Solid State Physics
Lecture 22
Hall Effect and Magnetoresistance

Hello, we have started discussing Drude model and within Drude model we have already discussed the assumptions behind underlying assumptions behind this Drude model and also how electron transport occurs and the difference in momentum of the electrons. Now, we are going to discuss the Hall Effect and Magnetoresistance, how what we understand about this from Drude model. (Refer Slide Time: 01:02)

So, what is Hall effect? Let us consider that first we have a Hall bar of metal which looks somewhat like this. Here we have this kind of a coordinate system this is say x-direction, this is y-direction, this is y-direction and this is z-direction; that means, this one is x direction, this one is y-direction and this one is z-direction. This kind of a situation we have. Now, this is a bar of metal if we apply an electric field along the x-direction and we call it E_x there would be a current along the x-direction we call that j_x and now, if we apply a magnetic field H along the z-direction positive z-direction then what happens? Then, we have a Lorentz force acting on the moving electrons. The move the electrons are moving along the negative z-direction in this kind of a situation in any how the charges are moving along x be it positive or negative and there is an applied magnetic field along the z-direction. So, the Lorentz force coming from this is that will be given as $-\frac{e}{c}\overrightarrow{v} \times \overrightarrow{H}$ this is the expression of the Lorentz force in CGS units. In SI unit it is a bit different, but since we are having, we are considering everything in CGS in it let us stick to that. Now, this one applies on the electrons and it deflects the electron along the y-direction. When does it reach an equilibrium? When some Hall voltage is developed that opposes the effect of the Lorentz force, then this arrangement comes into an equilibrium; that means, there is no current along the y-direction. So, when the Lorentz force is there we have positive charges accumulated here and negative charges accumulated here. This is the kind of situation we end up into. So, here we are interested in two quantities – the first quantity is called the magnetoresistance; it is $\rho(H)$ resistivity as a function of the applied magnetic field we can write which is $\frac{E_x}{j_x}$ this is the magnetoresistance and Hall found this quantity to be field independent; that means, this is the independent of the applied magnetic field according to Hall. And, the other one is the size of the transverse electric field E_y which is applying along this direction E_y. Now, because E_y balances the Lorentz force, you can expect it to be proportional to the magnetic field as well as the current along the x-direction. So, we can define a quantity called Hall coefficient this would be defined as $R_H = \frac{E_y}{j_x H}$ Hall coefficient and remember something interesting this Hall coefficient depends on the sign of the carrier. So, if your carrier is electron it will have certain sign; if it is hole it will have the opposite sign. Why that is the case? You can think about it in terms of Lorentz force the value the sign of j_x in terms of. So, in terms of E_y that is generated due to Lorentz force and in terms of the sign of j_x, in terms of these two things you can think about it why Hall coefficient is sensitive to the nature of the sign of the carriers be is it if it is electron or hole. And, this is something very important because this is how we in semiconductors we find out what the majority of the whether the majority of the carriers is electrons or if it is hole. Now, we want to calculate the Hall coefficient and the magnetoresistance. How do we do that? We first find the current density j_x and j_y in the presence of electric field. So, if we consider arbitrary components of E_x and E_y that is the components of the electric field and in the presence of a magnetic field H along the z-axis we can then write down the force acting on each electron which is position independent it does not depend on the position of the electron. (Refer Slide Time: 08:58)

So, the force acting on each electron this can be given as $\overrightarrow{f} = -e(\overrightarrow{E} + \overrightarrow{v} \times \frac{\overrightarrow{H}}{c})$ of course, in CGS unit. In CGS unit this is the expression for force and the rate of change of momentum that is not force there is another damping term that we have found out earlier. So, the rate of change of momentum $\frac{d\overrightarrow{p}}{dt}$ in this case can be written as the force $-e(\overrightarrow{E} + \frac{\overrightarrow{p}}{m} \times \frac{\overrightarrow{H}}{c}) - \frac{\overrightarrow{p}}{\tau}$. The last term is the damping term that we have worked out earlier. Now, in the steady state the current is independent of time. If the current is independent of time that means, this quantity $\frac{d\vec{p}}{dt}$ goes to 0 and if this quantity goes to 0 we can write down $0 = -eE_x - \omega_c p_y - \frac{p_x}{\tau}$ considering the electric field is applied externally along the x-direction and also the momentum of the electron is along the x-direction, and in the steady state there is no motion of the electrons no resultant motion of the electrons along the y-direction because the Hall voltage and the Lorentz force they have balanced each other. Now, if this is the situation; that means, we can simplify it by writing this $0 = -eE_y + \omega_c p_x - \frac{p_y}{\tau}$ which is just changing the sign changing E_x for E_y and E_y for E_x p_x for p_y, p_y for p_x and if we do this we have introduced ω_c here a new term. So, ω_c would be given as $e \times \frac{H}{mc}$. So, if we now multiply this equation by $\frac{-ne\tau}{m}$ both equations both sides. (Refer Slide Time: 12:57)

And, if we rewrite the components of the current density we will find that $\sigma_0 E_x = \omega_c \tau j_y + j_x$ and from the other equation $\sigma_0 E_y = -\omega_c \tau j_x + j_y$ what is σ_0? It is the DC conductivity that we found in Drude model, ok and this is the DC conductivity when there is no magnetic field in the system. The Hall field that is E_y, this quantity is determined by when we have it is determined when we have no transverse current; that means, along y-direction there is no current $j_y = 0$. This is the time when we determine the Hall field because that is the saturation of the Hall field. So, if we set $j_y = 0$ in the second equation, what do we obtain? We obtain the value of $E_y = -(\frac{\omega_c \tau}{\sigma_0}) \times j_x$. This is writing the value of each quantity $-(\frac{H}{nec}) \times j_x$. How did we define the Hall coefficient? $\frac{E_y}{j_x H}$. Here we have the expression for E_y as this. So, $\frac{E_y}{j_x H}$ will give us R_H the Hall coefficient $= -\frac{1}{nec}$, it is simply this. This is the Hall coefficient. We have found that for metals Hall coefficient depends on nothing except the density of the carriers that is n it depends only on n. The other two quantities the charge of a proton and the velocity of light these are universal constants. So, the Hall coefficient only depends on the density of carriers and it is inversely proportional to the density of carriers.