open where I1 equal to 0 at this terminal and then provided the current I2 of the antenna 2 is equal to I1. In the equation form we can write Z21 is equal to V2 OC over I1 where I2 equal to 0 and Z12 is V1 OC over I2 where I1 equal to 0 okay. If the medium between the two antennas is linear, passive, isotropic and the waves are monochromatic then because of reciprocity you have Z21 is equal to Z12.

If a current $I_1$ is applied at the terminals 1–1 and voltage $V_2$ (designated as $V_{2oc}$) is measured at the open ($I_2 = 0$) terminals of antenna #2, then an equal voltage $V_{1oc}$ will be measured at the open ($I_1 = 0$) terminals of antenna #1 provided the current $I_2$ of antenna #2 is equal to $I_1$. In equation form, we can write

$$Z_{21} = \left.\frac{V_{2oc}}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_{1oc}}{I_2}\right|_{I_1=0}$$

If the medium between the two antennas is linear, passive, isotropic, and the waves monochromatic, then because of reciprocity

$$Z_{21} = \left.\frac{V_{2oc}}{I_1}\right|_{I_2=0} = \left.\frac{V_{1oc}}{I_2}\right|_{I_1=0} = Z_{12}$$

If in addition $I_1 = I_2$, then $V_{2oc} = V_{1oc}$

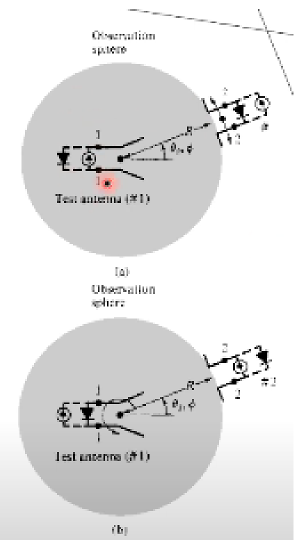



Image Credits: Antenna Theory (By C. Balanis)

If in addition I1 is equal to I2 then V1 open is equal to V2 open okay. So just to mention it one more time it may be confusing so we have what we have done we have one antenna under test which is the antenna number 1. Current flowing through that is I1 and self-impedance is Z11. Current flowing through antenna 2 the probe is I2 and self-impedance is Z22. The mutual impedances between 12 and 21 are Z12 and Z21 okay.

So based on the if all the radiation emitted by test 1 or all the radiation emitted by test 2 is captured by 1 then the following has to be true okay. V1 which is in antenna 1 is equal to Z11 I1 which is the self and Z12 the mutual times I2 because because I2 is generating the radiation which is coming in from 2 and incident on 1. Similarly V2 which is here if now the this in the second case where I is transmitting and 2 is receding in case of that the V2 is Z21 I1 times Z22 I2 okay this is clear. So then just revisit this part again we have the same setup and now we are just considering that what is Z21 and Z12 will be. Now Z21 is the V2 open over here across this 2 divided by I1 because in this case the all the thing is coming from here okay that is the impedance Z21.

And then in the second case Z12 is the reverse so V1 open circuit okay divided by the I2 is the current in the from number 2 where I1 equal to 0 because here it is open circuit in 1 so that is why this equation is valid. So now if Z21 and Z12 are equal based on the reciprocity then and even I1 and I2 are also equal then V2 open and V1 open circuit are same. Moving on reciprocity will now be reviewed for 2 modes of operations in 1 mode antenna number 1 is held stationary while number 2 is allowed to move and on the other on the surface of number 2 is maintained stationary while number 1 pivots about a point as shown is figure number B. So 1 this is held stationary and this moves across the periphery in the second situation this is stationary and this rotates around this okay. In the mode of figure A antenna number 1 can be used either as a transmitter or a receiver in transmit mode while antenna 2 is moving on the constant radius surface sphere surface the open terminal voltage V2 OC is measured in the receiving mode the open terminal voltage V1 OC is recorded okay either way if this is transmitting then this V2 OC is recorded if this is transmitting then V1 OC is recorded.

The three-dimensional graph of V2 OC is identical to that of V1 OC due to reciprocity theorem the transmitting V2 OC and the receiving V2 OC field patterns are also equal. The same conclusion can be arrived at if antenna number 2 is allowed to remain stationary while one rotates as shown in figure B okay. So this is the reason of the reciprocity of the theorem. Now if we connect what we learned from this and connect it with the definition of antenna temperature then it means that the gain of the receiving antenna will be same as the transmitting antenna. Also reciprocity theorem relates that the angular dependencies of the transmitting power pattern and the receiving collecting area of any antenna the power pattern of an antenna is the same for transmitting and receiving okay.

So the gain is proportional to the effective area we have already learned so effective area is over lambda square g over 4 pi. Also we know that there is another relationship with effective area and reactivity is given by this.

$$G(\theta,\phi) \propto A_e(\theta,\phi) \Longrightarrow A_e(\theta,\phi) = \frac{\lambda^2 G(\theta,\phi)}{4\pi}$$

- As we know that,

$$A_e(\theta,\phi) = \frac{e_{cd}\lambda^2}{4\pi} D(\theta,\phi) \text{ and } G(\theta,\phi) = e_{cd} D(\theta,\phi)$$

So now we come to another concept called antenna temperature but before that just think about it this entire power pattern demonstration the measurement they are possible in the transmit mode but they also give an effective idea of what the gain of the antenna will be or the response of an antenna will be in a receiving mode at the same far field. So when we say the antenna is radiating with this kind of efficiency this is the directivity this is also the gain in that particular direction which actually translates to the fact that the sensitivity of the same antenna in a receiving mode towards the same direction is is similar or same okay. And that helps us in characterizing our antenna for radio astronomy in the receiving mode by measuring the radiation pattern of the antenna itself okay and that is possible just because of this reciprocity theorem which holds so well.

So this is a very important concept and that's why we try to stress on it a little bit more. It might arrive a little bit confusing so but the take-home message should be that antenna properties from the transmit mode is equal to the receiving mode and that's how we able to do a lot of measurements and characterization of the antenna particularly in case of radio astronomy. Now coming to antenna temperature okay before we proceed forward to internal temperature we will discuss about little bit about noise. We talked about signal so much of things now noise what is noise? Noise is the unwanted signal which we do not like.

It kind of adds to the to the budget and our signal should be much higher than the noise in any system.

How does noise comes into picture?

$$NF = \frac{SNR_{input}}{SNR_{output}};$$

In radio astronomy we can talk about different noises like environment noise, noise due to thermal variations of the environment, man-made noise due to transmission of other unwanted signals in other bands creates a noise in our band, lightning, sparks, several unwanted other emissions, interferences which causes noise and lastly but not the least the system noise. If any electronic system has itself generated noise and that also creates trouble because remember we are looking at a very very faint signals on the sky so we should be able to do the best we can in reducing the noise in our system. The noise power, noise sources are typically listed in terms of temperature. The noise power is compared to a resistor at a temperature T whose thermal noise would produce the same power per bandwidth as the source. Nyquist formula says in the limit of h nu is very very less than kT like the same as the relevance noise limit.

The noise power per unit bandwidth of a resistive element at a temperature of T is given by p nu equal to kT. Thus the noise temperature is given by p nu over k which is a Boltzmann constant. Noise is quantized in receiver system as noise factor SNR input over SNR output. What is SNR? SNR is nothing but signal to noise ratio average signal power over average noise power so noise power then the noise figure reduces to noise power at the output over noise power at the input. This is a term called noise figure which is often heard when talking about noise,

$$N = 10 \log_{10} NF.$$

It is nothing but the noise factor in decibel scale and n is equal to 10 log 10 of nF. So this is a noise factor which we talked about and finally actually in decibel scale it's called noise figure. So now we come to the antenna temperature and we take the clue from the definition of the noise power per unit bandwidth and that is equal to k times the TR.

$$p = kT_r \quad (\mathrm{W\ Hz^{-1}})$$

The TR is the low temperature of the resistance. And so now if we take the same concept as in figure number A and we can say clearly that this noise power per unit bandwidth remains the same.

If we now replace R with a lossless antenna of the same resistance R, now put inside a anechoic chamber of temperature Tc. The same noise power per bandwidth will be available again for this antenna if Tc is equal to TR. Similarly now we replace the anechoic chamber with the sky with a temperature of Ts. Again the same noise power will be effective at the end of the terminals for this lossless antenna provided the Ts is equal to the TR. Okay so this Ts equals to the TR and instead of the R we now say this is the antenna temperature because that is relevant to the antenna.

So the antenna temperature is equals to the sky temperature provided this matching happens. So this impedance matching is actually very important in case of the passive remote sensing. That is what comes out of this analysis. So to measure the sky temperature the antenna noise temperature may be compared with that of a resistor at an adjustable temperature of TR sub R by alternately connecting antenna and resistor to a receiver. When the receiver detects a node difference, Ta is equal to Ts the sky equal to TR.

Assuming that an antenna has no thermal losses and also is directed towards the region of interest then total acquired power can be expressed by the multiplying receiver bandwidth that is P is equal to K times the antenna times the bandwidth B. If you are observing astronomical radio source then the received power per unit bandwidth can be expressed in terms of flux density provided the source size is greater than antenna beam. So this is measure the flux density is can be expressed in terms of the antenna temperature. In practice the antenna temperature may be may include contributions from several sources or the source under the observation may be superimposed on the background temperature region. In other words source size is less than the antenna beam.

Suppose the beam size is larger than the source size then they can multiple sources coming through that beam and so it does not differentiate between different sources within that beam and it measures temperature entire sky which is visible or which is accessible to that particular beam. So source flux density can be written as K delta Ta.

$$p = SA_e = kT_A$$

Or $$S = \frac{p}{A_e} = \frac{kT_A}{A_e}$$

So the measure the difference between of a source under the these circumstances the radio telescope beam is moved on and off source and an incremental difference temperature delta Ta is measured. These are the techniques we will be again discussing more in week number five where how the measurement of antenna temperature differential temperature actually becomes a measurement of the sky. So supposedly there is a sky which a source as well as the background.

Now we typically choose some other background where there is no source of interest and then we subtract out the background from the main observations that is what this is talking about. Only delta Ta is due to the source and the above equation gives the correct flux density  but delta Ta is not equal to the source temperature. However if the source solid angle  omega s and the antenna beam solid angle omega a are known the source temperature  T sub s is given very simply by the ratio of the both. So T source is given by omega a  divided by omega s times delta Ta which the delta T is a differential between the on and off where  on is the position where there is a background along with the source of interest within coming  through the field of view of the antenna and the off is only the background is visible there is no  source of interest in the field of view. Okay so here are this there is the finally this is the  equation of the antenna temperature as a function of the sky temperature and the power pattern of  the antenna.

$$S = \frac{k\Delta T_A}{A_e}$$

So Ta is the total antenna temperature and not delta Ta. ds is theta phi is the  brightness temperature of the source or the sky pn is normalized antenna power omega a is the antenna beam solid angle and d omega is the infinitesimal solid angle. Okay so this is the final relationship between the antenna temperature and the sky temperature.  As we are coming close to this particular lecture we close it with discussing different types of antennas mostly used for radio astronomy purposes. These are mostly wire antennas you can see the  single monopole there is this log periodic dipole antenna there is YAGIs etc which are  very important we have some in our laboratory also this clipole.

$$T_s = \frac{\Omega_A}{\Omega_s}\Delta T_A$$

- Till now we have considered 2 extreme cases – $\Omega_s \geq \Omega_A$ and $\Omega_s \leq \Omega_A$.
- Let us consider now the general situation for any source beamwidth–size relation. For this general situation, the total antenna temperature $T_A$ will be

$$T_A = \frac{1}{\Omega_A}\int_0^{\pi}\int_0^{2\pi} T_s(\theta, \phi) P_n(\theta, \phi)\, d\Omega$$

So different kinds of antennas  are used for different purposes wire antennas comes in different shape we have shown you before  loops helix dipole antennas are very important. HARS dipole we have already discussed before.  Another important thing that previously that this tv stations used to come through this  Yagiura antennas the dish tv was not there before and this this were the directors  and this was a reflector at the end and so that was the capture. This is one of the most  popular wire antenna in 1990s it has only one active element that is the one radiative element and the others are passive element in the image there on the right the second last is the radiating element or the receiving element the rest of them are directive and the reflective.

Okay the reflector is a metal rod placed at 0.25 lambda such that the em field generates by the dipole gets reflected and creates a positive interference thereby directing the beam in the forward direction. So in a transmit mode is the directors this is the reflector and this is the radiator in a receive mode of course this is the receiving antenna. The director is also placed strategically having 0.

3 lambda to 0.4 lambda spacing and the length of 0.35 lambda to 0.45 lambda this allows directors to become passive antennas on it boost the electromagnetic field even further thereby making it a very directive antenna. This type of antenna also used for S3RAR I mean in fact there is a few S3RARs in in Kolkata as well as in in Garangi in Tirupati there is a similar kind of designs are available for those kind of radars where it is actually transmitting as well as receiving in both modes operates. Remember when we talked about this bandwidth of the antenna okay there is really a need for radio astronomy that we build antennas which are really available over large bandwidths and in this case this log periodic dipole antenna it really comes into picture very very handy. Okay we have a few in our IIT but it is definitely a design where each of this kind of length defines their resonant frequency and culmination multiple of them assembly of multiple of them defines the the bandwidth of the operation of a single dipole a log periodic dipole antenna.

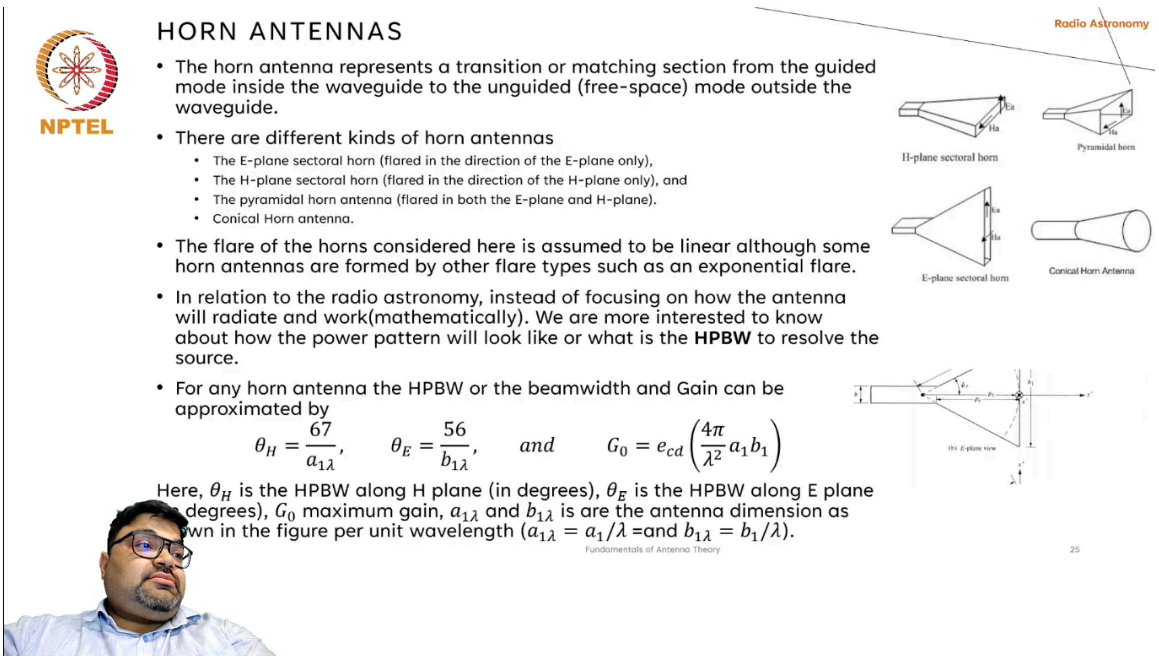
Okay there are some design constants which are which are here but majorly these are used to extend the bandwidth of the operation of that particular antenna. There are other antennas like rectangular waveguides although becoming not very well used nowadays but it's still very popular there are other horn antennas or planar antennas like we have shown you earlier the pyramidal horns conical horns and sectional horns which have their own designs and their planar antennas like microstrip antennas which depends on the previous and other antennas were depending on the the wavelength the the dimension has a relationship with the wavelength here in this case the substrate the dielectric constant substrate has a relationship with the wavelength of radiation. So now we end this with a couple of examples which will explain the things better.

For LPDA Array antenna,

Design constraints:

$$\tau = \frac{l_{n+1}}{l_n}$$

$$\sigma = \frac{d_n}{4l_n} = \frac{1}{4}(1-\tau)cot\alpha$$




• For any horn antenna the HPBW or the beamwidth and Gain can be approximated by

$$\theta_H = \frac{67}{a_{1\lambda}}, \qquad \theta_E = \frac{56}{b_{1\lambda}}, \qquad and \qquad G_0 = e_{cd}\left(\frac{4\pi}{\lambda^2}a_1 b_1\right)$$

Question number one the incremental antenna temperature for the planet Mars measured with the US Naval Research Laboratory 15 meter radio telescope antenna at 3.15 millimeter wavelength was found to be 0.24 kelvin. Mars subtended an angle of 0.005 degree at the time of measurement the antenna half power beam width is 0.116 degree and find the average temperature of Mars at 31.5 millimeter.

Question 1:

The incremental antenna temperature for the planet Mars measured with the U.S. Naval Research Laboratory $15\ m$ radio telescope antenna at $31.5\ mm$ wavelength was found to be $0.24\ K$. Mars subtended an angle of $0.005^\circ$ at the time of the measurement. The antenna $HPBW = 0.116^\circ$. Find the average temperature of Mars at $31.5\ mm$ wavelength.

So um omega a is the half power beam width whole square omega s is the subtended angle so the Ts the sky temperature in this case is omega a over omega s times delta Ta.

Delta Ta we noted is 0.24 and these two angles are known so hence this Ts comes out to be 129.18 kelvin. The temperature is less than the infrared temperature measured by measured for the sunlit side 250k. So we find the average temperature of Mars at 31.5 millimeter which is a simple formula which we have um given before.

Solution:

Let us consider the antenna beam is symmetrical then by solid angle approximation we have $\Omega_A = (\mathrm{HPBW})^2$ and $\Omega_s = (substended\ angle)^2$, then

$$T_s = \frac{\Omega_A}{\Omega_s}\Delta T_A \approx \frac{(0.116)^2}{(0.005)^2} \times 0.24 = 129.18\ K$$

This temperature is less than the infrared temperature measured for the sunlit side (250 K), implying that the 31.5-mm radiation may originate farther below the Martian surface than the infrared radiation. This is an example of passive remote sensing of the surface of another planet from the earth.

Question number two a circular reflector antenna of 500 millimeter square effective aperture uh operating at lambda equal to 20 centimeter is directed at zenith. What is the total antenna temperature assuming the sky temperature is uniform and equal to 10 kelvin. Okay take the ground temperature equal to 300 kelvin and assume that the half the minor low beam area is in the bad direction towards the ground beam efficiency is 0.7.

Question 2:

A circular reflector antenna of $500\ m^2$ effective aperture operating at $\lambda = 20\ cm$ is directed at the zenith. What is the total antenna temperature assuming the sky temperature is uniform and equal to $10\ K$? Take the ground temperature equal to $300\ K$ and assume that half the minor-lobe beam area is in the back direction (toward the ground). The beam efficiency is 0.7 ($= \Omega_M/\Omega_A$).

Let's see assuming that the internet aperture efficiency is 50 percent its physical aperture is thousand meters square and its diameter is uh 35.

7 meter um at 0.2 meter 20 centimeter diameter is 179 lambda implying that the half power beam width is 0.4 degree thus the antenna is highly directional with the main beam directed entirely at the sky close to the zenith. Since main beam efficiency is 0.7 uh 70 percent of the beam area omega a is directed at the 10 degree sky half of the remainder or 15 percent at the sky and the other half 15 percent at the 300 kelvin ground thus integrating ta from 0 to pi and 0 to pi in three steps. So first step is um in the main beam you get 1 over omega a 10 times 0.

7 omega a and that is 7 kelvin side lobe contribution is given by the side lobes okay and that is calculated like this and so so 10 k is the sky temperature which is uniform which is coming through the side lobe the back lobe is sensitive towards the 300 kelvin ground okay and to 10 kelvin okay so the the front lobe is seeing the 10 kelvin through 0.7 uh which is the 70 efficiency in the main lobe that gives you 7 kelvin then 10 kelvin is coming through

the side lobes with 0.3 that is giving 1.5 and the back lobe is seeing 300 kelvin which is giving 45 kelvin so overall that antenna temperature becomes 7 plus 15 7 plus 1.

5 plus 45 which is 53.5 kelvin 45 of the 53.5 kelvin or 84 percent total antenna temperature results from the back lobe pickup from the ground with no back lobes the antenna temperature could ideally be only 10 kelvin so that is that in that in this example the back lobes are very detrimental to the system sensitivity it is for this reason that the radio telescopes and space communication antennas are usually designed to reduce back and side lobe responses to a minimum there's a design constraint and we should be understanding and driving more on that.

Solution:

Assuming that the antenna aperture efficiency is 50 percent, its physical aperture is $1000\ m^2$ and its diameter $35.7\ m\ (= 2\sqrt{(1000/\pi)}\ \ m)$. At $\lambda = 0.2\ m$ the diameter is $179\lambda$, implying that the $HPBW \simeq 0$[illegible] $(= 70^\circ/179)$. Thus, the antenna is highly directional with the main beam directed entirely at the sky (close to the zenith).

Since the (main) beam efficiency is 0.7, 70 percent of the beam area $\Omega_A$ is directed at the 10 K sky, half of the remainder or 15 percent at the sky and the other half of the remainder or 15 percent at the 300 K ground. Thus, integrating $T_A = \int_0^{\pi}\int_0^{2\pi} T_s(\theta,\phi)P_n(\theta,\phi)d\Omega$ in 3 steps, we have

1. Sky contribution = $\frac{1}{\Omega_A}(10 \times 0.7\Omega_A) = 7\ K$
2. Side-lobe contribution = $\frac{1}{\Omega_A}\left(10 \times \frac{1}{2} \times 0.3\Omega_A\right) = 1.5\ K$
3. Back-lobe contribution = $\frac{1}{\Omega_A}\left(300 \times \frac{1}{2} \times 0.3\Omega_A\right) = 45\ K$

And $T_A = 7 + 1.5 + 45 = 53.5\ K$

Note that 45 of the 53.5 K, or 84 percent, of the total antenna temperature results from the back-lobe pickup from the ground. With no back lobes the antenna temperature could ideally be only 10 K, so that in this example the back lobes are very detrimental to the system sensitivity. It is for this reason that radio telescope and space communication antennas are usually designed to reduce back- and side-lobe response to a minimum.

The information given regarding aperture and wavelength is relevant to the problem only to the extent that it [illegible]icates that the main beam is directed entirely at the sky.

so as always uh we have created the materials from different books and other other resources so that's all are acknowledged over here um thanks for joining us for this lecture and see you next week hope you like these videos and the material i hope this will clarify your doubts and good luck with the assignment for this week i hope all the material which is shared will help you to sort out the problems in the assignment thank you see you next week bye