Radio Astronomy

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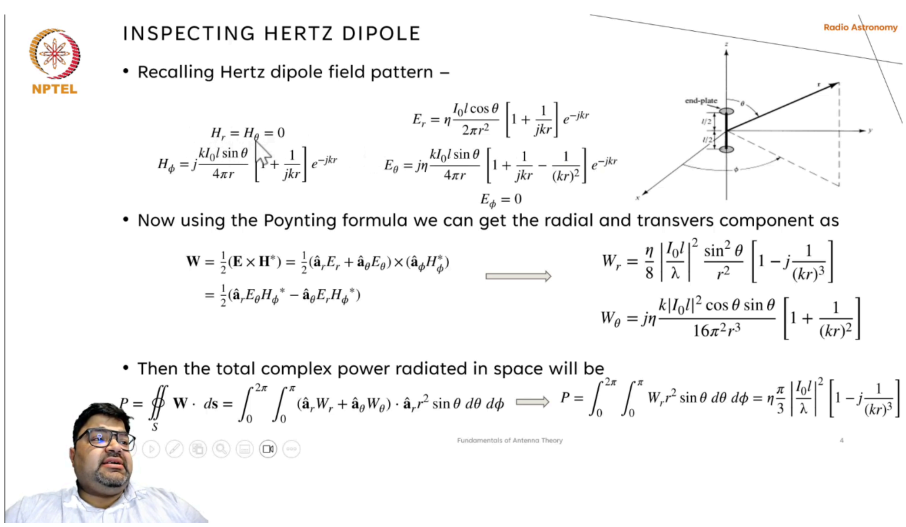
Lec-10

Fundamental of Antenna Theory

Hello, welcome to this lecture number two of week three of this course radio astronomy. We will continue discussing on the antenna theory. Just to give you a brief overview of what happened in the last class. We started on antenna, we basically emphasized on the fact that antenna is a very integral part of a radio telescope and radio telescope is one of the most important, if not the important component of the entire receiving system of the radio astronomy. So there are state of the art low noise receivers, but antennas also the design, the engineering marvels that antenna stands for is absolutely wonderful. We showed couple of different kind of antennas, the wire antenna, the aperture antennas and patch antennas.

Before we continue on several different antennas and other definitions, properties of antenna, we started discussing about the most idealized Hertz dipole and how to evaluate the electric and magnetic field as a function of different distance from the antenna radiating. And we defined, we see there are three distinct regions. First is a very near reactive zone and then intermediate zone and or near field radiation radiating zone and a far field radiating zone. And we defined them and that was kind of where we stopped in the last class. This we continue exactly where we left.



So this is the Hertz dipole on the top right corner of there, the same one, total length of the dipole is L, half of it is above the xy plane and half of it is below. The origin is exactly near the half midpoint of the dipole. The current flowing is along the z direction, it is I0 is a constant and we calculate electric and magnetic field at a distance r from there. We saw very soon that there is a spherical symmetry in this system.

So we converted the xyz coordinates into r theta and phi, the spherical coordinates. We figured out that the radial and the theta component of the magnetic field is equal to zero. Only the azimuthal component survives in this case. For electric field the azimuthal component goes to zero and the radial and theta component survives. So that is the thing and we also derived couple of other important notions.

Like at large r the second term in the er radial component goes to zero because r tends to infinity, this is r cubed, 1 over r cubed, this goes to zero. For the e theta components the second and the third term are 1 over r squared 1 over r cubed so they go to zero because there is one r, another extra r is over here and so this second term becomes 1 over r squared, third term becomes 1 over r cubed and they go to zero. Similarly only term survives the first term for both er and e theta. But even the er the first term is also r squared that also tends to zero so er fully vanishes. For e theta only the first component which is 1 over r that exists and for h phi the first term which is 1 over r that survives, the second term also goes to zero as r tends to infinity.

Now that is where we stopped essentially last time and then we defined like several different regimes of r where k r is less than 1, greater than 1 and much greater than 1. Okay, so now we proceed. The very next step we learned from the electromagnetics is we have e and each h field so we compute the pointing vector and that finally turns out so we have a wr and w theta component and from there we can easily figure out what is the p, the complex power radiated. We figure it out and then finally after integrating total power comes out of theta and phi, theta goes to zero to pi and phi goes to zero to 2 pi and so the final expression of radiated power comes out to be this which still has a 1 over r cubed dependency. So that is the radiated power half of e cross h dot ds and that comes out to be having a real and imaginary part.

Now we have $$P = \frac{1}{2}\iint_S \mathbf{E}\times\mathbf{H}^* \cdot\ d\mathbf{s} = \eta\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2\left[1 - j\frac{1}{(kr)^3}\right]$$

This can be rewritten as $$P = P_{rad} + jP_{E\&H}$$

So the p radiated is the real component and imaginary component is represented by p e and h which is the time average imaginary power radiated radially outward. So finally the p radiated one is given by the eta times pi l squared over 3 lambda squared into I naught squared. So as the r tends to larger and larger this value is we can see it's 1 over r cubed so the effect of this in the far field radiation pattern is almost zero. Whereas the effect of the p radiation is kind of it remains. So in the near field where r is less than 1 this reactive component of p e and h still remains but as r tends to infinity grows larger for the far field this p radiation component dominates and the other one almost negligibly equal to zero.

$$P_{rad} = \eta \frac{\pi l^2}{3\lambda^2} I_0^2$$

$$P_{\mathrm{rad}} = \eta \left(\frac{\pi}{3}\right) \left|\frac{I_0 l}{\lambda}\right|^2 = \frac{1}{2} |I_0|^2 R_r$$

$$R_r = \eta \left(\frac{2\pi}{3}\right) \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

So for far field pattern we can concentrate only on the p rad term which is equal to this half I naught squared r radiation resistance where r is given by 80 pi squared l over lambda whole square. So we go back again and revisit this entire calculation once more. We had from the I which is running along the z axis we calculated the vector potential. From vector potential we got the h by following Maxwell's equation and from h we derived e and so this is the components of e and h we replace the xyz components with r theta and phi components. We got only one component of h field is there whereas two of the e fields are there.

Then we compute the pointing vector from there we compute the power and then finally we end up having this expression of power in terms of both real and imaginary component. We investigate the imaginary component and see this 1 over r cube so at the far field this component almost goes to zero. In the near field of course it remains effective that is why it is a reactive near field. For the far field component it only P that survives so we express that in terms of I naught and radiation resistance where radiation resistance is given by this formula. Now let us look at the radiated power in the far field.

And that if you plot it basically will show a structure like the donut. Now remember this is the radiation pattern in a free space which is a tremendously idealistic situation. Okay in a real case we will show you the patterns it is not that symmetric and there will be something called the main lobe and the side lobes which will start appearing because there will be some kind of reflection from a nearby what we call ground. So if the antenna is on top of just mounted on the ground the ground will reflect it can be mounted on top of other things they will also reflect so this reflection causes creation of side lobes and if you see but it is not present in this particular idealized radiation pattern because this is assumed to be in free space. So half wave dipole is just like a length of lambda by 2 the concept is there we already have gone through it and I naught is there so we will come and visit again Hertz dipole later on in this week when we talk about other different wire antennas.

$$[I] = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \pm z\right)\right] e^{j\omega[t-(r/c)]}$$

So we saw the radiation pattern we show that some general steps how it appears so we got the W radiation in terms of R theta and Phi so W is not just but real component of the pointing vector of real of E cross H and so we can write the since we know the E theta and E Phi H theta and H Phi components can be written in terms of that. So the W rad can be written in terms of E theta square plus E Phi square and so and so but in case of Hertz dipole we know that E Phi does not exist so we can write it in form from that for any other dipole any other antenna sorry these are terms which will basically survive okay. So from there we can write U which is kind of R square W rad and then from there we also determine P of radiation which is W rad integrated over the entire theta and Phi that is what we did for the Hertz dipole and then finally yeah so the detectivity is basically U theta Phi in any direction over the U naught okay and or you can find out 4 pi U theta Phi which is given by this expression on top over P radiation is defined as a directivity in theta and Phi. Directivity is basically how much it is boosted in a particular direction radiation in compared to an isotropic distribution. So this will come later on as well I am just mentioning it here for completeness.

For normalized amplitude pattern we use the P n theta Phi which is the normalized radiation pattern is U theta Phi in any direction over the U max which is the maximum mean the boresight direction or the central portion and input the radiation and input resistances are also written in this format in terms of P rad and I naught square and finally the effective area is also determined by lambda square over 4 pi times D naught okay. This we will again come back D naught is nothing but the D maximum which is given over here.

Then by following steps one can determine the antenna characteristic theoretically

- Specify electric and/or magnetic current densities J, M.
- Determine vector potential component in far field.
- Find far-zone E and H radiated fields ($E\theta$, $E\phi$; $H\theta$, $H\phi$)

$$\mathbf{W}_{\text{rad}}(r,\theta,\phi) = \mathbf{W}_{\text{av}}(r,\theta,\phi) = \frac{1}{2}\text{Re}[\mathbf{E}\times\mathbf{H}^*]$$

$$\simeq \frac{1}{2}\text{Re}\,[(\hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi)\times(\hat{\mathbf{a}}_\theta H_\theta^* + \hat{\mathbf{a}}_\phi H_\phi^*)]$$

$$\mathbf{W}_{\text{rad}}(r,\theta,\phi) = \hat{\mathbf{a}}_r \frac{1}{2}\left[\frac{|E_\theta|^2 + |E_\phi|^2}{\eta}\right] = \hat{\mathbf{a}}_r \frac{1}{r^2}|f(\theta,\phi)|^2$$

$$\boxed{U(\theta,\phi) = r^2\, W_{\text{rad}}(r,\theta,\phi) = |f(\theta,\phi)|^2}$$

- Determine either

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} W_{\text{rad}}(r,\theta,\phi) r^2 \sin\theta\, d\theta\, d\phi \quad \text{or} \quad P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

- Find directivity using

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{\text{rad}}}$$

$$D_0 = D_{\max} = D(\theta,\phi)|_{\max} = \frac{U(\theta,\phi)|_{\max}}{U_0} = \frac{4\pi U(\theta,\phi)|_{\max}}{P_{\text{rad}}}$$

- Form normalized power amplitude pattern: $P_n(\theta,\phi) = \dfrac{U(\theta,\phi)}{U_{\max}}$
- Determine radiation and input resistance: $R_r = \dfrac{2P_{\text{rad}}}{|I_0|^2}$; $\quad R_{in} = \dfrac{R_r}{\sin^2\left(\frac{kl}{2}\right)}$
- Determine maximum effective area $A_{em} = \dfrac{\lambda^2}{4\pi}D_0$

This is just to capture the entire picture at once we will keep coming to this gallery again and again as we proceed okay. So as we understood there was a couple of this terms which came in as we discussing the radiation pattern. The radiation pattern is an important terminology radiation power density, beam width, radiation intensity, effective area, antenna equivalent area, and in a vector effective length, bandwidth, directivity, beam efficiency, radiation efficiency, input impedance, antenna efficiency, HP BW or FWHN sorry are basically half power beam width or full width half maxima, gain and realized gain and polarization.

These are very important parameters to understand, characterize a particular antenna, its performance, its radiation pattern, its gain in a particular direction. Supposedly sometimes we need an antenna to radiate equally in all direction, isotropic radiation. That antenna design will be different from when I need absolutely high gain only towards a particular direction along line of sight. The usage are various. For example I know that I have to there is a downlink set up from a satellite in a particular direction.

So I will we will design an antenna which has a maximum gain or maximum sensitivity towards that direction. In contrary I we would like to radiate in all possible direction, collect information about various communication over the you know almost horizontally and for that I need a isotropic radiator. So the design of these two antennas are quite different from each other. Hence these terms are important which are used for characterizing the antenna. So antenna design essentially depends on all these terms.

Okay so we will see them as we progress. Yeah by this time you have already seen the radiation pattern. We have discussed the ideal radiation pattern in free space from a Hertz dipole. It looks like a doughnut. But as I said that is not how the real things look like.

And so there we have major lobe or primary lobe and minor lobes or side lobes. The field pattern typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular coordinates, angular space. Power pattern is represents a plot of the square of the magnitude of electric or magnetic field as a function of the angular space. They are both in linear units, the field pattern and power pattern. The power pattern can also be expressed in logarithmic scales in dB, in decibel scales.

Okay that is mostly what we have. So here are the three different plots. The first one is the electric or magnetic field pattern. You can see the main lobe is in the center, the most prominent one and the smaller ones are called side lobes. And exactly the opposite one to the main lobe is called the back lobe.

Okay and this arises because that in reality there is nothing called free space. No matter where you place your antenna there will be some reflecting body nearby. Supposedly I put an antenna on the ground. The ground itself acts as a reflector and it produces these side lobes.

Okay. So that is the case. On the second plot we are showing the power pattern in the linear scale and it is really difficult to see the as compared to the main lobe. It is very difficult to see the contributions from the side lobe and the back lobe. So it is desirable to plot this in a different way in a logarithmic scales so that all the contribution of the back lobe and the side lobes and the respective nulls are very prominently visible. Okay. So we also plot this in a dB scale where now you see the main lobe is there but also the back lobe which was almost invisible in the linear scale now becomes very prominent.

Okay in the power pattern. And the other ones are called side lobes or minor lobes. Now if I want to create an antenna which is highly directive then this main lobe has to be the major portion of the entire power where the power is concentrated on. Okay. All the side lobes has to be extremely minimized with respect to them.

Okay. That is the design study. Okay. To find the points where the pattern achieves its half power or minus 3 dB points is where so basically what is the half power being with? I go from the central portion where the power is maximum to where the power is half of its maximum value on both these directions and those are the two half power points. So the angular distance between these two half power points is called half power beam width or HP BW. Similarly in the dB scale those half in power goes to minus 3 dB in each side from the maximum. And there it calculates the entire half power beam width.

This HP BW is actually very important term when we come to defining our field of view of a particular antenna. Because these radiation patterns actually are pasted on the sky and they are the major portion through which the sky is observed. Because as we go away from this point five portion the power really reduces a lot. That means power reduces in the transmit mode that means the receiving mode the sensitivity of the antenna outside this half power beam with positions are very minimal.

Okay the efficiency goes down. We already have very faint signals coming from the sky. If they're further reduced because of the inefficiency of the antenna that is not desirable. So we try to concentrate on the region where the antenna is most sensitive. Okay an antenna is more sensitive within this HP BW region. Of course we have sensitivity towards this first null.

We will come to this little bit later. But I mean the efficiency becomes much much lower. Okay so that's one thing to remember and this HP BW actually forms our field of view for a particular antenna. So for the radiation pattern this is again shown in this top right picture. Actually it's a spherical symmetry so if you rotate this structure you will see the entire 3d view.

Unfortunately this is not present here. So you can see the main lobe, the major lobe, the side lobes, the back lobe etc. Now for our benefit and try to understand it we take a slice. Okay and if you take a slice you can see these structures placed in the theta axis. And you can essentially be able to understand each of these features. So what is this? The main lobe, the maximum power radiation point at the center and it goes in either side of this axis from 0 to pi by 2 to pi.

So near the pi the portion the back lobe maximum happens and in the zero point the front lobe or the main lobe maxima happens. Apart from that there are a few other components like at the half power beamwidth, the position where in each side of the main lobe the power comes to the half of the maximum power radiated and the distance angular distance between the both of them is equal to the half power beamwidth. When the power goes down to zero completely it's called null. First null because it goes to zero multiple times. So this is the first null then second then third and fourth and henceforth.

So the distance between these two first nulls is called first null beamwidth or FNBW. Okay then comes the first side lobe then second then third and etc etc. So these are called also minor lobes and finally reach to the back lobe. This half power beamwidth is very important as we just discussed and yeah that's mostly what we have to discuss in this particular slide. Now come to the next slide we have mainly three different categories of antennas based on the radiation pattern.

First is the isotopic antenna which is we have shown the dipole Hertz dipole in a in a vacuum or free space. It's a hypothetical lossless antenna having equal radiation in all directions. In other words an ideal antenna which can radiate in all direction in a perfect spherical pattern. The second one is omni-directional antenna an antenna having essentially non-directional pattern in one plane and a directional pattern in any orthogonal. For example dipole antenna has equal radiation in azimuthal plane whereas directional pattern along the elevation.

A directional antenna all non isotropic antenna is directional antenna. Okay the electromagnetic wave can be polarized or non polarized so that it is also true for the antenna that means an antenna can radiate polarized waves. So an electromagnetic wave is polarized so does that different antenna design can induce polarization in the wave which is radiated out. Okay to understand the polarized antenna performance E and H field pattern has to be studied. This is also termed as principle E and H plane patterns.

Okay so we sometimes we also need to look at the difference between the E field and the E plane and the H plane radiated power to understand the polarization characteristics of the antenna itself. Now this is a thing which we have discussed earlier but we again come back and discuss it. So there are three different what it appears from the radiated power formula also there is a reactive component which acts at the short hours or shorter distances. As the distance from the antenna grows this reactive component goes to zero because it is a 1 over r cube dependency. Okay so that's what exactly is even again summarized in this particular schematic is that from the antenna the origin to a particular value 0.

62 square root of d cube over lambda this is called near field reactive zone. Okay then from the 0.62 d cube over lambda square root of that to up to 2d square over lambda this region is called Fresnel zone or radiating near field zone. So the reactive term is almost vanishing and you have a radiating zone. The third one is anything above beyond point d square 2d square over lambda is called far field radiating zone or front offer zone.

Okay so depending on these three different zones antenna react behave differently. For our case when we are studying the power patterns we are mostly studying them in the front offer zone. So you can see that these side lobes and everything appearing over here in the bottom right figure in the front offer zone itself. Now coming back to the same slide again we just wanted to make a couple of points in this power pattern plot that this distance between the half power points is called Hb Bw and between the two nulls is called Fn Bw right. This is very important in terms of a radio telescope because it also defines the field of view.

Okay as we said earlier that below this the sensitivity of a given telescope goes down, reduces also this angular width between two half power points gives us another very important aspect towards the resolution. What is resolution? We understood in the first lecture the ability of our telescope to distinguish between one star in this direction and the other star in this direction. The ability to decipher the angular separation between two stars is called resolution. So if they are very closely spaced within this half power beam width this particular telescope will not be able to distinguish them between them.

They will see them as a single star. But if they are apart by angular separation more than the half power beam width then there is a possibility of distinguishing between these two stars. Okay so this again this particular component with respect to the radio astronomy point of view is very important. Now coming to the revisiting the power density as we know for any given setup we have shown the example for the Hertz dipole. Now we generalize this. So for any given antenna we first calculate the electric field and the sorry we first calculate the current density then from there we get the magnetic vector potential and for vector potential we get the H or the magnetic field from H we get E or the electric field.

Once we get the electric field and the H we can choose the light coordinate system which describe the system at hand. Sometimes it has a spherical symmetry. Sometimes it may have a cylindrical symmetry. Okay cylindrical symmetry for example like you know for a for a thin wire for example it can have a cylindrical symmetry. Now in those cases you exploit that symmetry and move from the Cartesian coordinate to that particular coordinate system and redefine your E and H in that coordinate system.

After that is done then you calculate what is called the W or the pointing vector. So when pointing vector is defined then from there you can finally calculate the power density which is W dot ds or the normalized normal component of the pointing vector to the entire surface. So once that is done you can essentially calculate the real that goes to the real value of E cross H and then you know the E you know the H so you can calculate the real value of H E and H and then finally integrate and you get the W average. Now what is radiation intensity? It is basically the power radiated from an antenna per unit solid angle. The radiation intensity is the far field parameter and it can be obtained by simply multiplying the radiation density by the square of the distances.

Okay so we define it as U which is R square times the radiated power W. So U is the radiation intensity in W per unit solid angle and W rad is the radiation density in watt per meter square. Now for radiation intensity we can also express that in terms of theta and phi and that will be just R squared over 2 eta where eta is intrinsic impedance of the medium and E square. So if as we know the R component of the electric field vanishes so theta and phi component exist and so it can be written in this format. The power pattern is also measure of the radiation intensity and so total power radiated by integrating the radiation intensity is obtained by the radiation intensity over the entire solid angle.

$$U(\theta,\phi) = \frac{r^2}{2\eta}|\mathbf{E}(r,\theta,\phi)|^2 \simeq \frac{r^2}{2\eta}\left[|E_\theta(r,\theta,\phi)|^2 + |E_\phi(r,\theta,\phi)|^2\right] \simeq \frac{1}{2\eta}\left[|E_\theta^\circ(\theta,\phi)|^2 + |E_\phi^\circ(\theta,\phi)|^2\right]$$

Okay so for isotropic source U 0 is period over 4 pi. radiation directivity is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all the direction. Okay so to make it simpler you have the reduction radiation intensity over the isotropic radiation intensity is given by the direct is equal to the directivity of the antenna. So as we know from the previous slide that the for a isotropic antenna it is period over 4 pi is U 0 so U by U 0 is given by 4 pi U over period where U 0 is the radiation of the radiation intensity of isotropic source and and period is total radiated power. Okay and U is the radiation in a particular radiation intensity in a particular direction. If the direction is not specified it implies the reduction of maximum radiation intensity.

$$P_{\text{rad}} = \oiint_{\Omega} U \, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

For antennas with orthogonal polarization components we define partial directivity of an antenna for a given polarization in each direction. So then in each direction total directivity is sum of the partial derivative directives in for any two orthogonal polarization. So like D 0 is equal to D theta plus D Phi where D theta is given by 4 pi U theta over period theta plus period Phi and the other one is 4 pi U Phi over period theta by P Phi.

$$U_0 = \frac{P_{\text{rad}}}{4\pi} \quad D = \frac{U}{U_0} = {4\pi U}/{P_{rad}}$$

Mathematically $D_0 = D_\theta + D_\phi$ where $D_\theta = {4\pi U_\theta}/{[(P_{rad})_\theta + (P_{rad})_\phi]}$ and $D_\phi = {4\pi U_\phi}/{[(P_{rad})_\theta + (P_{rad})_\phi]}$

These are just great details you don't have to worry about all these details too much for this particular course but just for completeness we are putting it here. So radiation intensity for all other sources the maximum directivity will always be greater than unity and it is a relative figure of merit which gives an indication of the directional properties of the antenna as compared with those of an isotropic source.

Directivity is important as I said I mean some antennas really need to be very specifically designed that it has a maximum gain or maximum sensitivity towards a particular direction. For example for satellite communication where isotropic source is the ideal thing if you really want the radiation to be in all directions only direction. So the values of directivity will be equal to or greater than 0 and equal to or less than the maximum directivity. For example let the radiation intensity of an antenna be of the form U as given by this where B naught is a constant, U maximum is given by of course this functional form maximum of that and P rad is given by this. So if you find really want to find the direction directivity expression as a function of theta and Phi this is given by this whole expression over here where the top one is directional property and that bottom one is integrated property or response over all theta and Phi.

$$U = B_0 F(\theta,\phi) \simeq \frac{1}{2\eta}\left[|E_\theta^0(\theta,\phi)|^2 + |E_\phi^0(\theta,\phi)|^2\right]$$

$$\boxed{D(\theta,\phi) = 4\pi \frac{F(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi}}$$

- Directivity for the major lobe or direction of maximal radiation can also be written as

$$D_0 = 4\pi \frac{F(\theta,\phi)|_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi} \Rightarrow D_0 = \frac{4\pi}{\left[\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi\right] \Big/ F(\theta,\phi)|_{\max}} = \frac{4\pi}{\Omega_A}$$

- where Ω_A is the beam solid angle, and it is given by

$$\Omega_A = \frac{1}{F(\theta,\phi)|_{\max}}\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

- Where,

$$F_n(\theta,\phi) = \frac{F(\theta,\phi)}{F(\theta,\phi)|_{\max}}$$

- Here, $F(\theta,\phi)|_{\max}$ merely normalizes the radiation intensity $F(\theta,\phi)$, and it makes its maximum value unity.

Directivity for the major lobe or direction of maximum radiation can also be written for the major lobe it can be written as for that particular beam solid angle where theta sigma a is the beam solid angle and is given by this particular form. Instead of using an exact expression of directivity to compute the directivity it is often convenient to drive simpler expressions even if they are approximate to compute the directivity value. So essentially for a non symmetrical pattern directivity can be approximated to D naught is equal to 4 pi over sigma a where the beam solid angle and if you have a objective along beam width along X and Y that also can be written. So then D naught becomes equal to these values.

The next parameter we are discussing today is the antenna efficiency. Now we know that any real antenna cannot be completely lossless okay there will be losses due to several reasons I mean the design may not be exactly matching with the radiation wavelength. There can be losses between the mismatch of impedances in the transmission line and the antenna. There are ohmic losses also. Okay so overall the entire antenna inefficiencies can be combined and can be written in terms of this E naught okay the antenna efficiency can be in terms of ER EC and ED where E sub R is a reflection efficiency E sub C is the conduction efficiency and E sub D is a dielectric efficiency okay. If I put in the value of something called a voltage reflection this is something completely different if you know already know it it's fine just these are the if the any any network any electrical system can be different in terms of a two-port network.

- Let us consider the total antenna efficiency is e_0 then, the overall efficiency can be written as

$$e_0 = e_r e_c e_d$$

- Where, e_r is the reflection (mismatch) efficiency, e_c conduction efficiency, and e_d dielectric efficiency.
- Usually, e_c and e_d are very difficult to compute, but they can be determined experimentally. Even by measurements they cannot be separated, and it is usually more convenient to write it as

$$e_0 = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2)$$

- Where, $e_{cd} = e_c e_d$ and is known as antenna radiation efficiency, which is used to relate the gain and directivity.
- And Γ is the voltage reflection coefficient at the input terminals of the antenna and is defined as $\Gamma = (Zin - Z0) / (Zin + Z0)$

[where Zin = antenna input impedance, Z0 = characteristic impedance of the transmission line]

In that there is of course the characterization happens in terms of the input and output impedances. So in that case there is a reflection where one component ends another component begins and this reflection coefficient is measured in terms of the antenna input impedance and the characteristic impedance of the transmission line. You know when the transmission line ends and the antenna begins there can be a junction where there may be an impedance mismatch for which the amount of voltages which was flowing in to the transmission line may not be efficiently hundred percent converted into radiated power because the impedance mismatch some of the incoming current can be reflected back. Then those are standard things you don't have to worry too much about this because this is just for completeness over here this won't be part of your assignments and other things. Gain and realized gain so of course this is the system it is radiating power it is receiving power in the reception mode and so there should be a gain involved okay.

Now what the gain looks like of course it looks like similar to what directivity is. So radiation intensity over total input accept or accepted power okay that the ratio of this is called the gain okay.

$$\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input (accepted) power}} = 4\pi \frac{U(\theta, \phi)}{P_{in}}$$

In most cases we deal with relative gain which is defined as a ratio of the power gain in a given direction to the power gain of a reference antenna in its reference direction okay. So G is given by 4 pi U the radiation intensity over the input power for a lossless isotropic source.

$$G = \frac{4\pi U(\theta, \phi)}{P_{in}(\text{lossless isotropic source})}$$

$$P_{rad} = e_{cd} P_{in}.$$

If you rewrite this that so the gain or the relative gain can be written in terms of the efficiency of the antenna radiation efficiency times the 4 pi over U theta 5 over P radiation or there is a directivity.

$$G_{re}(\theta, \phi) = e_{cd} 4\pi \frac{U(\theta, \phi)}{P_{rad}} = e_{cd} D(\theta, \phi)$$

So reactivity and gain are almost proportional and multiplicative term is the efficiency. Another term is called beam efficiency that is the ratio of the power transmitted or received within a cone angle of theta 1 divided by the power transmitted or received by the antenna total okay. So that causes the beam efficiency along a particular direction or throughout the main major lobe or primary lobe. Bandwidth it is defined as the range of frequency within which the performance of the antenna is sensitive with respect to some characteristics confirms a specified confirms to a specific standard.

- For an antenna with its major lobe directed along the z-axis ($\theta = 0$), the beam efficiency (BE) is defined by

$$\text{BE} = \frac{\text{power transmitted (received) within cone angle } \theta_1}{\text{power transmitted (received) by the antenna}}$$

- where θ_1 is the half-angle of the cone within which the percentage of the total power is to be found.

$$\text{BE} = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$

The bandwidth is the range of frequency on either side of the central frequency where the antenna characteristics like input impedance pattern, power pattern, beam with polarization etc are within an acceptable value.

So we cannot design an antenna which is monochromatic means it only offers a single wavelength. Whenever we do some radiating antenna there will be a range of frequencies over which it is sensitive to. So this range of frequency where the important characteristics like the efficiency, the primary beam pattern, radiation pattern, the antenna impedances, polarization etc characteristics are within a desired value. That bandwidth is called that frequency range is equal to the bandwidth of the antenna okay. So that brings us to the close of this particular lecture of course we have taken a lot of material from different books particularly for the antenna theory we have taken it from Balanis the book of Balanis and book of Krauss fantastic textbooks if you want you can please go ahead and pick them up and read it at your own time. Thanks for joining us see you in the next lecture. Thank you very much.