Non Equilibrium Statistical Mechanics
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Lecture 33
Critical phenomena (Part 5)

Right so today we will go on with our study of phase transitions and we will have a gentle introduction to what is called the Landau theory. Now I am not going to do this we do not have time to do this in great detail or in at any length but what we are going to do is to take from familiar model namely the ising model which we already looked at and then I will simply make a statement that a very large class of phase transitions will fall under the so called ising universality class.

So whatever happens here is generic to whatever happens in general and I will use the same symbols that we use for the ising problem in other words the order parameter is a magnetization I used length m for that you continue to do that but the physical significance of this m could change from problem to problem and you will see that the structure of the theory is more of less generic it is quite general.

And then we will go on to some introduce fluctuations and go on to what is called the Ginzburg Landau theory and hopefully finally we will talk about dynamic critical phenomena. So we take this in several steps let us start with some familiar material and it is as follows. I will draw a number of pictures today is to show you what the phase transitions what various diagrams phase diagrams look like.

So wherever the algebra gets little unnecessarily complicated we will simply draw a picture and go ahead the things and there will be a lot of hand waving arguments which can be made rigorous but I want to communicate the essential physics of what we are talking about.

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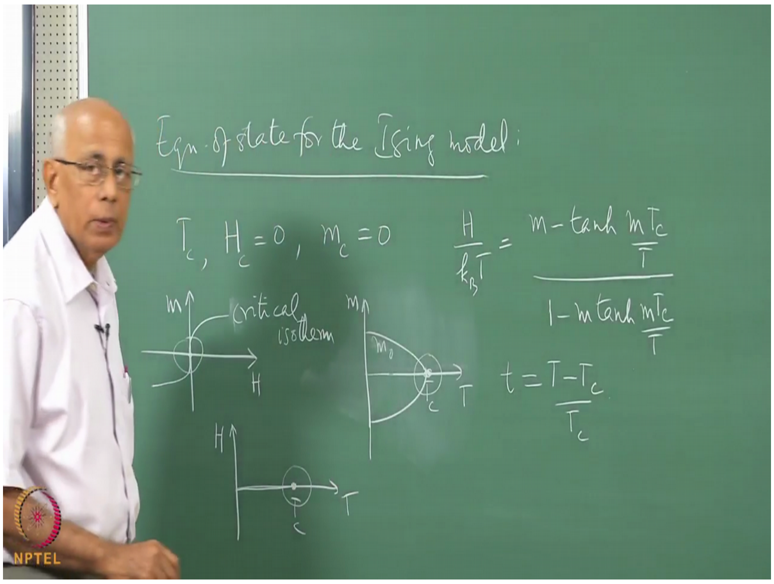

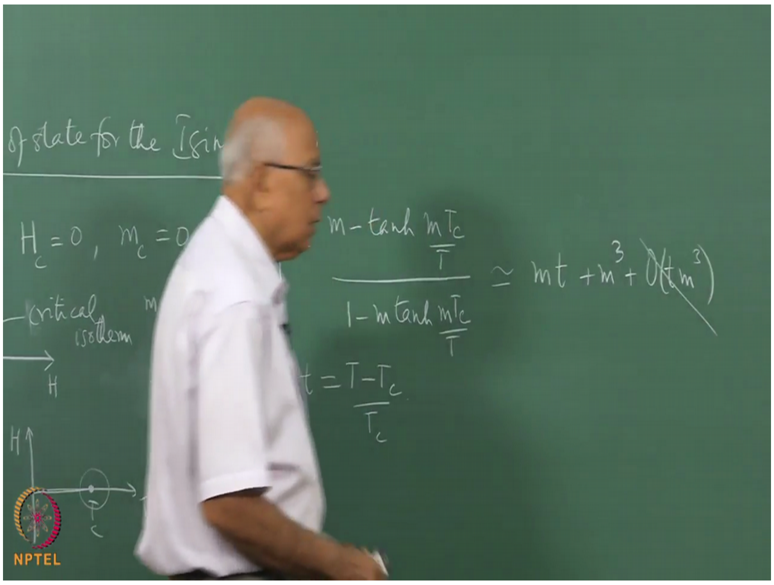

So we start again with the ising model for which if you recall we had an equation of state. So we had an equation of state for the ising model and we are dealing now with the critical region we are looking at the region near the critical point, okay if you recall the critical point is characterized by a temperature T c the query temperature at which the system goes from paramagnetic to ferromagnetic.

The critical value of the field which is equal to 0 in this case because it is at 0 field that you crossover from paramagnet to a Ferro magnet spontaneous magnetization is in the absence of the field and of course the critical value of the magnetization is also 0 because it takes off from 0. Now just to recall to you what these what the diagrams where in this case I plot an H versus m then the critical isotherm did this isotherm critical isotherm so H c was 0, m sub c was 0.

And then we also had the figure let me draw this again just to recall to you we had m versus T and here is T c in this case and the magnetization in the absence of a field m not was 0 beyond T above T c and then it went down like this or the Ferro down was a branch like this and this branch become one stable. So this was m not, the spontaneous magnetization and again the critical region I mean this region just as I mean this region critical region.

And finally there was the H versus T graph and the H versus T graph was a line like this and then a T c, so again this is the critical region. So H c was 0 and T c is at this point here, okay. So it is this circle region that we are dealing with we are talking about and what happen in the circle region was that in ising model in main field theory we ended up getting an equation of state and that equation of state read like this we have Tan hyperbolic H over k T but essentially in the critical region H is near 0. So it was H over k T was equal to m minus Tan hyperbolic m T c over T divided by 1 minus m tan hyperbolic m T c over T.

Now we are going to focus on the critical region namely T near the critical temperature. So let us introduce the reduced temperature t equal to T minus T c over T c then I can write this equation here in terms of critical quantities very close to the critical region in terms of little t and little m, m minus m c is same as m because m c is 0, H minus H c is again just H because H c is 0, okay.

So this equation can be written in the form on this side mt that is the leading term and then the next term is of order m cube and then plus order tm cube so we will drop this it is a higher order term, the spontaneous magnetization is found by putting H is equal to 0 in this and then the equation is m cube plus mt is equal to 0, so you have m cube is minus mt, m equal to 0 is always a root but for t less than T c that is unstable and for t negative namely t less than T c out here this region m squared is equal to t mod t was a solution and therefore you got plus or minus square root of little t, so that was the mechanism.

Now we want to make this systematic and one way to do this is to argue and this is Landau's argument it proceeds in a large number of steps so let me say what it was to start with. One way to look at it is to say there is (propbab) this is possible to introduce a potential after all this equation of state has essentially a reason by saying some thermodynamic potential is at a minimum so that you are in thermally equilibrium.

In this case essentially the equilibrium of the Gibbs free energy because we have an H here the analog of the pressure and we have the analog of the volume out here and the temperature here that is the equation of state. So at a given value of the magnetic field H 0 or otherwise it is equivalent to saying at a given value of the pressure and temperature so it is equal to the Gibbs free energy.

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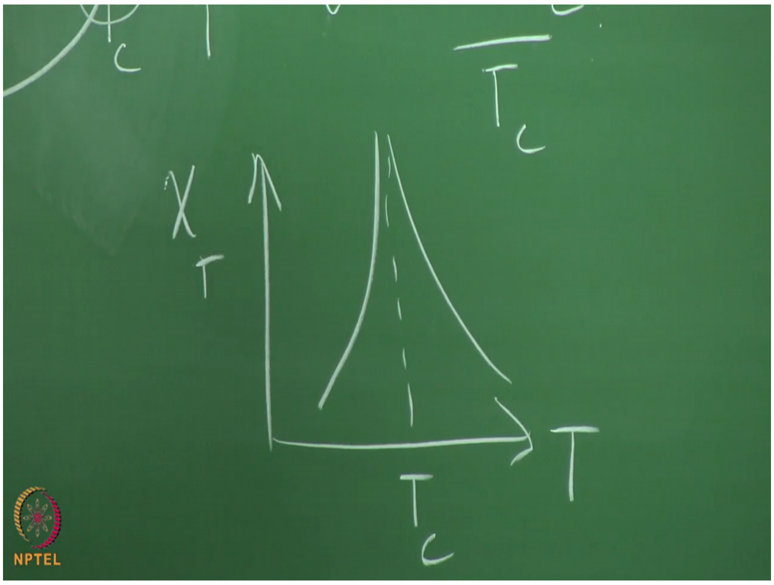

We are not going to write that down let us go back and do a bit of phenomena you can say essentially this is the equation that I have. So I understand from this equation how you get the square root here, the same equation gave us this cubic here, the same equation gave us the susceptibility also remember that we plotted the susceptibility as a function of T, I plotted the isothermal susceptibility and it diverged at the point T c like 1 over little t all these things came out of this equation of state here.

So let us do the following let us introduce some potential on differentiating that potential with respect to m and setting the result equal to 0 I get this equation of state you could call it the Landau functional or something like that. So this potential it is not exactly the Gibbs free energy it is not exactly Helmholtz free energy because those things have to satisfy certain convexity properties these potentials known this is just an empirical this is an equation of state at the critical region.

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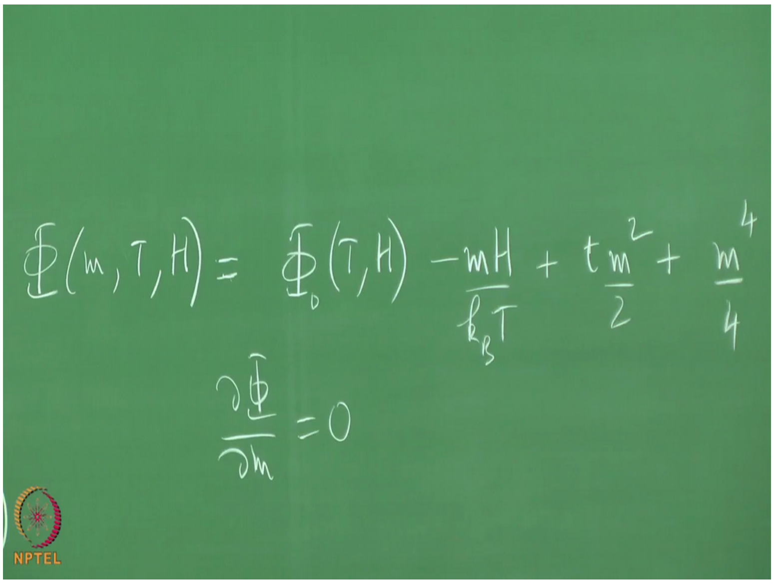

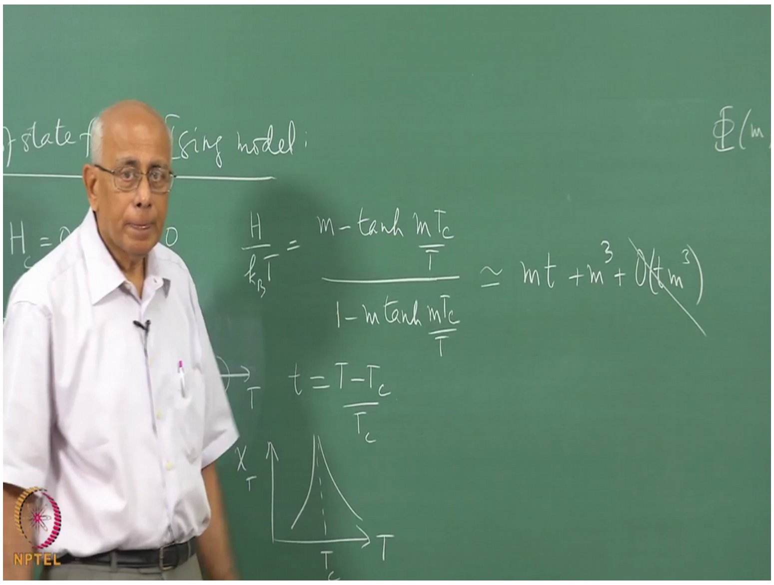

So let us define a Phi which is a function of m, T and H equal to some constant Phi not which is a function of T and H does not involve m does not involve m and I want to produce this equation of state so minus mH. Let me just for convenience you could write this T as T c plus small correction the correction is the higher order correction so it is essentially T c here and we can (())(10:06) in it so it is not an essential factor here.

I have already done that see I have already got a T c here this is essentially T c because if I multiply through it going correction T minus T c is going to give me higher order terms. No give you order T squared it will be of order T squared because if I write this H T c plus little t essentially and I take that little t across then it goes away only the T c part contributes, the next term is of order little t square which are neglected.

Ya, right you need that yes okay you have okay good question you have to check and I am not going to do this here you have to check that is consistent to subsume this T here and call it as T c, yes good exercise check that like I have neglected this term (())(11:28) okay exactly exactly ya exactly. So this is the consistent this term is not relevant I call it H, okay. So if you do this minus mH plus tm squared is this term here plus m 4.

And then you compute delta Phi over delta m and you said it equal to 0 you get preciously this okay the 2 here, put a 4 here if you like these are constant which are not paying attention. So do you agree if I take this Phi and I differentiate with respect to m and said it equal to 0, if you like ya if I said delta Phi over delta m equal to 0 I get this equation here, right.

So my claim is that this potential whatever it is this Phi has a certain geometric shape and its minimum will give me the equilibrium state we can plot that we can plot that fellow in order I am going to generalize that this is the original Landau functional we are going to the point is eventually what I am going to argue is that the factors here the signs are very important they could be numerical factors which are not bothered about.

So I am going to have a term which is proportional to the product of the order parameter and the field then a term proportional to the square of the order parameter with the little t in the coefficient as a leading coefficient and then a term which is essentially a constant coefficient times the fourth power of the order parameter that will generically give me this second (order) this continuous phase transition and it will generate for me all these diagrams so that is the idea.

And then I am going to argue that these coefficients could in general be temperature dependent but we have looked at the leading temperature dependence near the critical point, okay.

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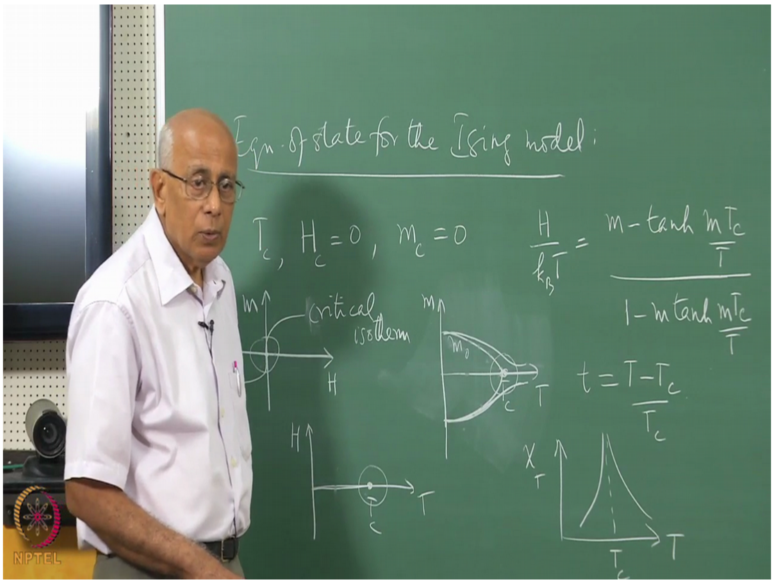

So let us draw some pictures and see what happens notice also that you could do the same thing with fixed field. Suppose you plotted the magnetization versus T for small positive field then the curve would be this is the paramagnetic region and then this would be Ferro region eventually it would saturate. Similarly on the negative side a small negative field if you do not switch it off will behave in this fashion.

So what you are doing is drawing this isotherms for nonzero value these various isotherm and looking at what happens as you change keep H fixed and you decrease the temperature from above T c to below T c. So as you go up here or as you go down here in this. In the case of the magnetization versus temperature it means if you keep the field fixed at some positive value you are here at this point and as you change the temperature you come down for a fixed value of the field you are going to jump this and make a transition here as you change the field from positive to negative values keeping the temperature constant you are going to jump here that is equivalent to saying if I go from here to here keeping the temperature constant I cut across this phase transition line that is like the liquid gas coexistence line and there is a discontinues change in the magnetization from this point to this point.

So this is a line of discontinues transitions ending in the critical point where the transition becomes continuous, in other words the discontinuity in the magnetization keeps decreasing till it vanishes at the critical point which is why it is called a continuous transition, okay. Now that is going to be encapsulated in the figures that we can draw we can now draw ask what is this potential look like in various cases so let us draw this potential.

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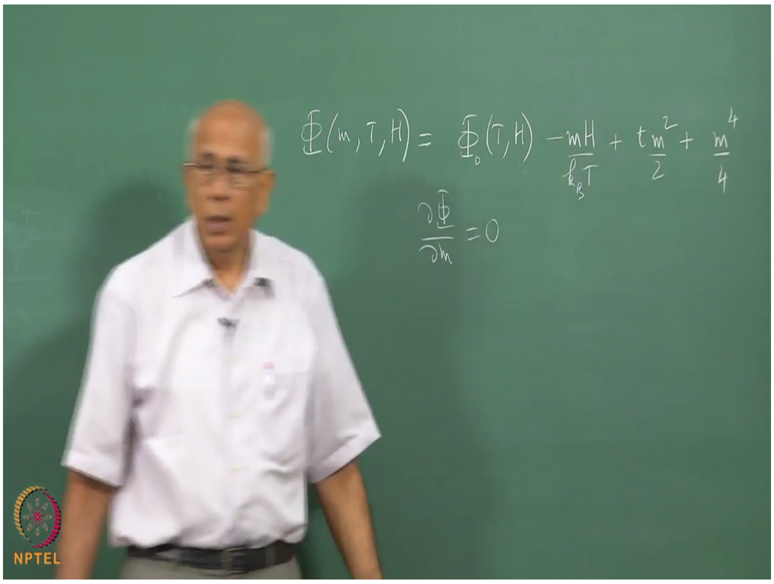
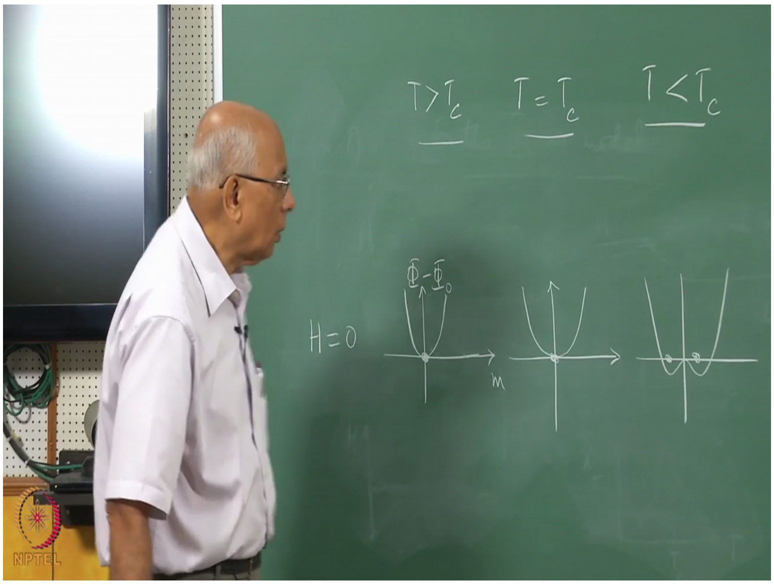

So let us draw let us have T greater than T c, T equal to T c and then T less than T c let us draw these three cases separately and let us draw them for H equal to 0 0 field. So this fellow is gone and I am going to draw Phi minus Phi not so this is just a constant as far as the m is concerned I move it aside and plot this graph here. So I have tm square plus m 4 and that is now easy to see what is going to happen for T greater than T c.

So I plot Phi minus Phi not always and what is the potential look like as a function of m always the order parameter m then at H equal to 0 this is gone, T greater than T c this is positive this coefficient starts with a parabolic behaviour and then takes of like a 4th power. So the minimum is a simple minimum it is not quite a parabola but it is like this for T greater than T c.

At T equal to T c this same thing as a function of m little t is 0 now and the field is 0 so you have a pure quartic term. Therefore this looks like a very flat and T less than T c you have an inverted parabola the field is 0 and then you have the m 4 which takes you up. So let me plot with across a dot the equilibrium value which is the state of thermally equilibrium and we will give you the order parameter at thermally equilibrium, it is obviously 0 here, it is still 0 here but now it is either this or that these are the two values of spontaneous magnetization that you saw in the m versus T m not versus T graph either up or down, Ferro up or down this is for H equal to 0.

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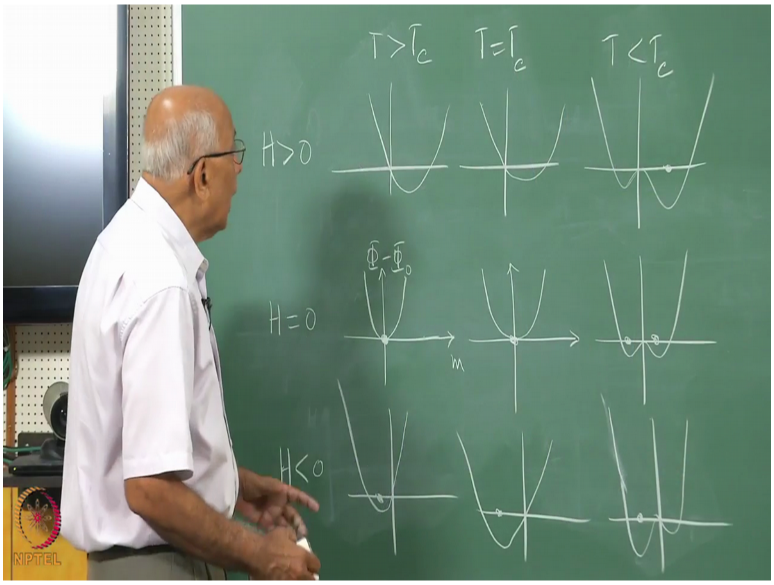

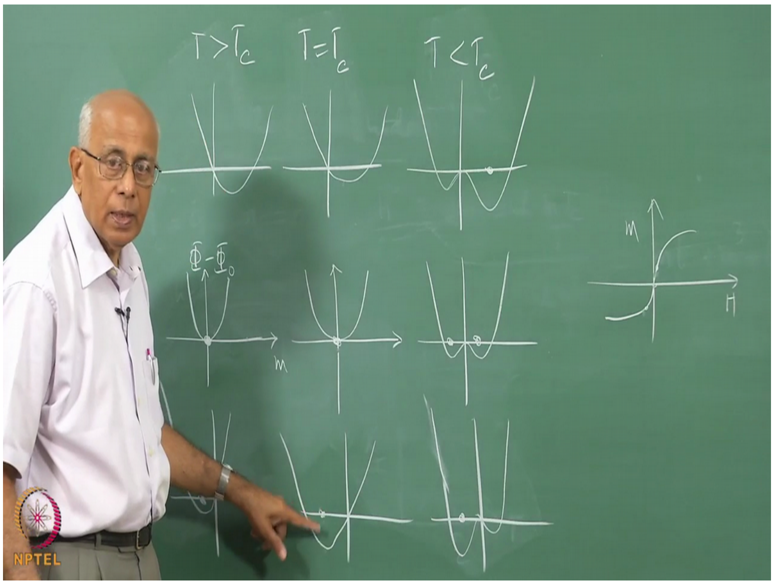

So let us plot the same things that means take this upper bit T greater than T c, T equal to T c, T less than T c. Let us plot the same thing for H small positive on this side. Now H is positive this fellow is positive so near the origin you have a linear behaviour with a negative slope. So this curve for T greater than T c looks like something like this because near the origin you have this thing then we can find the minimum by working that out by taking its minimum (()) (19:38) T equal to T c what happens at T equal to T c in this case but H is positive, T equal to T c this fellow goes away and you still have a curve which looks like this it change its shape here a little bit so this guy gets a little flatter and then it does this.

And for T less than T c what does it do? It will still be like that but it will be it will buy say towards this so near the origin if H is less than 0 there is a linear term. So essentially there is a thing like this we know something like that bias towards the positive side. So the absolute minimum is still here on the positive side, on the other hand for H less than 0 for H less than 0 for T greater than T c you have a positive slope here and T greater than T c that follows still is square and so on it will be simpler reflection something like that (())(21:23) and it will do the same thing it will come down and then broad (())(21:29) again it is on the negative side.

But now for H less than 0 and T smaller than T c we will have this slope but you will have a deeper deeper minimum and you will have a negative thing here. So at T equal to T c we have seen what happens already positive slope I mean H positive you have a minimum at a positive value, H negative you have a minimum at a negative value, at T c you have this 0, right. So this corresponds to what do these three graphs corresponds to they correspond in this H versus m graph on the critical isotherm you are describing this point, this point and this point that is the critical isotherm.

So on this thing case you are describing this point here corresponds to this minimum, this value passes through corresponds to this flat minimum and this value here corresponds to this minimum negative.

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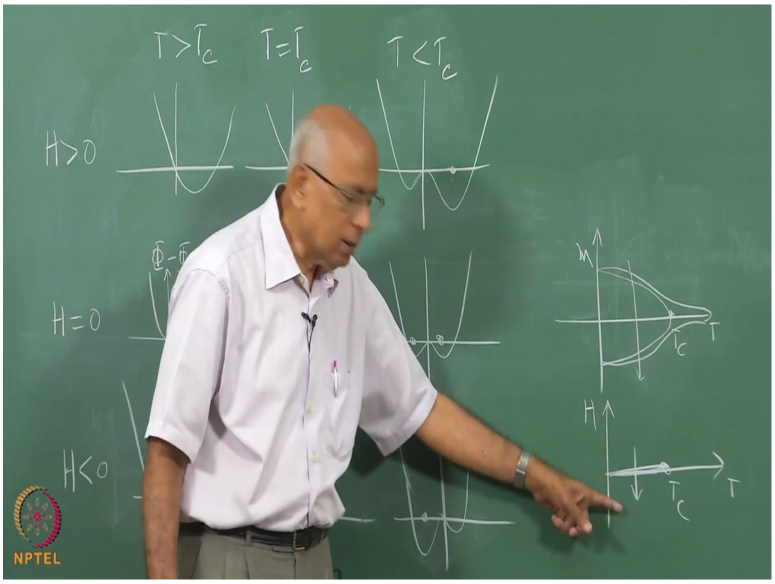

On the other hand when you go to T less than T c here you are going through a first order phase transition because now you are looking at this here is T, here is T c and you had the spontaneous magnetization curve like this and then you had the other curves in the presence of small positive and negative fields like this. But now T is less than T c so you have to the left of this curve you are going like this as you are going from positive to negative values of the field.

So you are really crossing this is m if you do this in the T versus H graph you had this graph with T c you are crossing this line you are going through a first order phase transition from a positive magnetization to a negative magnetization. So the way the free energy graph or the energy function or the Landau functional changes shape is first you have a graph with 2 minima but this is the global minimum therefore the equilibrium state is a positive value of magnetization at the critical value at T c the 2 minima are equal and below for negative fields the left hand side minimum becomes more pronounced in the top so this is exchange of stability that has happened in some sets.

So from this minimum you went okay where equally probable and this became lower. So you went from here to here discontinuously crossing this line here, okay. So you can see that this generic form already encapsulates in it all this is happening here both the first order transition below the critical point as well as the critical region itself within main field is the simplest of approximations.

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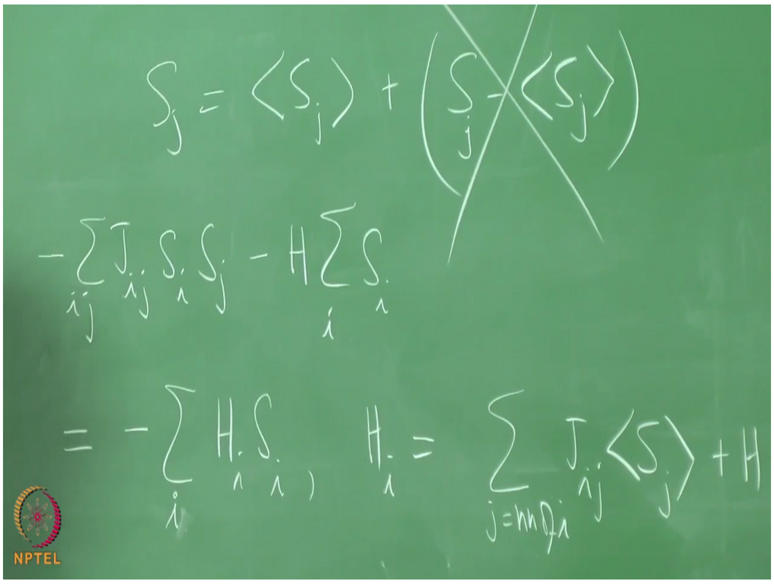

So the way to generalize this is now deep very deep this is such a generic thing that one starts by saying that now let us look at this picture in general we still not introduced any fluctuations remember that the crucial point the assumption was that we had a term in the Hamiltonian which look like ij J ij S i S j so the Hamiltonian was this minus H times summation over i S i and I wrote this as equal to minus summation H i S i over i where H i was equal to summation j equal to nearest neighbour of i J ij expectation is j out here plus H applied field, okay.

So essentially I replace the expectation of this term by S i by this term by S i times expectation S j. In other words I neglected the fluctuation in S j. So essentially I wrote S j equal to S j plus S j minus S j and neglected this in main field that is really what has happened I neglected the fluctuations, okay. So still we have not taken that into account (()) (27:12) within main field theory this in this model main field theory simply means this nothing more than that.

And we got an m which is specially independent of where you are in the (())(27:23) it is completely homogeneous etc, etc. We will subsequently put in special dependence little bit later because we want to include fluctuations in some sense, okay.

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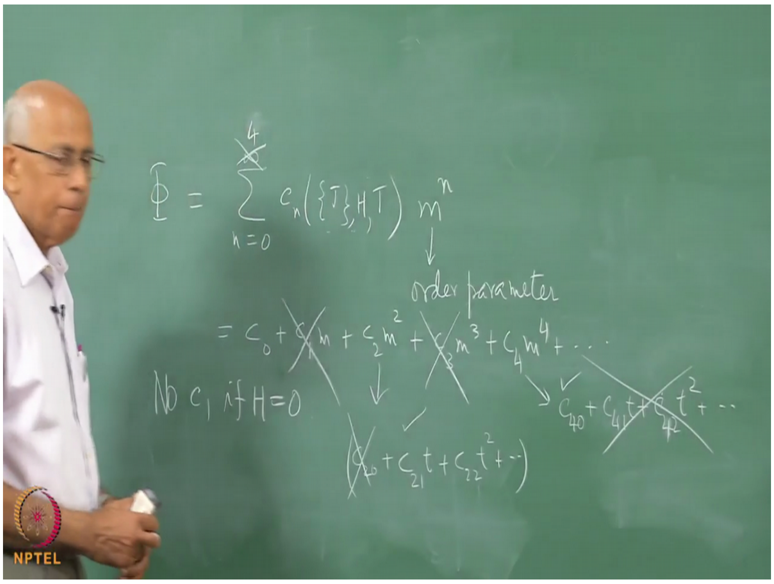

But the original land of theory itself starts by saying that for a given problem within order parameter I introduce a functional Phi which as a power series in the order parameter is something like summation n equal to 0 to infinity in principle, some coefficients C n which of functions of set of all the exchange constants the field and all the other parameters. So it is functions of all these quantities multiplied by temperature principle multiplied by m to the power n this is the order parameter.

So I construct such a functional out here and now I ask what are the possible values of m, okay so if I read over to write this out this will be of the form C not plus C 1 times m plus C 2 times m square plus C 3 times m cube plus C 4 times m 4 plus dot dot dot. Now as far as the critical region is concerned we have seen that all that you need is to keep up to the fourth order terminate do not need anything else. So we are trying to describe that sort of phase scenario here.

So first step this is replaced by 4 so we do not have to go beyond that, second point this is a constant and we can drop it it is not doing anything when I differentiate etc, etc. So this is a harmless constant but this term here cannot exist because if it existed it would say that there is a solution m not equal to 0 above T c because if I differentiate it this fellow here I end up with constant term here, okay.

So in the potential you cannot have a term which is linear in the order parameter look at what happened there because if you did you can easily check for yourself that if you had such a term in the potential if I differentiate delta Phi over delta m and put it equal to 0 I get a constant C 1 independent of m which would not go away even above T c so to end up with a solution for m not equal to 0 above T c which we do not want because we started by defining out phase transition as such that the order parameter 0 above the critical point and nonzero below the critical point so this term goes away.

These terms exist but now the argument is now this is with H not equal to for H set equal to 0, no C 1 if H equal to 0 ya you should say that if H is not 0 then of course there is a linear term that very much there that what is giving you is the susceptibility and so on yes we are going to do that in a second. The reason I am singling this out is because you want this if H is equal to 0 this is definitely going to be 0 it is by the very statement that the order parameter 0 above the critical point.

Then the next point is that in the absence of the field I want to have symmetry between m and minus m you say all these figures all that happened was m minus m got exchanged as you made the first order transition. Since you want to retain that symmetry this term is 0b you want this in to be invariant under m goes to minus m you can couch this in fancier language you can say that the probability of the given configuration is either the minus greater times Phi and Phi of m must be equal to Phi of minus m otherwise the probabilities will change in the absence of a symmetry breaking field, okay.

So then you have this and you have this and now we have seen that these coefficients could be temperature dependent themselves. So this term here would be some C 2 (2) not plus C 21 t plus C 22 t square plus dot dot dot. Yes we have assumed that it is analytic in the coefficients in all these exchange constants in the field and then the order parameter I am not saying this is a convergent power series you are going to truncated with a finite time infinite time and 4 impact etc.

So this term has to be 0 because the only way in which you can get this phase transition critical point is for this term to change sign as you cross little t equal to 0 as you cross T c which means the leading term must be 0 and it must start with the term proportional to T as indeed this is the case here is because this term change sign that you had solutions for m which are not equal to 0 plus or minus square root of minus mod T to the half of mod T to the half you got that because there was the linear term there so this term is 0 in this case and we retain only this term.

Then similarly C 4 will also have a C 40 plus C 41 times t plus C 42 times t square plus dot dot dot. Now the only role of that fourth power term is that you want stable minima so when m becomes very large in the positive or negative side you want the (())(34:27) to point up we concave upwards or even downwards. So for stability thermodynamic stability you want this to have a positive coefficient the temperature dependence of it is not very significant.

So you may as well retain only this term and throw out all these terms, okay and you end up with preciously this.

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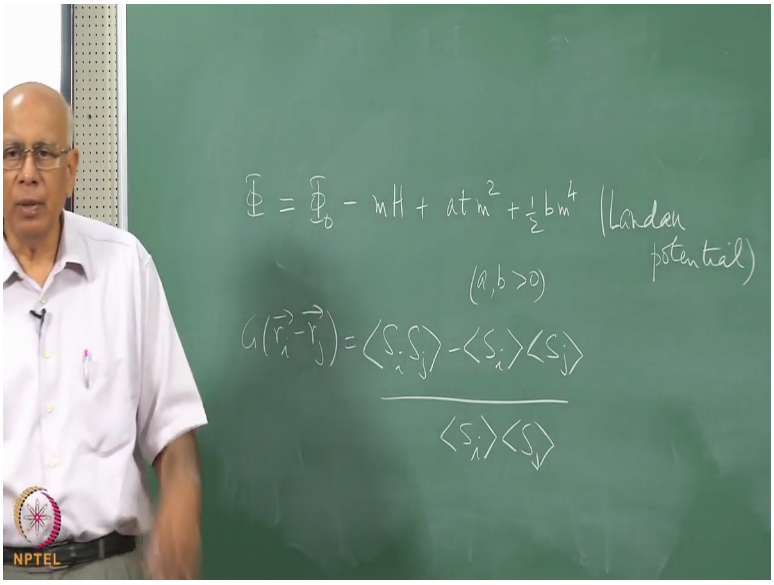

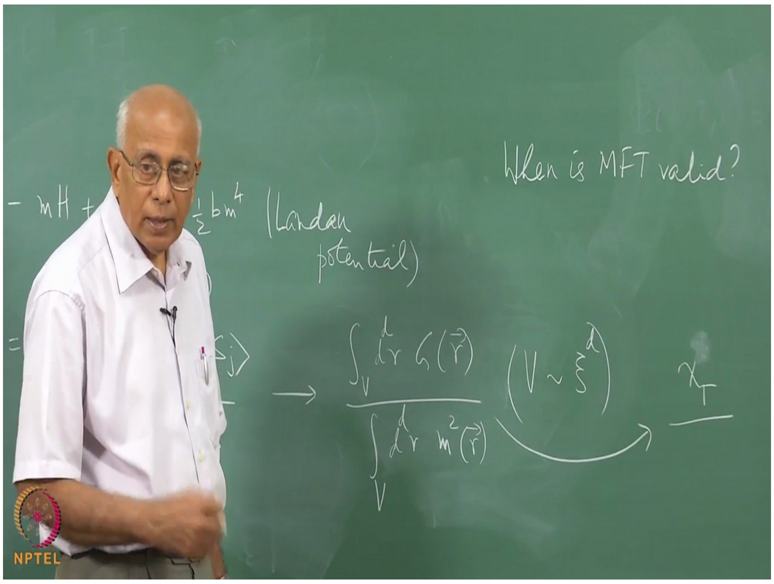

So now let us write that look at fancy notation so we want to take Phi equal to some constant Phi not minus mH plus standard notation a t m square plus half bm 4 this is the Landau potential and all the information I want comes out of this potential alright. Now the question is what is the guarantee that this main field is valid so when is it valid you need to know when this (())(36:05) is valid well let us ask that question separately let us ask okay.

We are going to put in all the information that we already have in this business I have already said that all these critical exponents come from just the correlation function essentially the behaviour of the correlation function that is the correct way to look at this. So I am going to use that information without actually proving it we have said certain things about this correlation function I am going to use that without actually without actually proving it in some sense.

So I would say that it is valid as long as you can neglect fluctuations, okay. Now when can you neglect fluctuations because what is really happening is that the system is correlated you cannot say all the spins are independent within some correlation length Xi and we also I mentioned that the Xi diverges as you hit the critical point, right. So if you recall we had a G which I called r i minus r j and I call this S i S j minus S i S j this fellow here, right ah sorry product of expectations and the nonzero nature of this G proves fluctuations correlations because if this is completely uncorrelated then this thing would be 0 identically, okay. So good measure of this and we also saw several properties of this I related this to the susceptibility static susceptibility and so on, right.

Now a measure of this would be to say that if within a correlation length you compare this quantity to the magnetization itself namely to the factored form below. So divide this by S i S j and compare how big this is compared to this guy that will give you some idea of how accurate this main field theory is. So this ratio is much much less than 1 and say this is the error relative error in this quantity and that is a good measure of the fluctuations, right so that would be one way of doing this let us (())(39:00) with this i and j in any case.

So finally this whole thing reduces to the following an integral in d dimension so let me be working in d dimensions special dimensions d d r and then this is G of r which is mod S i minus S j, etc we computed this quantity, okay divided by integral d d r m squared of r. So I have coarse grained in the volume of size linear size the correlation length and I am comparing the two.

So I got to integrate over a volume over a volume V where V is of order Xi to the power d ya we are going to sum over i and j on top and then below and find out what is the relative error in this whole business, okay so that is how I get this and the volume of integration is not the whole sample I do not need to do that I find out within one correlation length because outside it the correlation is anyway essentially.

So I need to compute this but notice that we computed this quantity and showed that it was essentially the susceptibility, okay. So this thing goes to Chi T and susceptibility on top this is essentially what it is, okay divided by the magnetization it could be taken to constant inside a correlation length and it is this value m whatever this is and we know that in the critical region it goes like (())(41:19) that is the magnetization index.

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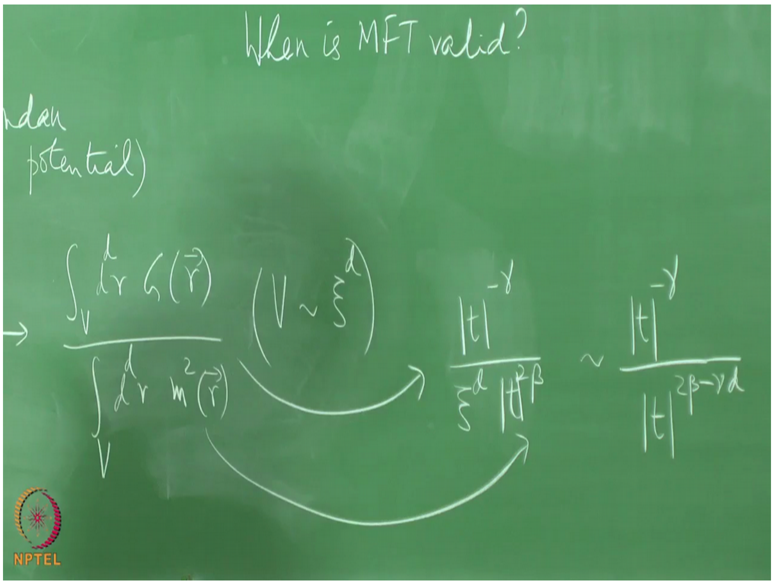

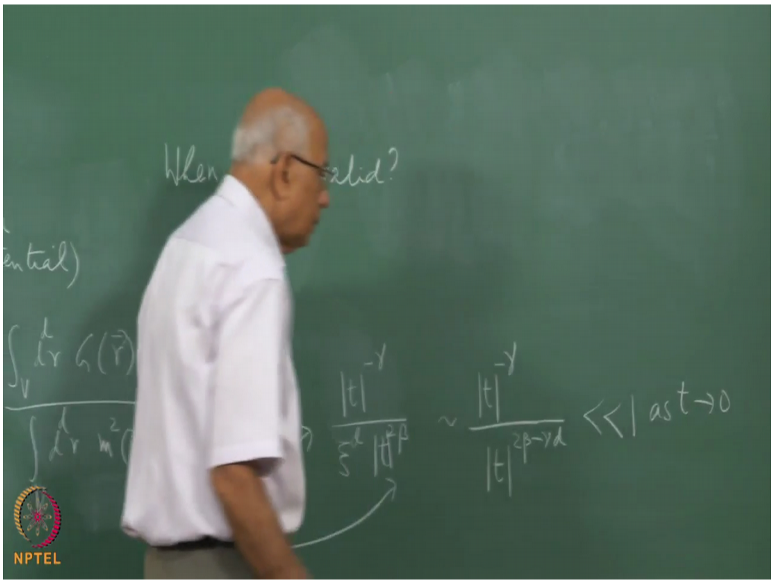

So let us write that out effectively this is t to the minus gamma mod t let us go to modulus is everywhere mod t to the minus gamma divided by this factor is t to the 2 beta because the magnetization exponent was greater the square root of half in main field theory and this gives me t to the 2 Nu and then a volume d dimensions of linear size Xi, so this is Xi to the d so apart from constants of order 1 this thing is mod t to the minus gamma divided by we know that Xi itself goes like t to the minus Nu it diverges so mod t with the 2 beta minus Nu d this must be much much less than 1 t goes to 0 then you are safe in neglecting fluctuations.

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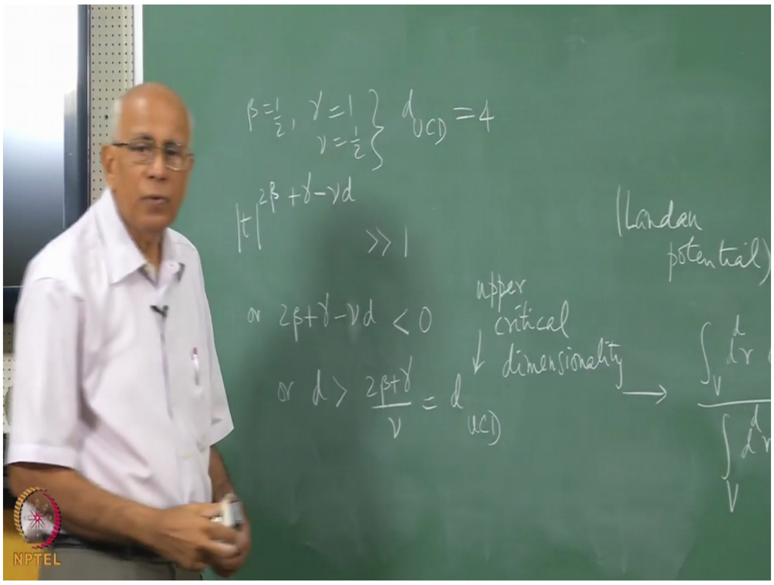

So (())(42:50) get us it says 1 is much much greater than 1 over mod t to the power or rather let us write it like this mod t to the power 2 beta plus gamma minus mod t to the 2 beta plus gamma minus Nu d must be much much greater than 1 as t tends to 0 that means this exponent must be negative Nu d, okay or d greater than 2 beta plus gamma over Nu.

So only the special dimensionality is high enough is this correct, okay this must be high enough in any given problem if it is equal to or less than fluctuations are very important and the smaller it is the fluctuations get more and more important, okay this quantity this is called the upper UCD it is called the upper critical dimensionality not yet not yet yes. So it says that any of this critical phenomena whenever you have a critical point then depending on what kind of universality class you have their existence upper critical dimension above which you may as well work in main field theory because the fluctuations are relatively unimportant and vanish in the thermodynamic limit, okay they do not affect the exponents or anything like that, okay.

Now let us check what it is for the ising problem because we have a non-set in main field theory in main field theory in our problem we had beta equal to half, gamma equal to 1 because magnetization went like plus or minus square root of t minus T c minus T e in the critical region, the susceptibility went like 1 over t minus T c this is the wise theory wise law, so gamma was 1 and Nu was equal to half in this case.

So here 1 plus 1 divided by half which is 4 so that is the reason we find the statement made in standard textbooks in critical phenomena that in d greater than 4 dimensions the fluctuations are unimportant, okay obviously we live in 3 and we have 2 dimension magnets. So it gets more and more significant the deviation from the main field exponents will become more and more significant as d become further and further away from the upper critical dimensionality that is however a lower critical dimensionality below which the critical point itself vanishes I mean we do not have a critical point, we do not have a phase transition at all because the disorder can never be overcome by the interaction the effect of entropy is too strong.

In the ising model the d lower critical dimensionality is 2 so less than 2 dimensions we do not have a phase transition in greater than 4 special dimensions main field theory is good enough but the interesting physics lies in 2 and 3 is very very nontribal in 2 and 3, okay that seems to be more or less the case always (())(47:27) right exactly so you have to so the point is when we computed main field and we found the phase transition it said that it predicted the phase transition in every dimensionality it did not say anything at all I mean it said the independent of d d did not appear at all so that is wrong main field theory overlooks that we know the critical exponents are very dependent on the dimensionality, whereas the main field theory completely ignores that all together, okay.

So experimental evidence rules out the possibility that main field theory is correct for the ising model we know that it is wrong we can compute in 2 dimensions we can compute ya ya ya if you do the renormalization group you will see the thing at 4 and above at greater than 4 main field exponents are exact exponents oh ya exactly. Now it is a question of whether you know what sort of accuracy you want we have perturbation methods, we have this (d minus) 4 minus epsilon expansions and so on we need to keep the sufficient number of terms to get these into results.

So the competition of the critical exponents for d less than upper critical dimensionality is hard you could hard what happens at the critical dimensionality itself typically of all these cases very typically there will be logarithmic corrections. So the corrections are not power law corrections but log corrections, etc. Similarly you could ask what is the critical region how small should t be and said t going to 0 but how small should it be let us call the Ginzburg criterion and I will talk about it next time we will see how small it should be in order for this to happen.

By the way this whole thing is the whole renormalization group approach to critical phenomena started off with lot of empirical observations on scaling exponents. Now let me just write those down since we are not going to use them anywhere I need to write them down so that you have at the back of your mind for the ising model for the ising universality class yes for other universality class is all the main field itself is different the ising universality class includes the (())(50:19) kind of model because those are identical to this they are in the same universality class.

So they all have to do with an order parameter which is a scalar if you look at the Heisenberg Ferro magnets the order parameters are 3 dimensional vector and then you finished it is a different class, if you look at the XY model the order parameter is two vector moving in a plane with two components and that is different. So I said that it depends on on the special dimensionality, it depends on the dimensionality of the order parameter number of components and it depends on the range of the interaction basically these are the only things that govern which universality class a given Hamiltonian belongs to.

Now what is being found is that in the critical region and this is how it started in the 1960's the whole theory of scaling a lot of these functions of several variables where found to be scale invariant in the sense that they were generalize homogeneous functions, they were functions of combinations of the independent variables, okay and they let to a lot of empirical scaling relations which today we can justify by some out more rigorous methods.

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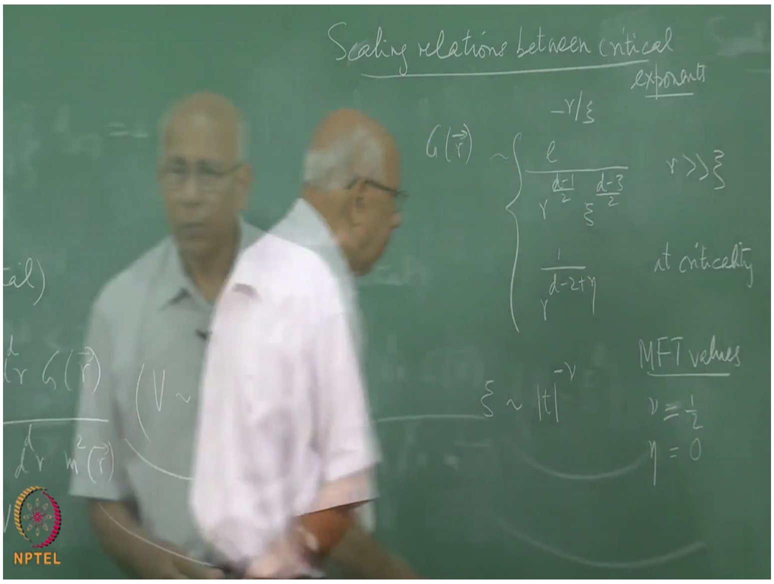

Let me just write them down so that you have some information the first of so let me call it scaling laws between critical exponents. First of all I said that everything is dependent on the correction function so we really need to see what the correction function does so that we can have this quantity G of r basically S i S j minus etc, etc this quantity went like e to the minus r over Xi and then I mentioned what it did at the critical point this was Xi to the power d minus r to the power d minus 1 over 2 Xi to the power d minus 3 over 2 this is for r much much greater than Xi and at the critical point at criticality it goes like 1 over r to the d minus 2 plus Eta.

And remember we define the critical exponent for this Chi we also found that Xi goes like mod t to the minus Nu so two exponents are introduced in the correlation function one is the way it diverges at the critical point the critical region and the second is what happens to the correlation which changes from an exponentially damped function to a pure algebraic function a power law inverse power, this Eta is called the non-classical because if you did main field theory Eta is 0.

So let me also write down main field values in this problem MFT values Nu equal to half, Eta equal to 0, okay.

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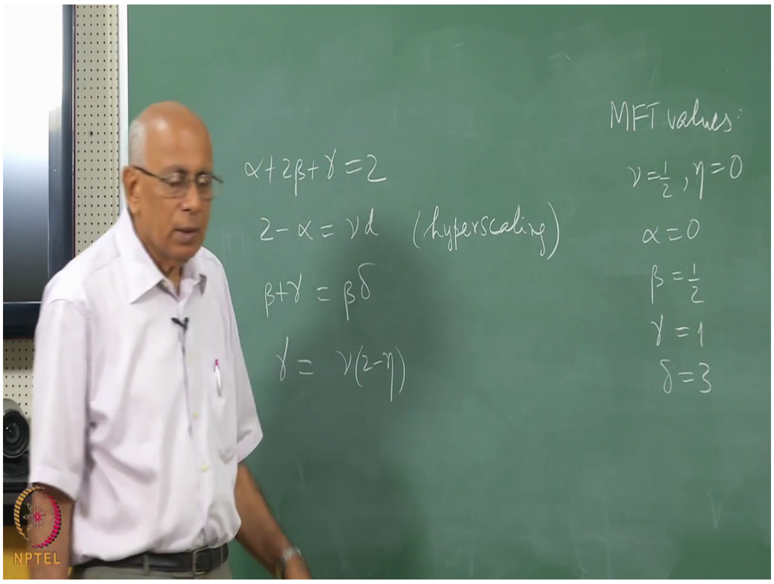

Now what the scaling relations tell you are the following is the set of 4 relations so that all the exponents are written in terms of Eta and Nu so the first is called the (())(54:21) equality in this case alpha plus 2 beta plus gamma is equal to 2 this is the specific heat exponent specific key goes like 1 over mod t to the power alpha, this is the magnetization or the order parameter exponent, this guy here is it is the susceptibility exponent.

Second relation says 2 minus alpha equal to Nu d this is called hyper scaling this is for a reason just mentioned in a second. So if you give me the dimensionality of space and you give the of the system that you are working with and Nu then I give you alpha specific heat exponent.

And then beta plus gamma equal to beta delta delta is the critical isotherm exponent, okay and then the next one is gamma equal to Nu times 2 minus Eta. So with the help of these relations you can express everything in terms of Nu and Eta, okay and what do the main field values do MFT values it says Nu equal to half, Eta equal to 0, alpha equal to 0 it is a finite discontinuity main field produce a finite discontinuity in jump discontinuity in the specific heat beta equal to half, gamma equal to 1, delta equal to 3 what happened here you notice that all these relations are valid with these values alpha plus 2 beta plus gamma is 0 plus 1 plus 1 is 2, beta plus gamma half plus 1 is 3 halfs this is 3, this is half, this guy is 1, this is 0, this is half, this is 2 (())(56:36).

But this this it is violated by these numbers notice that this is the only law only scaling relation which is dependent on the dimensionality all the others are independent of the dimensionality but this depends on the dimensionality. Now these main field values are valid only above the critical dimensions upper critical dimensionality. So unless you put d equal to upper critical dimensionality this relation will not be valid with those values, right and then indeed it is valid because if you put this is equal to half and that equal to 4 then and then this equal to 0 that is perfectly consistent, okay.

Ya but the well it will work okay two statements here one is that this relation here for it to be valid with main field values you got to put the upper critical dimensionality here. Second point is it is a separate problem to show separate thing to show that the effect of fluctuations is negligible above the upper critical dimensionality that is a separate story all together, okay. So this is not on the same footing as the rest of it that is what I am trying to point out.

So what we need to do is to pay more attention to fluctuations this means that our original Landau functional where we just took m to be independent of r is not good to include fluctuations you have to allow the facts that the order appears in patches of size Xi typically it gets larger and larger as you approach criticality. So we need to create a functional that will be the Ginzburg Landau functional in which we have to put in the special dependence of the order parameter.

In some coarse grained fashion not at each let aside but when you coarse grain it into blocks of say size equal to the correlation length then we should be able to get the effective free energy functional or Landau functional which include special variations and we will see what happens when we do that. Again we use very general principles we will use a symmetry arguments, we will use the fact that we want a scalar etc, etc.

Ya they will be dependence on the correlation length in direct kind of way ya sure it will certainly appear so that will be the next step before we do time dependence we will do that, okay. Then the kinetics of this transition is something we have not talked about it all that will be the last thing we do where critical slowing down will appear just as the lengths or the correlation lengths diverges the correlation time also diverges so things slow down at the critical point and it will create its own exponent we need to get we need to deal with that as well ya that will be the time dependent Ginzburg Landau and then we have to include fluctuations in it which we will with a random force and we will be back to our Landau's model but this time for not a single particle but for order parameter, etc.

So the basic ideas are very straight forward very simple but the implementation and then the renormalization group is something I have not talked about here at all we do not have time for that but that is a separate story all together, okay all right so let me stop here today.