Solid State Physics
Lecture 69

Single Particle Tunneling and Josephson Effect

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Hello, now we shall discuss about Tunneling through Josephson junction. Josephson junction is a junction between two superconductors and an insulator at the junction there is a thin layer of insulator. Before discussing that let us try to understand what tunneling is. Let us discuss single particle tunneling. Let us consider two metals separated by a thin insulator layer. This is the first metal, this is the second metal, we call this one M_1, this is M_2 and in between them there is a thin layer of insulator shade it like this and marked with I. The insulator normally acts as a barrier to the flow of conduction electrons from one metal to another. If the barrier is sufficiently thin if the insulator is pretty thin, there is a significant probability then that an electron will pass from one metal to another through this insulator. Although, a classically that is never possible in quantum mechanics there is always a finite probability of tunneling provided this insulator is thin. So, this kind of phenomenon is called tunneling. The existence of an energy gap E g in case of superconductor, centered at the fermi level requires that an at absolute 0 temperature no current can flow until the applied voltage is sufficient. If the applied voltage is $V = \frac{\epsilon_g}{2e}$ where e is the charge of a proton that is equals $\frac{\epsilon_g}{2e} = \frac{\delta}{e}$. This is the minimum voltage that is required to make sure that a current goes through a superconductor otherwise there would be no current because of the existence of this gap at absolute 0 temperature. If the temperature is beyond absolute 0 more than absolute 0 then there would always be a current then the situation will not be like this and a voltage greater than this amount will definitely induce a current even at absolute 0. The gap E_g corresponds to the breakup of a pair of electrons in the superconducting state with the formation of two electrons or an electron or a whole in a normal state. The current starts when $eV = \Delta$. At finite temperature there is a small current flow even at low voltages occur that is because of the thermal excitation owing to the thermal excitation in the superconductor that is always there in case of more than absolute 0 temperature. Now let us consider Josephson superconducting tunneling. For Josephson superconductor tunneling we will consider $M_1,\ M_2$ as $S_1,\ S_2$ superconductor 1 and superconductor 2. Under suitable conditions we will observe remarkable effects associated with the tunneling of superconducting electron pairs from a superconductor through a layer of insulator into another superconductor such a junction is called a weak link. The effects of pair tunneling includes three different kinds of phenomenon phenomena one is called the DC Josephson effect. Here we observe a DC current flowing across the junction even in the absence of any electric or magnetic field from outside. There is no electric field and there is no magnetic field even in that kind of a situation there is a DC current across the junction that is the DC Josephson effect. There is an AC Josephson effect. AC Josephson effect is we apply a DC voltage across the junction and that causes the radio frequency oscillating current across the junction. The effect has been utilized in high precision determination of some fundamental constant $\frac{e}{\hbar}$. Further, an a radio frequency voltage with a DC voltage can cause a DC current across the junction. So, here we have a DC voltage applied that leads to a radio frequency current and there is another thing an interference that is a macroscopic long range quantum interference. This is also quite interesting if we have a DC magnetic field applied through a super conductor DC magnetic field not electric field, the superconducting circuit containing two junctions that will cause maximum super current to show interference effect as a function of magnetic field intensity. Now, this effect this interference effect is used in high sensitivity magnetometers let us discuss these effects in some more details. (Refer Slide Time: 08:27)

Let us start with the DC Josephson effect. Our discussion of Josephson junction phenomenon that follows the discussion of flux quantization that we have learnt earlier. Now we consider ψ_1 to be the probability amplitude of electron pairs on one side of the junction. And similarly ψ_2 we consider the probability amplitude of electron pairs in S_2. For simplicity we consider that both superconductors are identical for the present we suppose that they are both held at zero potential. The time dependent Schrodinger equation can be written on this ψ_1 and $\psi_2 = i\hbar\frac{\partial\psi}{\partial t}$ for any such probability amplitude equals $H\psi$ where H is the Hamiltonian. Now this applies to the probability amplitudes ψ_1 and ψ_2 that gives us $i\hbar\frac{\partial\psi_1}{\partial t} = \hbar T\psi_2$. Similarly, $i\hbar\frac{\partial\psi_2}{\partial t} = \hbar T\psi_1$. So, we have these two Schrodinger like equations where $\hbar T$ this represents the effect of the electron pair coupling or transfer interaction across the insulator T has the dimension of a rate or a frequency. It is the measure of the leakage of ψ_1 into region 2 and of ψ_2 into region 1. If the insulator is very thick t = 0 and there is no pair tunneling. So, that is not the situation we are going to consider we are going to consider when there is a tunneling. Now if we assume some form of ψ_1 and ψ_2 , $\psi_1 = n_1^{1/2}e^{i\theta_1}$ where θ_1 is the phase and $\psi_2 = n_2^{1/2}e^{i\theta_2}$. (Refer Slide Time: 12:25)

Now, these notations have their usual meaning as we have been continuing, then with this these values we can write from our Schrodinger like equations $\frac{\partial\psi_1}{\partial t} = \frac{1}{2}n_1^{1/2}e^{i\theta_1}\frac{\partial n_1}{\partial t} + i\psi_1\frac{\partial\theta_1}{\partial t} = -iT\psi_2$. Let us mark this as equation 1 and for $\frac{\partial\psi_2}{\partial t} = \frac{1}{2}n_2^{1/2}e^{i\theta_2}\frac{\partial n_2}{\partial t} + i\psi_2\frac{\partial\theta_2}{\partial t} = -iT\psi_1$ we mark this equation as 2. Now, if we multiply equation 1 with the factor $n_1^{1/2}e^{-i\theta_1}$. Then 1 what would be obtain? We will obtain $\frac{1}{2}\frac{\partial n_1}{\partial t} + in_1\frac{\partial\theta_1}{\partial t} = -iT(n_1n_2)^{1/2}e^{i\delta}$. We have introduced some new quantity delta which is obvious from this expression that δ would be equal to $\theta_2 - \theta_1$ the phase difference between the probability amplitude function in both sides of the junction. We mark this as equation 3 and then we multiply equation 2 with the factor $n_2^{1/2}e^{-i\theta_2}$ that gives us $n_1^{1/2}e^{-i\theta_1}$. Then 1 what would be obtain? We will obtain $\frac{1}{2}\frac{\partial n_2}{\partial t} + in_2\frac{\partial\theta_2}{\partial t} = -iT(n_1n_2)^{1/2}e^{i\delta}$ let us call it equation 4. Now if we equate the real parts in equation 3 with the real parts real parts on both sides and similarly imaginary parts on both sides by doing that we will find $\frac{\partial n_1}{\partial t} = 2T(n_1n_2)^{1/2}\sin\delta$. (Refer Slide Time: 16:25)

This comes from the real part from imaginary part we get $\frac{\partial n_2}{\partial t}$. No, from equation 4 by comparing the real part we get $\frac{\partial n_2}{\partial t} = -2T(n_1n_2)^{1/2}\sin\delta$. We call this equation 5 and similarly from the imaginary parts of equation 3 and 4 we can obtain $\frac{\partial\theta_1}{\partial t} = -T(\frac{n_2}{n_1})^{1/2}\cos\delta$; $\frac{\partial\theta_2}{\partial t} = -T(\frac{n_1}{n_2})^{1/2}\cos\delta$ we call it equation 6. If we have identical superconductors, then we will have $n_1 \approx n_2$ the pair density and if we have that then equation 6 that is this equation here that gives us $\frac{\partial\theta_1}{\partial t} = \frac{\partial\theta_2}{\partial t}$ and the time variation $\frac{\partial}{\partial t}(\theta_1 - \theta_2) = 0$. And by putting $n_1 \approx n_2$ in equation 5 we get $\frac{\partial n_1}{\partial t} = -\frac{\partial n_2}{\partial t}$. With this we can say that the current flow from superconductor 1 to superconductor 2 is proportional to $\frac{\partial n_2}{\partial t} = -\frac{\partial n_1}{\partial t}$. We can conclude from 5 by using this fact that the current J of the superconductor pairs across the junction depends on the phase difference δ and that can be written from equation 5 as we obtain as $\overrightarrow{J} = J_0\sin\delta$ which is nothing but $J_0\sin(\theta_2 - \theta_1)$ which is the value of δ. J_0 is proportional to the transfer interaction capital T; the current J_0 is maximum 0 voltage current that can pass through the junction that will happen provided $\sin\delta$ has the maximum value. With no applied voltage we will have a DC current across the junction with a value between $J_0 \; and -J_0$ because $\sin\delta$ can have values between 1 and -1. So, this value of the current that depends on the phase difference $(\theta_2 - \theta_1)$. So, $(\theta_2 - \theta_1)$ this is a constant therefore, $\sin\delta$ is a constant and the current would be a constant although it is a function of sin, it is not an oscillatory function it is a DC current. So, this kind of effect is called DC Josephson effect.