Electromagnetism

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Lecture – 09
Vector integral calculus: Line integral

After working some problems related to gradient divergence curl and the second order derivatives. Let us now move onto the Vector Integral Calculus.

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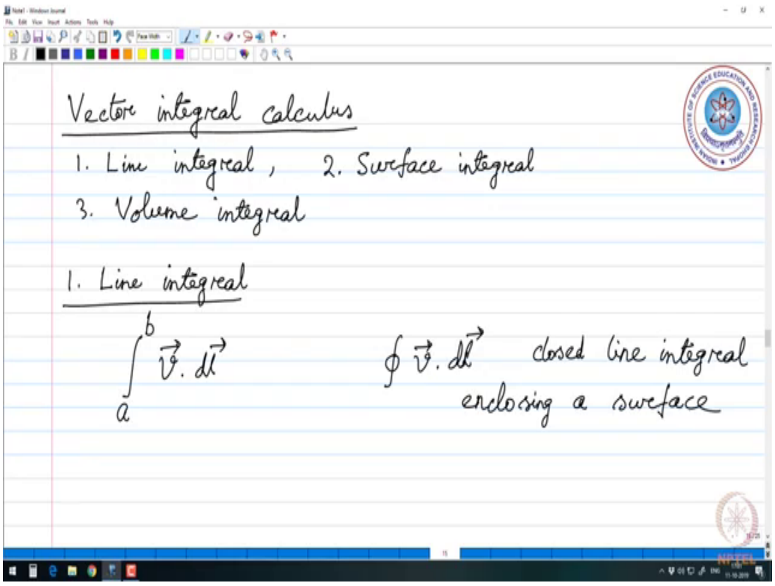

There are three different types of vector integrals; line integral, surface integral and volume integral. Let us start with the first one that is line integral; as the name suggests it this integral is performed over a line. So, if we have a line starting from point a to point b this may be any line a straight line a curve or something else we have a vector v we have a dot product of this vector with a line element dl and that represents a line element line integral.

We can often have another type of line integral that is over a closed line; closed line means this line encloses a surface v dot dl over a closed line. This is this type of integral is also quite useful in the context of electro magnetism that we will see later on. Now, let us have an example to understand how line integrals are calculated and what it means.

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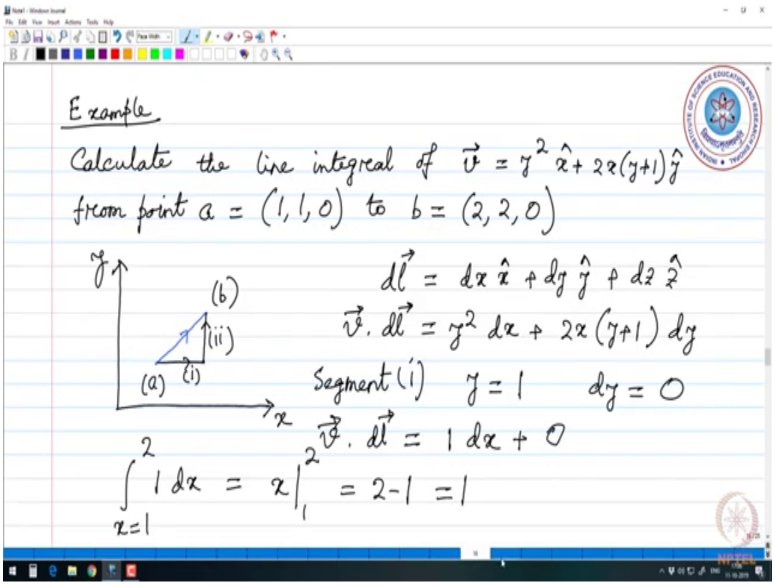

So, the example says calculate the line integral of the vector function v that is expressed as y square x cap plus 2 x y plus 1 y cap. You can clearly see that this vector v is a 2 dimensional vector and start from point a that is given by 1, 1, 0 and end at point b that is given by the coordinates 2, 2, 0.

So, what can be the paths for these two points and for this line integral there can be many paths connecting these two points. So, the first point is 1, 1, 0; let us say that point is here the second point is 2, 2, 0 let us say that point is here. So, this is point a this is point b and we will consider two different paths here; one path we will take us this way the black line. The other path will take us along the blue line.

So, let us consider the black line first. The expression for dl the line element is given as d x x cap plus d y y cap plus d z z cap this is always true. And for the given vector v dot d l can be given as y square d x plus 2 x y plus 1 d y. Now, in the black path we will have in the first segment, we will have y equals 1 all this is x axis this is y axis and that means, d y equals 0.

So, over the first path we have v dot d l equals y square d x y is 1. So, this is 1 d x plus because d y is 0 the second term becomes 0. So, the integral over x equals 1 to 2 1 d x is given as x from 1 to 2 that is 2 minus 1 equals 1. Now we will have to go for the second segment.

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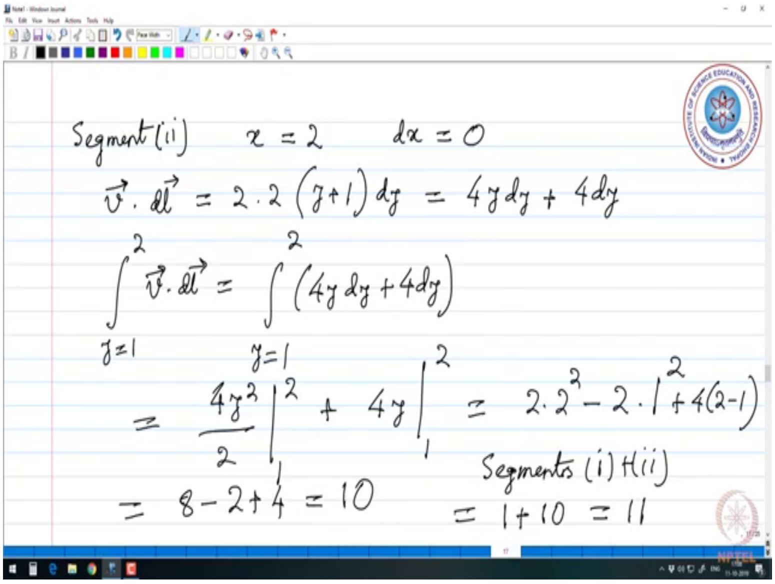

In the second segment we have x equals 2 which implies d x equals 0; for x being constant. So, v dot dl this quantity will be 2 times 2 times y plus 1 d y that is 4 y d y plus 4 d y and the integral will be over y only; because only d y term survive here. So, integration over y equals 1 to 2 v dot dl equals integration y equals 1 to 2 4 y d y plus 4 d y equals 4 y square over 2 the limit being 1 to 2 plus 4 y the limit is 1 to 2.

This is equal to 2 times 2 squared minus 2 times 1 squared plus 4 times 2 minus 1. And this is 8 minus 2 plus 4 equals 10. So, the sum of segment 1 and segment 2 becomes 1 plus 10 equals 11.

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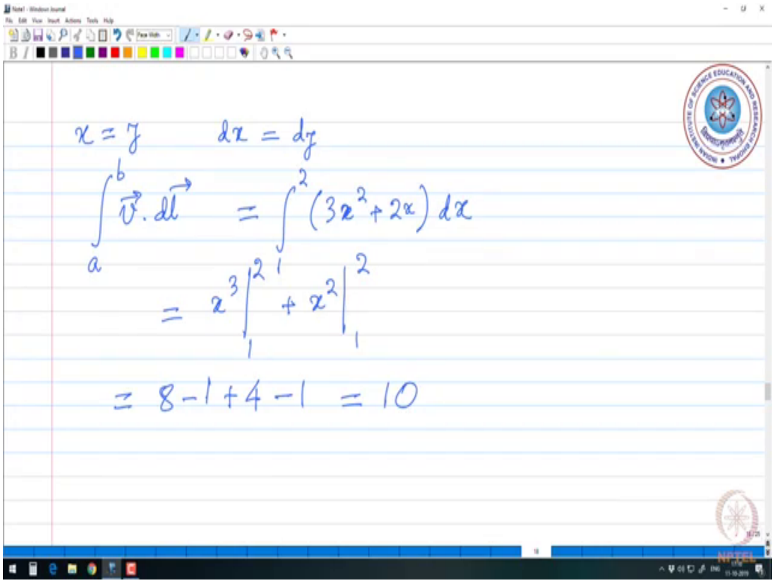

Now, let us move onto the blue path in the example. On the blue path the equation of that path is x equals y so, d x equals d y. If we have that then integration from point a to point b v dot dl this can be expressed as integration from 1 to 2 we will substitute y by x we will have 3 x square plus twice x d x and we are supposed to perform.

The integral this will become x cubed, the limit being 1 to 2 plus x square the limit is 1 to 2; that is 8 minus 1 plus 4 minus 1 equals 10. So, we can see that the integral along the black path is 11 and along the blue path is 10. So, its not necessary that the line integral along every path will be same. If you connect 2 points in the line integral for a vector it can be path dependent. There are certain cases where the vector integration line integration of a vector is not path dependent.

We will come to that situation later.