Fluid Dynamics for Astrophysics
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Lecture - 43
Particle acceleration in astrophysical settings: Diffusive shock acceleration

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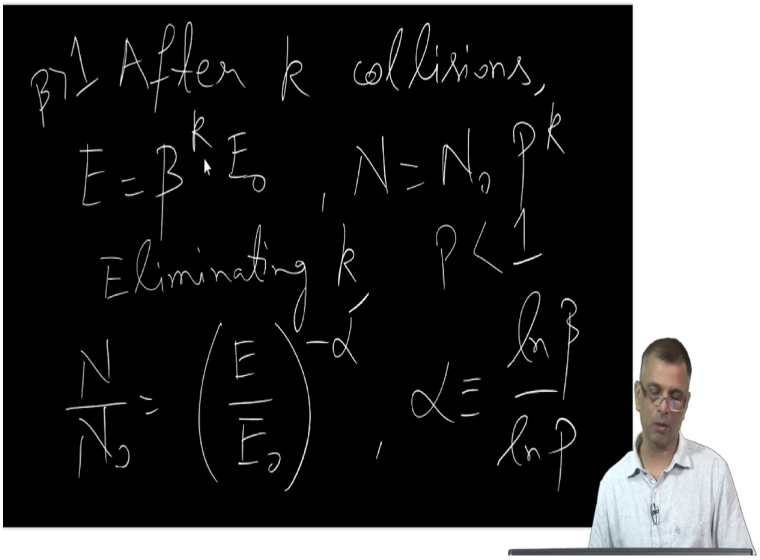

So, we are back. And so, if you recall we were discussing this subject, which is how is it that you get non thermal particles. Non thermal particles meaning, this is the number of particles as a function of energy and this is clearly not a Maxwellian. This is a power law right and these are what are called non thermal particles this is what we were discussing.

So, what we said was that these non thermal particles are produced as a result of collisions in a phenomenological acceleration region ok. And the requirement is that in each collision, there is an energy gain and this beta is larger than 1, maybe I should write it here beta is larger than 1

So, this would be the energy gain after k collisions; however, its not as if the particle if the given particle which is getting accelerated can just get accelerated indefinitely its not like that. What is in fact, happening is that after each collision? There is a certain probability P which is less than 1 that the particle can actually escape the acceleration region ok.

So, the particle can escape the acceleration region with the probability P less than 1 and so, after k collisions what happens is the energy of the particle is beta raised to k. And after k collisions, the number of particles remaining in the acceleration region is given by this. And taken together, it results in this kind of a power law distribution of particles which is clearly non thermal and which is what we are talking about right.

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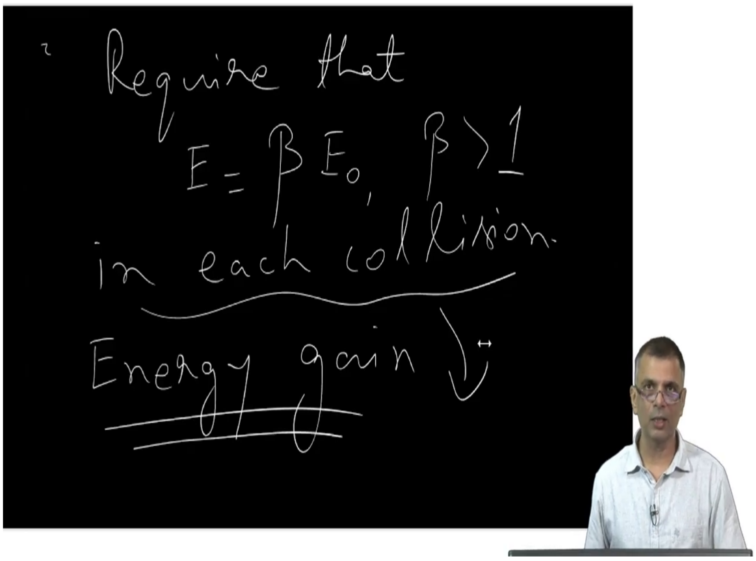

Now, one important thing to remember is that you require that E is equal to beta E naught, beta greater than 1 in each collision. In other words you require an energy gain in each collision ok.

So, whatever the collisions with these scattering centers are you want to ensure that there is an energy gain in each collision. So, that with each successive collision the energy of the particle keeps increasing. Now, let us think a little bit about what kind of situation would ensure that there is an energy gain in each collision ok.

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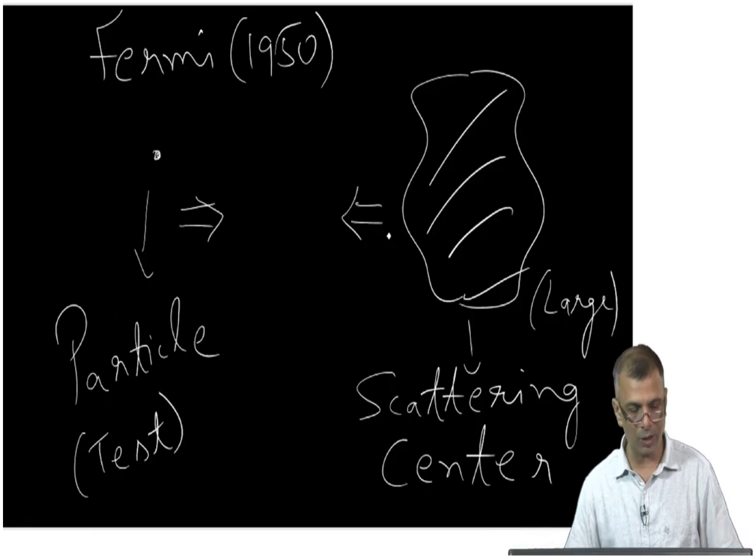

So, now, let us think of a particle and a scattering center I have, so this would be the particle which is destined to gain energy. And, this would be a scattering center in fact, in the original in the original formulation by Fermi I believe it was in 1950 if I am not mistaken, but in the original formulation by Fermi. Fermi simply Enrico Fermi the famous Italian astrophysicist.

He thought of the particles as some phenomenological particles and the scattering centers as large immobile clouds ok.

So which is why I have drawn it this way, here is a particle and here is a scattering center. And the particle is essentially a test particle. What do I mean by that? Itss a test particle in the sense that whatever happens to this particle hardly affects the scattering center. The scattering center is some sort of a large cloud its a huge rock for instance. And its hardly affected by what happens think of the particle as a light, table tennis or a ping pong ball. And think of the scattering center as a large table tennis racket ok.

So, you know when you hit a little table tennis ball with the table tennis racket, there is hardly any recoil on the racket whereas, the light ball it goes bouncing off right. So, this is what happens right. So, that is exactly the kind of situation we are envisaging. So, this would be a large a large scattering center ok, but even so I think this is 1950 I would urge you to you know check this; this treatment of you know scattering ok.

So, now consider an approaching collision consider a situation, where either the particle or the scattering center or both are approaching each other like this ok. So, consider a situation where the particle and the scattering center are approaching each other. So, consider an approaching collision right.

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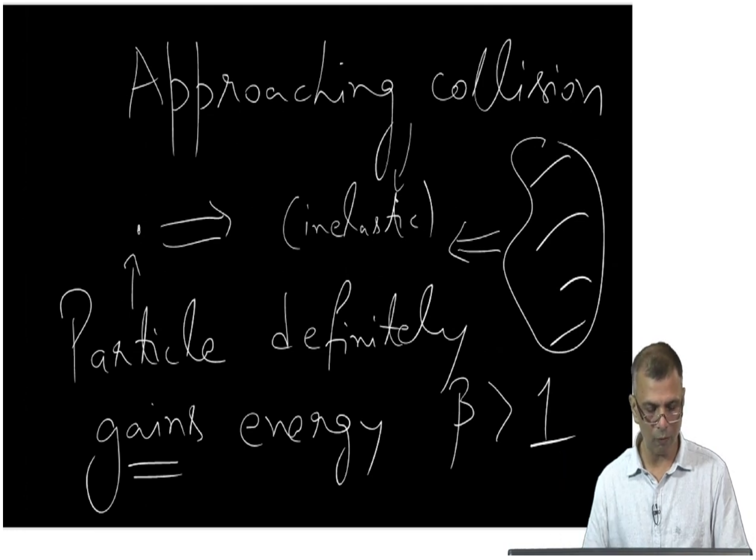

And, in such a situation in such an approaching collision kind of situation, where this guy is moving this way and the scattering center is also moving this way ok or you can transform frames such that and maybe one is stationary and the other one, either way you understand what an approaching collision is.

An approaching collision would be one where, there is definite the particle definitely gains energy ok. And I would also say an approaching inelastic collision inelastic right. The collision is inelastic that is why there is an energy gain. So, in an approaching collision the particle definitely gains energy this fellow, this particle definitely gains energy.

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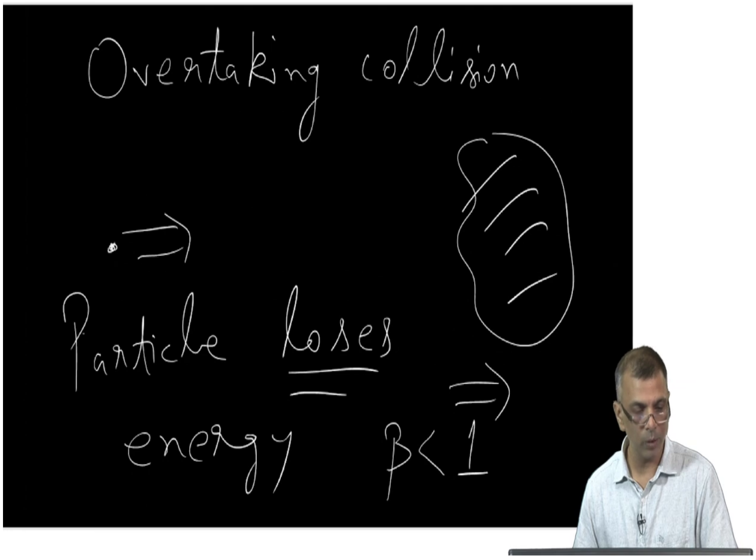

Whereas, in an overtaking collision, in an overtaking collision one where the particle is here and the scattering center is here and the particle is moving in this direction. And the scattering center is also moving in this direction and the particle is trying to overtake the scattering center right. The particle loses energy right.

So, in this case the E equals you know if you write down the energy of the particle as this, E equals beta E raised to k or in a single collision E is equal to beta times E naught. The beta would be larger than 1 in an approaching collision right and in an overtaking collision the beta would be less than 1 ok.

So, this is clear and clearly in our treatment here, we have assumed we want we want beta to be larger than 1. In other words you know we want approaching collisions ok.

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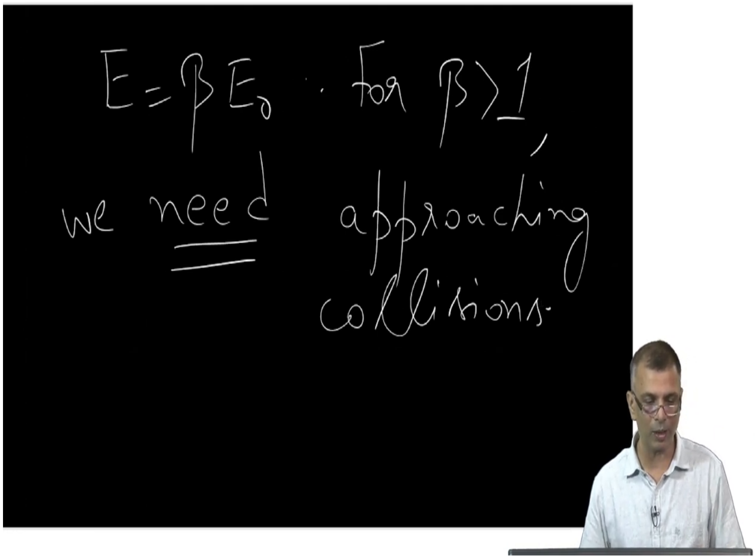

So, we essentially since we need we need in E equals beta times E naught for each collision ok. For beta larger than 1 we need approaching collisions ok. So, this is the situation that we are talking about, we would like a situation where the scattering center is always approaching the particle ok.

And the particle always gains energy with each collision and of course, it is not as if that the particle will keep encountering more and more and more scattering centers as time goes by otherwise you know the particle would just simply, you know amass infinite amounts of energy and that would you know that would create problems. So, we say that yeah that the particle gains energy with each successive collision approaching collisions to be precise.

And, you know as time goes on and the particle has a finite probability of leaving the scattering center, leaving this region filled with scattering centers. And therefore, the number of accelerated particles goes down ok.

So, this is the kind of situation we are talking about. Now, what does this have to do with shocks you might ask? Because, you know that is a whole point, we are after talking about shocks as agents of particle acceleration. So, what does this have to do with shocks? So, let us now consider a shock like this yeah.

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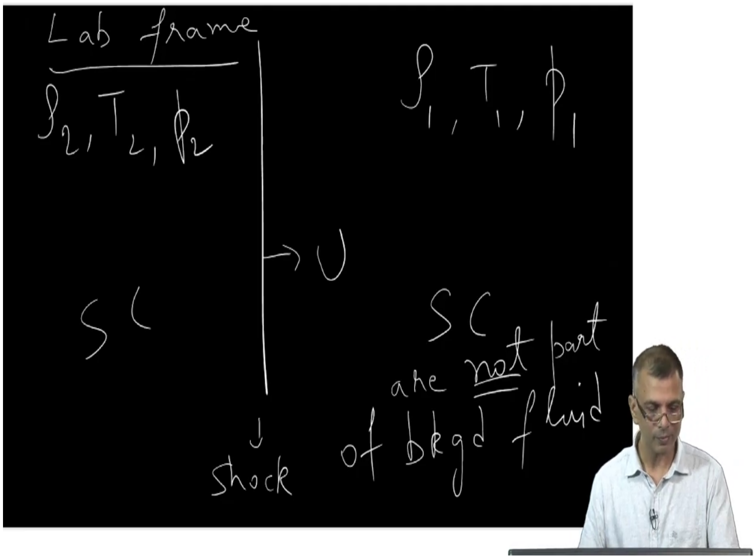

So, this would be the shock and now we are not talking about particles anymore we are talking about the fluid. The low energy the background fluid which forms a shock ok so, this is the shock and this is the frame of the lab frame this is as observed in the lab frame. As observed lab frame the shock is moving ahead with a velocity U ok.

And, there is a downstream region characterized by some you know some density rho 1, some temperature T 1 and some pressure p 1 and there is an upstream region sorry yeah. So, this would be the downstream region and this would be the upstream region and this is characterized by some density rho 2 T 2 and p 2 right.

And we know that the whole point of a shock is that there is a very definite relationship there is a very definite jump right here at the shock for the time being we just idealize the shock. There is an infinitely thin discontinuity ok, and there is a very definite relationship between rho 1 and rho 2, T 1 and T 2 p 1 and p 2 we know this. And so, let us consider a shock yeah which is moving like so.

And you have got scattering centers embedded in both these. So, you have got scattering centers here, and you have got scattering centers here as well. The scattering centers are not part of the fluid are not part of the background either of the background fluid. Neither the scattering centers nor the energetic particles are part of the background fluid ok.

But, the scattering centers are you know are carried along by the fluid ok. The scattering centers are frozen into the fluid, if the fluid is moving the scattering centers also move with the fluid ok. So, you have got scattering centers both in the downstream region as well as upstream region, and they are simply moving along with the fluid the scattering centers are not part of the background fluid.

Maybe think of the background fluid as something blue, the scattering centers as something black and maybe the particles which are destined to get accelerated as red ok.

And the scattering centers are also few and far between and so, are the accelerated particles, particles which are destined to get accelerated they are also few and far between. Whereas, the background fluid there are so many background you know low energy particles that there you just observe them as a blue continue ok. So, here you have you have a shock moving like so, and this is how this picture looks like in the lab frame. Now, let us consider how it would look like in the shock frame.

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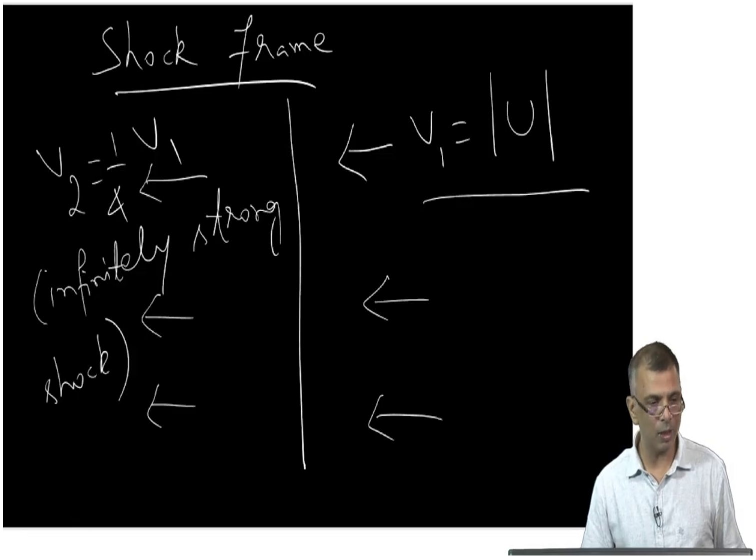

In other words you climb onto the shock, in the frame of the shock in other words here is an observer who is sitting on the shock right. So, as far as this observer is concerned the shock is stationary yeah. So, in the shock frame the shock is stationary; obviously, yeah. So, you have got a shock which is stationary yeah. And, because the shock is stationary it does not have a velocity U, but for the observer sitting on the shock, he or she will find that the fluid in here is rushing towards the shock with a velocity U.

So therefore, the V 1 the V 1 here is equal to like that moving this way ok. And we know that there is a very definite relationship between the velocity here and the velocity here yeah. And, let us consider an infinitely strong shock for which the Rankine Hiergonio conditions are satisfied right. If the Rank Hiergonian conditions for an infinitely strong shock is satisfied.

So, we know that there is a factor of four difference between the velocity here and the velocity here right. So, here what happens is you have the fluid is moving, this is all in the shock frame the fluid is moving with a velocity V 2 which is equal to 1 4th of V 1 right.

And this is assuming an infinitely strong shock, but the point is you have V 1 which is U and V 2 which is 1 4th of U 1 this is in the shock frame ok. Where are we going with all this you will see in a minute, ok alright. So, let us now go into the frame of the downstream frame ok.

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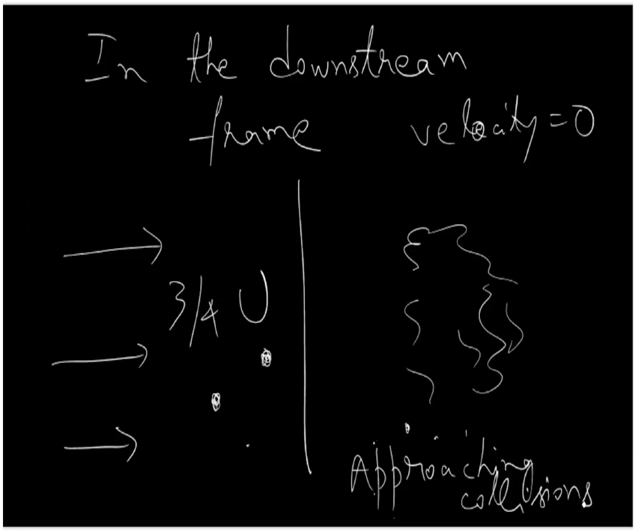

In the downstream frame; however, in other words let us go back here in this frame. So, we now want to climb into the frame, where the fluid on the downstream side is at rest ok. So, how do I do that? I go from the shock frame I subtract a velocity V 1 ok and that brings me into the frame, where you know this stuff is at rest right. So, I subtract the velocity V 1 and so, you have got the shock here. And here you know the velocity is 0 because I am at rest, but here you see I have got fluid rushing ok, towards me with three quarters U why is that you can see that from here.

So, you see you have got v one coming to the left right and so, what I want to make I want to make this 0 right. So, I add a velocity U rightward velocity U here that makes it 0 and if I add a rightward velocity here equal to U ok. So, I have U minus 1 4th of V 1 which is U and that gives me a three fourths U coming towards the right. Now, what do I have here? I have got lots of scattering centers I have got scattering centers which I am going to denote by the squiggly lines ok.

So, consider accelerated particles which are adverted along with the so, these would be particles which are destined to get accelerated ok. They are being swept from the left to the right and the reason and here is a reason for calling this diffusive shock acceleration. What happens is? In the vicinity of the shock there are enough scattering centers so, that these particles can diffuse over from this side to this side once they diffuse over to this side ok. So, or for that matter they can diffuse over from this side to this side ok.

So, once they diffuse over to this side ok the particles are sitting here ok, and they have equilibrated with the fluid here ok. So, they are at rest but you see there are scattering centers embedded with the fluid on both sides ok. So, they are at rest essentially and what they observe is that there are scattering centers from the left approaching them with a velocity three quarters U.

So, they are approaching the particles that are destined to be accelerated. So, you have got approaching collisions you have got. So, you have got particles that used to be here, which diffusively crossed over to this side. And once they have crossed over to this side, they equilibrate and they become part of the downstream fluid ok.

And they are just sitting there happily just waiting around what they see is that they are approached by scattering centers, which are approaching them at a speed 3 quarters U. And, they experience approaching collisions and we know what happens during approaching collisions the beta is larger than 1 and the particles gain energy ok.

So, this is very important. So, they gain energy when they are embedded in the downstream fluid. So, they gain energy as a result of these approaching collisions ok fine. So, the gain energy they got one hit from these paddles which were which are approaching them. So, fine ok.

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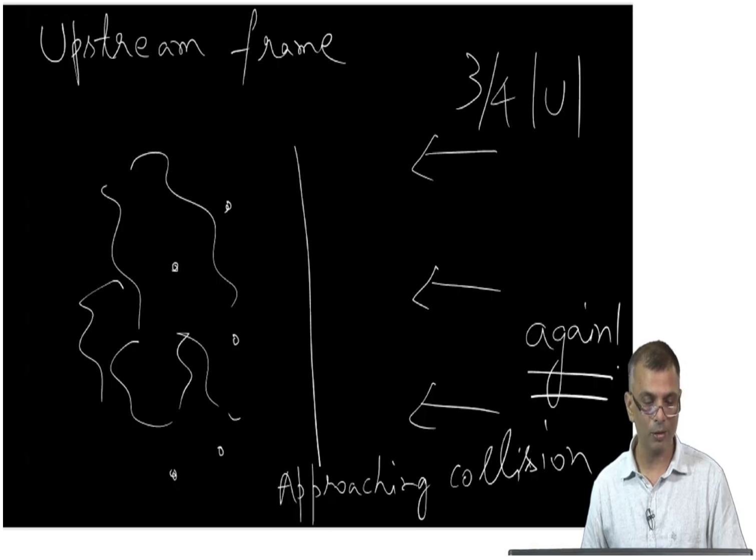

Now, what happens is? Let us now consider, let us now draw this shock again, except now we want to draw this in the upstream frame ok. This was in the downstream frame ok, now I want to remain in the upstream frame. In other words I want to make this velocity 0; I want to make this velocity 0. So, that I can be at rest in this frame that is what upstream frame means right how do I do that, well I add a leftward directed velocity equal to 3 quarters U.

If I do that this becomes 0, but of course, this out here I have to do the same on both sides right. So, out here I get a 3 quarters U moving to the left right and that is what happens here. So, here I have got rest frame that is what I mean by the squiggly rinds the fluid is not moving. But, on the other hand here I have got fluid moving at 3 quarters U ok, maybe I should have a mod sign here too.

So, here I have got fluid as far as upstream frame glows, if I was an upstream frame I would see the fluid rushing towards me with a velocity 3 quarters U ok. Now, what happened to these accelerated particles? Here, it gained energy yeah it gains energy. So, that is fine, but after it gained energy due to the approaching collisions from scattering centers on the you know from the upstream side. It was on the downstream side and scattering centers from the upstream side, approached them yeah and they hit them bam and they gain energy

Now, after that what happens is they diffusively cross over to the up to this side, and that is the whole point diffusion is a very important. The ability to diffuse on both sides of the shock is a very important aspect of this problem. If you do not have diffusion this whole process will not work ok. So, the diffuser may cross over to this side. And, now they are like this yeah they have diffusively crossed over to the upstream side, and they equilibrate with upstream frame in other words in the upstream frame they are at rest.

So, the particles are here they are happily sitting in the upstream frame, they have already gained energy due to one collision and they are still they are happily at rest. Now, what do they observe well as we said there are scattering centers on both sides right?

So, even on this side there are scattering centers and these guys which have already gained energy due to one collision right. They observe scattering centers again moving towards them at a velocity 3 quarters U.

And remember this was also 3 quarters U and this is also 3 quarters U ok. They observe since they have equilibrated here, you know they now observe scattering centers moving towards them at a velocity 3 quarters U. And, again you have got approaching collisions, approaching collision again ok, they diffuse to the upstream side and they again experience you know an approaching collision. So, they gain energy again.

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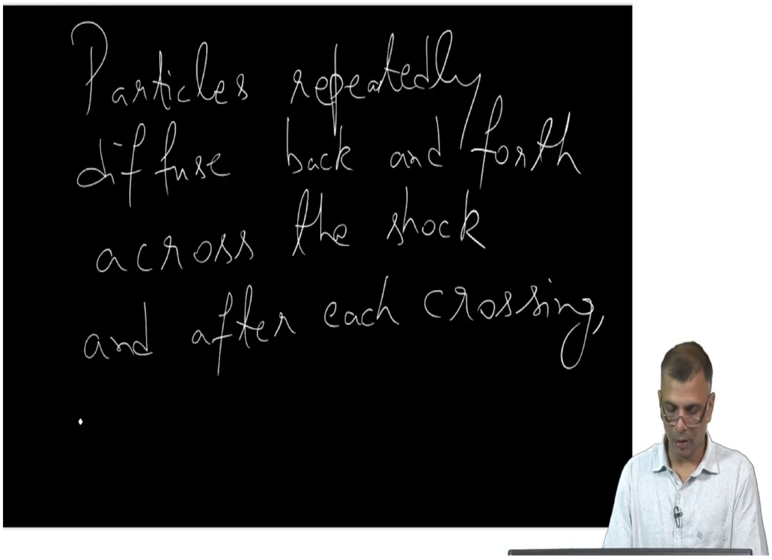

So what's really happening here is that particles repeatedly diffuse back and forth across the shock right, in other words from the upstream side to the downstream side and vice versa they keep diffusing back and forth ok.

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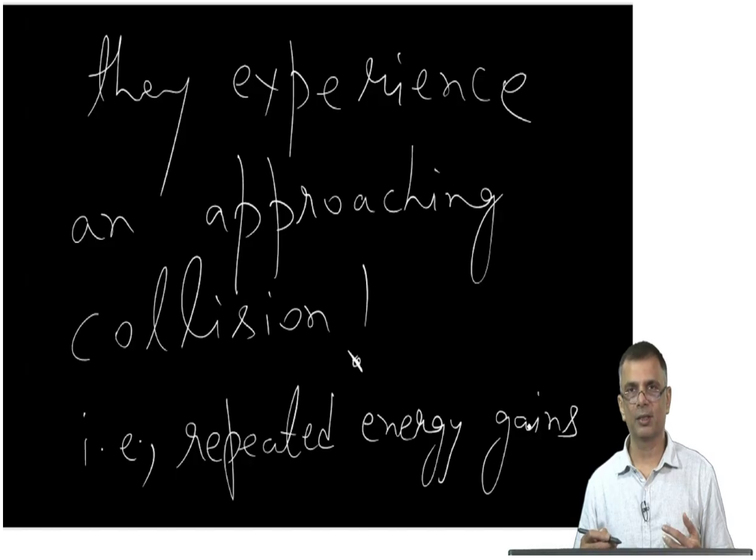

And in each and after each shock crossing, after each crossing they experience an approaching collision that is a whole point right. They experience an approaching collision after each shock crossing i.e, repeated energy gains right. So, they repeatedly experience energy gains. So, they diffuse back and forth across the shock and after each crossing they experience an approaching collision, from who? From the scattering centers that are embedded along with the flow ok.

So therefore, you experience this kind of a situation with a beta greater than 1, because on each side of the shock you have an approaching collision. So, a beta is greater than 1.

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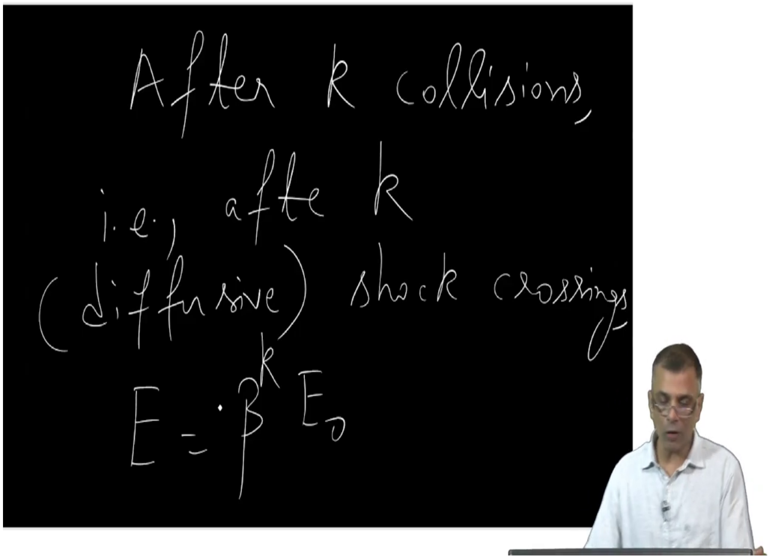

And since they are repeatedly doing this. After k collisions so, after k collisions i.e, after k diffusive shock crossings, the energy of a given particle is beta raised to k, E raised to 0 as we wanted. So, here is a physical situation where this condition is satisfied. And of course, since the whole thing is diffusive it is not as if particles can you know sometimes you know, they diffuse back and forth.

But, they do not they really do not have to remain in the vicinity of the shock always; they can diffuse a little farther way. Once they diffuse a little farther way, they will find it difficult to come all the way close to the shock.

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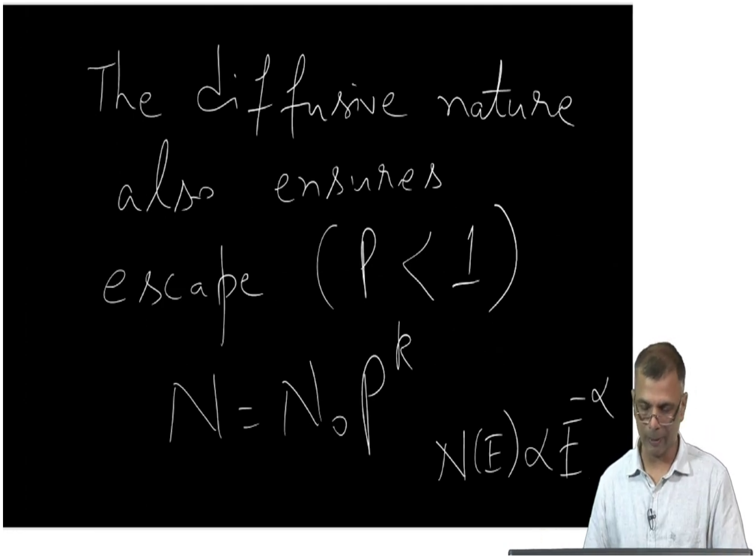

And therefore, there is also escape I should say. The diffusive nature also ensures, escape with a probability P less than 1. In other words after k crossings the number of particles is equal to N naught P raised to k, where p is less than 1.

And, remember these were the two ingredients that we needed, we needed this and we needed this right and you eliminate k between this and this and you get a power law distribution in the number of N of E, these two result in an N of E which goes as E raised to minus alpha in other words a non thermal particle distribution ok.

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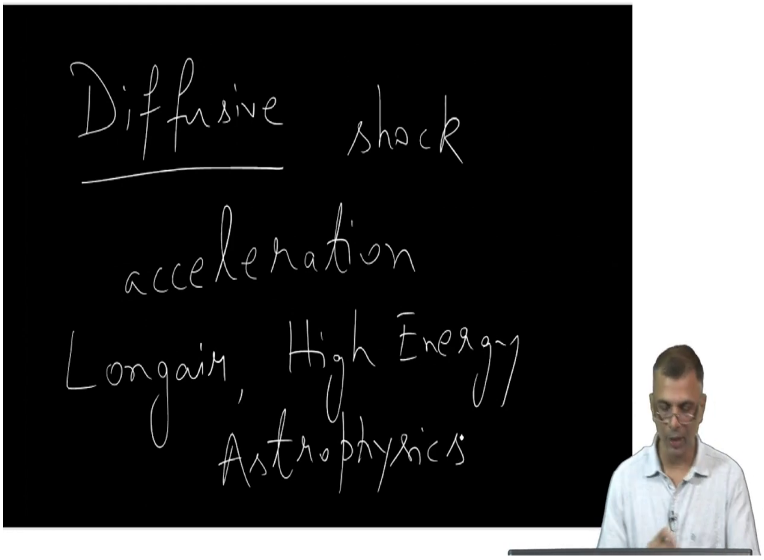

So, this is a is an explicit demonstration of the phenomenon of diffusive and diffusive is very important shock acceleration. Diffusive shock acceleration of who of the test particles ok, by whom by the particles are accelerated by who well they are accelerated via encounters head on encounters with scattering centers, which are embedded along with the fluid which flows with the shock ok.

So, the scattering centers are embedded in the fluid and therefore, they just simply obey whatever the fluid tells them to. If the fluid is flowing and it if the flow has a certain discontinuity well, they also flow with the same velocity ok. So, its important to clearly understand these frame transformations, this would be the lab frame and this would be the shock frame.

And this would be the downstream frame, and this would be the upstream frame. Really in order to understand the fact that there are approaching collisions all you need really is a downstream frame and upstream frame. But, in order to get to the downstream from the upstream frame, you need to go via the shock frame. So, I urge you to turn this over in your head a little bit to understand how exactly the shock acceleration happens.

And this treatment is taken from Longaires book, high energy astrophysics I believe this is by Cambridge university press. So I urge you to look up this book further details and so, this particular treatment is taken from there.

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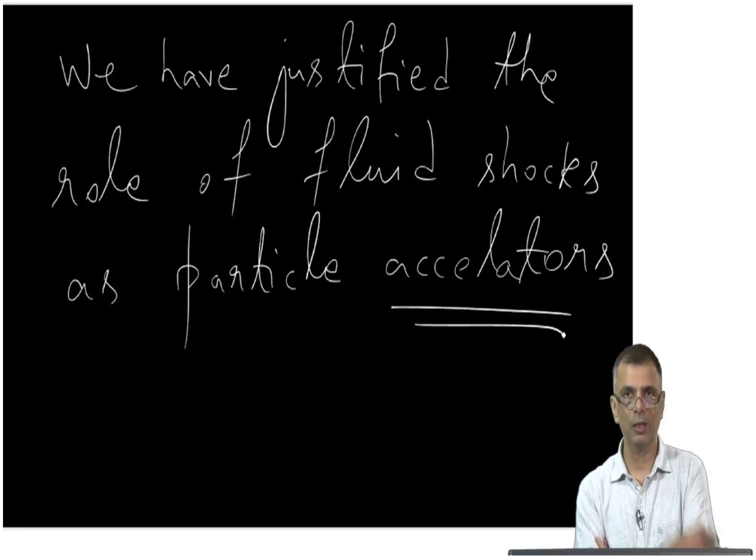

And so, what we have now done is we have we have we have shown not shown, we have justified to some extent, we have the role of fluid shocks as particle accelerators. And, I thought it was important to demonstrate this, even though even though particle acceleration as such is not really part of a fluid dynamics course, but in astrophysical fluid dynamics, shocks are simply agents for accelerating particles.

So, I thought it was important to sort of give you at least a brief flavor of how particle acceleration takes place, before going on and examining, how you know how shocks arise in astrophysical contexts and so on so forth. So, when we meet next we will consider the formation of shocks in supernovae, supernovae shock or very well studied phenomenon in astrophysics. And the predictions of the theory match the observations very well indeed.

So, when we meet next we will consider spherically symmetric shocks in supernovae, and I also wanted to reemphasize that shocks are to be found in all kinds of other astrophysical situations. Such as near earth shocks the earth's bow shock you know shocks and accretion flows, shocks in supernovae and so on. So, forth we will simply consider one of these situations that of spherically symmetric shocks in supernovae. So, that is it for now and thank you.