Radio Astronomy

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Lec-11

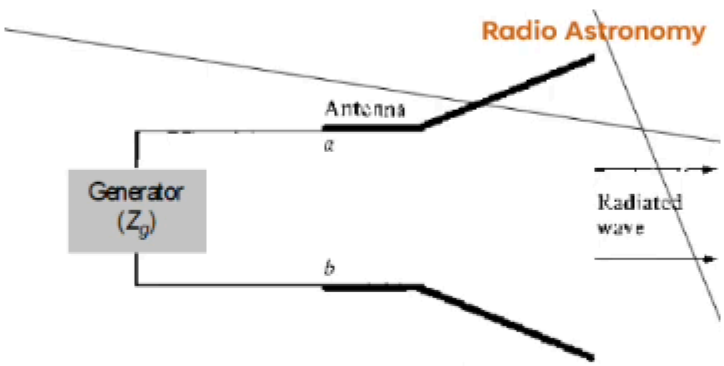
Fundamental of Antenna Theory - Part 02

Hello, welcome to this third lecture on week number 3 on radio astronomy. We are talking about antenna theory and other fundamentals. So, we have mostly covered in this particular week details about radio antenna, why it is important. Started with an ideal Hertz dipole radiation pattern, the measurement of the electric and magnetic field at different distances from the antenna, how do we calculate radiation pattern, far field radiation pattern, what are different parameters of a radiation pattern, half power beam width, first null beam width and directivity radiation impedances etc. So, in this particular lecture, we will carry that discussion onward with mostly discussing about the impedances and also solving some problems which will help you to understand what is going on. So, the next thing is polarization.

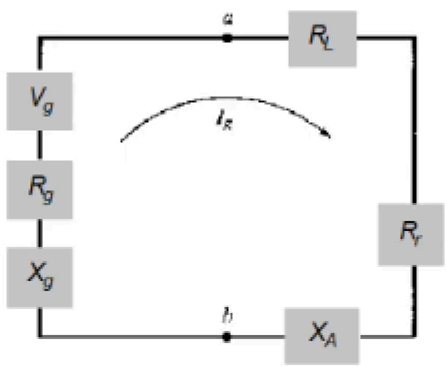
Polarization of an antenna in a given direction is defined as the polarization of the wave transmitted or radiated by the antenna. Polarization of radiated wave is defined as a property of the electromagnetic wave describing the time varying direction and relative magnitude of the electric field vector. Specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space and the sense in which it is stressed as observed along the direction of propagation. The polarization characteristics of an antenna can be represented by its polarization pattern which is the spatial distribution of the polarizations of a field vector by an antenna over its radiation sphere.

So, as the wave propagate, the polarization vector will make a motion and that defines the polarization pattern of the antenna. For one thing we have to understand here and we will talk about it more is for radio astronomy when we are mostly receiving the radiation from the cosmos, the radiation itself can be polarized, ok. An antenna itself by its characteristics also have a polarization property. Now, these two have to be entangled disentangled to understand the inherent radiation the polarization property of the radiation coming from the cosmos itself. We will talk about it little bit when we come, but this has to be nevertheless you please note it down that this has to be done when you actually analyze.

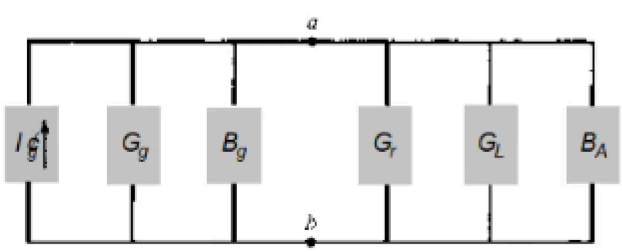
Otherwise, you will get a wrong answer or will measurement of the polarization of the source which you are observing in the far field sky will become erroneous. So, the next important parameter to discuss will be input impedance. We started little bit and this is mostly for completeness of the discussion that we are going into this. It is defined as impedance present by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a given point. Now, CPL is a generator and you see a transmission line ending up into a antenna, ok.



(a) Antenna in transmitting mode



(b) Thevenin equivalent



(c) Norton equivalent

Image Credits: Antenna Theory (By C. Balanis)

$$Z_A = R_A + jX_A$$

$$I_g = \frac{V_g}{Z_t} = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)}$$

If you try to draw a equivalent circuit, Thevenin equivalent circuit, the generator of the voltage can be divided into its voltage, its resistance and reactance and similarly there is a load resistance and the radiation resistance is there and the reactive part, ok. So, the antenna impedance can be broken up into this R sub R and X sub A. There is a R L which is another load resistance. So, we can break this up into this format by following the Thevenin equivalent circuit. If you have already done it, fantastic.

I will follow this. Otherwise, just note it down and we can discuss it more later most likely. So, these terminals between, so the impedance is measured between the terminals A and B in this particular figure and where the antenna impedance is defined by Z sub A is R A plus J into X A, where Z A is the antenna impedance and R A is the antenna resistance, X A is the reactance between terminals A and B. In general, resistive part consists of two components, R A is R sub R and R sub L. R sub R is the radiation resistance and R sub L is the loss resistance of the antenna.

And if you assume that the antenna is attached to a generator with the internal impedance of Z sub G, then there is a R sub G and X sub G of the voltage source itself. Hence, to find the amount of power delivered to R sub R, the radiation resistance for radiation and the amount of dissipated in R sub L, the loss resistance as heat or the ohmic loss, I squared R L over T, we apply Kirchhoff's voltage law in circuit showing in B. Okay. So, this Kirchhoff's voltage law is applied and I sub G can be in terms written in terms of V G over Z A and Z G. So, after the solving for the power, we get the power radiated by the antenna and the power dissipated by heat at the antenna due to the internal resistance R G is given by this expression.

Okay. Now, by following something called a maximum power transfer theorem, it appears that the impedance of the this internal voltage source and the impedance of the antenna, they should be conjugate of each other to be able to do the maximum power transfer. What it means is that the resistive part R sub R plus R sub L should be equal to R G and X A should be negative of X G. Okay. That is how it becomes conjugate, right? Because it is complex in nature. Thus, when the maximum power transfer is the power is delivered, the power radiated or lost in the heat will be given by this.

$$P_r = \frac{1}{2}|I_g|^2 R_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right]$$

$$P_L = \frac{1}{2}|I_g|^2 R_L = \frac{|V_g|^2}{2}\left[\frac{R_L}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}\right]$$

$$P_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

$$P_L = \frac{|V_g|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right]$$

$$P_g = \frac{|V_g|^2}{8}\left[\frac{R_g}{(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{1}{R_r + R_L}\right] = \frac{|V_g|^2}{8R_g}$$

This V G square over 8 R R over R R plus R L whole square. Okay. And P L is given by V G square over 8 and R L over R L plus R R whole square and P G is given by this V G square over 8 R G. So, P here P G is equal to P R plus P L that is the total power generated by the generator and dissipated as heat and radiated finally. So, this is the maximum you can get if everything is matched.

$$P_T = \frac{|V_T|^2}{8}\left[\frac{R_T}{(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8}\left(\frac{1}{R_r + R_L}\right) = \frac{|V_T|^2}{8R_T}$$

$$P_r = \frac{|V_T|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

$$P_L = \frac{|V_T|^2}{8}\left[\frac{R_L}{(R_r + R_L)^2}\right]$$

Quite valuable to note because you sometimes have a budget of radiated power, particularly in case of transmission, the communication. There is a power radiated and it has to be matched with because it has to go over a long distance etcetera, etcetera. For remote sensing also this radiated power has a valuation. For radars, atmospheric radars, Doppler radars, this power has a value. So, this maximum power transfer theorem it has to be applied and the design of the internal circuitry etcetera has to be done properly to maintain this.

For radio astronomy, it also is important because finally it is going to you know, we already have a very less amount of signals coming in from the cosmic sources and we do not want to lose any of them. Okay. So, what happens is that if we match the internal impedances in case of us, it is in the receiving mode. It is not a generator, it is a receiver which is followed and so receiver impedance has to be matched with the antenna impedance. Otherwise, there will be some amount of signal will be reflected back.

Okay. So, we do not want to do that also. So, this particular analysis is extremely important while designing an antenna, whether in a transmit mode or a receive mode. Okay. Thing to note here is that in the power by the generator, half is dissipated as heat due to its internal resistance and other half is delivered to the antenna if and only if we have a conjugate matching. The power that is delivered to the antenna, a part of that is radiated through the mechanism provided by radiation resistance and the other is dissipated heat which influences part of the overall efficiency of the antenna.

If antenna is lossless and matched to the transmission line that is not equal to 1, then half of the total power supplied by the generator is radiated by the antenna itself. So, if you consider the inverse case where antenna is acting as a receiver as we just mentioned, then in similar way, we will arrive at the following terms showcasing the power of matching impedances. Okay. Antenna radiation efficiency, we have already discussed it. The conduction dielectric efficiency ECD is defined as a ratio of the power delivered to the radiation resistance to the power delivered to R and R L.

$$e_{cd} = \left[\frac{R_r}{R_L + R_r}\right] \quad [\delta = \sqrt{2/\omega\mu_0\sigma}].$$

Okay. That is the ECD. We mentioned briefly in the last lecture. Here we define it. Okay. If we consider a metal rod of length L and a uniform cross section of any of A, the DC resistance is given by L over sigma A and having a skin depth of delta. Then the high frequency resistance can be written in terms of these values.

$$R_{hf} = \frac{l}{P}R_s = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}}$$

So, high frequency resistance is coming up to be L over P root over of omega mu naught over 2 sigma. P is the perimeter of the cross section and R s is the conductor is surface resistance. Omega is the angular frequency and mu naught is a permeability of free space. Sigma is the conductivity of the metal. The high frequency resistance can in turn provide the R L based on the current distribution in the antenna.

So, that was the radiation efficiency and the dielectric efficiency particularly. Next term we concentrate on is the antenna effective length and equivalent area. So, antenna has different geometry. So, to come up with an effective length, if it is like a bucket, then the effective area is understandable. But if it is like, like simply like dipoles, what is the effective area and as such.

So, this is actually important. So, what we start off is that define a quantity called vector effective length L sub e for an antenna, which is kind of a complex vector quantity and is represented by L e theta phi is given by a theta, L theta and a phi L phi, where L theta and L phi are the length of the antenna along theta and phi directions of course.

$$\boldsymbol{\ell}_e(\theta,\phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta,\phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta,\phi)$$

in its terminals

$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi = -j\eta \frac{kI_{in}}{4\pi r} \boldsymbol{\ell}_e e^{-jkr}$$

$$V_{oc} = \mathbf{E}^i \cdot \boldsymbol{\ell}_e$$

Now, in the far field quantity, it is related to the far L e is also referred to as an effective height in some sense. Okay, it is a far field quantity and it is related to the far field electric field radiated by the antenna with the current I in in its terminals. So, you can write the e sub a in form of that, that will give rise to the open circuit voltage value in terms of effective length and the e.

Additionally, we see there is a number of equivalent areas associated with an antenna. One of those equivalent areas is an effective area or aperture area and is defined as the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction. The wave being polarized, polarization matched to the antenna, okay. If the direction is not specified, the direction of the maximum radiation intensity is implied. So, the effective area is the power flux density of the incident wave over the radiation density, okay.

So, that gives you the this expression. So, W is the power density of the incident wave and P t is the power delivered to the load of this incident wave and effective area is given by a sub e. Using total power generated by the antenna in reception mode as discussed when we are discussing the antenna impedance, we get the effective area to be given by this. So, there are various ways of defining this effective area. One by the power transmitted to the load and another one is in the reception mode, the power generated, okay, by the antenna.

Both of them can be used to define the effective area. So, maximum effective, if we consider maximum power transfer condition, then the maximum effective area can be reached by following the maximum power transfer conditions where the source voltage, the source impedance or the voltage source impedance has to match with the antenna impedance in conjugation.

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i}$$

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right]$$

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_L + R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right]$$

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L + R_r)^2}\right]$$

$$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_T + R_r + R_L}{(R_L + R_r)^2}\right]$$

$$Capture\ Area\ =\ Effective\ Area\ +\ Scattering\ Area\ +\ Loss\ Area$$

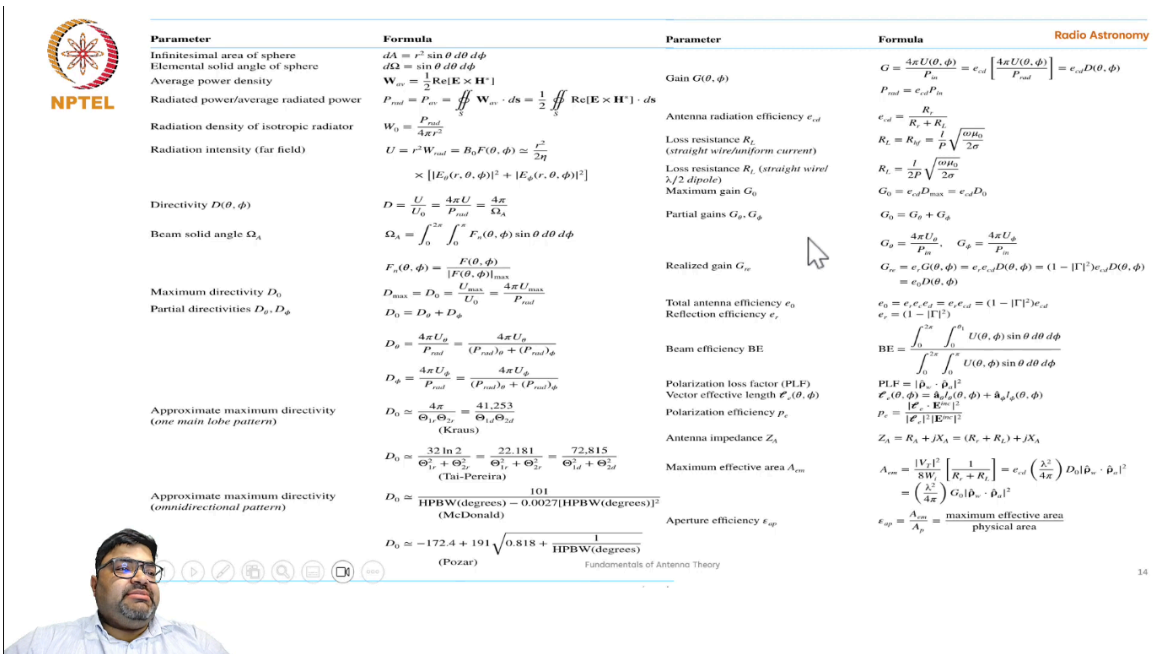
So, there are several other definitions like scattering area can be also defined as the equivalent area given by this formula. There is loss area can also be given, captured area can also be written. So, it can be observed that the captured area which is the equivalent area which were multiplied by the incident power density, digital power captured, okay.

The captured area is like effective area plus scattering area plus loss area. So, it is a superset kind of a thing. Now, we define a term called aperture efficiency, which is defined as a ratio of the maximum effective area of the antenna to the physical area, okay. So, this maximum effective area is kind of linked to the maximum power transfer. So, basically the efficiency, okay, to the physical.

So, physical area is a measure of the antenna, like if you have a dish, the dish diameter is known, so you can calculate the area of the dish. And electromagnetic, magnetically, the that doesn't really matter. The what matters is that maximum effective area, okay. So, this ratio kind of gives you an efficiency of the aperture which is radiating or receiving. For aperture type antenna such as waveguides, horns and reflectors, the maximum effective area cannot exceed the physical area, but it can be equal to it.

$$\varepsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}$$

So, it cannot be higher than the aperture in the physical aperture area, but it can be less than or equal to that. That is a maximum efficiency cannot be exceeding 100%, which is in unphysical. Further, one can introduce another area called partial effective area of an antenna for a given polarization in each direction. So, for any given polarization, you can define a term of partial effective area because we just have to do that particular polarization. So, that brings us to the close of all the definition of this, all the parameters which are involved in this.



So, here what we do is we summarize a bunch of parameters which are important for our discussion henceforth. This first two, we haven't yet come, which is just here for completion. But the next bunch of the details are actually many of them we have actually gone through. This is just for completeness and for your reference, it will be part of the lecture material distributed.

So, that is a summary of all the things. We will come back to these two in the next few lectures. Before we proceed further, I think there is a lot of material we have covered.

Brightness temperature $T_B(\theta, \phi)$ (K) $\quad T_B(\theta, \phi) = \varepsilon(\theta, \phi) T_m = (1 - |\Gamma|^2) T_m$

Antenna temperature T_A (K) $\quad T_A = \dfrac{\int_0^{2\pi} \int_0^{\pi} T_B(\theta, \phi) G(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_{0\pi}^{2\pi} \int_0^{\pi} G(\theta, \phi) \sin\theta \, d\theta \, d\phi}$

So, I think it is time to go through a few sample questions and so that we can understand the concepts more clearly. So, first one we are taking for a sphere of radius R, find the solid angle omega A in square radians or steradians of a spherical cap on the surface of the sphere over the North Pole region defined by spherical angles theta 0 to 30 degree and phi 0 to 360 degree. Find the solid angle. A exactly, B using omega A is equal to delta theta 1 and delta theta 2 where delta theta is theta 2 are two perpendicular angular separations of the spherical cap passing through the North Pole compared between the two.

Question 1:

For a sphere of radius r, find the solid angle Ω_A (in square radians or steradians) of a spherical cap on the surface of the sphere over the north-pole region defined by spherical angles of $0 \leq \theta \leq 30^\circ, 0 \leq \phi \leq 360^\circ$. Find the solid angle Ω_A.

a. exactly.
b. using $\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2$, where $\Delta\Theta_1$ and $\Delta\Theta_2$ are two perpendicular angular separations of the spherical cap passing through the north pole.

Compare the two.

So, for the first one, we use the formula. The steradian, the infinitesimal solid angle is defined by the area over R square that is given according to the spherical coordinate because dA is R square sine theta d theta d phi. So, R square cancels. So, this becomes d omega becomes sine theta d theta d phi.

Solution:

a. Using formula $d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$, we can write

$$\Omega_A = \int_0^{360°} \int_0^{30°} d\Omega = \int_0^{2\pi} \int_0^{\pi/6} \sin\theta \, d\theta \, d\phi = \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta \, d\theta$$

$$= 2\pi[-\cos\theta]|_0^{\pi/6} = 2\pi[-0.867 + 1] = 2\pi(0.133) = 0.83566$$

b. And by using approximation, we get

$$\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2 \underbrace{=}_{\Delta\Theta_1 = \Delta\Theta_2} (\Delta\Theta_1)^2 = \frac{\pi}{3}\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} = 1.09662$$

It is apparent that the approximate beam solid angle is about 31.23% in error.

So, we can write omega A is equal to the double integral of d omega and that can be written in terms of 0.83566 that is in radian. Okay, by using approximation, we get omega A is equal to delta theta 1 times delta theta 2. If they were same, then it becomes delta theta 1 whole square and that gives you 1.09662. So, by this approximation, if you follow these approximation rules which we have defined earlier, then this relativity can have an error of, approximate beam solid angle can have an error up to 31% which is quite high.

Coming to question number 2, the radial component of the radiated power density of an antenna is given by omega W that is equal to A R unit vector times W sub R given by A 0 sine theta over R square watt per meter square. Where A 0 is the peak value of the power density, theta is the usual spherical coordinate and A R is a radial unit vector. Define total radiated power.

Question 2:

The radial component of the radiated power density of an antenna is given by

$$\mathbf{W}_{rad} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\mathrm{W/m^2})$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the total radiated power.

Total radiated power is given by this double integral of W rad dot n the direction normal to the surface area and times dA.

So, if you do that simple expression and you can, so three dimensional normalized plot of the average power density at a distance of r equal to 1.

For a closed surface, a sphere of radius r is chosen. To find the total radiated power, the radial component of the power density is integrated over its surface. Thus

$$P_{rad} = \oiint_S \mathbf{W}_{rad} \cdot \hat{\mathbf{n}}\, da$$

$$= \int_0^{2\pi} \int_0^{\pi} \left(\hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2}\right) \cdot (\hat{\mathbf{a}}_r r^2 \sin\theta\, d\theta\, d\phi) = \pi^2 A_0 \quad (\text{W})$$

A three-dimensional normalized plot of the average power density at a distance of r = 1 m.

The radial question number 3 is radial component of the radiated power of a given this thing is also given by just a second this is also yeah they are similar looking. So, A 0 is the peak value of the power density, theta is the usual spherical coordinate and A R is the radial unit vector. Determine the radiation intensity and using the relationship of radiation intensity and total power determine the total power.

Question 3:

The radial component of the radiated power density of an antenna is given by

$$\mathbf{W}_{rad} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{a}_r$ is the radial unit vector. Determine the radiation intensity and using the relation of radiation intensity and total power determine the total power.

So, it is a same setup. So, U is the intensity is R square times W rad. So, that is equal to A 0 sine theta because you have W rad is equal to A 0 sine theta over R square. So, R square cancels off this is the thing. So, P rad is in the double integral of U sine theta, d theta d phi which is given by pi square A 0.

Solution:

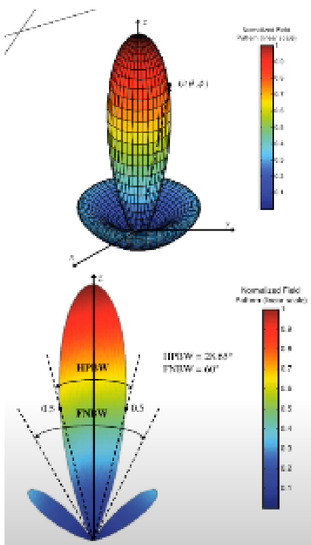
By using radiation intensity equation we have,

$$U = r^2 W_{rad} = A_0 \sin\theta$$

And by using the relation of radiation intensity and total power we have

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U \sin\theta\, d\theta\, d\phi = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2\theta\, d\theta\, d\phi = \pi^2 A_0$$

Now question number 4 the normalized radiation intensity of an antenna is presented by U theta is given by cosine square theta and cosine square 3 theta where theta goes from 0 to 90 degree and phi from 0 to 360 degree.



Question 4:

The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \leq \theta \leq 90^\circ, \quad 0^\circ \leq \phi \leq 360^\circ)$$

The three- and two-dimensional plots of this, plotted in a linear scale, are shown in Figure to the left. Find the

a. half-power beamwidth HPBW (in radians and degrees)

b. first-null beamwidth FNBW (in radians and degrees)

The 3 and 2 dimensional plots of this is plotted in a linear scale and shown in the left. Now what is to be found the please calculate the half power beamwidth in radian and degrees and f n B w or the first null beamwidth in radians and degrees. So, U theta is given you calculate H p B w and f n B w. So, from U theta the is represent the power pattern to find half power beamwidth you set the function equal to half of its maximum value ok and you finally find the theta H equal to this. Where ultimately you can calculate it is a is an equation of the transcendental function it can be solved iteratively after a few iterations you can find theta H to be equal to 0.25 radians ok or 14.325 degree. Since the function U theta is symmetrical about maximum at theta equal to 0 then the H p B w is twice of this theta H ok. This there is the first one the second one is to find the first null beamwidth. So, to set that U theta has to be equal to 0 at the first null. So, if you do that then this equal to 0 then theta n can be solved as theta can be 90 degree and 3 theta n can also be 30 degree. So, for both these two value 30 degree and 90 degree this particular beam pattern goes to 0.

Solution:

a. Since the U(θ) represents the power pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or $U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \Rightarrow \cos\theta_h \cos 3\theta_h = 0.707$

$$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$$

Since this is an equation with transcendental functions, it can be solved iteratively. After a few iterations, it is found that

$$\theta_h \approx 0.25 \ radians = 14.325°$$

Since the function U(θ) is symmetrical about the maximum at θ = 0, then the HPBW is

$$\text{HPBW} = 2\theta_h \approx 0.50 \ radians = 28.65°$$

b. To find the first-null beamwidth (FNBW), you set the U(θ) equal to zero, or

$$U(\theta)|_{\theta=\theta_n} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_n} = 0$$

This leads to two solutions for θn.

$$\cos\theta_n = 0 \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2} \ radians = 90°$$

$$\cos 3\theta_n = 0 \Rightarrow \theta_n = \frac{1}{3}\cos^{-1}(0) = \frac{\pi}{6} \ radians = 30°$$

The one with the smallest value leads to the FNBW. Again, because of the symmetry of the pattern, the FNBW is

$$\text{FNBW} = 2\theta_n = \frac{\pi}{3} \ radians = 60°$$

So, there are two solutions for theta n ok. However, because it is the first null so the first null will happen at the smallest value of theta right because there are two theta and this is the first null. So, first null will be at 30 degree not 90 degree. Hence one side is 30 degree for symmetry the other side is also 30 degree and so that becomes the F n B w becomes twice of theta n which is equal to 60 degree ok. Let me go through it once more. So, we have been given a U theta and we have to calculate the half power beamwidth and the first null beamwidth.

So, half power beamwidth is the place where the power becomes half of its maximum value ok. So, that means the maximum value is where is 1 cosine square theta and cosine square 3 theta this value is 1 maximum value is 1 of this function.

So, half of it is 0.5. So, basically then 0.5 you take a square root of that and then finally find that 0.707 is equal to the cosine theta h and cosine 3 theta h. So, that gives you to a theta h which is also a function of theta h basically these are called transcendental functions. Finally after solving this iteratively you get it is equal to the 14 degree 14.325 degree ok. After that if you go and find the first 14.325 degree is towards the half the other half has to be also calculated right. So, this is the from one side you are coming down to the 0.5 from the other side also you have to come down to 0.5 and the distance between the two half points is the half power beamwidth. So, it is effectively twice of this theta h is that may half power beamwidth which is equal to 28.65 degree. That is the half power beamwidth. Now for this particular beam pattern the f n b w is where the u theta comes to 0. Now u theta can be equal to 0 at two points but cosine of theta is equal to 0 that is 90 degree and cosine of 3 theta equal to 0 is theta equal to 30 degree. So, because it is the first null then the 30 degree is the value we should take. 30 degree is on the one side so the the full first null beamwidth will become 30 plus 30 is 60 degree.

So, remember the full width of maxima is a 28.65 degree and first null beamwidth is at 60 degree. First null beamwidth is a 60 degree.

Now next question is and as an illustration the find the maximum directivity of the antenna whose radiation intensity is that of example of question 2 which is a 0 sine theta is equal to u. Write an expression of the directivity as a function of the direction angles theta and phi.

Question: 5

As an illustration, find the maximum directivity of the antenna whose radiation intensity is that of Example question 2 ($U = r^2 W_{rad} = A_0 sin\theta$). Write an expression for the directivity as a function of the directional angles θ and ϕ.

So, we have u as r square over w rad is a 0 sine theta. Maximum radiation is directed along theta equal to pi by 2 because a 0 is the maximum. So, u max is a 0. P rad is integral of u sine theta d theta d phi that is equal to pi square a 0 we have already calculated that. Hence the directivity d naught is 4 pi u max over P rad that is 4 by pi or 1.27. Since the radiation intensity is only a function of theta the directivity as a function of direction angle is represented by d equal to d naught sine theta or 1.27 sine of theta.

Solution:

The radiation intensity is given by $U = r^2 W_{rad} = A_0 sin\theta$.

The maximum radiation is directed along $\theta = \pi/2$. Thus, $U_{max} = A_0$

And total radiated power is

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U \sin\theta \, d\theta \, d\phi = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2 A_0$$

Using the maximum directivity formula, we have

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4}{\pi} = 1.27$$

Since the radiation intensity is only a function of θ, the directivity as a function of the directional angles is represented by

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

If it would have been fine sine uh function of phi then the phi component will have been also included.

Question number 6.

Question 6:

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by

$$\mathbf{W}_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and âr is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and ϕ.

The radial component of the radiated power density of an infinitesimal linear dipole of length l less than less than lambda or the wavelength of radiation is given as w average is a radial unit vector times w r is given by a 0 sine square theta by r square.

a 0 is the peak value theta is the usual spherical coordinate and a r is the unit radial vector all these thing are there. Determine the maximum directivity of an antenna and express the directivity as a function of direction angles theta and phi. So, with that this sine square theta by r square. So, almost similar to the previous this is this is also function of theta only but instead of sine theta it becomes sine square theta. So, the you solve for the period you get the value of a 0 8 pi over 3 you get d naught is 4 pi u max over period just like the previous example if you substitute the value of period you get 3 by 2 as a directivity maximum reactivity and then you find that it is only function of sine theta sine square theta.

So, d theta d also is d naught sine square theta or 1.5 sine square theta. So, that solves the problem it is same as the previous problem only sine theta is replaced by sine square theta.

Solution:

The radiation intensity is given by

$$U = r^2 W_r = A_0 \sin^2\theta$$

The maximum radiation is directed along $\theta = \pi/2$. Thus

$$U_{max} = A_0$$

The total radiated power is given by

$$P_{rad} = \oiint_{\Omega} U \, d\Omega = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \, d\phi = A_0 \left(\frac{8\pi}{3}\right)$$

Then the the maximum directivity is equal to

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi A_0}{\frac{8\pi}{3}(A_0)} = \frac{3}{2}$$

which is greater than 1.27 found in Example question 5. Thus the directivity is represented by

$$D = D_0 \sin^2\theta = 1.5 \sin^2\theta$$

Next question we have a radiation intensity given by u as b naught which is the maximum value cosine square 4. The radiation intensity exists only in the upper hemisphere. So, theta is from 0 to pi by 2 and phi is 0 to 2 pi. It is also shown in this figure find a beam solid angle exact and approximate maximum reactivity exact.

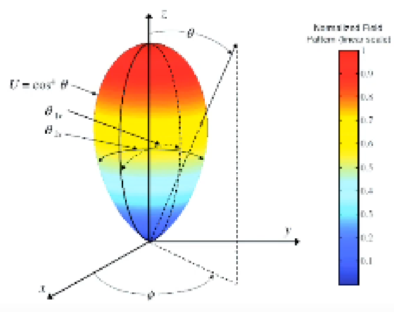


Image Credits: Antenna Theory (By C. Balanis)

Question 7:

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos^4 \theta$$

where B0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$), and it is shown in Figure. Find the

a. beam solid angle; exact and approximate.

b. maximum directivity; exact and.

The half power point of the pattern occurs at theta equal to 32.765. How do we do that? We again set this to be maximum is b naught. So, b naught over 2 happens when so we understand so cosine square cosine to the power 4 theta is equal to b naught by b naught by 2.

So, 2. Okay. So, that is the beam width in the theta direction is give is 65.53 twice of this value. So, theta 1 r is given by 1.1437 radians and the beam width for the other plane is also given by theta 2 r is 1.1437 radians. As a beam solid angle see omega a is given by integral of cosine to the power 4 d omega and that is 2 pi by 5 steradians.

Soluition:

The half-power point of the pattern occurs at $\theta = 32.765°$. Thus, the beamwidth in the θ direction is 65.53° or

$$\Theta_{1r} = 1.1437 \text{ rads}$$

Since the pattern is independent of the ϕ coordinate, the beamwidth in the other plane is also

$$\Theta_{2r} = 1.1437 \text{ rads}$$

a. Beam solid angle Ω_A:

Exact:

$$\Omega_A = \int_0^{360°} \int_0^{90°} \cos^4 \theta \, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta$$

$$= 2\pi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta = \frac{2\pi}{5} \text{ steradians}$$

And the approximate ones by following this expression omega is equal to theta 1 r theta

2 r is given by 1.308 steradians. So, 2 pi by 5 radian and 1.308 radian. Directivity is 4 pi by omega a is 10 dB and approximate is 9.83 dB. So, the exact maximum directivity is 10 and approximate value is 9.83 in this 9.68 one is dimensionless. Yeah. So, for in dB it is 9.83. So, this even better approximations can be obtained if patterns have much narrower beam widths. So, with narrower beam width this actually this approximation of solid angle actually works better that is what is demonstrated here.

Approximate:

$$\Omega_A \approx \Theta_{1r}\Theta_{2r} = 1.1437(1.1437) = (1.1437)^2 = 1.308 \text{ steradians}$$

b. Directivity D_0:

Exact:

$$D_0 = \frac{4\pi}{\Omega_A} = \frac{4\pi(5)}{2\pi} = 10 \text{ (dimensionless)} = 10 \text{ dB}$$

Approximate:

$$D_0 \approx \frac{4\pi}{\Omega_A} = \frac{4\pi}{1.308} = 9.61 \text{ (dimensionless)} = 9.83 \text{ dB}$$

The exact maximum directivity is 10 and its approximate value is 9.61. Even better approximations can be obtained if the patterns have much narrower beamwidths.

We go to the next one another illustrative problem where this u is given by b naught sine cube theta the input impedance of 73 ohms connected to a transmission line whose characteristic impedance is 750 ohms and assuming the u is given by b 0 sine cube theta find the maximum realized gain of this antenna. So, u max is nothing but b naught and p radian is given by b naught 3 pi square by 4.

So, we calculate this to we put it into the formula of d naught which is 1.697. Then what we do is we assume an efficiency radiation efficiency of equal to 1 because it is lossless and then calculate the gain g 0 as there is e c d times d naught. So, it is exactly equal to 1.697 and in dB it becomes 2.297.

Solution:

Let us first compute the maximum directivity of the antenna. For this

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4\theta \, d\theta = B_0\left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$. Thus, the total maximum gain is equal to

$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

$$G_0(\text{dB}) = 10\log_{10}(1.697) = 2.297$$

which is identical to the directivity because the antenna is lossless.

So, that directivity and the gain are identical because the radiation efficiency is 100 percent.

There is another loss factor which is not taken into account in the game that is the loss due to reflection and mismatch. Okay and that is equal to reflection mismatch is equal to 1 minus gamma square and gamma square is nothing but z in by z characteristics and what is z in plus z characteristics. So, this 2 becomes 73 minus 50 and 73 plus 50 this is equal to 0.965. So, the mismatch loss is equal to minus 0.155 dB. Okay. So, the overall efficiency even though the e c d is considered to be 100 percent but this e r contributes to that hence it little bit less than the previous one. So, we consider that 0.

965 into 1.697 and we get that the more realistic gain to be 1.6376 or 2.142 dB. Okay as compared to the previous 2.297. The gain in dB also can be obtained by converting the directivity and the radiation efficiency in dB and then by simply adding them it also can be done.

There is another loss factor which is not taken into account in the gain. That is the loss due to reflection or mismatch losses between the antenna (load) and the transmission line. This loss is accounted for by the reflection efficiency, and it is equal to

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73-50}{73+50}\right|^2\right) = 0.965 \longrightarrow e_r(\text{dB}) = 10\log_{10}(0.965) = -0.155$$

Therefore the overall efficiency is

$$e_0 = e_r e_{cd} = 0.965$$
$$e_0(dB) = -0.155$$

Thus, the overall losses are equal to 0.155 dB. The maximum realized gain is equal to

$$G_{re_0} = e_0 D_0 = 0.965(1.697) = 1.6376$$
$$G_{re_0}(dB) = 10\log10(1.6376) = 2.142$$

The gain in dB can also be obtained by converting the directivity and radiation efficiency in dB and then adding them. Thus,

$$e_{cd}(dB) = 10\log_{10}(1.0) = 0$$
$$D_0(dB) = 10\log_{10}(1.697) = 2.297 \quad G_0(dB) = e_{cd}(dB) + D_0(dB) = 2.297$$

which is the same as obtained previously. The same procedure can be used for the realized gain.

Okay. The question number 9 in this we are assuming a linearly polarized electromagnetic wave given by e i as e 0 times e to the power minus j k z in the direction of x is incident upon a linearly polarized antenna whose electric field polarization is expressed by e a as a x unit vector plus a y times e function of r theta and phi.

Find the polarization loss factor.

Question 9:

The electric field of a linearly polarized electromagnetic wave given by

$$Ei = \hat{a}_x E_0(x, y) e^{-jkz}$$

is incident upon a linearly polarized antenna whose electric-field polarization is expressed as

$$E_a \simeq (\hat{a}_x + \hat{a}_y) E(r, \theta, \phi)$$

Find the polarization loss factor (PLF).

So, incident wave is around x and for the antenna the radiation pattern is along x and y. So, the polarization loss factor is given by rho w dot rho a whole square and that is given by this. So, p l f in dB is 10 log 10 p l f in linear scale which is given as minus 3 dB loss. Okay. Um so this is about uh how much is it this is about 0.5 and in dB it is 0.5 log 10 10 times of that is essentially minus 3.

Solution:

For the incident wave

$$\hat{\rho}_w = \hat{a}_x$$

and for the antenna

$$\hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_x + \hat{a}\_y)$$

The PLF is then equal to

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\hat{a}_x \cdot \frac{1}{\sqrt{2}}(\hat{a}_x + \hat{a}_y)\right|^2$$

which in dB is equal to

$$PLF\,(dB) = 10\log_{10} PLF = 10\log_{10}(0.5) = -3$$

Question number 10 a resonant half wavelength dipole is made out of copper where conductivity is given by 5.7 10 to the power 7. Determine the conduction dielectric efficiency of dipole antenna at frequency of 100 megahertz is the radius of the wire is 3 times 10 to the power minus 4 lambda and radiative radiation resistance of the lambda by 2 dipole is 73 ohm.

Question 10:

A resonant half-wavelength dipole is made out of copper ($\sigma = 5.7 \times 10^7$ S/m) wire. Determine the conduction-dielectric (radiation) efficiency of the dipole antenna at f = 100 MHz if the radius of the wire b is $3 \times 10^{-4}\lambda$, and the radiation resistance of the $\lambda/2$ dipole is 73 ohms.

So, at frequency of 10 to the power 8 hertz or 100 megahertz uh we have lambda is equal to 3 meter and l is equal to 3 by 2 meter.

So, the c or 2 pi times b is 6 pi 10 to the power minus 4 lambda. For r l is half of the r high frequency and so it is given by the formula r l equal to 0.3 5 or 3.3 4 9 ohms. So, um the radiation the dielectric efficiency is given by 73 divided by 73 plus 0.

39 which is about 99.52 percent efficient and db loss is given by minus 0.02 db.

Solution:

At f = 10^8 Hz

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{10^8} = 3 \text{ m}$$

$$l = \frac{\lambda}{2} = \frac{3}{2} \text{m}$$

$$C = 2\pi b = 2\pi(3 \times 10^{-4})\lambda = 6\pi \times 10^{-4}\lambda$$

For a $\lambda/2$ dipole with a sinusoidal current distribution $R_L = \frac{1}{2}R_{hf}$. Therefore,

$$R_L = \frac{1}{2}R_{hf} = \frac{0.25}{6\pi \times 10^{-4}}\sqrt{\frac{\pi(10^8)(4\pi \times 10^{-7})}{5.7 \times 10^7}} = 0.349 \text{ ohms}$$

Thus,

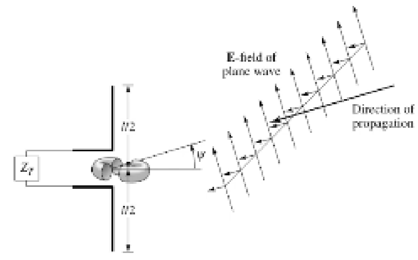
$$e_{cd}(\text{dimensionless}) = \frac{73}{73 + 0.349} = 0.9952 = 99.52\%$$

$$e_{cd}(\text{dB}) = 10\log_{10}(0.9905) = -0.02$$

Question number 11, uniform plane wave is incident upon a very short lossless dipole where the length is less than less than the lambda is shown in the figure. Find the maximum effective area assuming that the radiation resistance of the dipole is 8 t times pi l over lambda square and the incident field is linearly polarized along the axis of the dipole.

Question 11:

A uniform plane wave is incident upon a very short lossless dipole ($l \ll \lambda$), as shown in Figure (a). Find the maximum effective area assuming that the radiation resistance of the dipole is $R_r = 80(\pi l/\lambda)^2$, and the incident field is linearly polarized along the axis of the dipole.



(a) Dipole antenna in receiving mode

Image Credits: Antenna Theory (By C. Balanis)

So, for the maximum effective area by the maximum power transfer theorem gives you this v t squared over 8 wi times r r. Since dipole is very short the v t is equals to e times l and that gives you the incident power density is e squared over 2 eta and then the effective area is equal to e l squared over this denominator giving 2 3 pi lambda squared over 8 pi or 0.119 lambda square.

Solution:

For $R_L = 0$, the maximum effective area of reduces to

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r}\right]$$

Since the dipole is very short, the induced current can be assumed to be constant and of uniform phase. The induced voltage is

$$V_T = El$$

Where, VT is the induced voltage on the dipole, E is the electric field of incident wave, and l is the length of dipole

For a uniform plane wave, the incident power density can be written as

$$W_i = \frac{E^2}{2\eta}$$

where η is the intrinsic impedance of the medium ($\simeq 120\pi$ ohms for a free-space medium). Thus

$$A_{em} = \frac{(El)^2}{8(E^2/2\eta)(80\pi^2 l^2/\lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

So, that brings us to a close of this particular lecture. The there were quite extensive discussion on the solved problems. Please go through that. It is not it has to be noted that we haven't created this content out of from the vacuum. So, we have referred to the standard books particularly for this particular week. We are referring mostly to the Balanis and the Krauss's book books and so they are taken for this from this references mostly.

I hope you have enjoyed this particular lecture and got something useful out of it. Please go through the problems because they are linked also to the assignment from this particular week. So, see you in the next class. Thanks for joining.