Fluid Dynamics for Astrophysics
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Lecture - 46
Spherical blast waves: Sedov- Taylor Solution (contd.)

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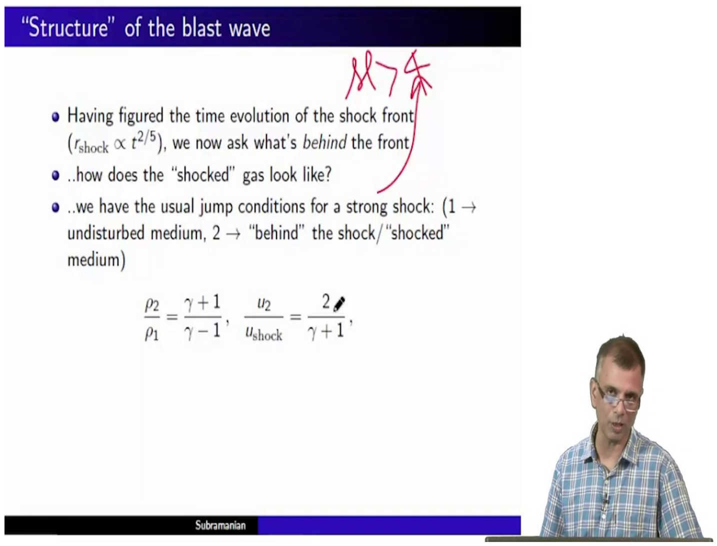

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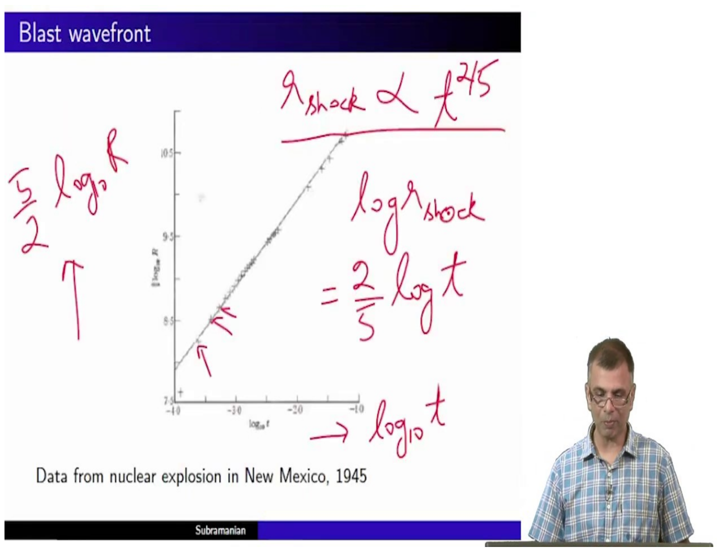

So, this plot that we saw is an amazing confirmation about the fact that we have essential physics of a blast wave shock propagation correct ok. The expectation of the theory which is that r should be proportional to t raised to 2 halves, which is represented by this line is amazingly consistent with the data. And there are some interesting there are some interesting stories about the very first nuclear bomb explosion, about how the great physicist Enrico Fermi made a very quick order of magnitude calculation of what the velocity of the shock wave ought to be right.

By even when he was standing very very far away from the blast site; obviously, you know he will not be standing close to the blast site, he would be standing several kilometers several tens of kilometers away and apparently what he did was he tore little pieces of paper and as soon as he heard the sound of the blast, he just he let those pieces of paper just go and he made a very quick rough, you know calculation of the velocity with which these pieces of paper were moving ok.

And from that he was able to say something about; you see the velocity would be like something like this right.

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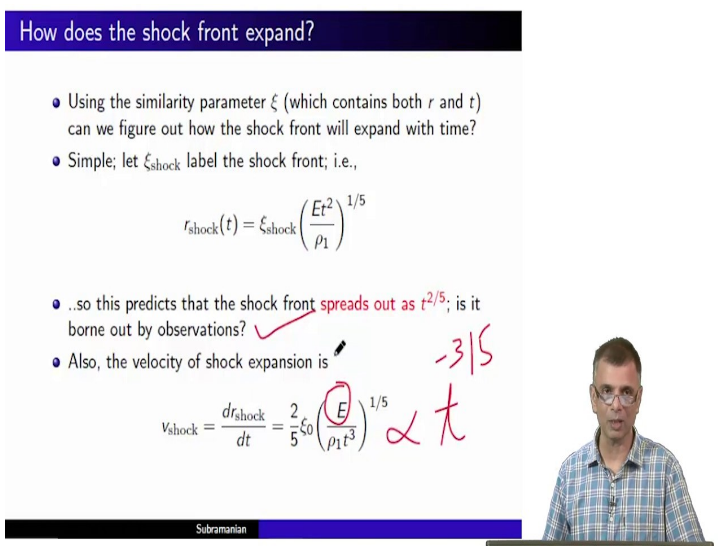

So, the velocity has the energy in it right. So, from the velocity of the pieces of paper at a certain time, he was able to say something about the energy of the original you know bomb blast and so, you see this is how this is how it is a amazing you know demonstration of how simple ideas can be used to arrive at conclusions that are very very important ok.

And of course, his answer was just rough but it was in the right ballpark and for such a simple experiment, it proved amazingly useful and he says his rough guess was you know confirmed with more accurate measurements later on ok. So, having now, figured out the time evolution of the shock front which is that r shock is proportional to 2 halves sorry, 2 5th's t raised to 2 5th, we now ask what is behind the front.

So, you see the shock has propagated, we need to know what is how does a shocked gas look like. Why is this why is this important? Because, all astrophysical observations like this;

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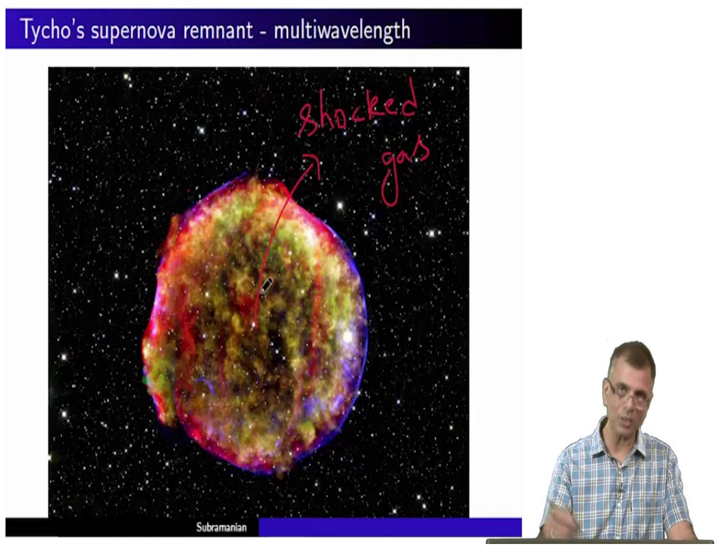

All of this is essentially shock gas, all of this is in a sense that the shock has passed through the gas and it is heated it and you know that is what is causing the particles to radiate and that is what we see right.

So, we would like to know what is the structure, what is the density structure, what is the velocity structure, what is the pressure structure behind the shock ok; this would be a shock front. So, behind it what is the structure right? So, that is what we would like to know and let us see how what does the shocked gas look like ok.

Now, we have the usual jump conditions for a strong shock. You remember this from our earlier discussion of shocks, the jump conditions for the strong shock the one the subscript 1 would describe the undisturbed medium, 2 would be behind the shock or the quote unquote shocked medium that we just saw in the beautiful picture.

So, this would be, this would be the shocked density and this would be the unshocked density and we know that for an infinitely strong shock in other words a strong shock would be you remember that the strong shock would be any anything with a mach number greater than approximately 4 ok.

For that for such a strong shock, the ratio of the density is this and the ratios of the velocity and this is not the ratio u 2 over u 1 it is slightly different ok. It is 2 over gamma plus 1, this these can be verified just from you know the shock jump conditions that we have derived earlier.

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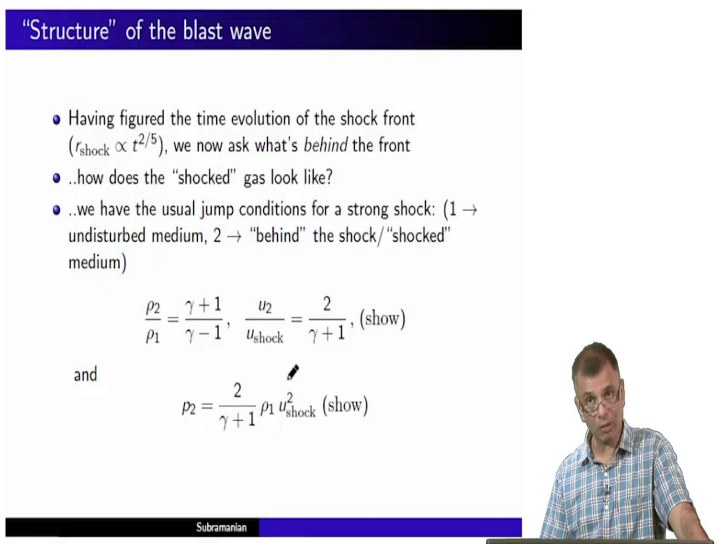

And I urge you to show this, it is a simple matter to show ok, and the pressure in the unshocked medium is related to the speed of the shock in this manner ok. All of these are fairly simple straightforward derivations that follow in a transparent way ok. So, that is that also I would like you to show this.

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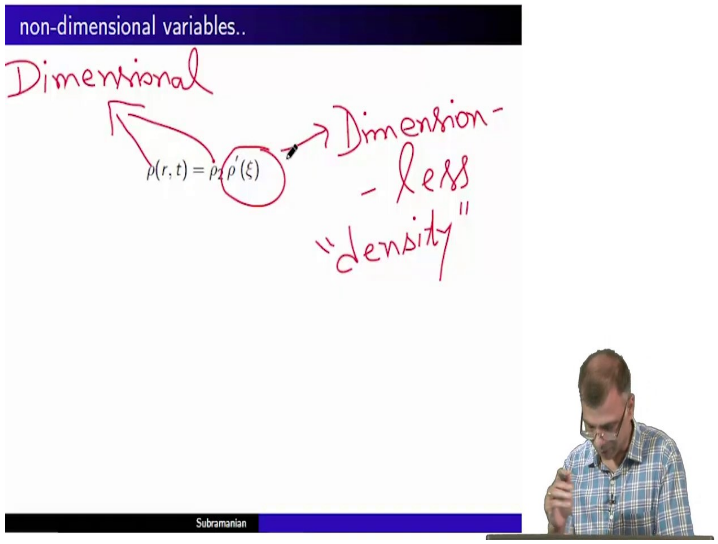

Now, you remember we made all these deductions, we made these brilliant deductions not we, but the people who figured this out, they made these brilliant deductions using these non dimensionalized variables right. So, xi this is what facilitated everything. So, therefore, we now too, we do not want to be working with dimensional variables such as density, velocity and so on so forth.

And pressure and everything we would like dimension less counterparts of the pressure of the density, of the velocity all of those ok, and so, what are the best ways of doing that? We say so; the primes would be the dimension less density and these are dimensional. So, for instance, this and this are dimensional whereas, this is dimension less.

This is dimension less density wherever you see a prime and when it is of course, when it is a when it is a function of the dimension less, you know distance xi it is a dimensionless quantity and simply from the symbol you should figure out its a dimension less version of what, in this case it is a dimension less I put this in quotes, because no density is really dimension less.

So, it is like a density ok, it is a dimension less counterpart of the density ok. So, this is how I construct a dimension less density. Similarly, and as it turns out rho 2 is related to rho 1 in this manner and therefore, you know this is how the dimensional density at a given radius in a given time is related to the dimension less density ok alright.

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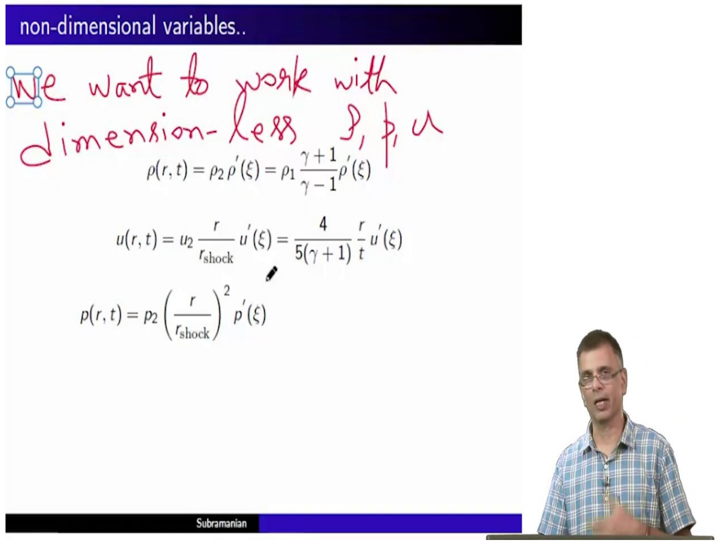

Similarly, for the velocity; so, this is how the dimensional velocity at a given radius in a given time is related to the dimension less velocity ok. This is the dimension less velocity and the radius and the time appear here, as would be expected u everything else is dimension less you see. So, the dimensions had better be r over t and that turns out to be right ok, alright, and that is how the dimensional pressure is related to the dimension less pressure ok.

The reason we are doing all this is because, we want to work with dimension less variables such as rho, p, u and so on, so forth from now on. And these are the dimensional this is the dimension less density, dimension less velocity, dimension less pressure. So, having related them to the dimensional quantities, we will see how they fit into the you know into the usual conservation equations.

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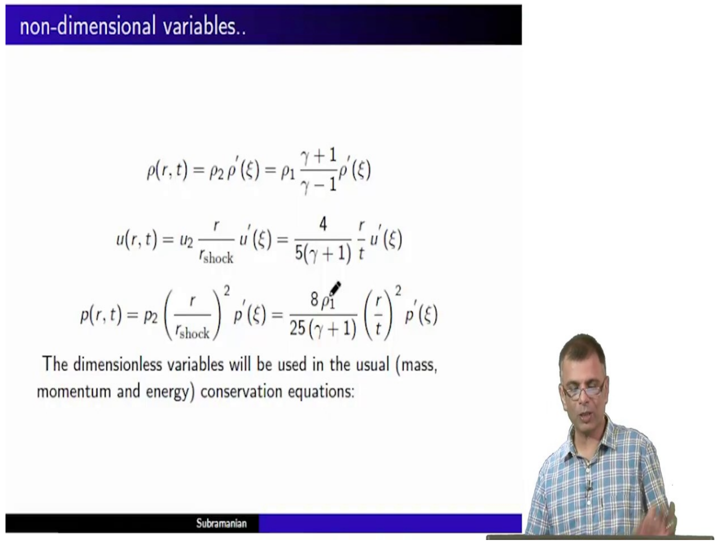

So, the dimension I mean here I, here all we have done is related the p 2 to the p 1 in more appropriately to rho 1 in this manner ok right. So, the dimensionless variables will be used in the usual conservation equations of mass momentum and energy right.

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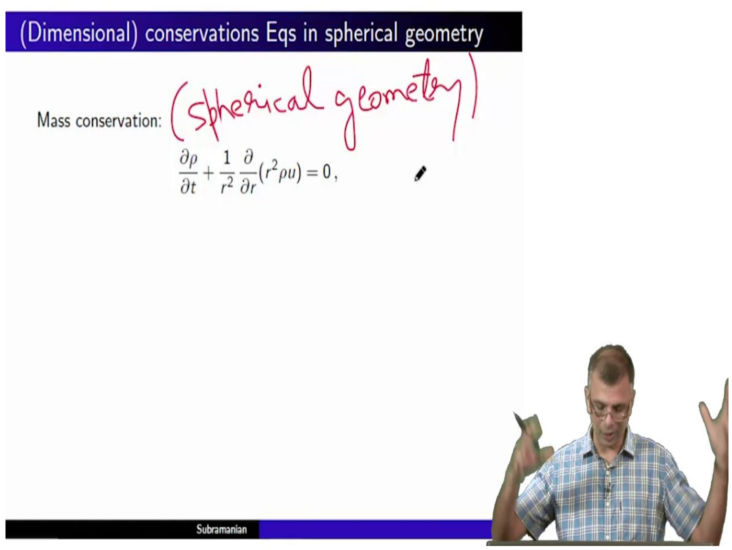

Mass conservation the usual in spherical in, so, in spherical geometry of course, because why because, you know the geometry spherical the whole assumption is that the blast wave is expanding outwards, you know spherically and momentum conservation we neglect viscosity ok.

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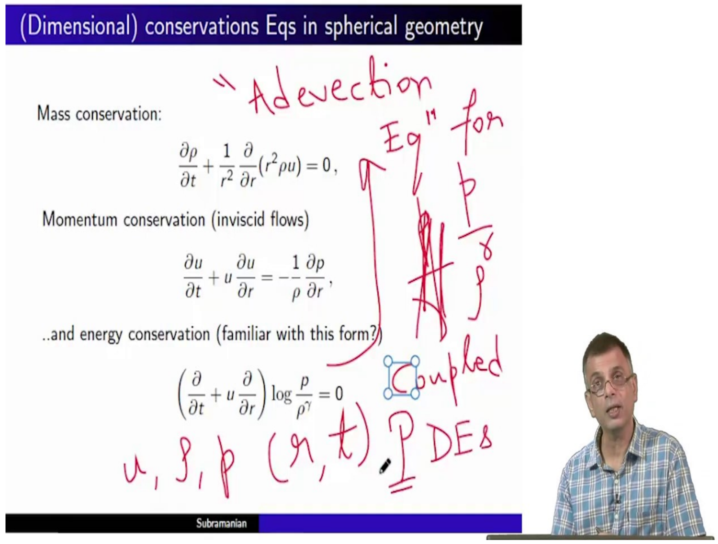

For inviscid flows is this ok, there is only one there is only one space variable and that is r and so, there is a gradient of pressure gravity does not matter ok. Gravity, body forces do not matter anymore. Of course, gravity is very very important in astrophysics, let us not get it wrong, but the point is gravity has already done its job ok.

It is caused gravitational collapse at the very at the at the very center of the exploding star and so, the masses has contracted to such an extent that has increased the density to such an extent that you know it is ignited runaway carbon fusion and because of this runaway reaction, there is a thermonuclear explosion and from then on, you know it is like a point deposition of that thermonuclear explosion that happened is like a point deposition of an enormous amount of energy and from then on, the fact that there is a massive code does not matter anymore.

In other words, the gravitational attraction of the massive code does not matter anymore and therefore, we do not have a term expressing the gravitational force in the momentum conservation equation, important to keep in mind ok. An energy conservation I would urge you to it is as if this is an advection equation. So, this is kind of a “advection equation” for p over for this is what it is ok. You see d over d t plus u d over d r.

So, the this quantity, this log p over rho raised to gamma, this is being a vector along with velocity u and that is the whole point is not it the energy and so, this would be essentially the if especially if you think about a polytropic kind of gas where p equals k times rho raised to gamma log p over rho this is just that k and the k does not change that is essentially what the whole adiabatic assumption is.

The energy does not change that energy is simply a vector along with the shock with the speed u that is what the energy conservation equation is telling you ok right ok.

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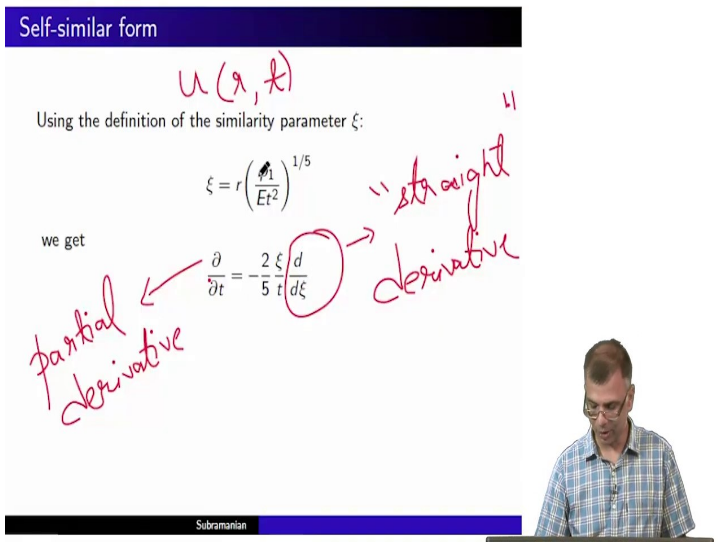

Now, using the definition of the similarity parameter xi, you remember this right. This is how xi is related to the dimensional radius r right, we get and this is very important. All of this you see here, there is a partial u partial t and there is a partial u partial r. So, in other words u, rho, p all of these are functions of r and t hence, the partial derivatives ok.

When you differentiate with respect to r you hold t constant, when you are differentiating with respect to t for instance, here you hold r constant that is why you have these partial derivatives. So, now, using this dimension less variable, you see the dimension less variable is a is a let me write this again, u for instance u or for that matter density and pressure all of those are functions of both r and t ok. And so, is xi is also a function of r and t in this particular combination.

Therefore, what happens is a partial over partial t would be something like this ok. So, this is a partial derivative whereas, what is this? This is a "straight" derivative not a partial one ok, and this is something that I want you to think about very carefully.

Similarly, and this just follows from here, follows directly from here ok. So, you could stick a u here, you could stick a p here you could stick anything here.

So, if you had d partial u partial t you would have a d u d xi, if you had a partial p partial t you would have a dp dx i so on so forth, and this follows directly from here.

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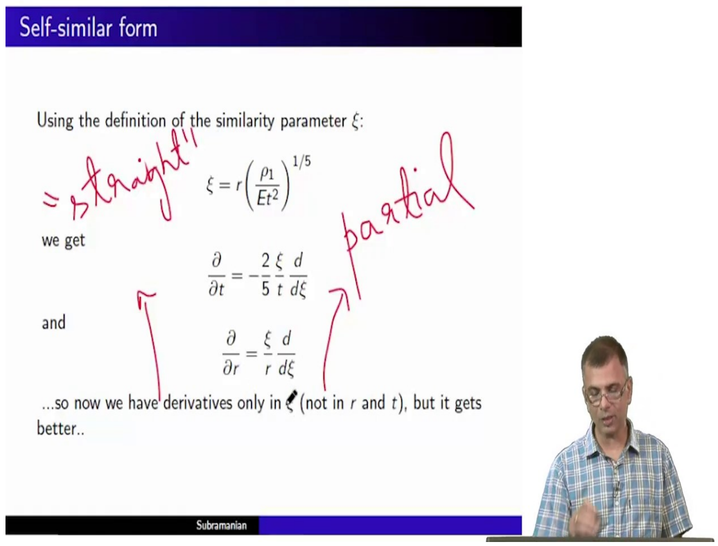

Similarly, a d over dr the partial over partial r is this. Again this is a partial derivative whereas, this is a straight derivative. Now, what are we getting it? We are getting at. So, we now have derivatives only in xi not in r and t ok, not I should say, not partial derivatives in r and t.

We have straight derivatives or this kind of straight ds in only in xi and not partial in r and t, but it gets better.

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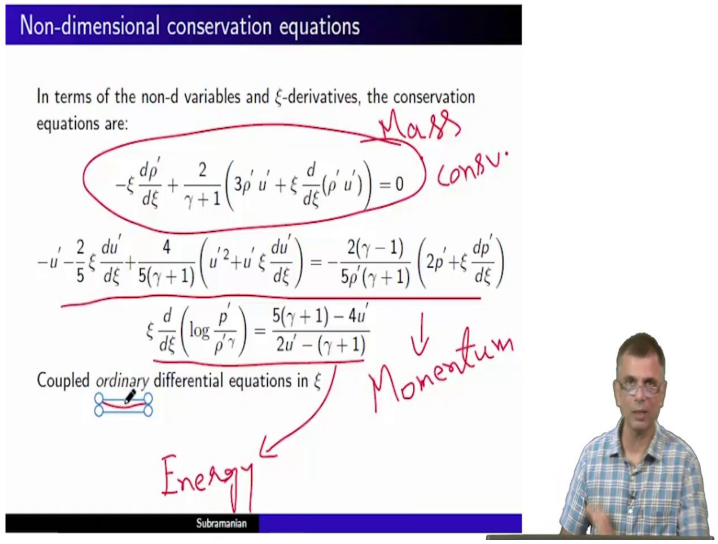

In terms of this is a lot of stuff, in terms of these non-dimensional variables rho prime u prime and so on so forth and these straight u derivatives ok. The conservation equations are; so, this mass conservation equation this guy it translates this guy essentially translates to this ok.

So, this is mass conservation in terms of rho prime and u prime and the partial derivatives with respect to time and space have disappeared and they are replaced by straight derivatives with respect to the non dimensional variable xi ok. And this would be momentum conservation and this would be energy conservation ok.

Now, the main thing to note here, this is not just to give you a whole bunch of mathematics, but the main thing to note here is that earlier you had a set of coupled PDEs these three were coupled partial differential equations PDEs whereas, now, the very same thing ok. You have coupled ordinary differential equations and there is a big simplification arising of course, from the assumption that the expansion and everything is self-similar, admittedly ok.

So, you have coupled ordinary differential equations and what is the big advantage with ODE is the number of boundary conditions goes down. We as we all know it is much easier to solve an ordinary differential equation than partial differential equations by extension it is much easier to solve a set of coupled ordinary differential equations, coupled ODEs than it is to solve a set of coupled PDEs ok.

In reality one really should be solving the set of couple PDEs, but if it is possible to make some reasonable assumptions about self-similarity or whatever if it is possible to play a few mathematical games, then the price of those, you know making those assumptions hugely borne out by this huge simplification. The fact that you now arrive at a system of coupled ODEs rather than a system of coupled PDEs which are much simpler to solve.

So, this is the final thing right, and one can solve this and find solutions for the shocked gas; what the you know how the medium behind the shocked shock looks like an instead.

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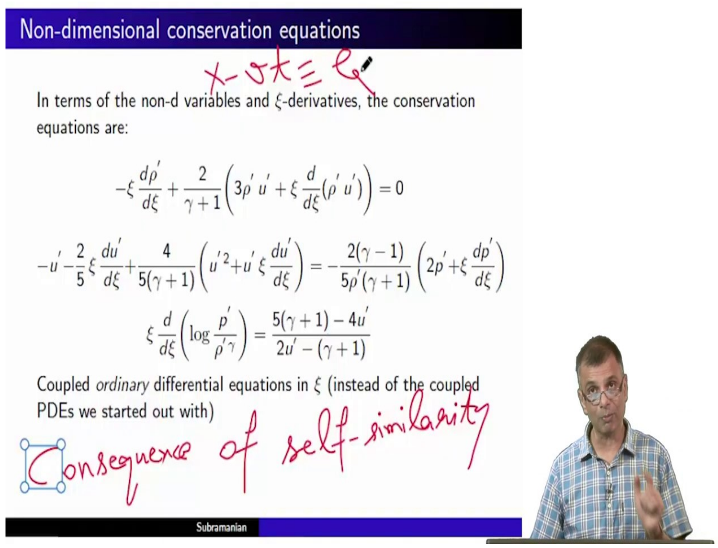

So, this is what I was just saying instead of the coupled PDEs we started out with we have ended up with coupled ODEs and this as a consequence this as a consequence of self similarity.

And this is something I would like you to keep in mind. As you go ahead you will encounter other situations where you will be using self-similar variables and this is one of the you can think in mathematical terms you can think of the motivation as just this. I try to choose a clever self-similar variable for instance, something like x minus v t. I call this is something some I do not know some zeta right,

I start working in terms of this similarity variable; so, that I will not have to now you know work in terms of partial variables with respect to x and partial variables with respect to t, maybe if I choose a clever combination of you know x and t. In this case this and in case of you know this particular problem, it was something else it was another combination of r and t, but nonetheless it was cleverly chosen so, that the set of coupled PDEs reduces to a set of coupled ODEs and that is a big simplification ok.

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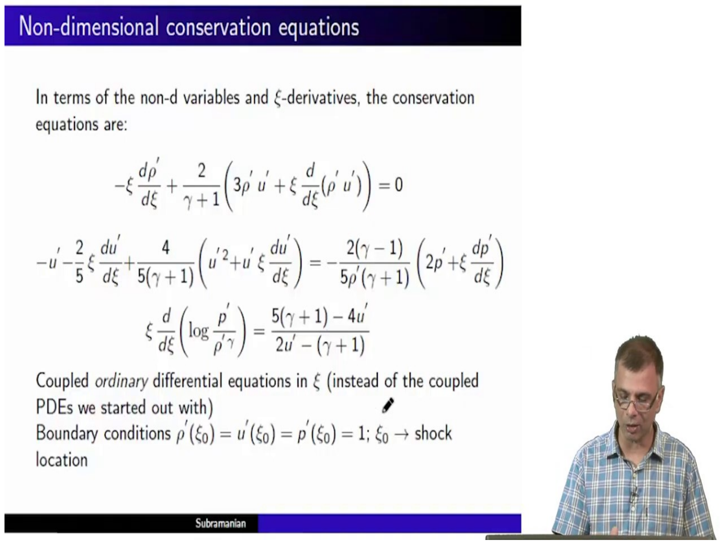

Now, the boundary conditions for solving these coupled ODEs or let us just say that at the some reference xi naught ok, the non-dimensional density, velocity and pressure are all set equal to 1. Anyway, we are non dimensionalized these are all non-dimensional quantities in order to get the dimensional quantities, we would have to we would have to multiply with appropriate normalizations ok.

So, the actual values will be taken care of by those normalizations, for simplicity we take the non dimensional condition and the non-dimensional boundary conditions to be just all equal to 1 ok, that makes things simple right; where of course, xi naught is a shock location, at the shock location all of these are equal to 1.

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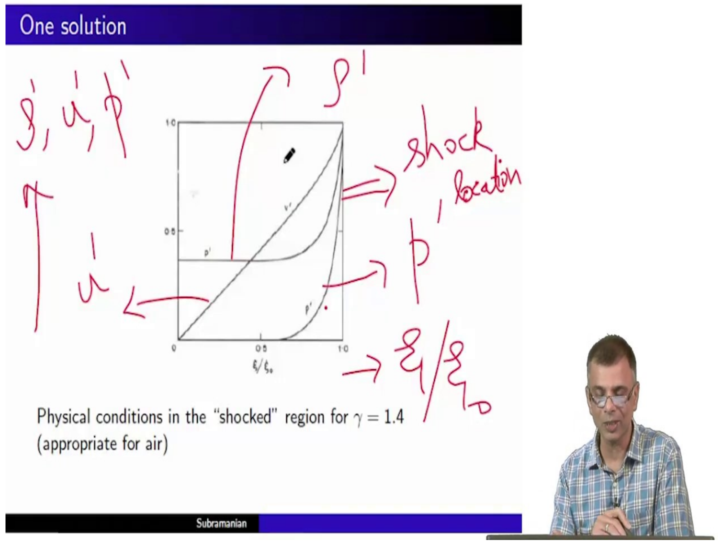

And this is how the solution to this set of coupled ODEs looks like ok. So, the x axis is the dimensional variable over xi naught ok, and this the y axis is essentially rho prime, u prime and p prime all of these.

So, this represents p prime, this represents rho prime and this is u prime and the shock location is here ok. So, the this is xi work xi naught and I work xi naught is equal to one which means that this is where the shock location is. So, behind the shock this is how the pressure varies, the pressure decreases in this manner the density decreases like. So, and beyond a point it flattens out the velocity keeps decreasing ok.

So, this is how you know the shocked medium looks. The shock is propagating into the interstellar medium like. So, it is going ahead what does the shocked gas look like, what is the density, velocity, pressure and other things in the gas that has been subject to it to the shock what does it look like? It looks like this, the density looks like this, the pressure looks like this, and the velocity looks like this ok.

Given these you can put this into a maybe a radiator transfer code or something and try to figure out say from the pressure and the density you can try to figure out the temperature of the shock medium. And from the temperature, you can say something about assuming of course, if you make the assumption that the radiating gas is a black body, you can say something about what regions of the spectrum it will be radiating in and so on, so forth.

So, you can make lots of interesting deductions about the shocked medium, about the medium that is been subjected to the shock in other words all of this stuff ok. So, the graph that I just showed you tells you the structure of the density, of the pressure and velocity and everything inside from here backwards.

And having known that one can put that into other radiation process is considerations in order to see, what to expect of the emitted radiation and this is the actual observation and you confront the observation with the expectations of from the theory, and try to see if we have an accurate depiction of what is going on.

So, you see this was you know an illustration of how fluid dynamics, fairly simple fluid dynamics ideas are used to attack a grand problem such as supernova explosions. So, we will stop here for the time being.

Thank you.