

Fluid Dynamics for Astrophysics
Prof. Prasad Subramanian
Department of Physics
Indian Institute of Science Education and Research, Pune

Lecture - 28
Shock thickness

(Refer Slide Time: 00:16)

Intersecting characteristics



- Forward and reverse characteristics intersect to yield a unique solution (as for the weak waves)
- But in this case, since the c_s from the peak of the pulse can be $>$ the c_s of a point *ahead* of it.
- So two *forward* characteristics can intersect
- indicating that the solution can be double valued \rightarrow discontinuity/shock
- Refer to Lax's article for a more general discussion

Mathematical discontinuities



Subramanian Fluid Dynamics

So, we stopped here. We started talking about intersecting characteristics and especially the fact that for non small. In other words for large disturbances the main difference the main interesting thing is that two forward characteristics are for that matter two reverse characteristics can intersect.

So, either way if two forward characteristics and two reverse characteristics if two like characteristics intersect then you have the possibility of a mathematical discontinuity ok, which is called the shock. So, we will consider shocks in a minute, but remember we are now

starting to talk about mathematical sorry discontinuities. Ideal I should say; mathematical or ideal discontinuities.

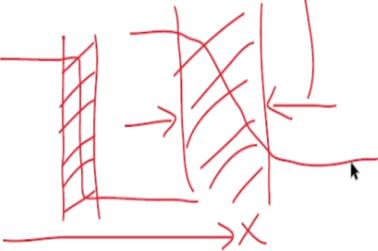
And an ideal discontinuity as you know in any say you are talking about say this is the X direction and you are talking about a discontinuity in pressure. So, let this be pressure. An ideal discontinuity in pressure would look like this right. Right here at this point the pressure is double valued. You do not know whether the pressure is this or this. It is double value. There is a jump and that jump has zero thickness right. It is a infinitely shock discontinuity.

(Refer Slide Time: 01:57)

Shock thickness - I

what is the thickness?

- Supersonic motion does allow for mathematically ideal discontinuities in the flow (such as shocks), but
- surely, there are no ideal (i.e., zero thickness) discontinuities in real life?



Subramanian Fluid Dynamics

So, supersonic motion does allow for mathematically ideal discontinuities in the flow such as shocks. And these are the not the only discontinuities. We will talk about another kind of discontinuity in a minute. Well, in a while I should say. After we finish talking about a few other things we will also talk about another kind of discontinuity.

But, the key thing is that so far we have been talking about mathematically ideal discontinuities ok. But, surely there are no ideal zero thickness discontinuities in real life. Surely, I mean this kind of a pressure jump or a density jump ok if this is X for instance this is not realistic.

In real life there is always a finite thickness. In other words, in real life this discontinuity surely will look like this. I mean this is the thickness we are talking about roughly. If you zoom in here it really surely it looks like this right. So, if you extend the scale it might look like a very sharp discontinuity, but if you zoom in surely the discontinuity is a little more.

So, what is the thickness? The question is how wide is the discontinuity? It is a very important question ok, very important practical question. Not so important from a formal mathematical point of view ok, but from a practical point of view it is a very important question.

So, what is the thickness? It is certainly it might well be zero from a mathematical point of view. But, from a practical everyday life point of view when you actually you know shocks have actually been observed in the lab and also in near earth outer space ok, spacecraft have traversed through shocks. They find sudden jumps, but they find that the jumps do not happen instantaneously. They are a little gradual.

So, exactly how much is the thickness and what is the thickness decided by what physical parameters decide the thickness? Turns out that this is a fairly complicated question to be fair, but we can understand this from certain basic considerations. We can understand this using stuff that we have done so far ok, alright.

(Refer Slide Time: 04:38)

Shock thickness - I



- Supersonic motion does allow for mathematically ideal discontinuities in the flow (such as shocks), but
- surely, there are no ideal (i.e., zero thickness) discontinuities in real life?
- At the shock, the inertial and viscous forces \approx balance each other (why?)



Subramanian Fluid Dynamics

Now, the main thing to realize is that at the shock the inertial and viscous forces roughly balance each other. And I urge you to think why? Remember so far in our entire discussion of shocks we have been neglecting viscous forces. We have been considering only the Euler equation. We have been neglecting the part of the Navier-Stokes equation that includes the new the viscosity. We have been neglecting it.

In other words, what this implies is that far away from the shock ok viscous forces are not important. So, this is a very high Reynolds number flow. So, there is a very high Reynolds number situation ok. Now, turns out that near the discontinuity viscous forces do start or viscous effects do start becoming important. They might not be important far away from the shock, but viscous forces even though the new itself might be very small.

Remember in the momentum equation it appears in this combination. This is the combination in which the viscous term appears right. And what is happening near the shock? Well, the shock is a large jump. Mathematically, it is a large jump. There is a large jump. This is jump in pressure, density, velocity, everything. In this particular case let us talk about velocity.

So, the jump occurred. There is a large jump here and that occurs over a very small thickness a very very small thickness. In other words the dv/dx is already quite large because the dx is very small right. So, dv/dx is very large and d^2v/dx^2 , which is what this represents is even larger.

So, therefore, even though ν might be small and as long as there are no large jumps or whatever in the flow as long as the flow is smooth this term might not be important. But at the shock very near the shock the viscous term starts acquiring importance ok. And I do not want to say exactly at the shock, but, let us.

So, it starts acquiring importance let us look at the point where the viscous term exactly balances the inertial term ok. Inside the thickness of the shock itself, the viscous term might well overwhelm the inertial term ok. And by the time you are done with the discontinuity it again goes away and the inertial term starts dominating again.

But, let us now start I mean since we this is just an order of magnitude kind of illustration, let us now look at the situation where the viscous term just about starts balancing the inertial term.

(Refer Slide Time: 07:50)

Shock thickness - I

- Supersonic motion does allow for mathematically ideal discontinuities in the flow (such as shocks), but
- surely, there are no ideal (i.e., zero thickness) discontinuities in real life?
- At the shock, the inertial and viscous forces \approx balance each other (why?)
- i.e., $\rho u du/dx \approx \nu d^2u/dx^2$

Inertial term

Viscous term



Subramanian Fluid Dynamics

In other words this so, this is the viscous term right and this is the inertial term. So, let us look at the situation where they are just about balancing each other like that right.

(Refer Slide Time: 08:19)

Shock thickness - I

$$\frac{\rho u}{dx^2} \approx \frac{u}{\Delta^2}$$

- Supersonic motion does allow for mathematically ideal discontinuities in the flow (such as shocks), but
- surely, there are no ideal (i.e., zero thickness) discontinuities in real life?
- At the shock, the inertial and viscous forces \approx balance each other (why?)
- i.e., $\rho u du/dx \approx \nu d^2 u/dx^2$ (verify!)

$$\frac{du}{dx} \approx \frac{u}{\Delta} \rightarrow \text{shock "thickness"}$$

Subramanian Fluid Dynamics



Viscous term just about balances the inertial term. And of course, the nabla square u is just replaced by $d^2 u/dx^2$. And you know $\rho u u \cdot \text{grad}$ I mean in the momentum equation it is this is essentially the one dimensional analog of that yeah. So, now let us do the usual thing. Roughly speaking let us replace du/dx by something like u/Δ , where this Δ is the shock thickness so to speak right.

And let us say this. So, by the same token this u/Δ^2 is very similar to what we used to do earlier. We used to write an l here remember, but that l was a microscopic quantity. Here we are really talking about a very small thickness and this Δ is that thickness ok. So, we replace this du/dx here by u/Δ capital Δ . And we replace this $d^2 u/dx^2$ by u/Δ^2 capital Δ ok and this is the important thing.

(Refer Slide Time: 10:01)

Shock thickness - I

- Supersonic motion does allow for mathematically ideal discontinuities in the flow (such as shocks), but
- surely, there are no ideal (i.e., zero thickness) discontinuities in real life?
- At the shock, the inertial and viscous forces \approx balance each other (why?)
- i.e., $\rho u du/dx \approx \nu d^2 u/dx^2$ (verify!)
- In other words, the shock thickness $\Delta \approx \nu/u$

"Thickness" is decided by the viscosity!



Subramanian Fluid Dynamics

And once you do that we immediately find that the delta the shock thickness is nothing but ν over u ok. It all depends upon the viscosity. In other words, the shock thickness is decided by the viscosity is decided by the microphysical viscosity. So, what this is saying is that, the is decided by the viscosity that is what this statement is saying.

It is a very important statement. It is also decided by the bulk speed of course, ok. The faster the speed the smaller the shock thickness right ok. So, let us go a little farther ok. So, that is one thing the viscosity is a transport coefficient, it is a property of the fluid, but can we now relate this to the microphysics a little bit? Let us see.

(Refer Slide Time: 11:14)

Shock thickness - II



$\nu \sim \text{cm}^2 \text{s}^{-1}$

- $\nu \sim \lambda \langle u \rangle$, where λ is the particle mean free path and $\langle u \rangle$, the mean particle speed is

Handwritten annotations:
- A circle around 'cm' in 'cm² s⁻¹'
- An arrow from the circle points to 'length scale'
- An arrow from 's⁻¹' points to 'velocity'
- The text 'cm/s' is written below the circle and 'velocity' is written below 's⁻¹'

Subramanian Fluid Dynamics



Now, remember the quantity ν is something like a diffusion coefficient centimeter square per second. In other words, this is some kind of centimeter times centimeter per second right.

So, this is a velocity and this is a length scale. So, it is some kind of length scale times some kind of velocity, some kind of length scale times some kind of velocity. That is the microphysical or origin of the viscosity coefficient ν right. And the relevant length scale in this particular situation is the particle mean free path.

(Refer Slide Time: 12:12)

Shock thickness - II

- $\nu \sim \lambda \langle u \rangle$, where λ is the particle mean free path and $\langle u \rangle$, the mean particle speed is
- $\langle u \rangle \sim c_s$

$$u_{rms} \sim \sqrt{\frac{2}{3} \frac{k_B T}{m}}$$
$$c_s \sim \sqrt{\frac{\gamma P}{\rho}} \sim \sqrt{\frac{\gamma n k_B T}{n m}}$$



Subramanian Fluid Dynamics

And the relevant velocity is the mean free speed, the mean particle speed. Mind you we are now we are now delving into the microphysics. This average u no longer represents the bulk fluid flow. This represents the mean speed of the particles that constitute the flow. In other words, we are not looking at the fuzzy picture or the fuzzy fluid picture anymore.

We are taking a lens and looking at an element of the fluid and trying to see what the fluid element comprises of. And now because we are holding a lens we are able to see the little the movement of the you know constituents of a fluid parcel, right.

Say molecules for instance and we are able to figure out that these molecules are whizzing about at a mean particle speed average u and the mean free path of this whizzing about is something like λ ok, alright. So, this is important. The other very important thing to

realize is this. The sound speed is nothing, but the mean particle speed to within a factor of unity.

Why is that? Well, you know what is rms speed right? Some u_{rms} you remember this? u_{rms} is something like $k_B T$, is not it? $3/2$ or $1/2$ depending upon the details and what is $c_{sub s}$? $c_{sub s}$ is after all some kind of γP over ρ ok. And once you write this as $n k_B T$ right, so, this is something like P over ρ which is some kind of $n k_B T$ over $n m_p$, right.

So, the n 's cancel out right and you have something like $k_B T$ here in the $c_{sub s}^2$ sorry and here too it should be u_{rms}^2 ok. So, both of these are something like $k_B T$, alright. So, this is one very important realization which essentially is saying that the mean particle speed is something like the sound speed not much of a difference between the two ok. And you can verify this we just showed how this.

(Refer Slide Time: 14:54)

Shock thickness - II

- $\nu \sim \lambda \langle u \rangle$, where λ is the particle mean free path and $\langle u \rangle$, the mean particle speed is
- $\langle u \rangle \sim c_s$ (verify!)
- Since the shock speed u is also of the order of c_s

$M \gtrsim 1$



Subramanian Fluid Dynamics

If that is the case and the shock speed u and now this u is different from this u , this is an average u and this is the bulk speed u . The shock speed is also the order of c_s . Why is that? Because in other words, the Mach number is it is greater than 1, but not that much larger than 1.

And in this spirit where we are neglecting quantities of order of unity whether its 2 or 3 or 5, it does not really matter ok. We are not talking about Mach numbers of hundreds or thousands or things like that. We are talking about mildly supersonic flows ok.

(Refer Slide Time: 15:45)

Shock thickness - II

Massy (many!) mfp within a fluid parcel

- $\nu \sim \lambda \langle u \rangle$, where λ is the particle mean free path and $\langle u \rangle$, the mean particle speed is
- $\langle u \rangle \sim c_s$ (verify!)
- Since the shock speed u is also of the order of c_s ,
- we find that the shock thickness $\Delta \approx \nu/u \approx \lambda!$
- The shock thickness is decided by viscous effects, and is around the particle mean free path
- There can be additional (interesting) complications when the gas is collisionless

$\lambda \rightarrow$ particle collisions can be $\sim L$



Subramanian Fluid Dynamics

So, that is why the u is also of the order of c sub s , which means that this quantity we said that the shock thickness was something like ν over u right. And we said that the ν is something like λ average u , which is something like λc sub s and this bottom u is also the order of c sub s . So, the two c sub s is cancel and we find that the shock thickness is roughly the mean free path. This is a very important result ok.

So, the sharp thickness essentially is approximately the mean free particle mean free path. So, the shock thickness is decided by viscous effects which is essentially saying. This is decided by viscous effects and its around the particle mean free path ok. Remember we are now mixing up the fluid picture and the particle picture ok.

This is I mean although we said very decidedly in the very beginning that you know when you start talking about fluid mechanics a fluid parcel is comprised of many many particles a

hundred or thousand or definitely many many more. So that the individual the trajectories of the individual particles and everything are sort of papered or smoothed over yeah.

And you remove this lens and you cannot see the individual particles whizzing around anymore and so. So, in that picture of course, and that is really the fluid picture and we are now you know starting to talk about the possibilities of discontinuities in macroscopic fluid quantities such as pressure, density, velocity and so on so forth ok. But, now, were starting to ask the question well ok. So these are idealized discontinuities, but surely in real life there is no such thing as an ideal discontinuity.

There is no such thing as a zero thickness discontinuity. We are asking the question what is the thickness of this discontinuity. If we want to answer that question we have to bring back the lens and we have to start looking at the fluid parcels yeah. And we have to start mixing up our understanding of particle effects and fluid effects and that is what we have done here. Specifically, we have to look into the microphysics of the viscosity ok.

Because we said that this we figured out that the shock thickness is decided by viscous effects. We already did that in the last slide and we furthermore we ask well what is the microphysics of the viscosity. If we look into that then we find that the shock thickness is roughly λ , where λ is the mean free path.

So, we are indeed mixing up the fluid and particle pictures a little bit, but we have to do that if we want to a investigate a small scale structure such as the shock thickness ok. Now, having said that we have to also say that there can be additional interesting complications, when the gas is collisionless.

What you mean by collisionless is I should elaborate that a little bit. What do you mean by collisionless? This entire discussion presumes that there are many many collisions there are many many mean free paths many mean free paths within a fluid parcel. That is what this entire discussion presumes. In other words, what is a mean free path? A mean free path is nothing, but λ .

The lambda is really small. There are lots and lots of lambdas within a fluid parcel that is what the entire fluid assumption is based on ok. But, this might not always be true. The lambda, the mean free path might occasionally be quite large in comparison to a macroscopic length scale. In other words, lambda can be that is what; this entire thing refers to this statement, where there are many many mean free path within a fluid parcel.

And in this situation the gas can be legitimately considered as collision collisional and this is the regular picture. This is what happens in everyday situations. Especially in astrophysics, you can run into situations where this assumption is violated and the gas is occasionally collisionless. What we mean by that is that the mean free path can be comparable to the macroscopic length scale.

Now, I know what you are thinking. You will immediately say that if that is the case then the fluid assumption does not simply breaks down. The reason I am saying all this is because you know this entire thing the shock thickness being decided by the mean free path. If you naively apply that to a collisionless situation then you will find that the shock thickness is huge because the mean free path is large in comparison to a macroscopic length scale.

So, you would expect that the shock thickness is huge in this kind of a collisionless situation. But, when you actually I mean in some astrophysical situation you will find that I mean even in a collisionless situation the thickness of the shock is actually quite small. It is not comparable to a microscopic length scale ok.

Now, first thing you might that is the reason I brought broad up this point. But, you might also think well if this is the case if the mean free path is comparable to macroscopic scale length. Then the entire fluid assumption breaks down and we should not be talking about fluid dynamics anymore.

We should really be investigating the trajectory of individual constituents of the fluids such as molecules or atoms or as a case might be protons and electrons. And whatever the individual

constituents are and we should not be applying the fluid picture at all ok. We should be following the individual particles around; yes and no ok.

The problem is you know from a practical point of view from a computational point of view following individual particles around is very very computationally expensive right. And you might ask well give me a practical example of a collisionless fluid. Well, the solar wind is an astrophysical example of a collisionless fluid. Why do I call it collisionless?

You know the distance between the sun and the earth is 1 astronomical unit. And within 1 astronomical unit an average solar wind proton ok, which is a solar wind you can consider it to be something like a million degrees hot an average proton. An average proton undergoes three collisions between the sun to the earth, ok.

Now, the distance between the sun to the earth is a representative macroscopic scale length you will agree with me half the distance or maybe one-fifth the distance not one-hundredth the distance. Would say half the distance or one-fifth the distance between the sun to the earth. Would be a representative you know macroscopic scale length and by the time this given solar wind parcel travels from the sun to the earth.

Roughly speaking, an average solar wind proton undergoes three collisions not many many collisions were just three and this is a situation where the mean free path is quite comparable right to the macroscopic scale length. So, this is an example of a collisionless fluid. And guess what? People have been applying fluid equations to the solar wind disregarding this very important thing.

And they have been coming up with pretty good put with results that match observations fairly well. So, what gives? There is something here. There is some funny thing here, which somehow enables the application of fluid equations to a situation to a collisionless situation, where the fluid equations really should not be applicable. So, what gives? The thing is I really should be writing λ not really λ , but λc , where λc refers to particle-particle collisions ok

So, this λ_c which is the mean free path governing particle-particle collisions right might well be comparable to the macroscopic scale length, but there can be other collisions ok. In other words the solar wind is turbulent and there are other kinds of scattering centers and the protons might well be colliding with each other in a particle-particle way very very infrequently ok.

But, they might be colliding much more frequently with these other scattering centers with these turbulent scattering centers, ok. So, the collisions of protons with turbulent scattering centers are much more frequent than the collisions with protons with each other ok.

So, in other words these turbulent scattering centers lend the fact that the turbulent scattering centers that present lends a fluid like character to the otherwise collisionless solar wind. So, this is the thing. So, therefore, when you are talking about shock thickness and things like that you will be talking about λ_c alright, but you should not be considering the particle-particle a mean free path which is huge.

You really should be considering an effective mean free path which gives you the average distance for collision between a solar wind proton and a turbulent scattering center, magnetic irregularity for instance. So, that turns out to be much smaller than λ_c which is of the order of main a macroscopically large distance and therefore, the shock thickness is really decided.

See essentially you are talking about a situation where the viscosity is not due to particle-particle collisions, but due to particle something else collisions.

And that something else can be a magnetic scattering center and that is what decides shock thickness. So, this is of course, you know an advanced topic which is why I said there can be additional interesting complications when the gases collision less. So, with that we will stop here and we will return to a discussion of idealized discontinuities when we meet next.

Thank you.