Fluid Dynamics for Astrophysics
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Module – 02
Dynamic similarity, dimensionless numbers
Lecture – 20
Reynolds number and dynamic similarity

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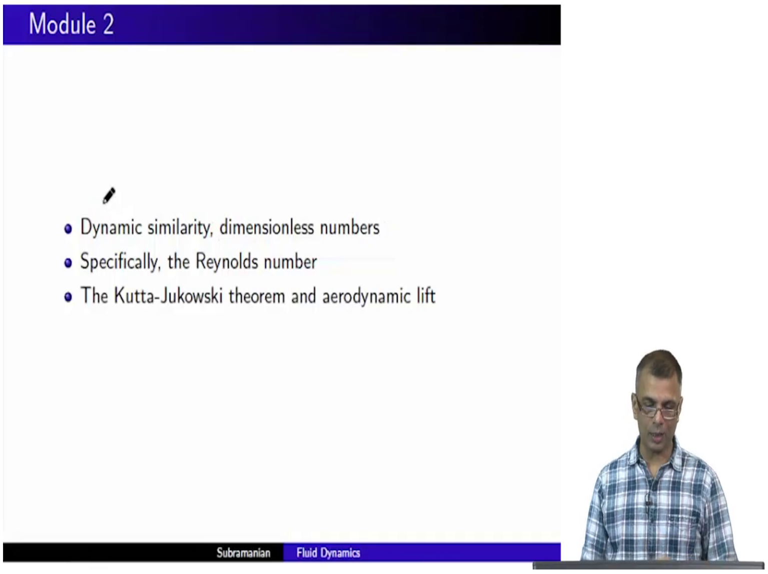

Hi. So, we are back and we are we will resume our discussion of dimensionless numbers and in fluid dynamics. We went through a quick survey of several of these when we met last and today, what we will do now is special attention just to the Reynolds number. Now, for two reasons, you will see we will encounter something called Dynamic similarity, which will illustrate the special importance of the Reynolds number, that is one thing.

Secondly, the Reynolds number is something that is the Reynolds number and the magnetic Reynolds number which we will not discuss today. These two are things that are especially important in astrophysics and so hence, you know my motivation in doing this. So, we will do the Reynolds number and we will introduce, we will try to understand what dynamic similarity is all about.

And in the same breath, having discussed dynamic similarity, we will also revisit the problem of lift on an aircraft wing that we discussed briefly you know some time ago. It is just a very interesting application of the dynamic similarity idea and we will revisit problem of lift. Specifically, we will outline a simple treatment of what is called the Kutta-Jukowski theorem. So without further I do, let us launch right into it.

So, we are talking about dynamic similarity, dimensionless numbers. We have already talked about this a little bit earlier dimensionless numbers; we have not yet talked about dynamic similarity; we will do that. Specifically, we will focus on the Reynolds number, a bit with a slightly different focus and like I said, we will also talk about the Kutta-Jukowski theorem and aerodynamic lift right.

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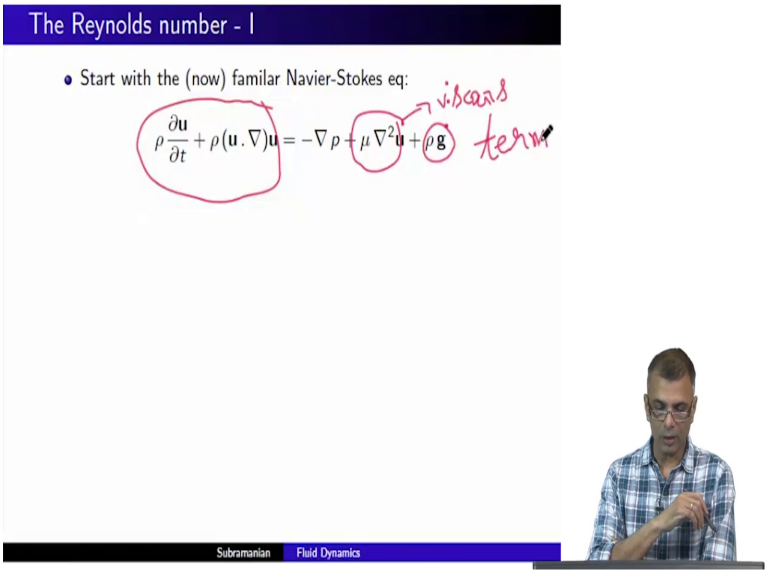

So, we will start with the now familiar Navier Stokes equation, what we were doing yesterday. Now, remember, this entire thing this is essentially m a and these are the different components of f and this is the f due to the pressure gradient, this is the f due to body forces and this is essentially the viscous term right.

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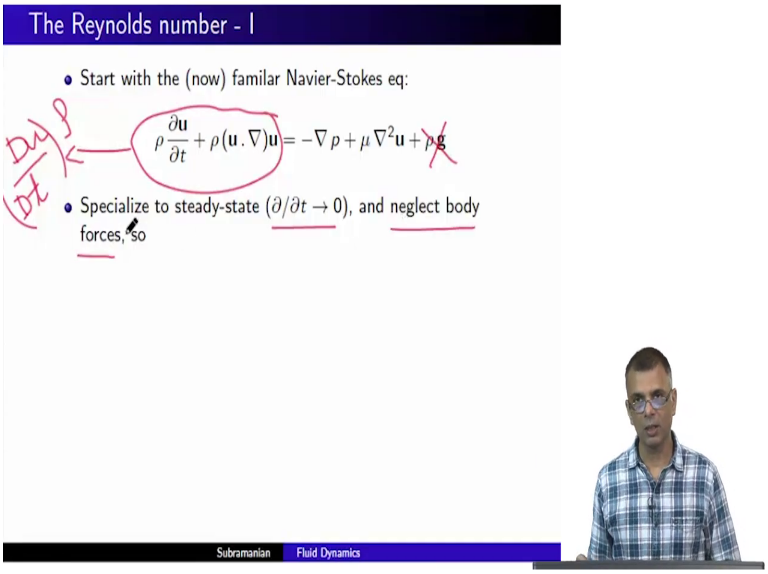

Yes and if you remember, the last time we did this; we were not writing it in this form, we were writing this entire thing in the Lagrangian form. We were simply saying like that, times rho of course, we were multiplying this whole thing by rho. That is what we are doing. In this case, we have written it out in Eulerian form and it should not be a big deal for you, you should be able to switch between one from the other right.

So, now, having written it this way, we will now specialize to there is a there is an advantage in writing it this way. We can specialize to steady state situations which is to say the partial d partial t is equal to 0. In other words, for an observer, who is the lab frame ok, he or she does not see an explicit time dependence and the other thing is we will neglect body forces; we will neglect body forces. We will not bother about you know just for simplicity.

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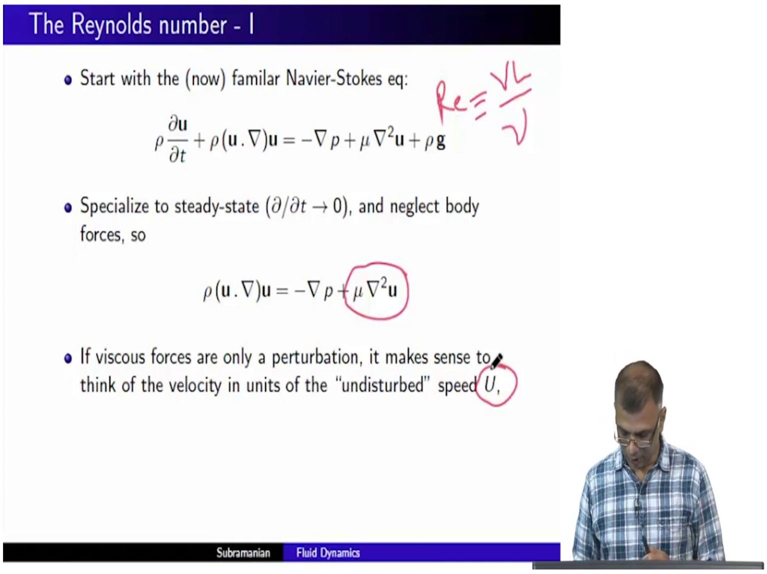

Having done that, so we are only left with so what are the terms we are left with? We are left with this, we are left with this and we are left with this and so, that is what this equation is all about right. Again, this is really just an f equals m a equation; in just a slightly specialized version of the Navier Stokes equation right.

Ok. So, now, this is a very important step. This marks a slight departure I mean it all boils down to the same thing at the end of the day. But the philosophy is important and it marks slight departure from the way we were doing things. This has to do with our discussion of characteristic, remember when we were talking about Reynolds numbers yesterday, we were talking about we were defining Re as V times L over nu. I beg your pardon for the slight change in notation from now on.

Today, when we were discussing, we will be replacing this capital V by capital U ok. So, this is something that you need to keep in mind. And so, when we discuss you know Reynolds number during the last session, we this is of course, the coefficient of viscosity; but the V was some characteristic velocity and the L was some characteristic length right.

Here, what we are seeing here is that if the viscous we consider a situation, where the viscosity is not; probably this is the, this is the better way to put it. At the back of our minds, we really should be thinking about a situation where the viscous term is only a perturbation. It is not the overwhelming, it is not the dominant term in the equation ok.

In other words, yeah, so this term is only a kind of a perturbation. Think of a situation, where you have say water moderately viscous fluid like a low viscosity fluid like water flowing around a sphere. So, you can imagine that you know there are any kind of sticking or anything viscous, viscous effects in other words happens only on the boundary on the surface of the sphere or the solid sphere.

Far away, the water flow is pretty much undisturbed, is not it? Far away from the surface, the water flow is pretty much undisturbed and water continues to flow as it were, as though the sphere was not at all there. And it is that is it is that velocity, where viscous forces or viscous effects are not important. Say far away from the boundary, that is the kind of you know velocity, we are talking about here as U.

So, it makes sense to think of the velocity in units. In other words, whatever velocity you see here, the small u, we normalize this with respect to a lot a capital U; where, the capital U is the velocity is sort of the undisturbed speed ok. If you want to think of a concrete situation, you can imagine this, where you have a low viscosity fluid flowing past a sphere and this capital U would be the speed of the fluid far away from the boundary of the sphere, sufficiently far away right.

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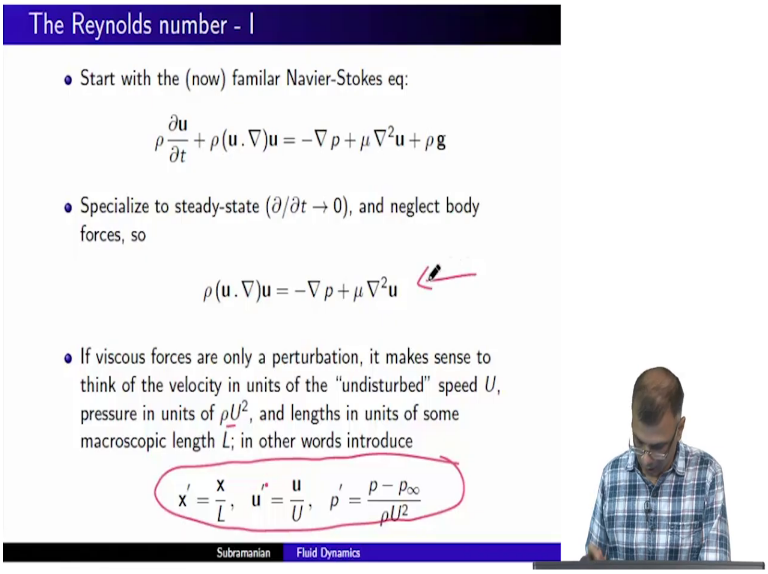

If that is the case, in other words, I am what I am saying is I am going to you know divide all; of course, divide both sides of the equation. So, the equation itself is undisturbed right. So, I am going to divide all velocities by U. In other words, I am going to consider velocities in units of this undisturbed speed U. Well, that is the case, then I can also consider pressure in units of rho U squared. You can verify that you know this as the units of pressure.

So, whatever wherever I see p, I divide by rho U squared and lengths in units of some macroscopic. This is the same, this is the same concept we talked about; in some macroscopic length L ok. For instance, if you had if you are thinking about flow past a sphere of diameter say 1 centimetre, the macroscopic length would be say 1 centimetre or 5 centimetres, certainly not 1 meter or certainly not 1 millimetre right. So, this is the spirit in which we define the macroscopic length L ok.

In other words, we introduce these dimension, these dimensionless variables x prime is x divided by L ok; u prime is u divided by this capital U, this undisturbed speed; p prime is well not quite p over rho u squared as this might have suggested, but p minus some pressure at infinity. Infinity meaning when you are sufficiently far away from the body, divided by rho U squared.

So, we are going to rewrite this entire equation in terms of these dimensions these prime variables which are dimensionless. You can you can check that these are dimensionless right. x and L both have units of say meter or centimetre as the case might be. So, x prime itself is dimensionless; same with u prime, u prime is dimensionless; p prime is dimensionless ok. So, this is dimensionless length.

Ok. I mean it is a bit of an oxymoron talking about dimensionless length, but length you know it is length, it denotes length; but it does not have the dimensions of length ok. Similarly, dimensionless velocity dimensionless pressure as elegant as these things might be we were really doing this for a certain purpose we will see. But you have to be very very careful while dealing with dimensionless numbers because there will not be any warning bells that go off if you make a mistake.

Many times, in an equation like this, if you are missing a factor of rho, if you are missing a factor of u or whatever you just look at the dimensions and you can you can figure it out; there is something wrong here and I need to set it right. When you are working with dimensionless numbers, there will not be any such warning bells ok.

So, you have got to be really really careful about your algebra. So, if I am going to introduce these dimensionless numbers, all of these, then and I urge you to do this yourself, then the steady state equation of motion which is just this.

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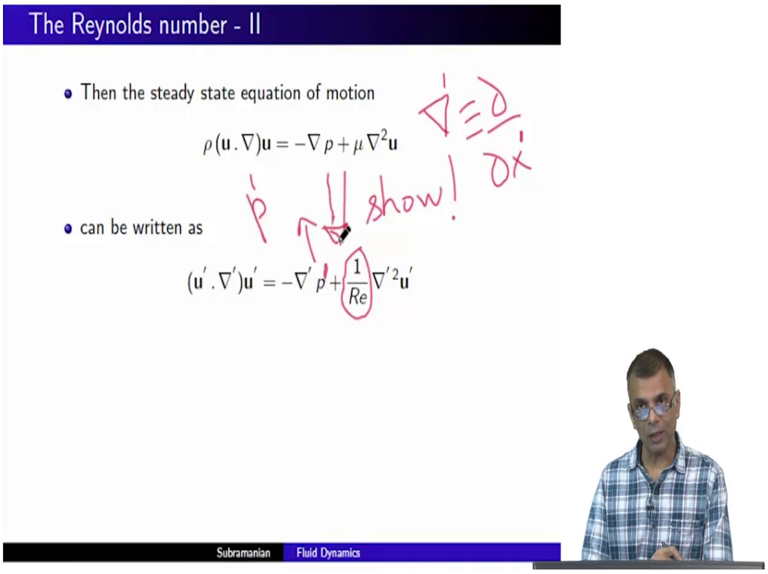

The steady state equation of motion again I reproduce this here, can be written as that and I urge you strongly to show this ok, where the definitions of u prime right, x prime which does not explicitly appear here and p prime and everything are there really should be a prime here; there should be a prime.

In other words, this is actually p prime ok. They are given as these are the definitions and what does this delta prime mean? The delta prime simply means and so forth ok, d over d x prime, d over d y prime so on so forth ok, that is what this delta prime means ok.

And I would like you to to you know to show this, to show how this comes from this. It is fairly simple algebra, but it is important. Now, the thing is when you derive this from here, there are some terms that are left over and all of those are bundled together in this term and you guess it.

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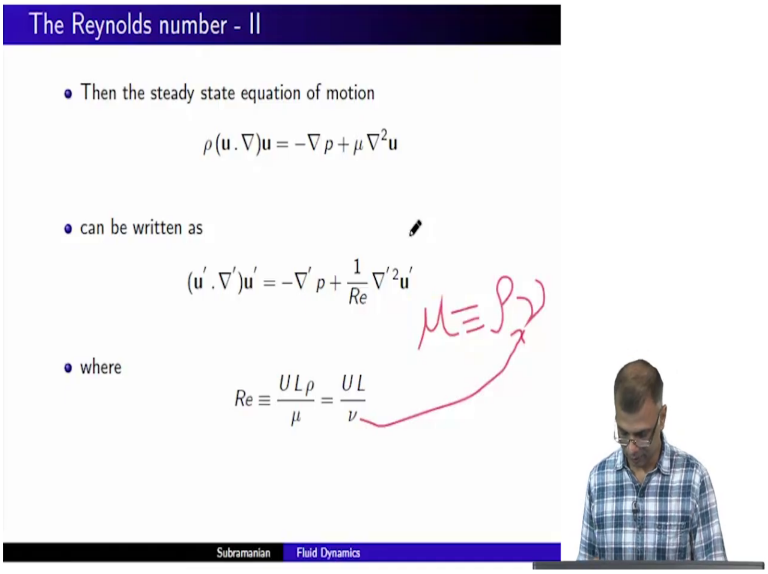

Here is where the Reynolds number Re makes an appearance; where Re is equal to and here, I am using the mu that we were always working with, the kind of viscosity coefficient that we were always working with before. But it is really the same as this because mu is nothing but rho times that ok, where this is what it is ok. So, it is the same definition that we had done earlier. So, it is essentially the ratio of terms that we discussed the last time we met.

Exactly the same definition, but this is how it comes about; it comes about in a slightly different way. I urge you to think about this in both different ways and familiarize yourself, make yourself comfortable. Because the reason I say this is because this version of the momentum equation or this version of the Navier Stokes equation will lead naturally to an understanding of the concept of dynamic similarity right.

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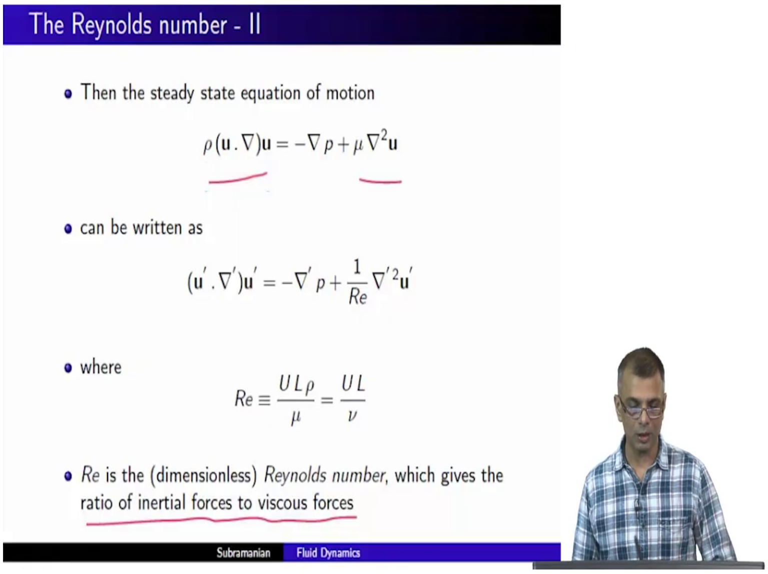

So, the Re is the dimensionless. You can verify that this is indeed dimensionless, you know nu has the dimensions of centimetre per second times centimetre. In other words, centimetres square per second and therefore, the Reynolds number is dimensionless. And as we discussed the last time we met, that there are the Reynolds number gives the ratio of the inertial forces to the viscous forces ok.

This is how we did it. I mean this particular statement is how we did it the last time we met. Today, we are simply saying that if when you non dimensionalized this equation using the prime variables, there are some terms left over which can be neatly bundled together into

Reynolds number. Two ways of looking at the same thing and both ways are important to understand ok.

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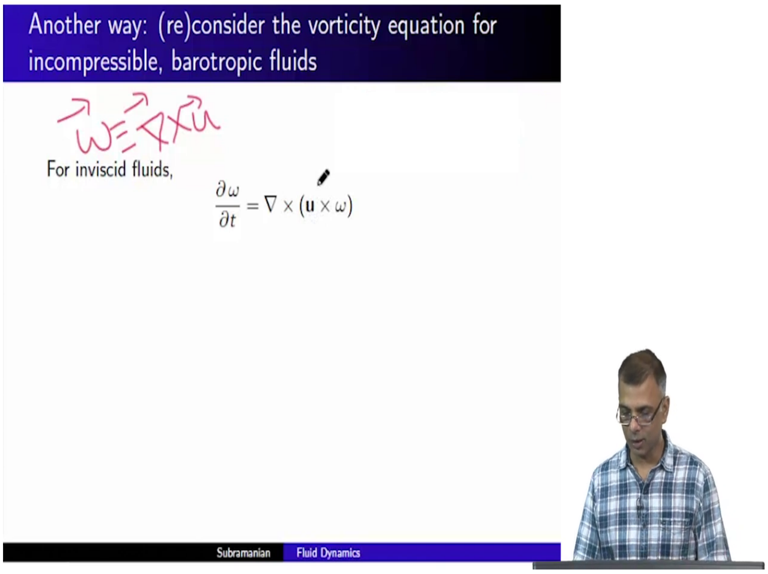

So, let us go forward right. So, this is one thing. This is about the momentum equation itself. Let us now go back to an equation, the vorticity equation that we discussed earlier. I am saying this just because you know just to make you comfortable with the fact that this non-dimensionalizing given by this kind of non-dimensionalizing is useful in different situations ok. So, remember the vorticity equation was essentially a dynamic equation for u ok.

You specify the u at a given time and this gives the evolution strictly speaking of omega, which is essentially omega which is the curl of u. If you recall, we remarked that this is a, this is a completely fully specified dynamical equation for the evolution of omega. Sure enough, there is a u there, but the u is uncurl of omega right.

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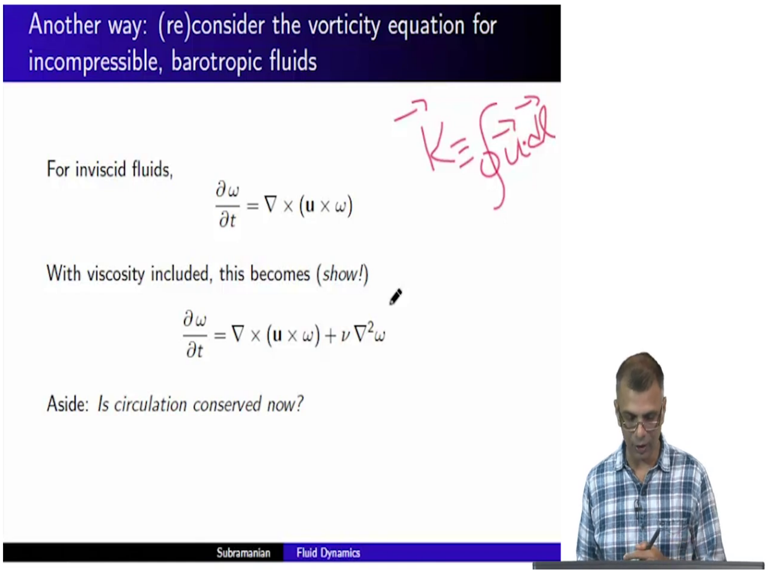

Now, so, this was for inviscid fluids, where you know viscosity was not important and with viscosity included this becomes and I would urge you very strongly to show this, to show that the vorticity equation with viscosity included becomes this. So, in other words, you have this additional term here.

Why am I talking about the vorticity equation? Because it is a dynamical equation, for you just like the momentum equation is. It is just a slightly different way of looking at things and there are some advantages to non dimensionalizing the vorticity equation also, which is what we are going to do now right.

So, at any way, with viscosity included you have this additional term which of course, completely alters the character of the equation here, you only had first derivatives right. Curl is just it just involves d over d x and so on so forth; whereas, here viscosity you have this has a Laplacian included.

So, you have d square x over d x square and things like this right. Not very surprising because you remember even in the Navier Stokes equation, the viscosity whenever you have the viscosity included, it involves second derivatives and that changes the character of the equation. If you just had these two terms, this would be the steady state Euler equation.

In other words, an equation that applies to inviscid; strictly speaking inviscid situations. But when viscosity is included, you have this term and you have the appearance of second derivatives and so, the same thing here too ok, second derivatives except there is a vector Laplacian, now acting on the vorticity right.

So, here is another interesting thing, we remarked earlier that that this quantity called circulation, which is defined as right. This is perfectly conserved for an inviscid situation. I would like you to think about whether the circulation be conserved in this situation now ok. The answer I will tell you. The answer the answer is no ok. Circulation is not conserved, when viscosity is included.

Circulation can be created or destroyed and we also talked about situations such as aiming a hose of water at a wall, a perfectly laminar flow of water. But when it hits the wall, you will see that the laminarity of the flow is destroyed and vortices start appearing.

And why is that so, it seemed like you know there was viscous effects were not important in the bulk flow of the water up until it hit the wall; but certainly viscous forces seem to be, circulation seem to be, seems to be generated when the jet of water is hitting the wall. So, why is that so?

The brief answer is that yes, viscosity is not important in the bulk of the flow; but just when the jet of water hits the wall, it comes to a crashing halt and therefore, either the vorticity or the or the velocity depending upon how you look at it has a large jump ok. So, the first derivative is quite large and second derivative is even larger. So, therefore, even though nu might be small, it is being multiplied by a pressure by a larger quantity.

And therefore, viscous effects become important in the boundary region, where the jet of water hits the plate of the wall and when viscous effects become important, circulation need not be conserved and in a situation where there are no vortices or no vorticity in the flow, need not remain the same. And its everyday experience that we see lots of you know we see the generation of vorticity, just where the jet of water hits the wall right. This is everyday experience right.

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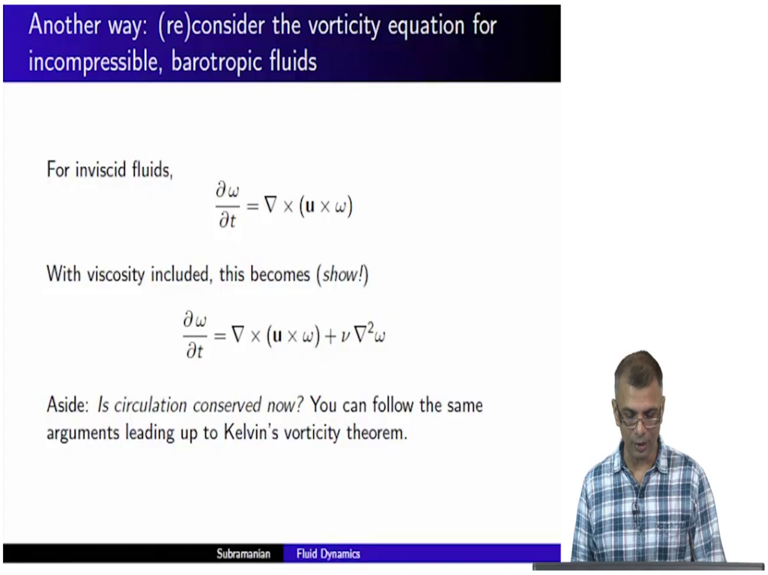

So, in order to answer this question, you can follow the very same arguments leading up to Kelvin's vorticity theorem which we discussed earlier, just a bit of an aside; interesting aside right.

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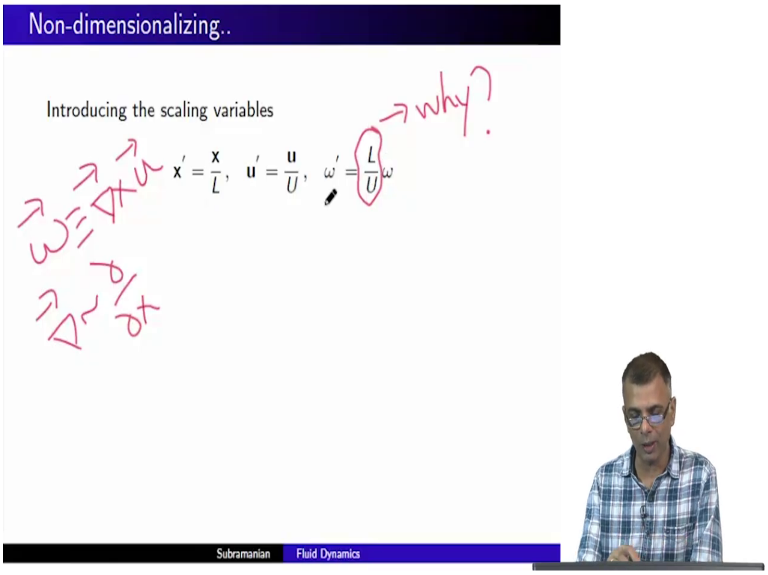

So, now we try to non-dimensionalize this equation ok. Non-dimensionalizing u is fine, we just introduce a u prime. In other words, we divide it by the large scale u, the undisturbed u; but how about omega? Well, that is not you know omega is non dimensionalized. Well, x prime is again you know non dimensionalized in the same way; omega is non dimensionalized this way ok. So, you have a L over U appearing. Why is that? I urge you to think about this also.

I urge you to think about the appearance of this term. It is just dimensional ok. You will find that L over U has the dimensions of well, I urge you to think about this. What are the dimensions of L over U? What are the dimensions of omega and what are the dimensions of L over U?

The dimensions of omega are easy enough to figure out omega is a is simply the curl of u. So, whatever the dimensions of u are say centimetre per second, you divide it by centimetre right because this is you know the dimensions of this are something like right. So, it is 1 over centimetre right. So, if you figure this, if you understand this, you will quickly understand why we are you know introducing this kind of a factor L over U, while non dimensionalizing omega.

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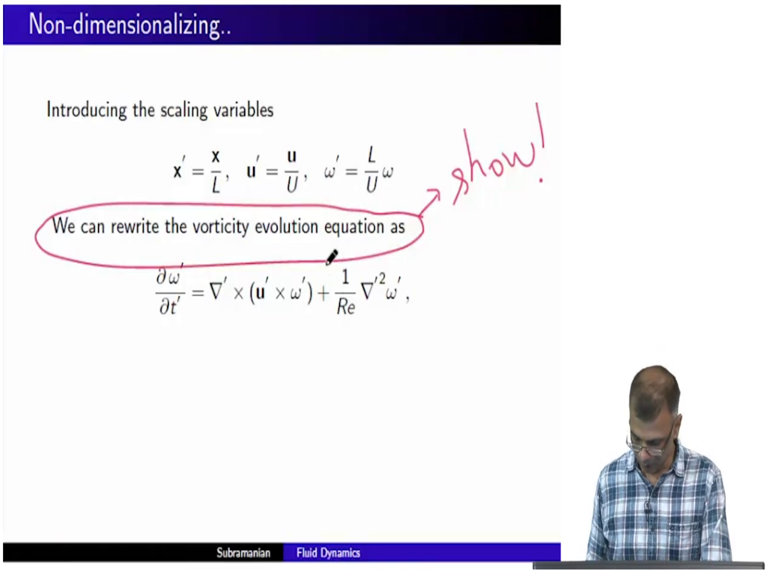

So, with this, we can rewrite the vorticity equation as like that and no surprise again I urge you strongly to show this ok. So, with this non dimensionalized variables the vorticity equation is written like so, and as before we have the appearance of this term 1 over Re where the Reynolds number as before is simply U L over nu the same thing right.

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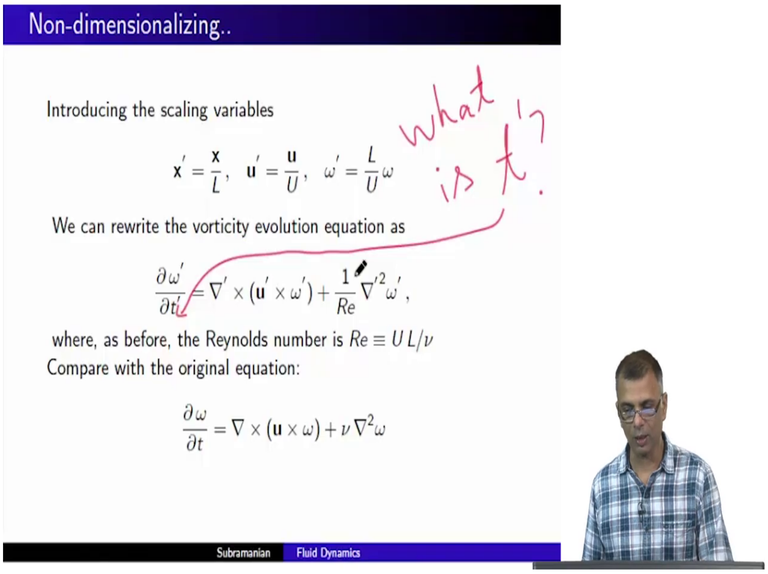

Compare this with the original equation that right, it looks exactly like the same with everything you know. So, I have not told you what the t prime is, I will leave that to you. I will say. What is t prime? I would leave you to think about that.

Again, you know so what is time non dimensionalized, you know what is a unit like just like here I this is you know in order to non dimensionalize the vorticity I use L over U; so, what should I be using to non dimensionalize t? It is not a difficult answer, it is some combination of U and L right.

So, having non dimensionalized, this you it looks exactly like this right, with just with primes on everything, primes even on the Nablus right, on the deltas right except for the appearance of this 1 over Re in the non dimensionalized equation right. So, it is very similar to the way in which we did the Navier Stokes equation and right.

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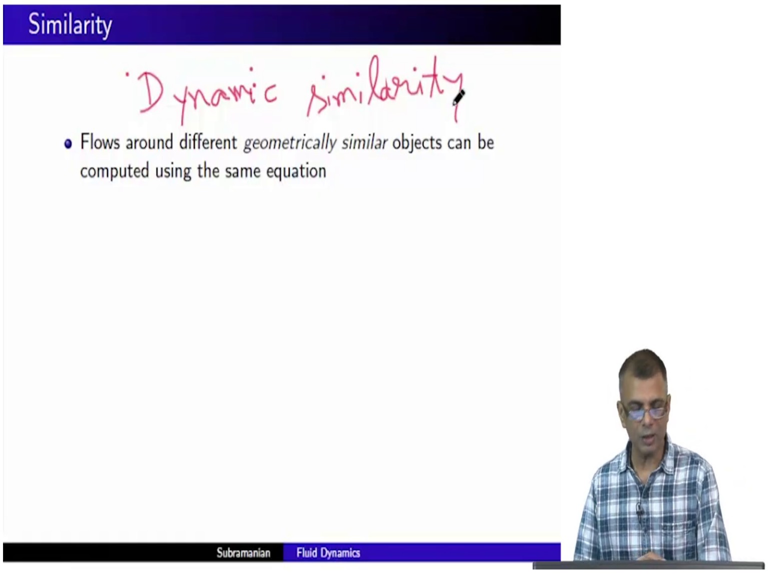

So, the advantage in doing this is that this is the concept of dynamic similarity, which I alluded to.

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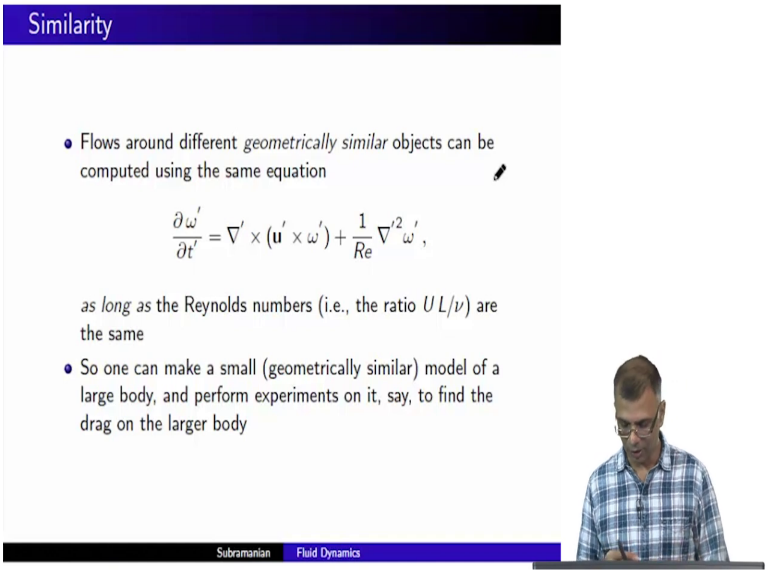

Flows around different geometrically similar objects can be computed using the same equation this or the non dimensionalized Navier Stokes equation, as long as the Reynolds numbers are the same. In other words yeah, so we will talk about this in a minute ok. So, I want you to remember this particular phrase geometrically similar objects ok. In other words, a small copy of a larger object say you have an aeroplane wing or a ship or whatever ok.

As long as I have a miniature model of that large ship; miniature model meaning a model that is in every aspect exactly similar to the larger ship except that it is all shrunken in size or larger as the case might be; but geometrically, it is exactly the same as my original object ok. I can investigate the dynamics of the flow equally well in both situations on the original object or on the miniature object as long as the Reynolds number in the two situations are the same.

This is the advantage in using a you know a non-dimensional equation like so ok. So, one can make small geometrically, a small geometrically similar model of a large body and perform experiments on it. Say for instance, why would I perform an experiment on it? For instance, to find the drag, to find the aerodynamic drag. This is a very practical application. You have an aeroplane moving through the air and I and as an aerodynamic aeronautical engineer.

You know if you were to try to figure out you know the fuel efficiency or so for that even a car for instance, you are very interested in the drag right that is experienced by the car. But you do not want to take the entire car into a wind tunnel, unless you are very sure of things and you are ready to invest that kind of money; instead you would prefer to perform experiments on a miniature model of the car or a miniature model of the aeroplane.

And the results are guaranteed to be the same as long as the Reynolds number in the two situations are the same ok. So, I want you to think about this carefully and this is at the heart of this concept called dynamic similarity. And then, one instance why one would want to perform such a you know such an experiment is to for instance find out the drag. I want to minimize the drag obviously to maximize the fuel efficiency or many other things right.

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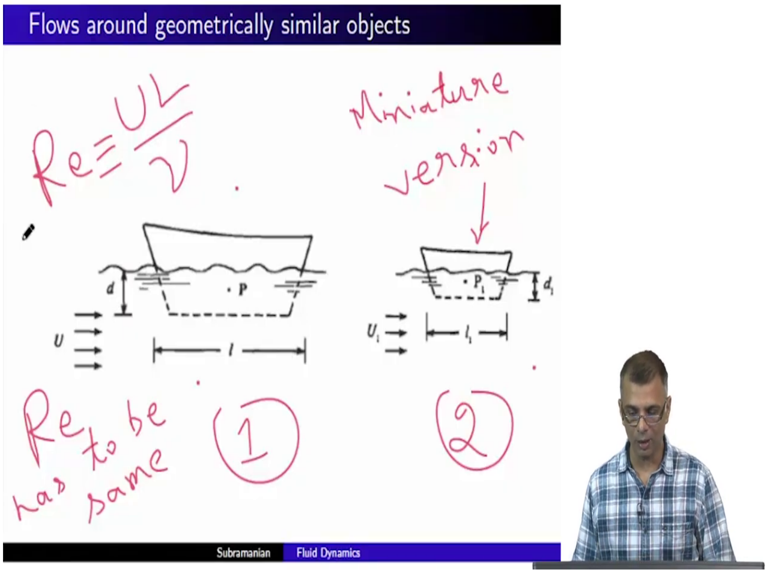

So, here is an example of geometrically similar objects. This is a, this is a miniature version right. It is a miniature version of the ship right. So, the l 1 is smaller than the l, so on so forth. So, I mean I have the ship and I want to find out the dynamics of the ship how fast it can move in water under certain conditions so and so forth; what is it is what is the drag many other things.

But I cannot afford to you know build the entire ship and then, do experiments on it. I would like a fair idea of things before I actually design the ship, I want to optimize the design.

So, I build a miniature version. Take it to maybe an artificial tank or something like that and perform experiments on it. But I can do this and I am guaranteed you know relatively accurate results, only as long as I am guaranteed that you know the Reynolds numbers are the same in the two situations.

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And so much of this work is due to Osborne Reynolds, I thought it is an interesting. It is always interesting to think I urge you to do a Google search on Osborne Reynolds to see the wide variety of his interests. For us I mean you know we are only interested in the Reynolds number, but you know people like this have done a wide variety of things. So, it is an interesting aside.

Having said that, so before we conclude this particular part, I just want to emphasize one thing. We kept saying that the Reynolds numbers in the two situations; situation 2 and situation 1, the Reynolds number in these two situations has to be the same. Now, remember what is the definition of the Reynolds number?

The Reynolds number is right. So, the U here is different from the U here; the L here is different from the L here right. This is a miniature version. So, the characteristic length is obviously different from the characteristic length for the bigger version right.

So, what does it, but I want to insist that the Re has to be the same in both the situations ok. So, what does this mandate? Can I perform experiments? Suppose, I was interested in the dynamics of water say sea water for instance, on the large scale ship; can I have the very same material, can I have the same sea water in this experiment that I am doing with the miniature object. The answer is no because the both the U and the L in this situation are different from the U and the L in the situation.

And if I have to keep the ratio constant, I have to change the nu right. So, the Re has to be the same in situation 2 and situation 1; but that means, that I have to perform my experiment with a different fluid in this situation as opposed to this situation, I have to think carefully about exactly what that fluid should be. Only then, I am guaranteed that the Re will be the same ok. So, with that we will end this segment.

Thank you.