Fluid Dynamics for Astrophysics
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Lecture - 38
Spherical accretion [contd], Disk accretion: Roche Lobe overflow

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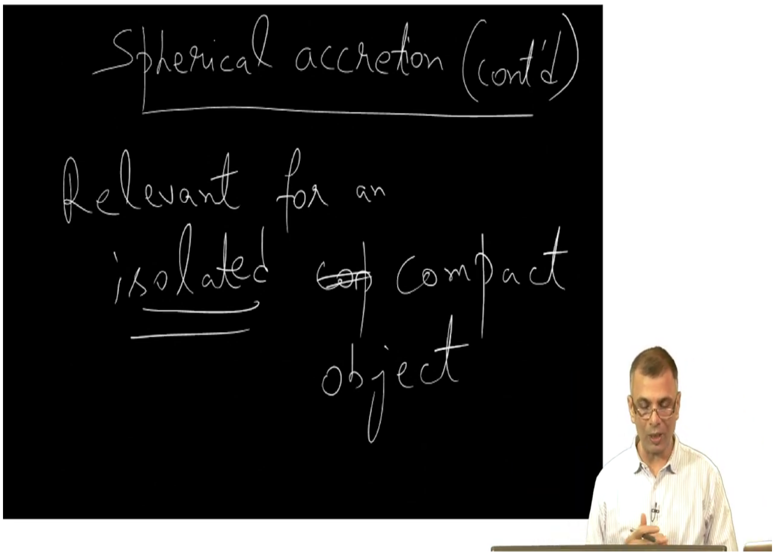

So, we are back. And let us wrap up our discussion of Spherical Accretion today. This is pretty much a quick wrap up of what we have done so far. And importantly, I will try to emphasize the kinds of points that we have not covered, our treatment does not cover certain important realistic astrophysical situations. So, these are the couple of things I want to you know emphasize.

So, spherical accretion just to repeat is relevant. When you have a compact object relevant for an isolated compact object, in other words a compact object which is just sitting by itself, ok.

So, in other words, it does not have a companion. It is just sitting there, yeah, and its accreting from the interstellar medium or maybe its accreting via a stellar wind there might be a companion, but the companion is far away and its emitting some kind of a stellar wind and this relatively isolated compact object is accreting.

So, there is really no reason to believe that there will be you know any disturbance to spherical symmetry which is why we consider spherical accretion.

Now, as we have seen accretion is all about M dot, M dot is a be all and end all of accretion, ok.

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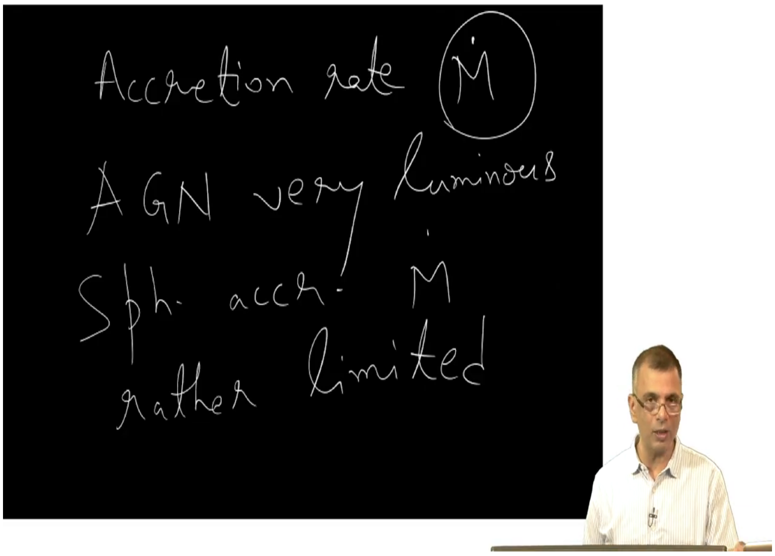

The accretion rate this is what accretion is all about. Why? Because the larger the M dot the more the energy the gravitational potential energy can be released into potentially of course, released into radiation that; it is, it is not always the case that all the gravitational potential energy will always be converted into radiation. But you know in principle it can be, right.

So, the larger the M dot the more you know the energy that can be released into radiation. And we want as much energy as possible to be released into radiation because it to be released as photons which is what we observe, because we are dealing with active galactic nuclei which are very luminous. So, we want to do our best by way of M dot, ok.

Now, in spherical accretion you see the M dot, for spherical accretion the point is M dot is rather limited. In comparison to what? Well, in comparison disk accretion which we will discuss soon, ok.

So, but nonetheless spherical accretion is a very important mode of accretion because many times astrophysical objects compact objects are just sitting by themselves and there is really no reason to believe that spherical symmetry is disturbed, right. So, this is one thing I wanted to just repeat before we went ahead and discussed other kinds of accretion, right.

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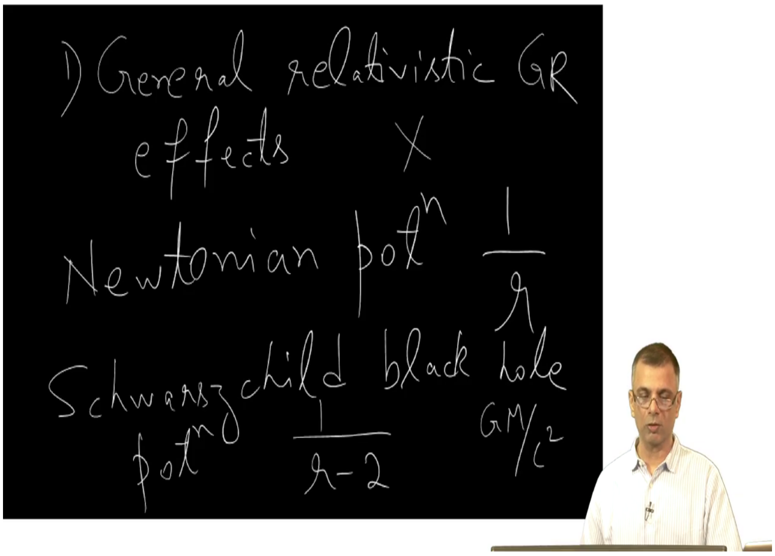

The one thing we did not really pay too much attention to is general relativistic effects. And I alluded to this towards the very end of our discussion when we met last. But general relativistic effects, we really have not considered much, we have not; when I put a cross here I mean we have not really paid too much attention to general relativistic effects.

Why? We have only been dealing with Newtonian potentials, Newtonian gravitational potentials which go as 1 over r. And anything that goes as 1 over r as you know it blows up at r equals 0, we know this, right.

Now, the general relativistic potential is different from the Newtonian potential most of the time calculations in GR are done for a point particle for a test particle not for a fluid, ok. So, treating a fluid in a general in a full GR metric is quite computationally quite intensive. So, one sort of cheap way of getting away with this is that at least for a non-rotating black hole, which is described by the Schwarzschild black hole which is essentially a non-rotating black hole, ok.

The entire GR metric can be approximated at least as far as fluid flow goes by a potential which does not go as 1 over r, but it goes as 1 over r minus 2, where now we are writing r in terms of the gravitational radius in in units of this, ok. So, this really should be 2 GM over r squared, but if you are writing r in units of GM over c square the potential can simply be written as 1 over r minus 2.

Now, notice the difference between this and that, ok. It seems like a small difference, but it makes all the difference in the word, ok as far as the inner boundary condition goes. So, really you can work through all the steps, the mass continuity equation and the momentum continuity equation these were the only two things that we considered.

And in the momentum continuity equation you had the GM over r kind of thing some something like this you know appearing. Instead of that you just use this. And for all practical purposes that is good enough to treat a Schwarzschild black hole, ok. It is good enough to treat a non-rotating black hole, ok.

This results in in some modifications to inner boundary condition, but not that much, ok. Essentially what will happen is the sonic surface will be shifted a little bit. So, this is one thing.

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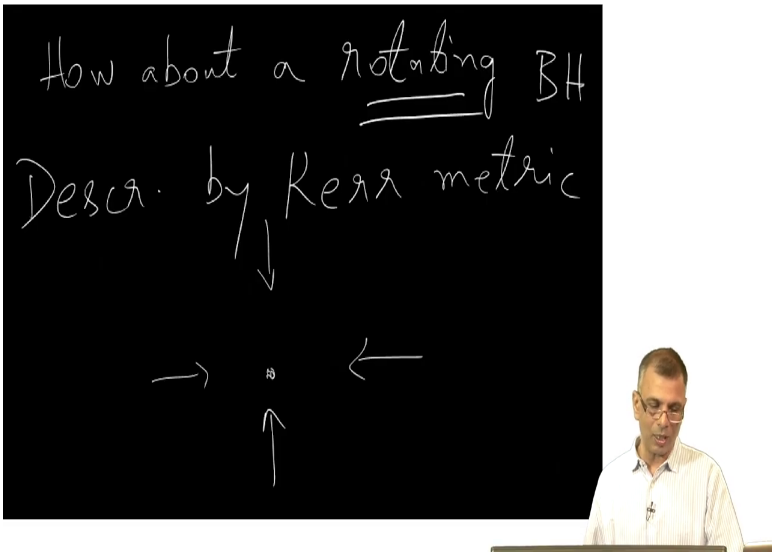

The other thing is how about a Kerr Black Hole? How about, how about a rotating black hole? Right, which is generally described by the Kerr metric, ok. In this case, generally there is no convenient parameter. This is what is called a pseudo Newtonian potential. This would be the full Newtonian potential this is a pseudo Newtonian potential that approximates the metric around a Schwarzschild black hole. For a Kerr Black Hole it turns out that there is no convenient, there are some pseudo Newtonian potentials, but they are not that convenient, ok.

So, there are people who have carried out full general relativistic calculations. The question really is you have a compact object here, you have a compact object here and its accreting you know in a quasi-spherical manner. Now, does it really matter and how does a rotation of the central object rotation about an axis how does it influence the manner of accretion this is the real question, ok.

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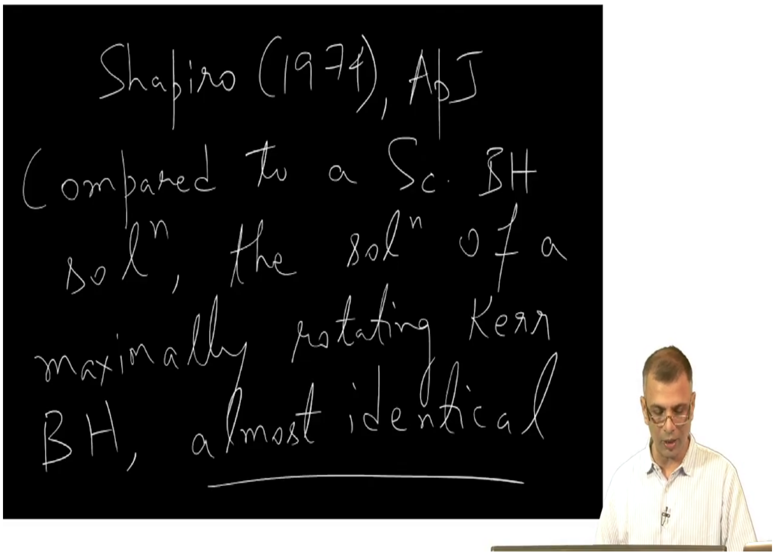

Now, turns out that and this go this what I am telling you the results go back to Shapiro 1974, astrophysical journal. You can search for this. And also some other later treatments. And so, they have essentially shown that in comparison compared to a Schwarzschild black hole solution, ok.

The solution for what? Solution for you know the velocity, density, so on so forth, everything, all the physical quantities solution for maximally rotating; the black hole can be rotating at various speeds and there is something called a maximally rotating Kerr Black Hole. And so, a solution for a maximally rotating Kerr Black Hole, ok.

Compared to a non-rotating black hole solution the solution for a maximally rotating black hole is almost identical. The fact that the central black hole is rotating makes hardly any difference to the actual accretion, ok.

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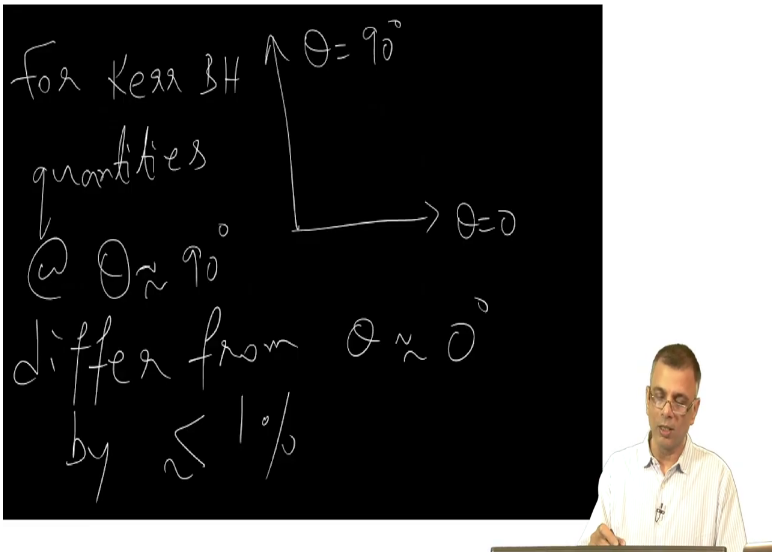

In fact, the difference and so, what happens is you see you have you have the equatorial plane which is characterized by theta equals 0 and you have the polar plane characterized by theta equals 90, right. So, the black hole is rotating along this axis, right. So, you would expect some if there was a difference you would expect some difference in going from theta equals from the equatorial plane to the polar plane, ok

The solution for a maximally for a Kerr Black Hole, I will not write maximally rotating you should understand that. The difference, the quantities at theta approximately equal to 90 degrees differ from those at theta approximately equal to 0 by about less than 1 percent.

So, if there was a big difference you would expect you know, by quantities I mean density velocity everything all physical quantities to be appreciably different along the rotation axis as compared to the equatorial axis. Turns out that there is a difference yes, but the difference is very small, it is just about 1 percent. And this was the finding of Shapiro. And there have been you know treatments somewhat more sophisticated treatments than that of Shapiro later on, and they have found essentially the same result.

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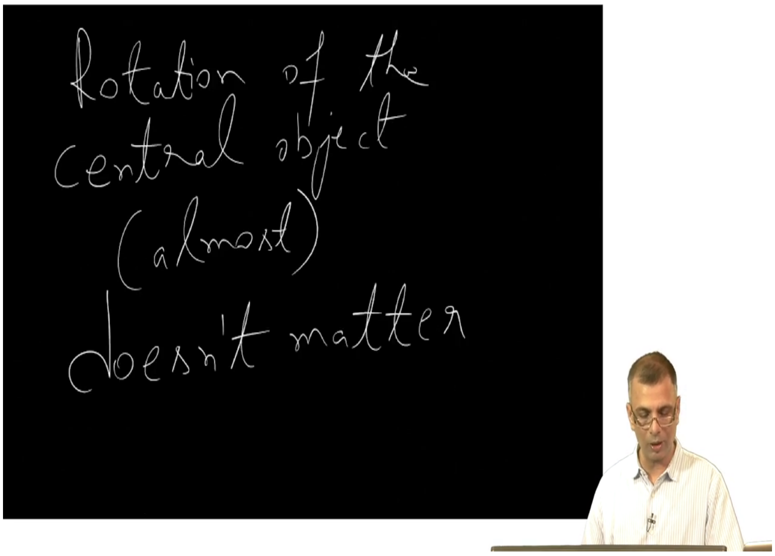

So, bottom line rotation of the central object does not make a big difference rotation of the central object, almost does not matter, almost does not matter. As far as what? As far as the geometry of the accretion flow is concerned, ok, yeah. So, I told you that we did not consider general relativistic effects and so, here is what we have described now is a very quick walk through some of the general relativistic effects, ok.

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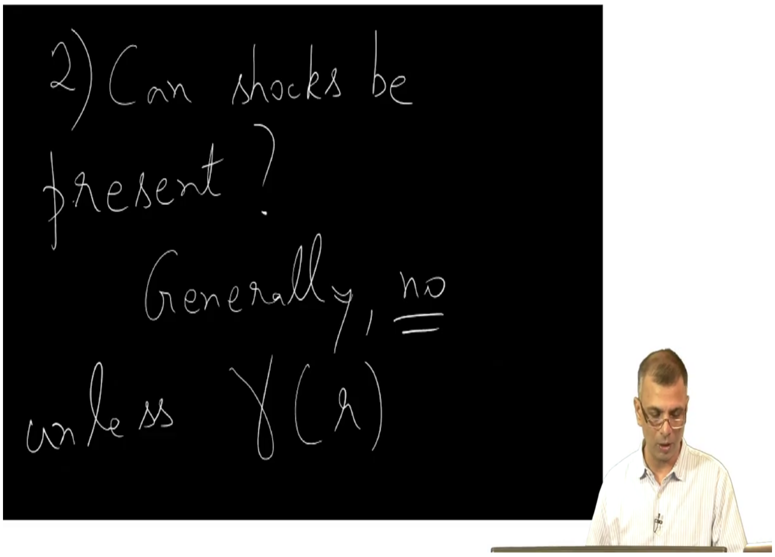

Now, there is another important thing that we did not consider and that is can shocks be present and we have already alluded to that, ok. Can shocks be present in quasi-spherical accretion? The answer to that is well not really, ok. Why? The main thing is you need multiple sonic points, ok in order to have the inner boundary condition such that the inner boundary conditions always supersonic. And in strictly spherical accretion you know, strictly spherical accretion also with you know a non-varying gamma which is the adiabatic index there is no scope for multiple sonic points. So, generally, the answer to this is generally no.

Unless you can engineer a situation where the adiabatic index is a function of the accretion radius, ok. And in in other words, you introduce new sources of heating and cooling such that the adiabatic index varies with radius and in that case there are ways you one can obtain shocks in spherical flows. And we are almost at the end of our discussion of spherical accretion.

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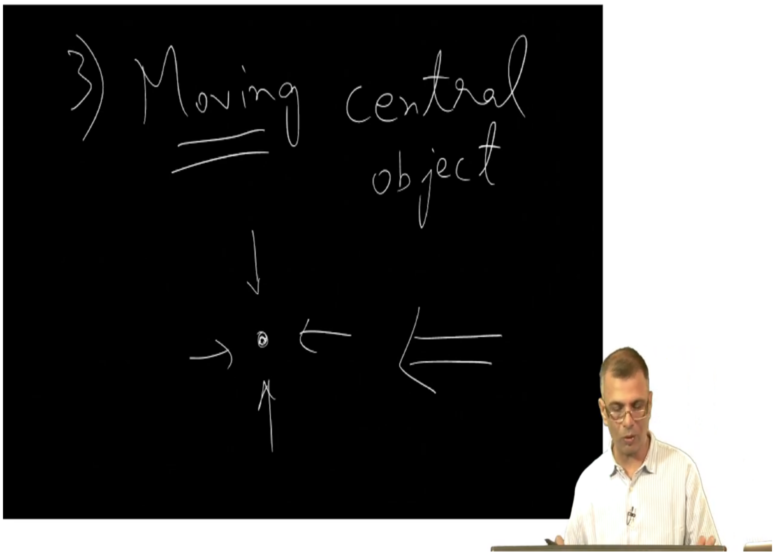

So, the final thing that we have not considered, but which is important is also a moving central object. You see like this. You have got the central object and its accreting. So, far our entire discussion has been in the context where the central object is just stationary, right. Now, what if the central object itself is moving? Ok. This is itself moving through the interstellar medium. This is an important question, ok and that is something that we have not discussed so far, I just wanted to make you aware of this, ok.

So, I think this is about enough about spherical accretion really. Again, like I said spherical accretion is an important astrophysical you know scenario relevant to isolated objects, but you know in in many cases the objects are not isolated, the central objects are not isolated.

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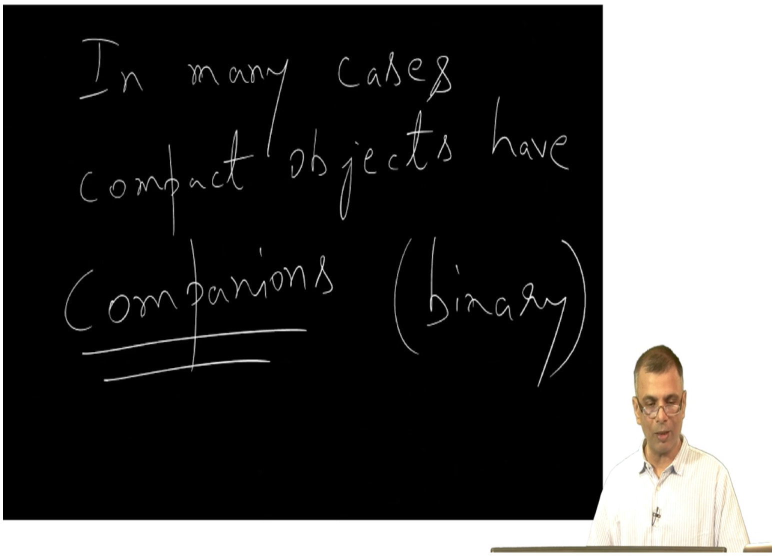

In many cases, compact objects come with, compact objects have companions. So, this would be what is called a binary system, two objects, ok. So, something like this you would have a compact object here, and then you would have a companion star.

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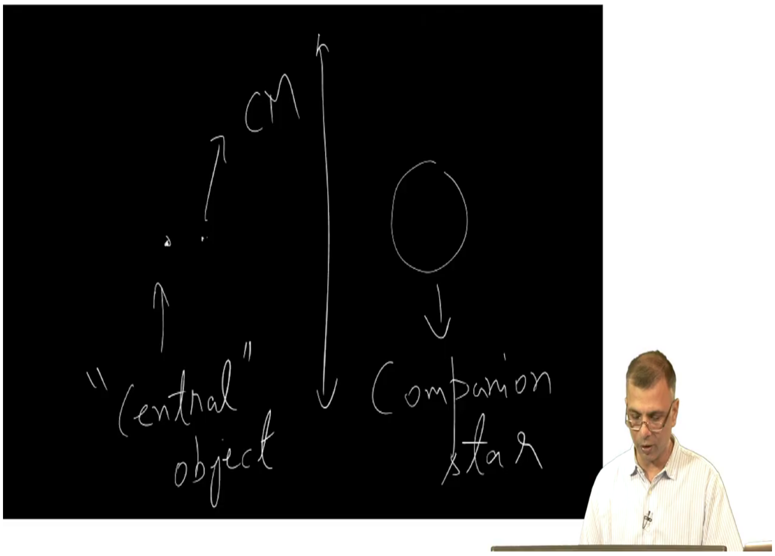

There is a reason I am drawing this compact object as just a dot. It might well be very very massive, but the actual physical extent is very small. So, this would be a very high density you know compact object, such as a black hole or a neutron star or something and it has a companion. So, this is a central object, in this case there is really no strictly speaking no real central object, but still in keeping with our previous terminology we would simply call this a central object and this is the companion star, right.

So, now what happens is these two will revolve around each other, right, these two will revolve around each other around their common center of mass. And since this is much heavier than this the common center of mass is very very much somewhere here, ok. The center of mass would be very much much closer to the central object to the compact central object than to the companion star, ok.

And in this case the mode of accretion is quite different. As you might well imagine you see now because of the mutual rotation, ok this is not spinning around its own axis, this is mutual rotation, rotation I know around this center of mass, there is a preferred axis, there is a preferred you know axis of rotation which would be along this direction, right. And therefore, there is a possibility of angular momentum transfer, ok. And so, this is what we will concentrate on.

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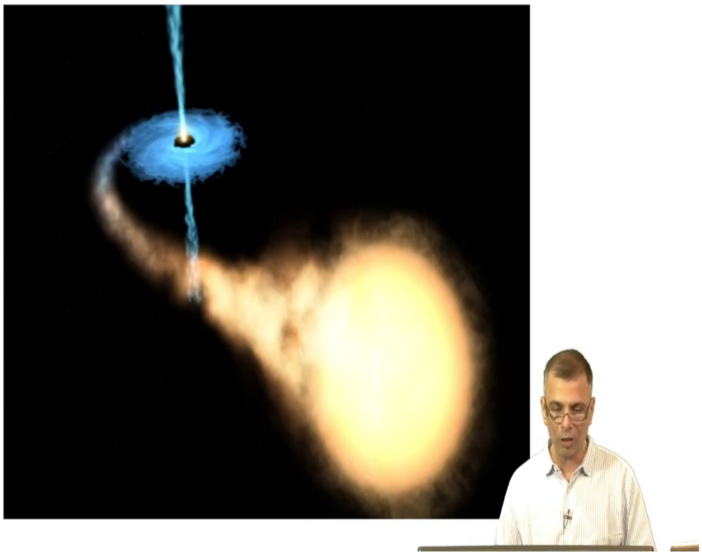

And this is the kind of you know cartoon that you normally see. I just want to caution that this is an artist's rendition. This is not a real astrophysical picture, ok. This is just a figment of imagination by an artist. So, this would be the companion star, this yellow thing, and that would be the compact object and that is at the very center, ok.

And here what is happening is matter from the companion star and so, these two are rotating around each other, ok around a common center of mass. And what is happening is matter from the companion star is stripped, is gradually stripped and it settles; it accretes onto the central object, but not in a.quasi-spherical manner, but in the form of a disk. You can see that this is idealized as a very thin disk, and hence the name accretion disk, ok. So, this is what we will be discussing.

And in addition to these accretion disks you also have these fantastic things called jets coming out. And so, we talked a little bit about jets some time ago. At the moment that is not what we were talking about, we are talking about the disk itself. We are talking about the fluid dynamics of the disk, ok.

How is you know how is disk accretion different from spherical accretion? And as you know you guessed it, you know at the end of the day our bottom line will be about the M dot, ok. Turns out that the M dot in a disk accretion geometry is substantially higher than that of the in a spherical accretion geometry and as you know we always want as high an M dot as possible, ok.

Of course, within the limits of the Eddington accretion limit, you cannot exceed the Eddington accretion limit otherwise you know matter simply cannot accrete anymore, matter simply cannot accrete at a rate that is larger than the Eddington rate; that is that that always you know. And of course, the concept of an Eddington accretion rate is predicated on a spherical accretion, but nonetheless it is a useful number to employ even for disk accretion, ok.

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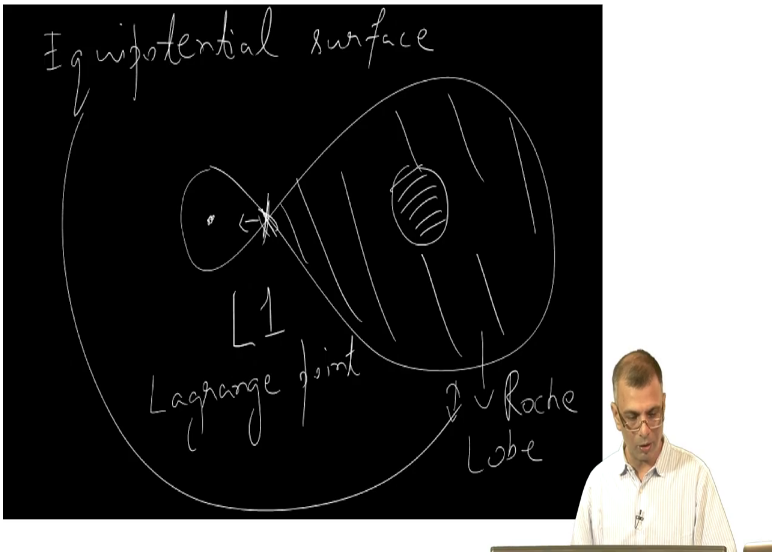

Now, a few things about disk accretion before we proceed. If you have a large object and a small object, the equipotential surfaces, you know where the equipotential surface as the name implies, in order to do this its best to climb into the rotating frame, ok into the frame of rotation.

And once you climb into the rotating frame as you know the rotating frame is a non-inertial frame. It is a frame where you know by virtue of the fact that the frame itself is rotating you will have a new force which is called the Coriolis force. It is not a real force in the sense that it disappears when you change frames.

But nonetheless, it is useful to climb into the rotating frame and draw these surfaces the equipotential surfaces, equipotential surfaces are simply surfaces where the total gravitational potential due to this object and that object is constant, ok.

Now, if you draw these equipotential surfaces, there are many such surfaces, but the inner most equipotential surface is what is of interest to us and it looks somewhat like this. It looks like a teardrop, ok. Now, it is the companion star and that is the central object right here.

So, this would be an equipotential surface, one of these. And this point here is called the L1 the Lagrange point, the first Lagrange. There are many other Lagrange points, turns out that there are there are about 5 Lagrange points for a binary system, but we are not concerned with all of those. We are only concerned with this Lagrange point. This is called the first Lagrange point.

And what is the significance of this Lagrange point? Well, it is a point where you know the gravitational potential due to this guy and that guy almost cancel, strictly speaking they cancel exactly, ok. And this turns out that there is a stable Lagrange point, in that if there was an object sitting here and an object sitting here would tend to just keep sitting there because the gravitational forces due to the two cancel each other, it will not be attracted towards this guy or that guy, ok.

It is a competing point. And turns out that the L1 Lagrange point is a relatively stable Lagrange point, you perturb you know an object sitting here at the L1 Lagrange point slightly it returns to the to that point, ok.

So, this is for instance satellites you know you can, suppose this was the earth and that was the sun and so, the L1 Lagrange point of course is much closer to the sun and satellites for the earth, earth orbiting satellites often like to be parked at the L1 Lagrange point. L1 Lagrange point for the earth for the earth sun system is about 1.6 million kilometers above the earth, ok.

So, that is pretty far away. These are not low earth orbiting satellites, low earth orbiting satellites orbit at about 800 kilometers, where 800 to 1000 kilometers. Whereas, the L1 Lagrange point is 1.6 million kilometers above the earth. But it is worth it. It is worth, it is worth sending satellites out there if you for instance you want to observe the sun all the time, ok.

You are not as in uninterruptedly, ok. So, that you do not have to be worried about you know day and night circles, ok. But it is much more expensive of course, to send a satellite to the L1 Lagrange point.

Anyway I just wanted to you know sort of bring that into focus that is for the earth's sun system and here that is not what we are talking about. We are talking about a black hole and a companion star. And so, what is the significance of the L1 Lagrange point? Why did we start talking about that in this context? The significance is that this is what is called, this region is what is called the Roche Lobe for the companion star.

So, matter is essentially due to the gravity of the of the compact object, matter is essentially you know attracted from the companion star until it fills up its Roche Lobe, ok. And through the L1 Lagrange point matter can kind of shootout, it is almost as if it is being squirted from the L1 Lagrange point, ok and it is this L1 Lagrange point of course is you know it is not constant.

So, so this in in this cartoon the L1 Lagrange point would be somewhere here, ok. And so, therefore, since the matter already has an intrinsic angular momentum when it reaches the L1 Lagrange point, it cannot simply accrete in a quasi-spherical manner, ok.

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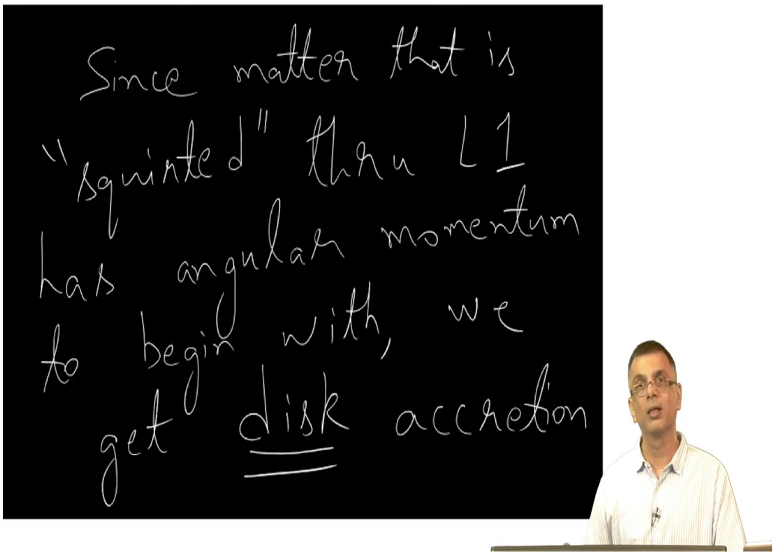

So, since matter or gas that is squirted, I want to use this word squirted because it is its very much like squirting through you know some kind of a, it is really as if you are squirting through some kind of a nozzle, ok. Through has angular momentum to begin with angular momentum you can think of it as angular momentum or you can say that the matter that is being squirted is already rotating and therefore, it has angular momentum, right.

We get disk accretion as opposed to you got it.quasi-spherical accretion that is why you form accretion disks like this because the matter that is being fed has an intrinsic angular momentum, ok. So, this is the reason you get accretion disks.

Often people argue that it is also true that the central object itself is rotating on its own axis. Yes it often is. But as we saw as far as quasi-spherical accretion goes the rotation of the central object makes very little difference, ok. Really the reason one forms accretion disks is because you have you know a binary system where you have a very compact object along with a companion star and these two objects are rotating around each other.

And you know matter is stripped from the companion star by what is called a Roche Lobe overflow, this Roche Lobe kind of overflows through the L1 Lagrange point to squirt matter off, to squirt matter onto the central object. And the matter that is being squirted is already rotating, ok.

It is already rotating, and so, it cannot, so it has a preferred axis of rotation. And sometimes that preferred axis of rotation can align with if the you know central object is rotating sometimes, the axis of rotation of the accretion disk aligns with the rotation axis of the central object. This is assuming that the central object is rotating and sometimes not, ok.

If the alignment is not proper you can have torques, you can have torques between the disk and the central object. And this is really fascinating stuff and so, sometimes the disk can be tilted and warped and so on so forth, there can be torques on the disk. But this is all advanced stuff. All of this is advanced stuff. We do not want to get into get into that at the moment. All we really want focus on is the geometry of an accretion disk, ok.

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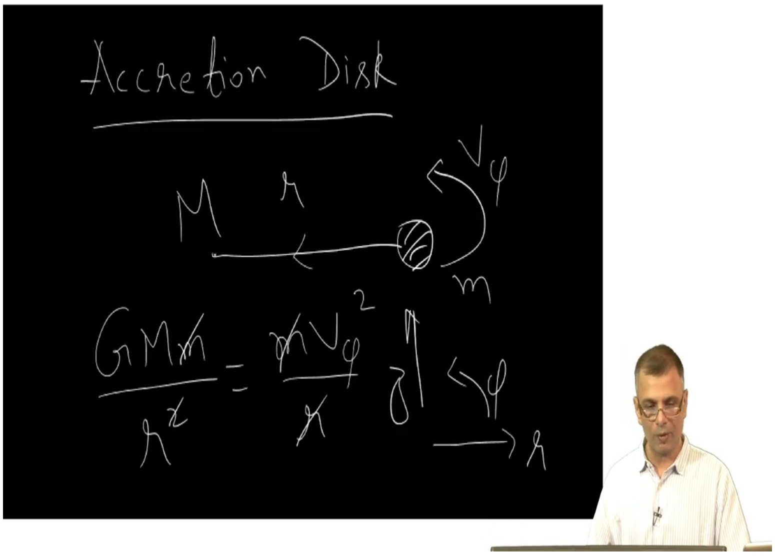

So, we will discuss an accretion disk in a minute. Now, the first thing I want to you know introduce to you is that you are already familiar with the manner in which objects rotate around a central object. Consider for instance you are holding a string and you have a stone whirling around your head, right. So, you know in this case what happens is that consider two objects point objects, ok and so, you have the gravitational attraction which goes as, so you have a central object and you have a smaller object.

Now, consider the gravitational attraction, right, and so, that would be some sort of GM M over r square where this is r, right. And if there is perfect balance between these two forces you would have and so, this would be an azimuthal velocity like a V phi. So, you would have an m V phi squared over r, right. So, this is what is called a Keplerian orbit, ok a perfectly Keplerian orbit.

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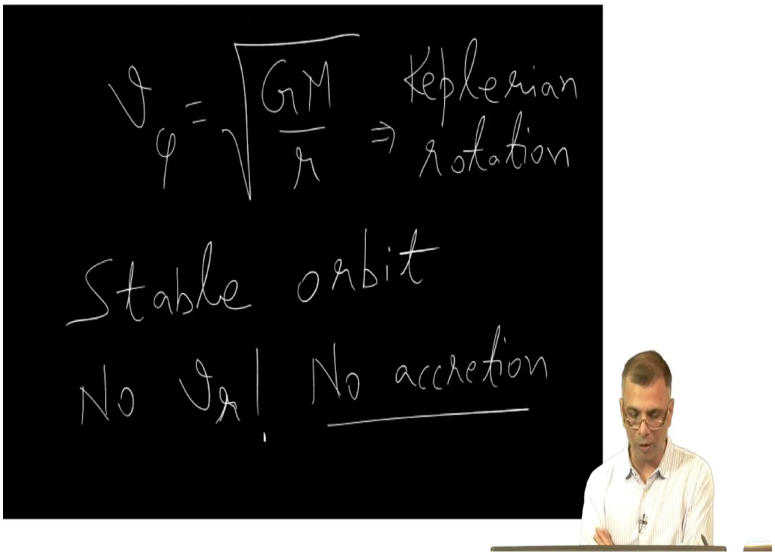

So, if this is the case, in that case the V phi is goes as square root of GM over r you agree that. You see the small m is cancel, and here one r cancels. So, all that is required is V phi squared equals GM over r, in other words the V phi the azimuth of velocity goes as square root of GM over r, ok.

And this is what is called Keplerian rotation. And this is our first, this is our first shall we say point of contact with an accretion disk, ok. So, you have an object, in Keplerian rotation around a central object with rotation velocity this, ok. And there is no other component of velocity there is only V sub phi, ok.

Now, turns out that an object in Keplerian rotation simply keeps it is in a stable orbit, ok, so this is a stable orbit. It simply keeps rotating over and over. And what is a accretion after all? An accretion is refers to a situation where this this fellow wants to accrete, it wants to settle down to the central object, ok. And so, that would imply then in cylindrical coordinates you would have as for instance in cylindrical coordinates if you have an r, phi, and z, it would imply you know a minus r directed velocity, right

So, in fact, this is not quite I should not have drawn it like this, I should have drawn the r like this, ok. So, it would imply V r, ok, but as we see a Keplerian orbit is a stable orbit, right, so there is no there is no V r, ok. So, no accretion. But of course, we want to study accretion, but this is our first jumping of point, ok.

It is important to be familiar with the you know concept of Keplerian orbits before studying disk accretion because you know an accretion disk essentially comprises of I think of an infinitesimally thin accretion disk.

And an accretion disk essentially would comprise lots of gas parcels in almost Keplerian orbits, orbiting around the central object, ok. But I just wanted to emphasize that a gas parcel in a perfectly Keplerian orbit around the central object is stable, ok. It does not sink into the central object. There is no radial component of velocity and therefore, there is no accretion, ok.

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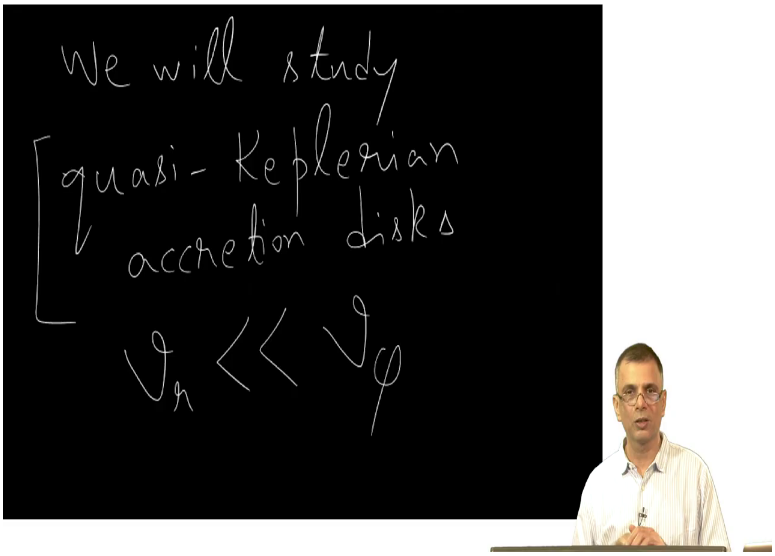

So, henceforth what we will do? We will study not quite Keplerian accretion because that is of no interest, ok that not quite Keplerian exactly Keplerian accretion disks, but we will study quasi-Keplerian accretion disks. And what is quasi about the quasi-Keplerian? These are disks where there is a certain inward component, there is a radial component of velocity, yeah, but it is much smaller than the azimuthal component.

So, you have a disk which is mostly Keplerian, ok. So, it is mostly rotating this way, but there is also a very small component of velocity inward component of velocity. So, you have gas that is slowly sinking in as it is rotating. So, this is what we will study from now on quasi Keplerian accretion disks.

And we will in particular, we will talk about thin disks and so on and so forth. This will be our idealized model of accretion disks and then as with spherical accretion we will we will sort of point out the deficiencies of this idealized model and try to see where additional physics can be brought in and so on so forth. So, that is it for the time being.

Thank you.