Non Equilibrium Statistical Mechanics
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Lecture 14
Linear response (part 8)

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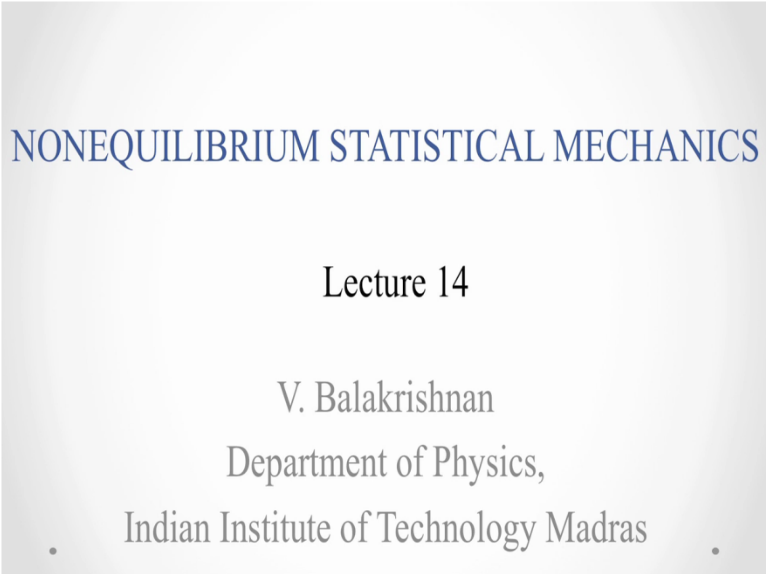

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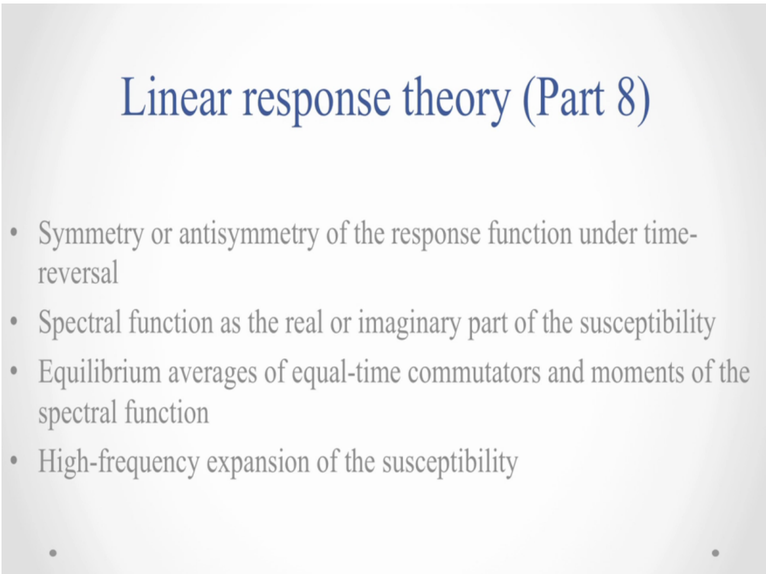

Right, so let us start with reviewing a few of the key formulas that we had come up to before we took a temporary break and this had to do with the way the spectral function entered in linear response theory.

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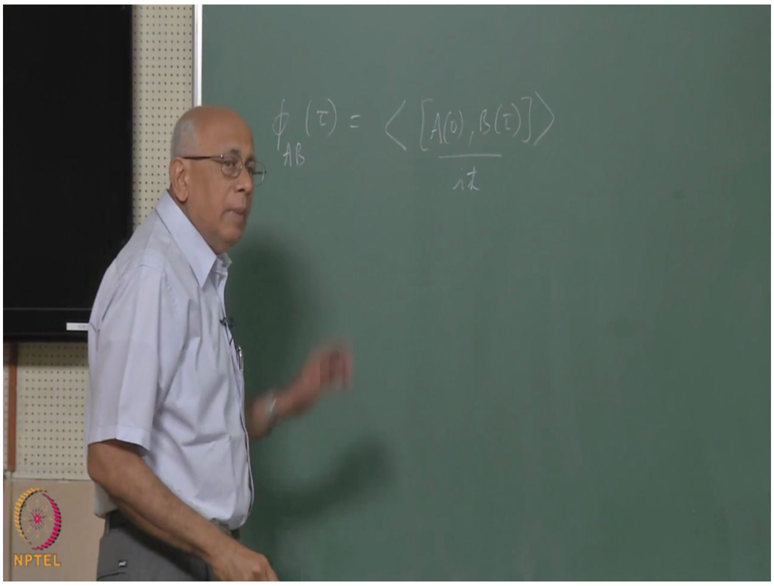

So just to recall to your memory for the fundamental formulas were, we have response function phi AB of tau which as you know was the equilibrium expectation value and let me write the quantum case for definiteness, this is equal to the commutator of A of 0 with B of tau over ih cross and this immediately led us to a formula for the commutator itself, the equilibrium expectation value of the commutator itself in terms of the spectral function.

Now I think there is a factor of 2 pi missing in one of the formulas we derive earlier, so to fix this properly, so that you can actually get the numbers correct the factors correct. B go back and remind you what our Fourier transform convention was?

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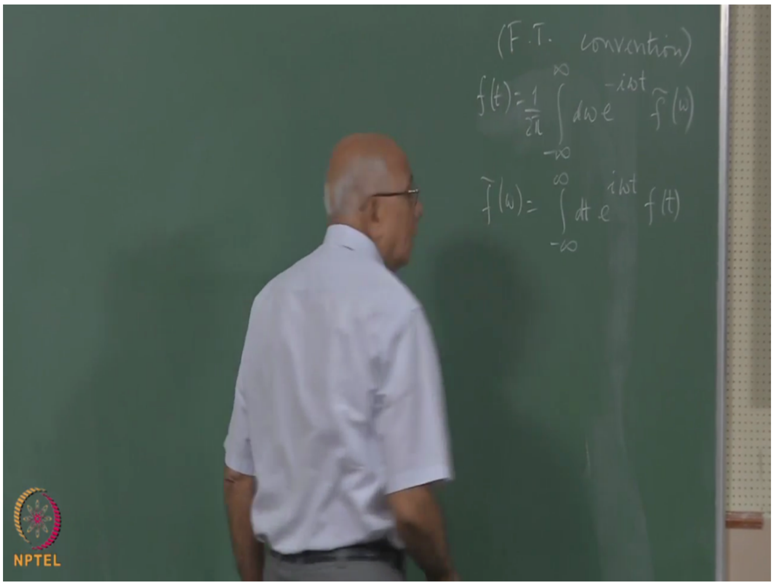

If you recall we said if you have a function f of t you expand it in a Fourier series as follows, so f of t is 1 over 2pi an integral minus infinity to infinity d omega e to the power minus i omega t f tilde of omega and correspondingly f tilde of omega is an integral minus infinity to infinity dt e to the plus i omega t f of t, this was our Fourier transform convention, okay. I guess this is the one we have been using throughout, so we should be consistent about it, right?

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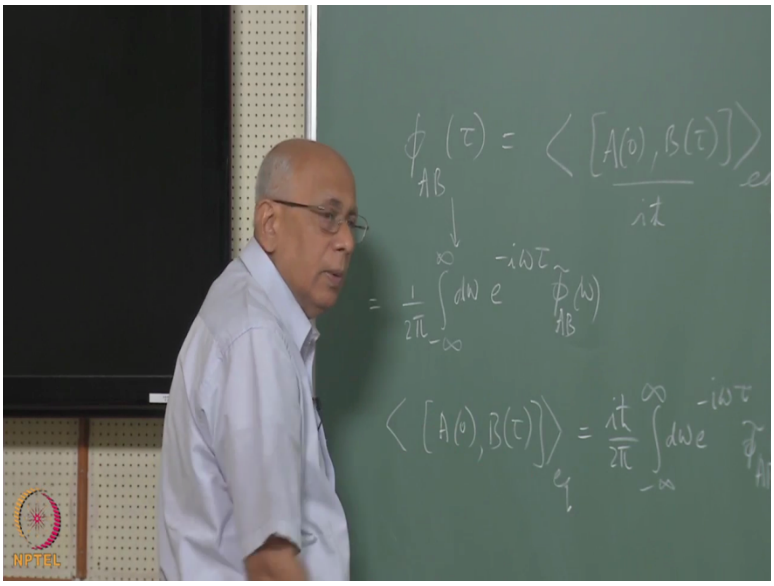

Then we discovered that the Fourier transform of this quantity is defined as phi tilde, so the definition of, so in other words this is equal to 1 over 2pi an integral minus infinity to infinity d omega e to the power minus i omega tau phi tilde of omega and that was the meaning of our the Fourier transform here AB of omega and we discovered there was a spec will representation for this quantity here.

In the sense that it had to do with the summation over matrix elements of A and B the Schrodinger of picture for instance over a basis set of states of unperturbed Hamiltonian h not (()) (3:17) different between 2 Boltzmann factors and then multiplied by Delta functions at all the transition frequencies of the system that is what this quantity had as a representation, okay. So the first fellow we derive was, what we wrote down was essentially that the commutator of A of 0 with B of tau this quantity in equilibrium was equal to ih cross times phi which is ih cross over 2pi's integral d omega e to the minus i omega tau phi tilde AB of omega. So that was the representation for this quantity here the commutator itself.

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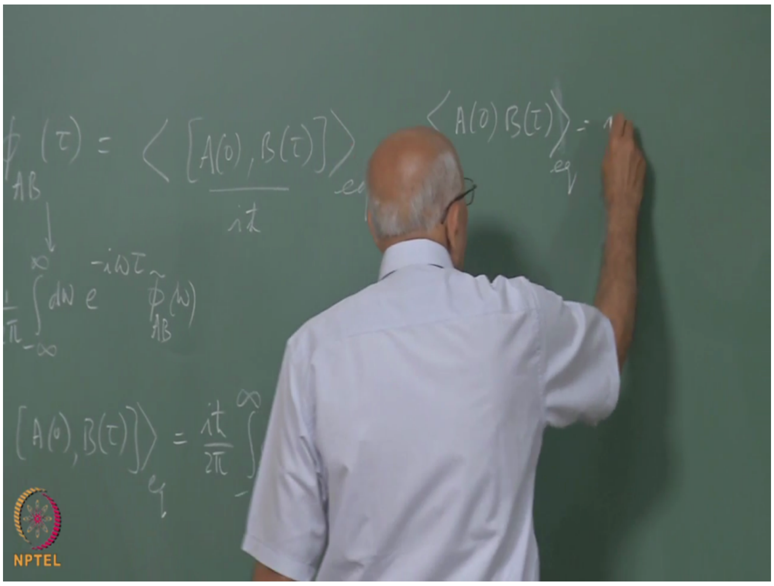

This immediately led us to expression for each of these terms A of 0 B of tau and the other way about and if you recall let me write those down, so A of 0 B of tau in equilibrium quite equal to the same thing but ih cross over 2pi an integral d omega minus infinity to infinity of course e to the minus i omega tau and then we had this factor phi AB tilde of omega.

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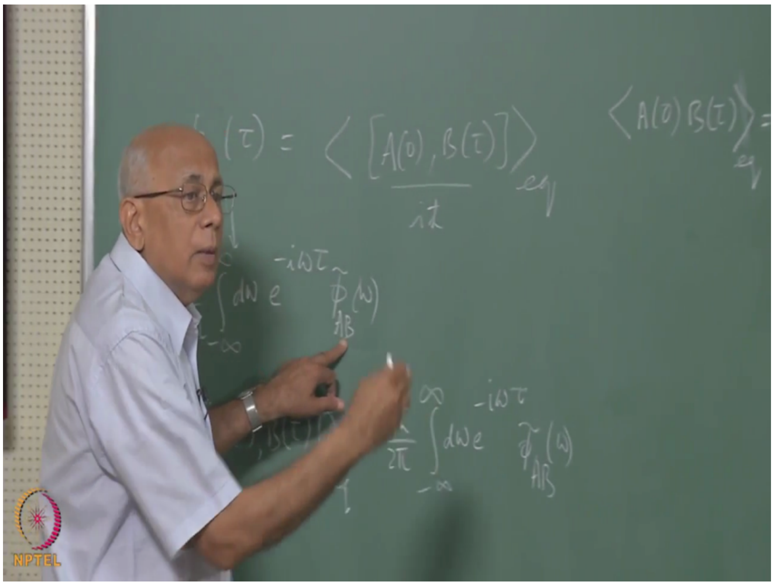

We exploited the fact that this quantity here as far as the omega dependence was concerned at the sequence of Delta functions, we use that to write the rest of the wherever this omega mn appeared we wrote it as Omega.

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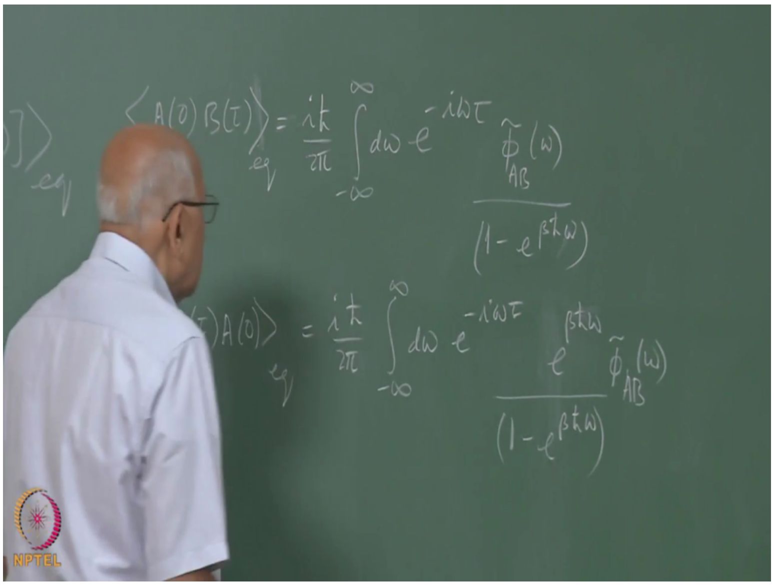

And we could pull it out of this series and that led to this representation 1 minus and similarly we found that B of tau A of 0 in equilibrium was equal to ih cross over 2pi integral d omega minus infinity to infinity again the same thing e to the minus i omega tau time,now we had an extra factor here e to the beta h cross omega phi tilde AB of omega divided by 1 minus e to the beta h cross omega, so that when you took the commutator the 1 minus this cancelled against the denominator and you are back to this here.

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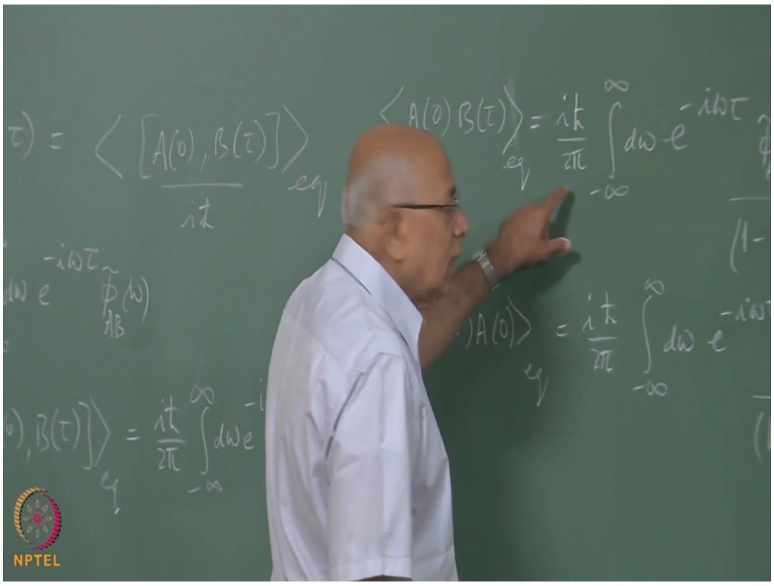

Perhaps I left this factor out, I am not sure if you put this 2pi in, we did put this factor in, okay. There was another place where I think we did leave out a 2pi factor and we will come to that in a short while. So the anti-commutator.

So this says A of anti-commutator shortly plus meaning the anti-commutator equilibrium this is equal to ih cross over 2pi integral d omega e to the minus i omega tau minus infinity to infinity and then of course you have this phi AB tilde of omega. So you have one last divided by 1 minus which was equal to.

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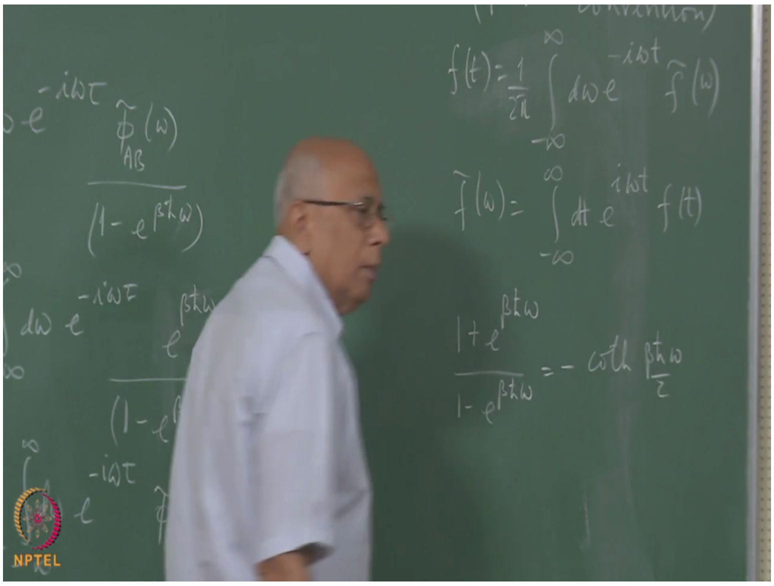

So we wrote 1 plus e to the beta h cross omega over 1 minus e to the beta h cross omega was equal to minus cot hyperbolic beta h cross omega over 2, minus because this fellow has a minus here this is e to the minus whatever it is.

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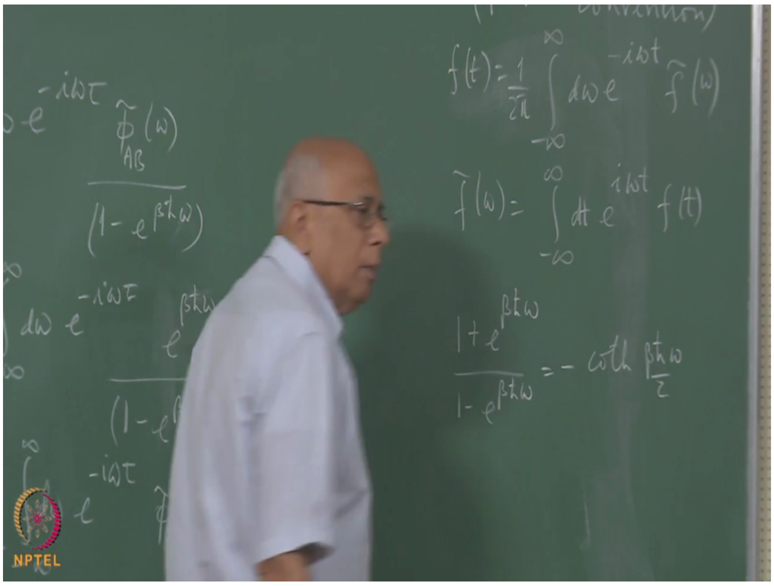

But then we also discovered that the average energy of a harmonic oscillator of natural frequency omega at temperature t in thermal equilibrium that we discovered we call that e beta of omega was defined to be the average energy of a harmonic oscillator at temperature t average though thermal fluctuations this was equal to h cross omega over 2 cot hyperbolic beta h cross omega.

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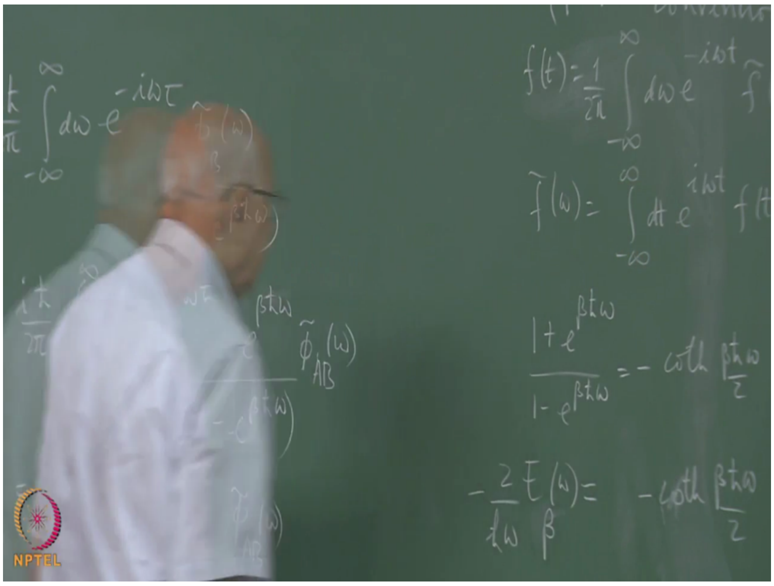

So if you multiply this you get to 2 over h cross omega is this fellow and therefore minus this fellow is minus this, so we can rewrite this now this quantity here, pardon me.

“Professor -Student conversation starts”

Student: You do not put that part.

Professor: Yes I am just going to write it, explicitly in terms of this fellow here.

“Professor-Student conversation ends”

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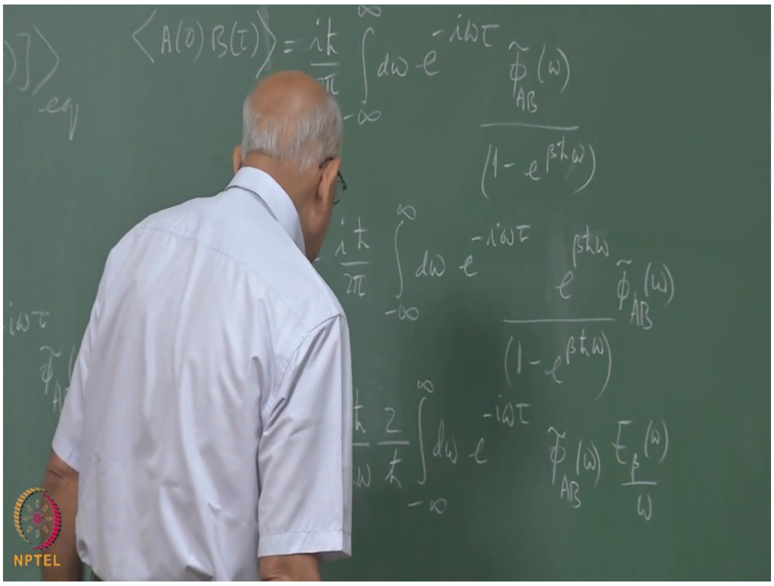

So what we have is 1 plus over 1 minus which is minus cot hyperbolic which I am going to writers 2 over h cross omega, so this is equal to ih cross Over h cross omega and then 2 divided by h cross e beta of omega divided by omega this is (()) (8:47) variable, so you have to have it inside the integral, you cannot pull it out and is this okay? Or is there a sign error is? Should I do?

“Professor -Student conversation starts”

Student: (()) (9:00)

Professor: There cannot be an...

Student: 2 should cancel with the 2, right?

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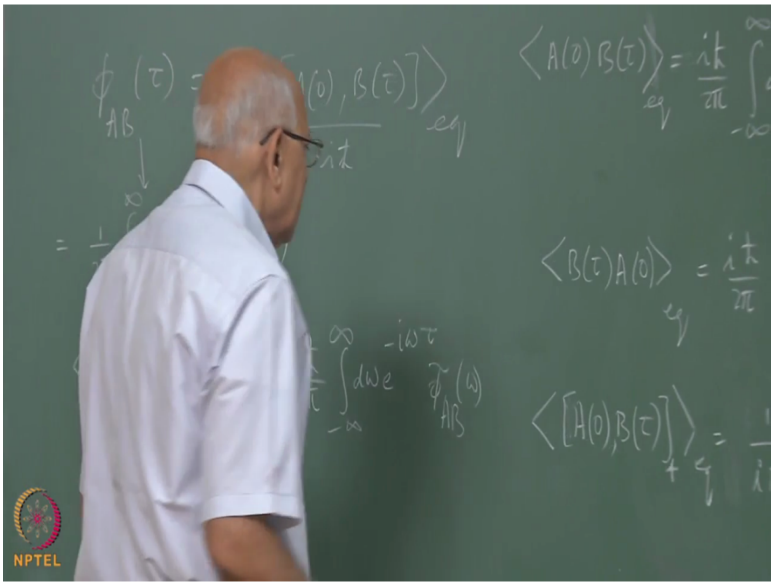

Professor: Oh! So it is at 2 pi, sorry? ih cross over 2 pi, so this was ih cross over 2pi and there was 2 over h cross, right? So this goes away, so it is i over pi minus i over pi, so this is 1 over i pi and that is even correct, okay. So that is a nice compact formula useful to remember.

"Professor-Student conversation ends"

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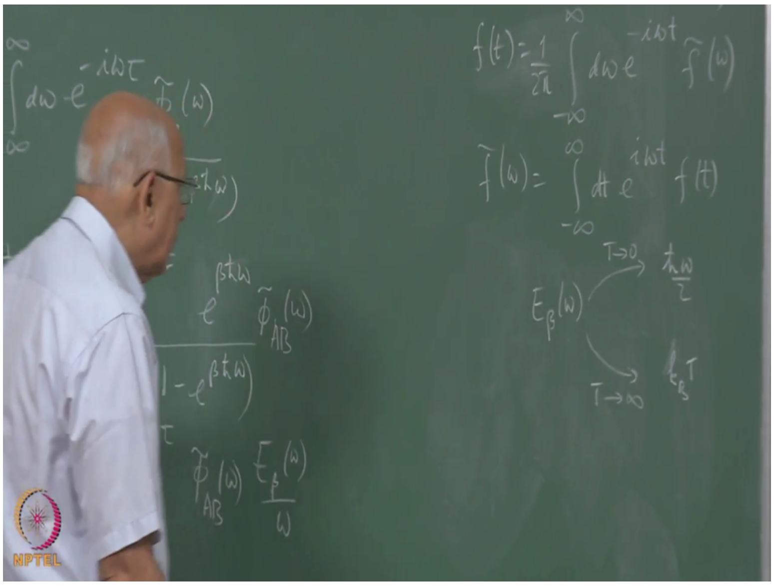

Remember that this quantity, remember that e beta of omega as t goes to 0 goes to h cross omega over 2 and for large T, T tends to infinity this goes like k Boltzmann T which is the classical value, okay. The average energy is average of the kinetic energy plus they have (())

(10:00) the potential and in each case the classical equal partition theorem says that it is half kt per degree of freedom, okay.

Actually it is half kt this is also for kinetic term also, so what is a correct statement of that equi-partition theorem? Just as a side remark. For every quadratic term in the Hamiltonian you get half kt, right? Because it is a Gaussian integral then and then you get a half kt, right? Right, so this quantity here will tell you the exact answer but in between for intermediate temperature but those are the limiting cases against with we are going to check various things as we go along.

Now this immediately proves by the way there was also a relation between the susceptibility and the spectral function itself and that is probably where we left our factor of 2pi, so let us fix that. Remember that let us forget about these subscripts for awhile.

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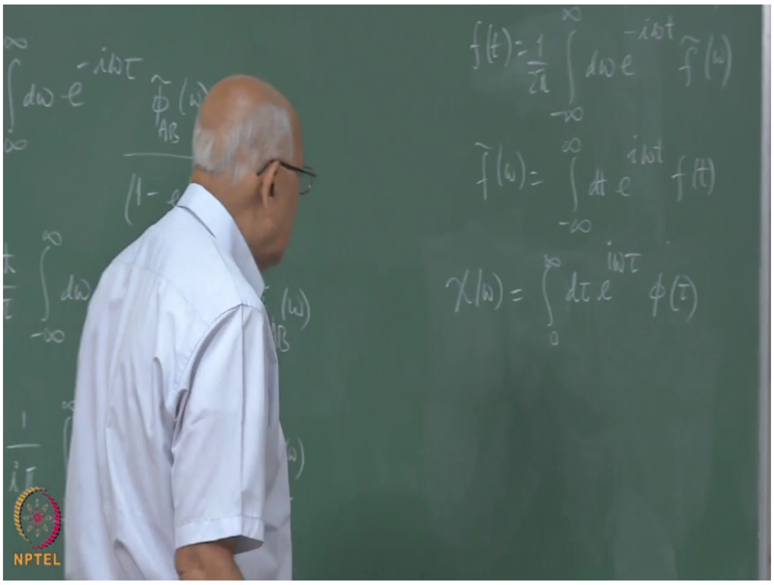

So remember that we had phi remember that we had kai of omega was an integral from 0 to infinity d omega e to the i omega tau phi of tau, sorry d tau this was the definition of the general i susceptibility, right?

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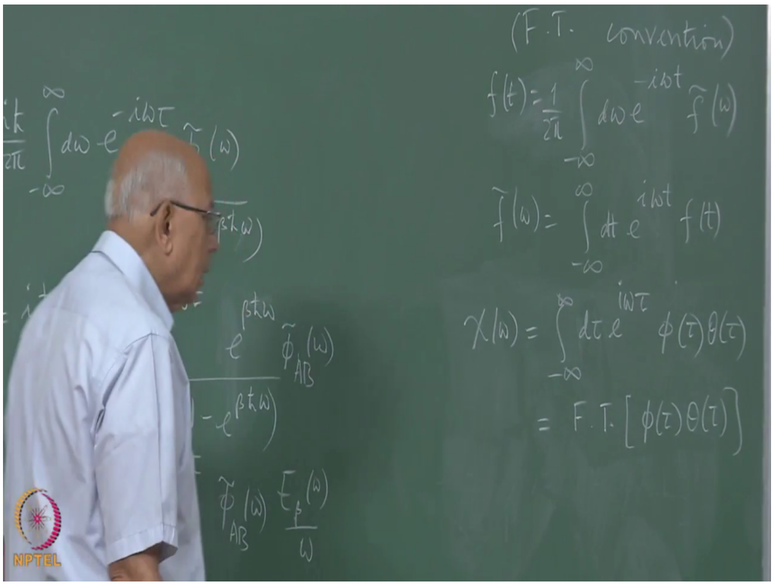

And the question was how is it related to the full Fourier transform of this fellow? So the argument was that we could write this as theta of tau from minus infinity to infinity and you want to make no mistake there. So this says, this is equal to the Fourier transform the quantity phi of tau theta of tau, just to recapitulate what we have already done.

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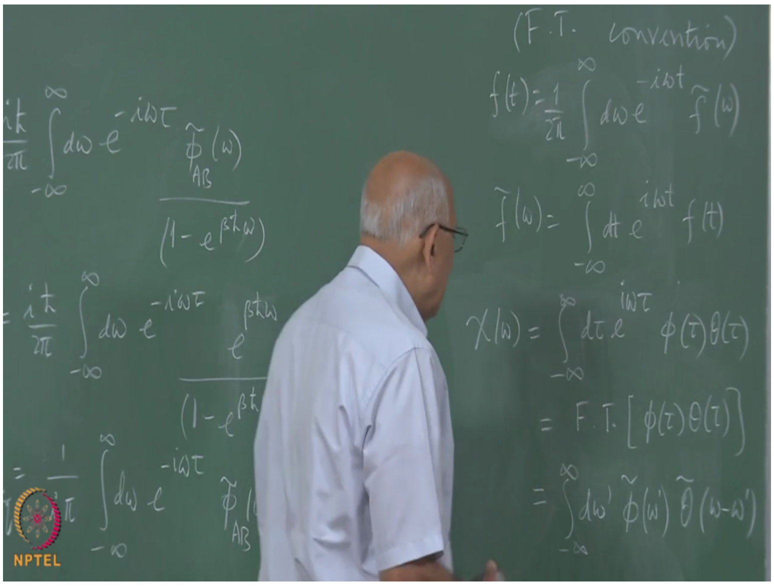

So we need to find the Fourier transform of this product here but that is equal to, if I call this phi tilde before you transform that is how definition, this theta tilde this is (()) (12:14) integral Of what, we now use the convolution theorem and I think that is where the 2pi got lost, what does the convolution theorem say? So this is equal to, it is a Fourier transform, right?

So this is equal to an integral d omega prime minus infinity to infinity phi tilde of omega prime theta tilde of omega minus omega prime, right? Divided by 2pi, there is an extra 2pi here because the way we did this was notice where the 2pi's are going by our convention.

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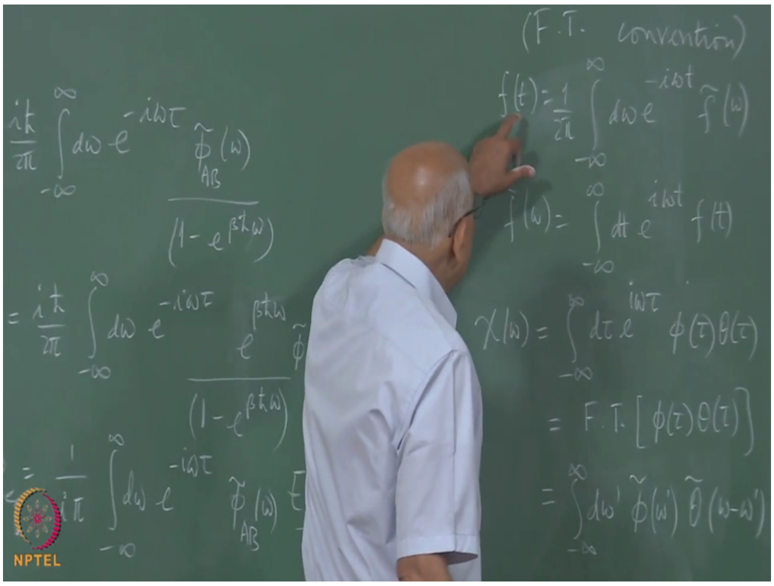

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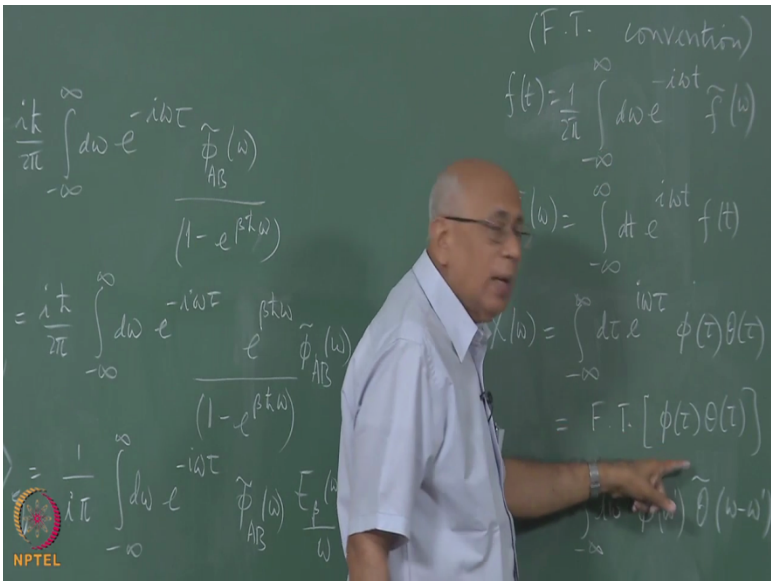

When you go from t, when you write t in terms of omega there is 1 over 2pi, so you are going to have to do that here and here when you are computing the Fourier transform, so there is a 1 over 2pi whole square and then you get an exponential which will kill one of the frequency terms.

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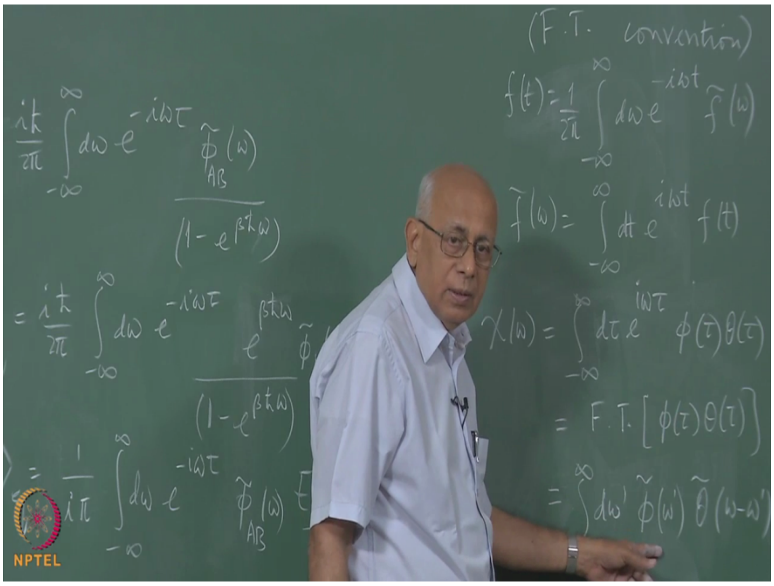

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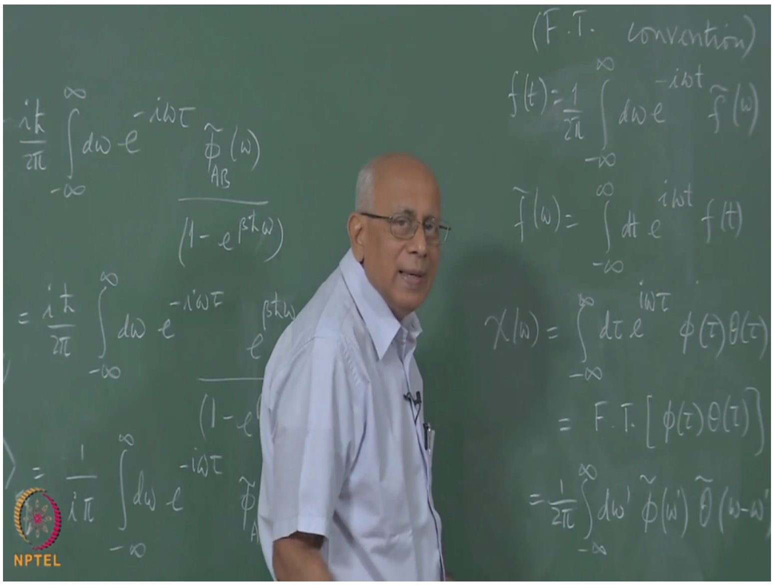

So when you integrate over time you get 2pi Delta function and the 2 pi will go against one of the 2pi's leaving a delta function you do the other omega integral and then you end up with this conversion. So there is 1 over 2pi, okay. And this is a factor writing we left out because I was playing around with this and then I was not getting, well, I will tell you where the inconsistency came up and where I discovered that this probably there was a factor missing but it is traced to this point, okay.

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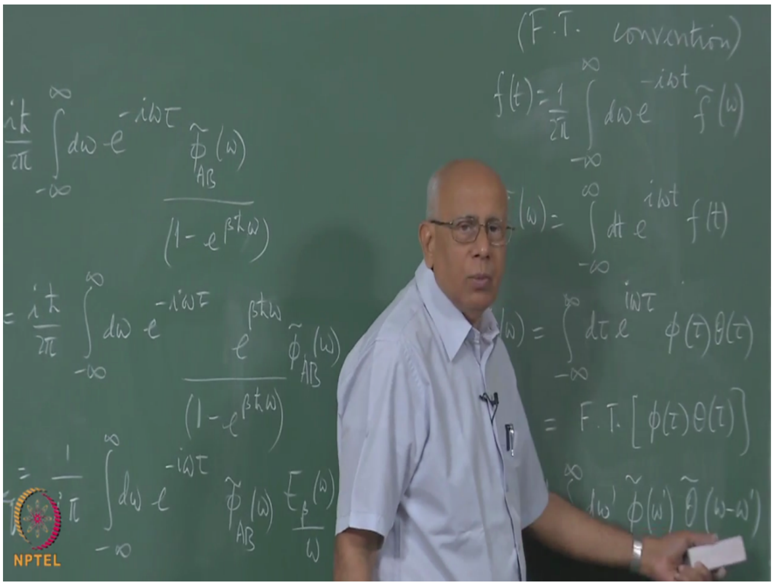

So, now all that remains is to find out what is this quantity and watch that quantity?

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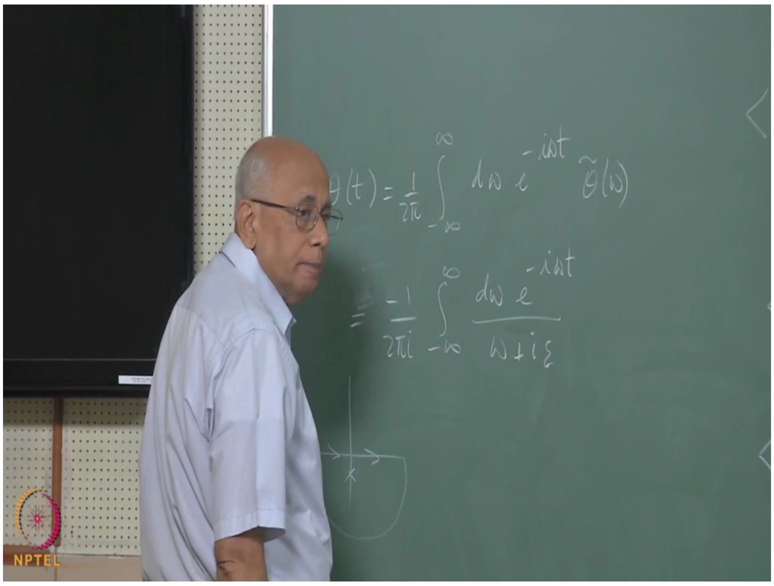

Alright, so let us do this again, so we do not make mistakes this time, we need a representation of theta of t, this is 1(()) (14:09) is greater than 0, 0 here less than 0 and our argument was its equal to minus infinity to infinity d omega with omega we had like to have e to the minus i omega, so e to the minus i omega t times theta tilde of omega 1 over 2pi that is the definition of this fellow here but what is this equal to?

You could also write this as an integral from minus infinity to infinity d omega e to the minus i, omega t divided by and you need to have the contoured closed in the lower half plane because there is a minus sign here, so if I close it in the lower half plane am going to get a damped exponential down there. So the pole should be in the lower half plane, so this is got to be omega plus i epsilon.

And when you compute it, you have a pole at this point and that is your contour integral and you are closing this by adding this well chosen 0, you are going to get minus 2pi i times the residue, the residue is one sitting over here, right? When you put omega equal to 0 you just get 1 there. So what is the fact that should multiplied? Minus 1 over 2pi i. So when I multiplied by minus 2pi i this is going to give you 1 that is going to give you e to the Epsilon t and Epsilon goes to 0 you get 1, so that is the representation therefore if you compare it with this you can read of what this fellow is.

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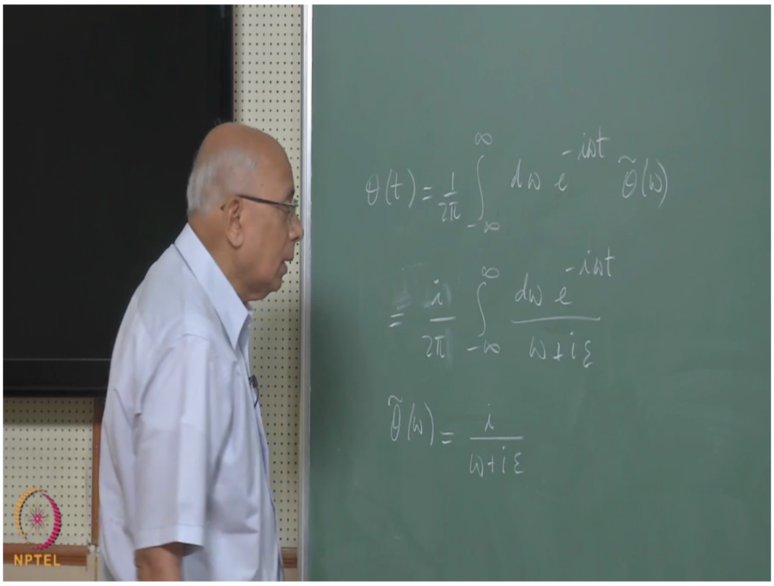

So it immediately tells us that theta tilde of omega equal to, let us write this as plus i over 2pi, so by comparison Theta tilde is equal to i times divided by Omega plus i Epsilon, there is 2pi but goes in the definition of theta tilde, right?

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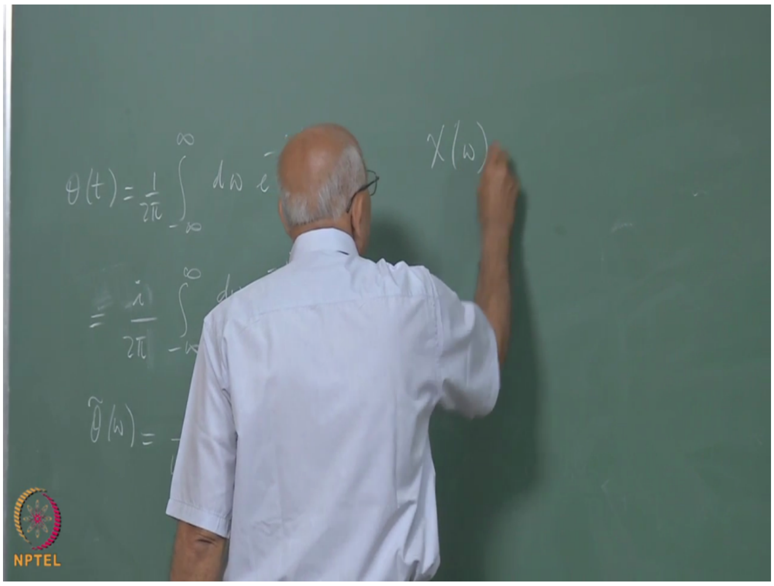

So now we have set, let us put that in here and therefore it says, we can remember this is now begin erase all this. So we have Kai of omega equal to 1 over 2pi and integral d omega prime phi tilde of omega prime minus infinity to infinity

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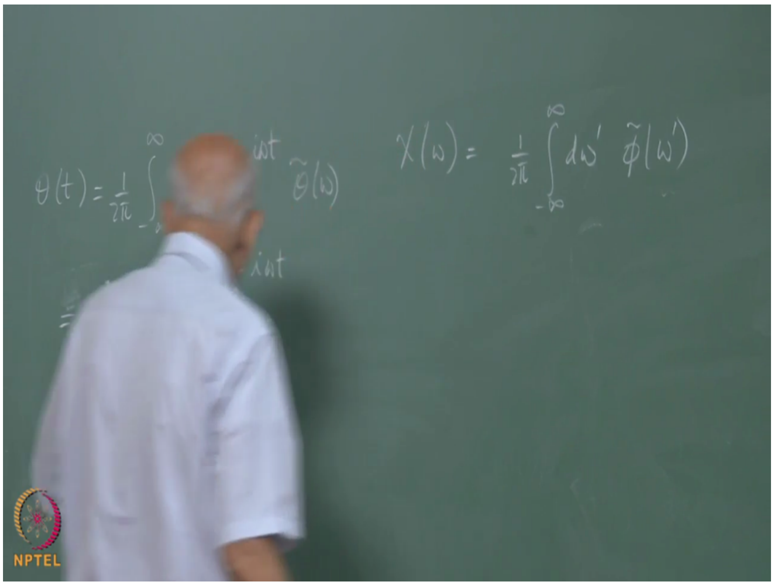

And then you have to multiply it by i.

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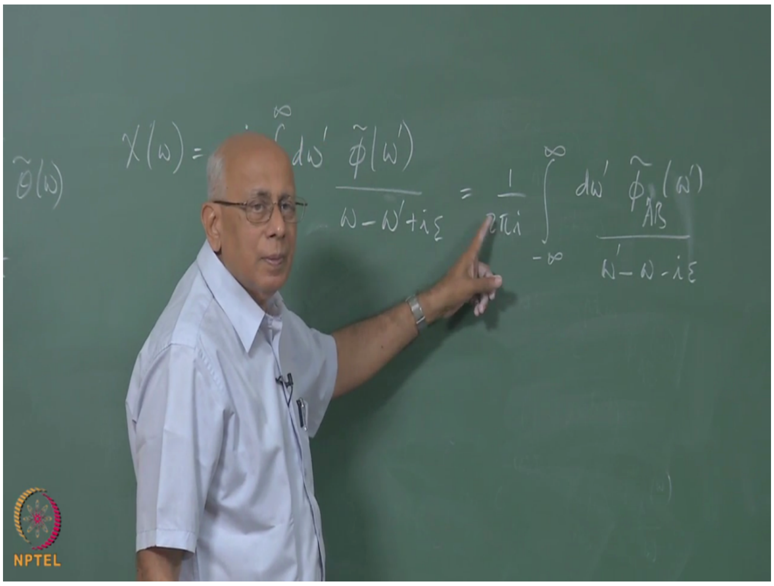

So it is i divided by Omega minus wherever omega appears you put omega minus omega prime plus i Epsilon, okay. Or if you like this is equal to 1 over 2pi i and this is the way it is convenient to use it, d omega prime phi tilde, now let us put all the AB's and so on, omega prime divided by omega prime minus omega minus i epsilon I took a minus sign out and brought this i downstairs, okay.

"Professor -Student conversation starts"

Student: 2pi.

Professor: This was the 2pi we missed we wrote minus i times the rest of it but this 2pi is in imp and you will see why in a minute. There is so many of these factors around that but this is the place where there was a mistake in here, okay.

“Professor-Student conversation ends”

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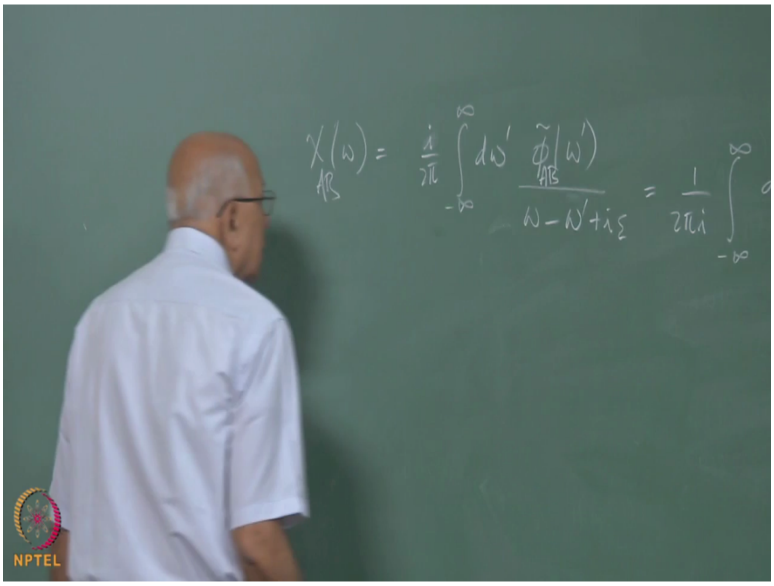

Now once we have this representation then we are in good shape, go back to the other statement we made now and look at this. We derive dispersion relation for this etc.

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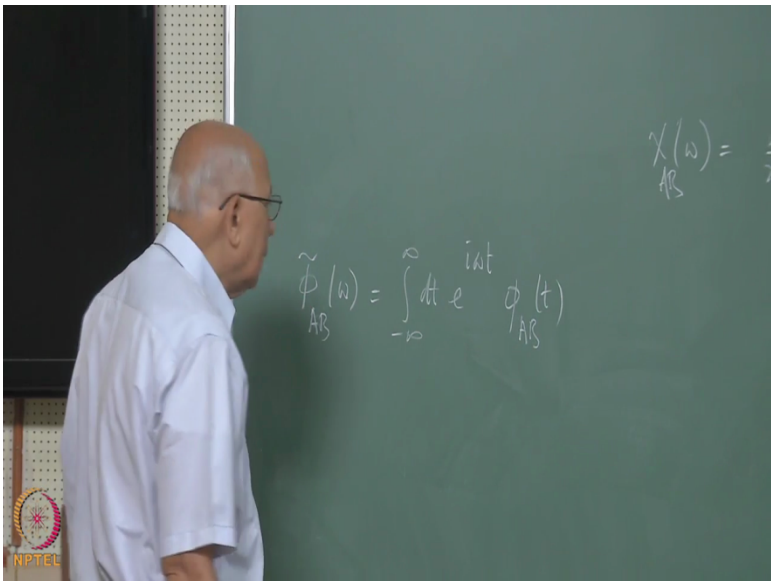

Now look at phi tilde AB of omega by definition this is equal to integral from minus infinity to infinity e to the power i omega t phi AB of t by definition, okay.

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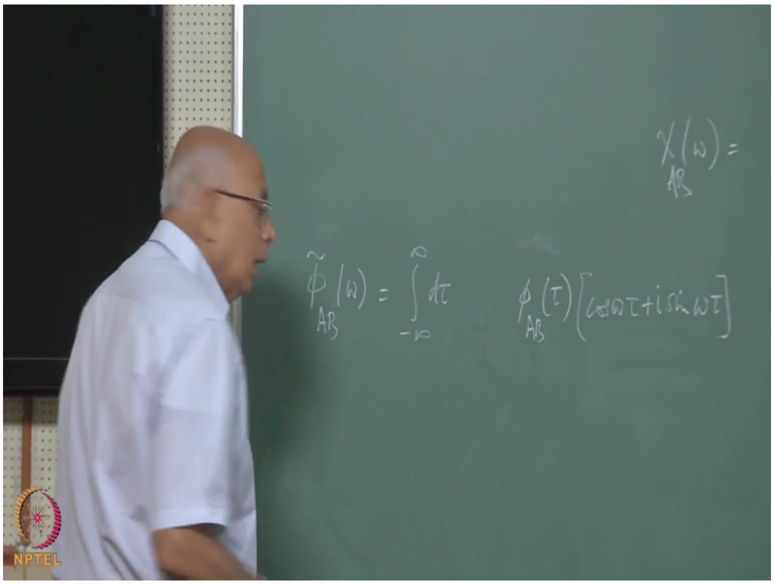

Or since we used tau let me use that for the time difference of d tau e to the i omega tau phi AB of tau I write this as cos plus i sine immediately, so you can write this as cos omega tau plus i sine omega tau.

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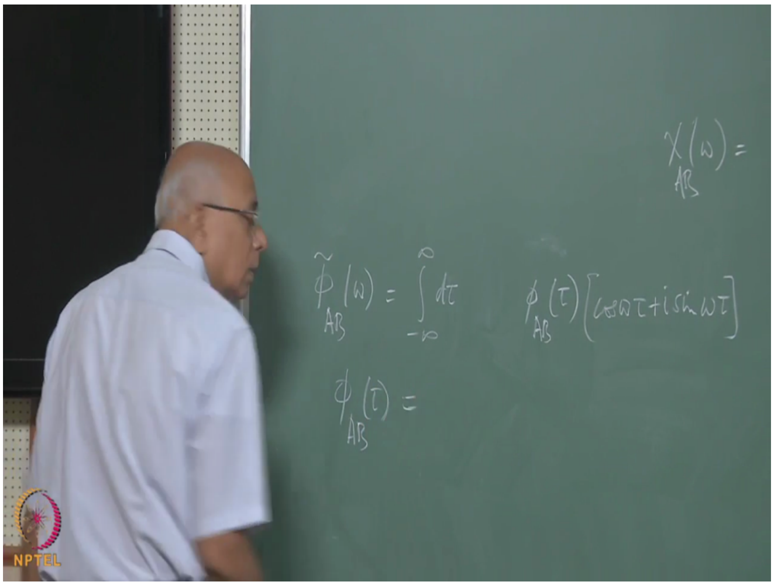

Recall that this quantity phi AB of tau both classical and quantum cases, this was equal to beta times A dot of 0 B of tau in equilibrium, okay. Where this is defined as in the classical cases just the product in the (()) (19:51) case this is an integral over from 0 to beta and then there was this other (()) (19:55) transform of this operator on either side, so sandwich between e to the lambda h not e to the minus lambda h not and then you take trace with e to the minus beta h not, okay.

Now by the way as an aside we did all those manipulations showing the reality of this correlation the in the case where AB are hermitian we showed it was symmetric.

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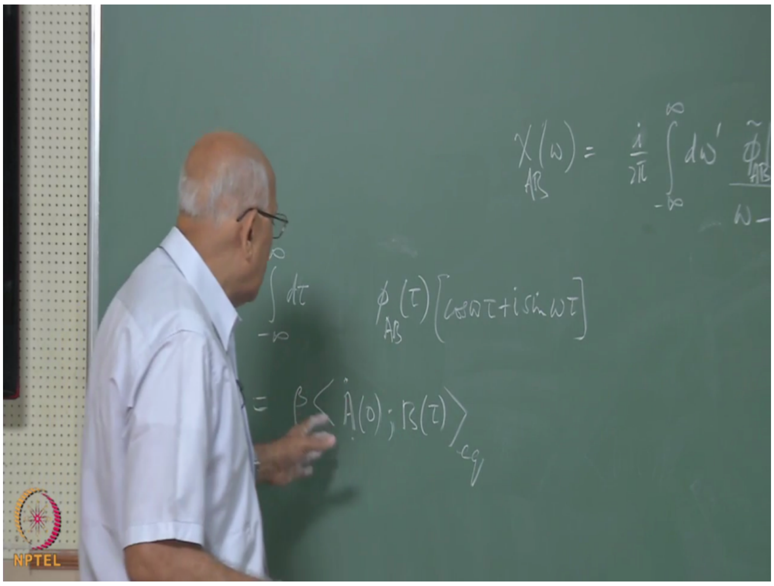

This quantity was met regardless of the time arguments of these quantities whether they were hermitian or not. We showed also that the reality and symmetry and the stationarity of this quantity regardless of what these time arguments were. Throughout we manipulated by taking the cyclic property of the trace with e to the minus beta h not. You could now ask I never normalised it, I never put anything in the denominator.

I assume trace e to the minus beta h not was 1 but we know you are in the oscillator case, it is not you have to explicitly divide by it. So does it make any of the conclusions wrong, does it invalidate? No because that is a say number it is a pure number on both sides. All the manipulations we did were with the operators can divide by a say number then get back to the correct answer right away.

or if I carelessly wrote or rather wrote in a slipshod way e to the minus beta h not as rho equilibrium instead of writing that divided by trace with the same operator it does not matter you can put that number and it will still, so the final formula this thing here involves taking a trace with respect to the normalised density matrix it reviews that fact, okay.

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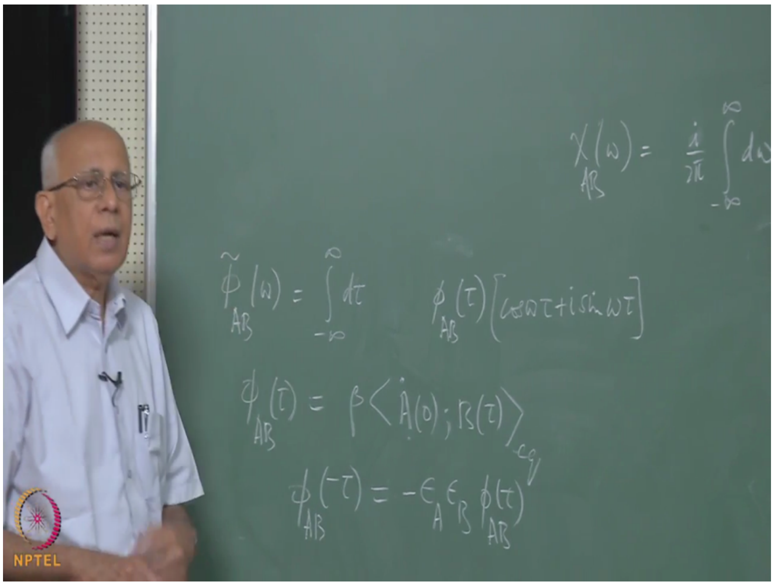

Now the statement was phi AB of minus tau was equal to Epsilon A Epsilon B phi AB of tau with a minus sign where Epsilon A if you like (()) (21:56) and Epsilon B is (()) (21:59) in a sense that you ask what happens to these operators under time reversal, how do they transfer? Position for example would not be changed momentum will change linear momentum will change, what is about angular momentum?

That will change to, it does not matter what sort of angular momentum it is, any angular momentum must transform in exactly the same way independent of the origin. What about more complicated combinations? What about the Hamiltonian itself? Does not change, so the point is you are looking at those Hamiltonians which do not change under time reversal because P square over 2m does not and therefore every other part of the Hamiltonian also should not.

Then in that case you have a well-defined Epsilon A, well-defined Epsilon B in this whole thing it is either plus one or minus one. So this quantity is either symmetric or antisymmetric, okay. You can artificially construct cases where it is not but then you will have to be careful, those response functions would not have well-defined symmetric properties.

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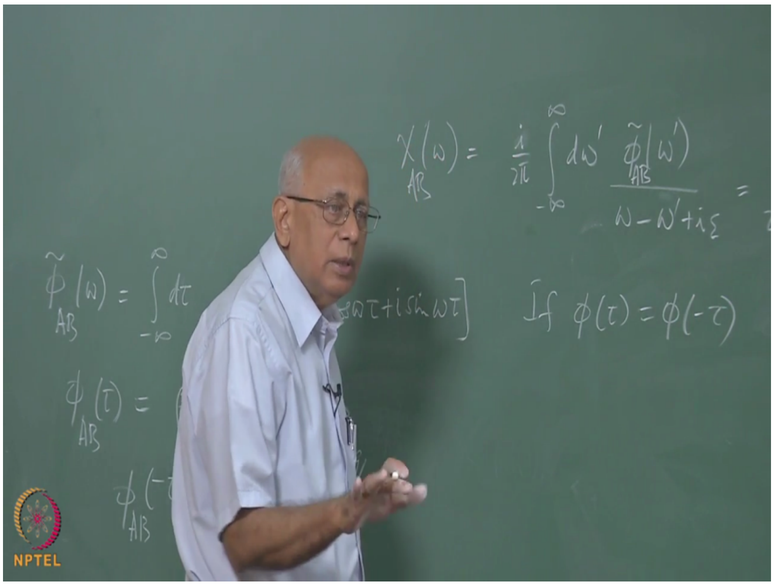

But in the cases that they do if this is symmetric, so we can now write this down immediately if phi of tau equal to phi of minus tau and now we will assume that A and B are hermitian operators, so these things are real phi AB is real quantity the Kai here has then appropriate transformation properties, okay.

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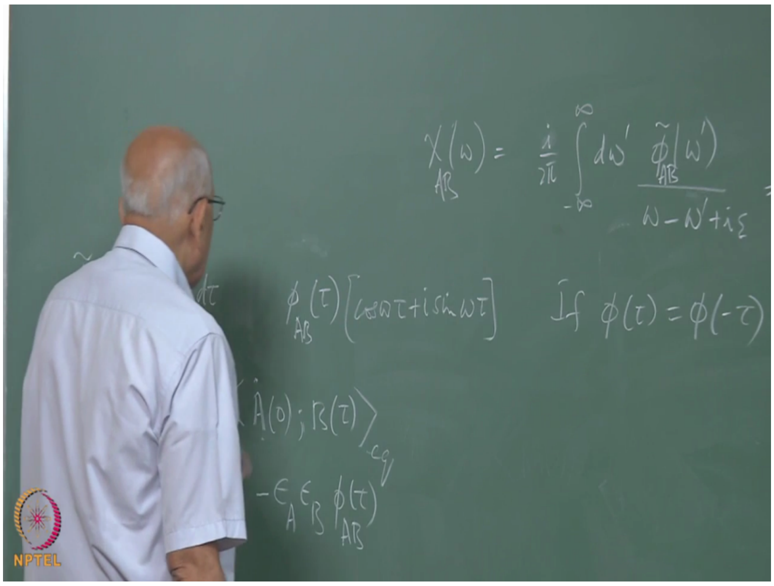

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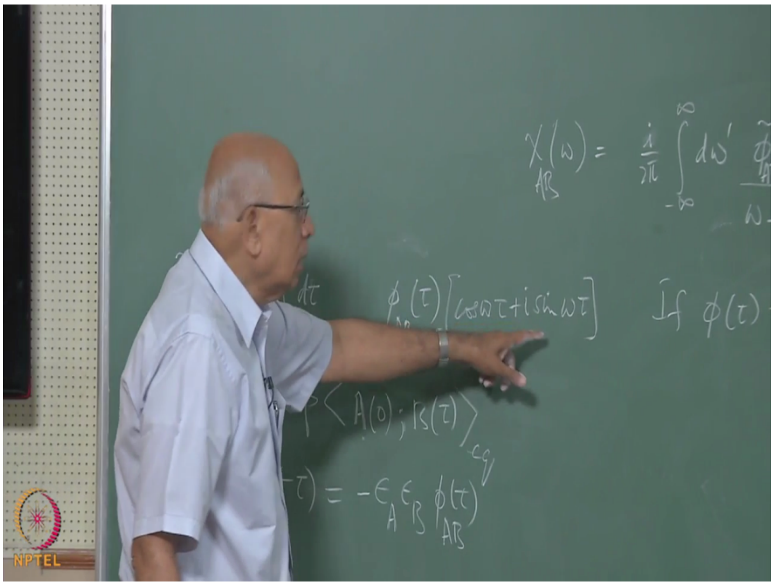

So we have shown the reality of this quantity then you can say that this portion will be 0 identically because it is an odd function and this is an even function, so this says phi AB tilde omega is equal to twice integral 0 to infinity dt cos omega tau phi AB with a factor of 2, right? And what is that equal to?

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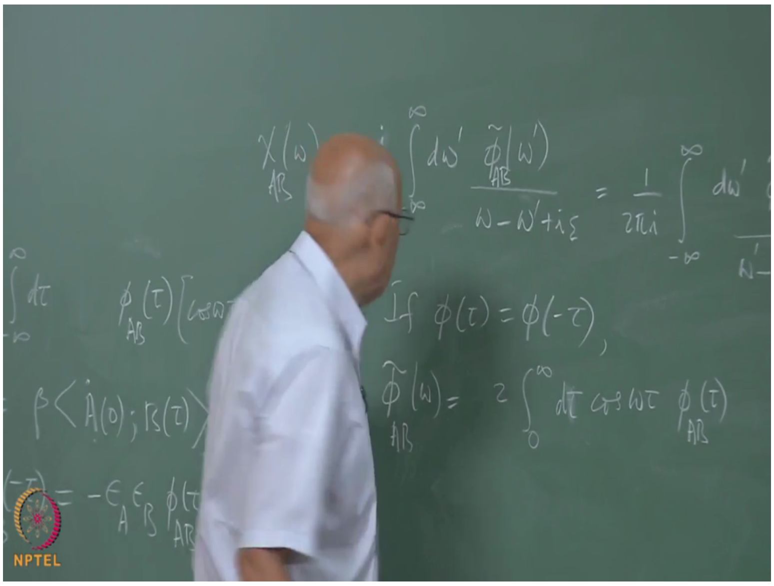

It is the real part of our susceptibility because the susceptibility is defined as 0 to infinity of that integral in the real part of e to the i omega tau is cos.

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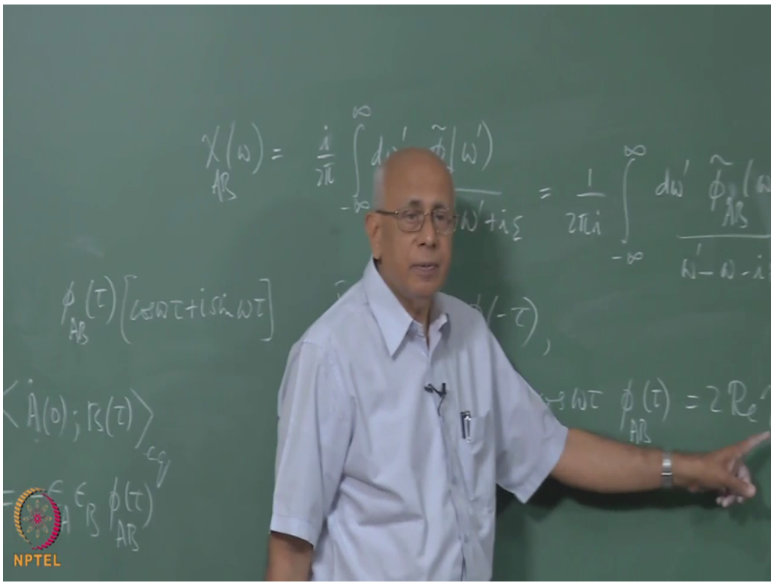

So this is equal to twice the real part of Kai of Omega. So there is a direct relation between the spectral function and at least part of the susceptibility.

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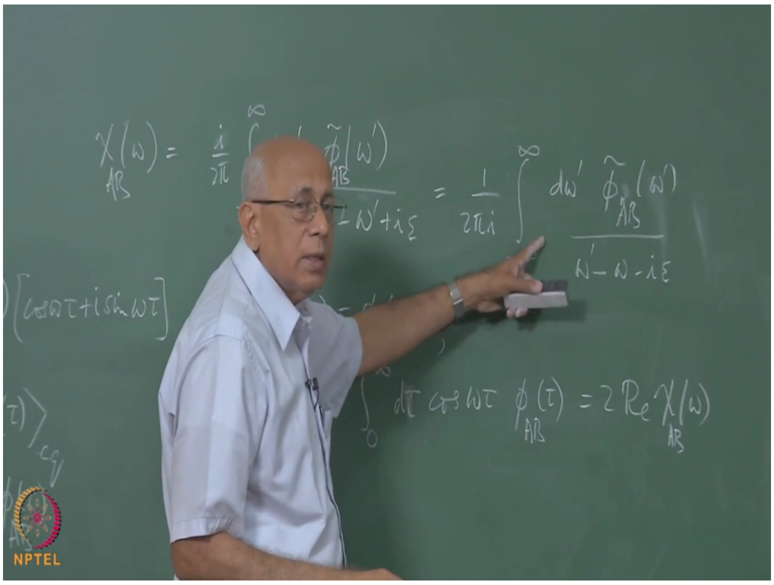

Now let us write that other relation we derive down, so that because why I need to write this down? This was equal to, okay here we are, we need to keep this in mind we are going to put this in here and see what happens.

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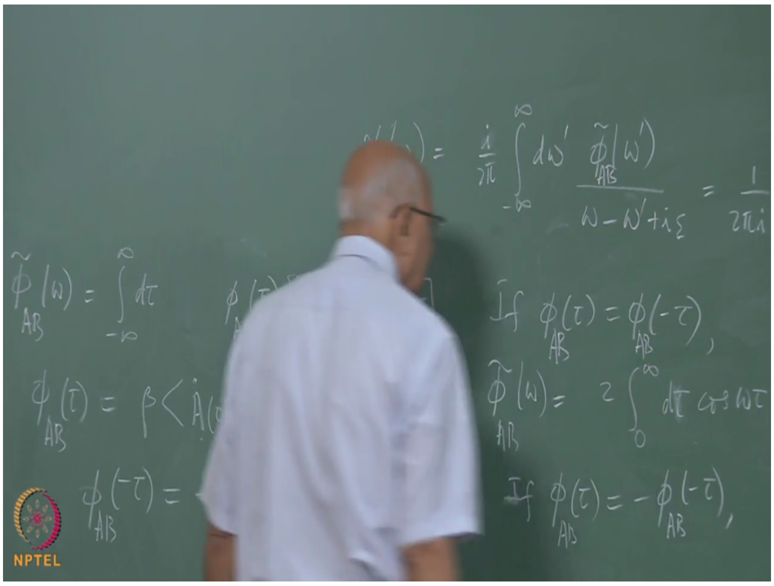

On the other hand if phi AB of tau equal to minus phi AB of minus tau is a AB, okay.

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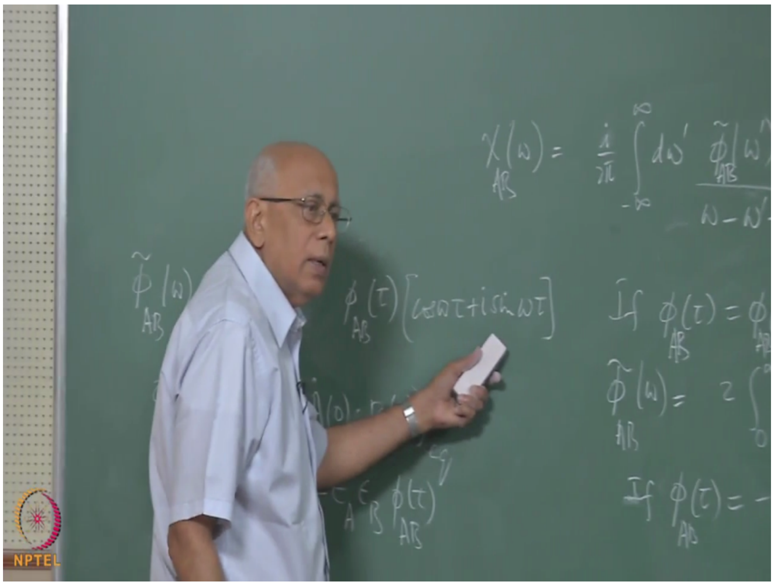

Then phi tilde is equal to only this quantity here and it is 2i sine whatever it is.

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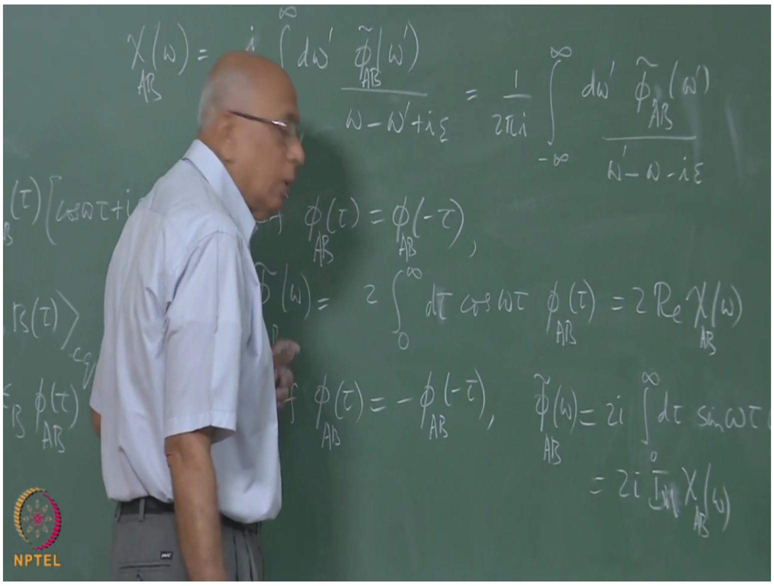

This is equal to 2i integral from 0 to infinity d tau e to the i omega tau phi AB of tau which is equal to i times the imaginary part Kai AB of Omega sine sorry tau omega tau that is 2i I can say imaginary part. So in either case you have a relation between the spectral function and either the real or the imaginary part.

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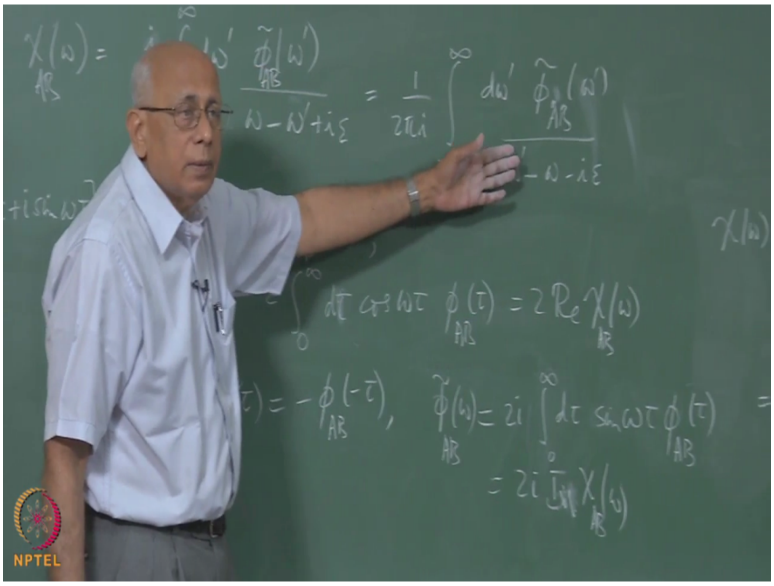

Now let us see how the tallies with this fellow here? We have a formula which says Kai equal to this fellow out here directly and now let us look at the case where phi happens to be a symmetric function then we know that this quantity here, this thing here can be replaced by twice the real part of Kai, okay. So what is it actually saying?

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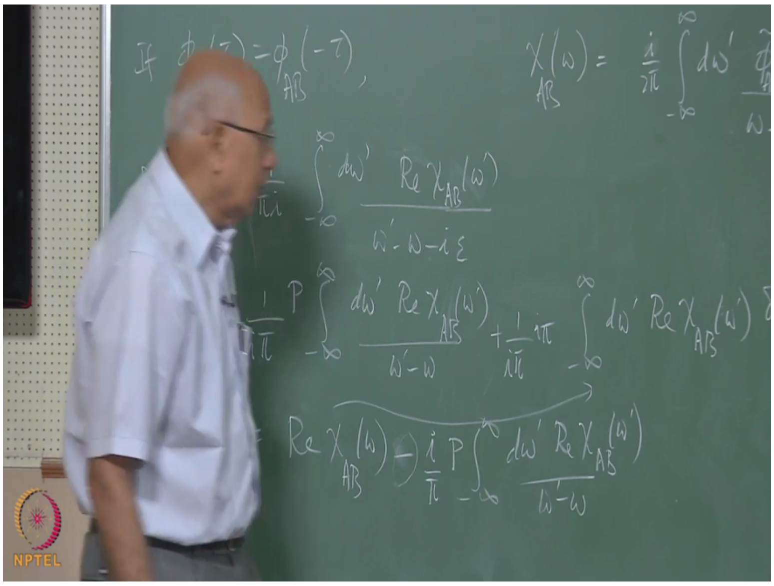

This saying that and we now must check consistency, so in the symmetric case phi AB of tau equal to Phi AB of minus tau. In this case we must have Kai AB of omega must be equal to what we are doing is right, 1 over 2pi i an integral minus infinity to infinity d omega prime and then twice the real part of the same object, the twice goes away, so real part of Kai of Omega prime divided by omega prime minus Omega minus i Epsilon Ab of omega prime.

The question is, does this make sense or not? Well, we should use the representation of this sum up the principal value plus i pi delta. If there is any justice in the world all the factors will come out right, okay. Equal to 1 over i pi principal value minus integral d omega prime real part kai of AB of omega prime minus omega plus i pi delta because there is a minus sign here with respect to the variable of integration.

So this pole is in the upper half plane and therefore you get plus i pi delta, so plus 1 over ipi ipi times an integral minus infinity to infinity d omega prime real part of Kai AB of omega prime but delta function of omega prime minus omega which is equal to the real part or write that term first real part of Kai AB of omega that is this portion the ipi cancels out minus i by pi can say principal part of integral minus infinity to infinity d omega prime Real kai AB of omega prime over Omega prime m minus omega But together with the minus sign and the P over pi this is precisely the imagine part with the minus sign.

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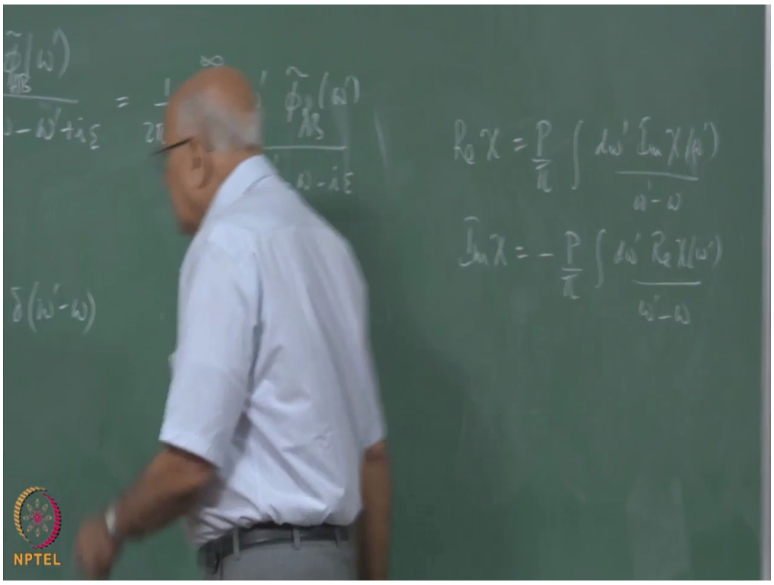

So recall the dispersion relation which we already derived which was real part of Kai or equal to P over pi integral d omega prime imaginary part of Kai of Omega prime over Omega prime minus omega and imaginary part of Kai equal to minus P over pi integral d omega prime real part of Kai of omega prime over omega prime. So there is an extra minus sign.

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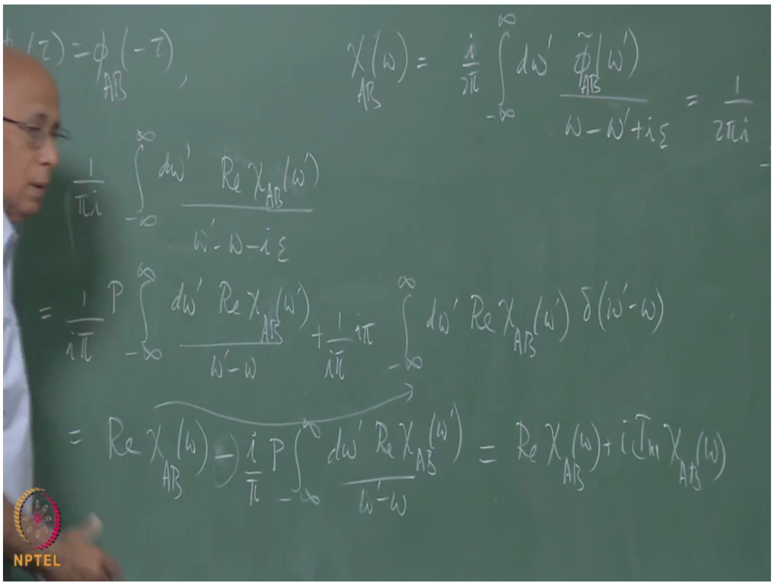

So this says rather obvious fact that this is real part of Kai AB of omega plus i times imaginary part and we have checked that it is indeed correct, so all the factors are in place now there is no mistake anywhere. Says Kai equal to real part of Kai plus i times the magnitude part of...

Similarly check out what happens if the odd function, again you will discover that there is an identity for the imaginary part and real part will be represented in terms of an integral over the imaginary part, make sure the factors are right in that case too.

Alright, I went through this lengthy exercise because this factor of 2pi bothered me and really could not figure out where we have lost this factor and it was not in the relation between phi and the kai the real or imaginary parts but rather convolution theorem. Okay, now let us get back to where we were let us go back to this representation that we had, let us see where it takes us. These are called moment relations, so let me write this down quickly.

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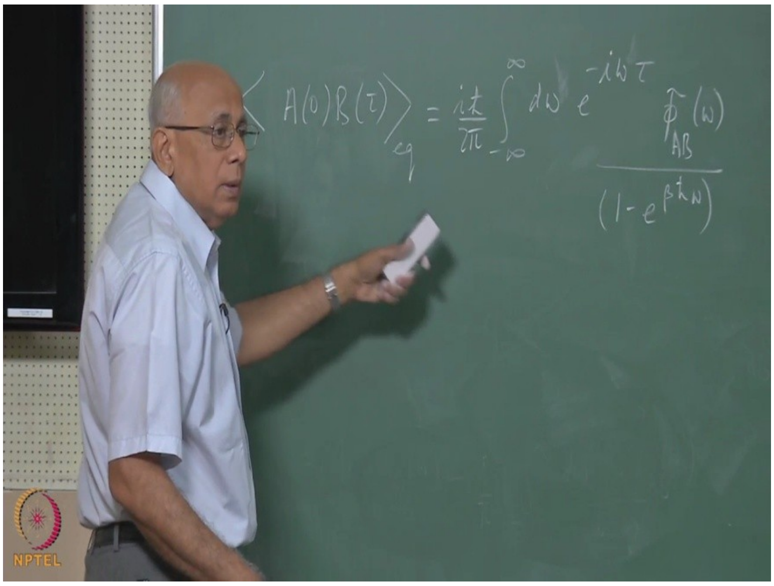

Let us begin with this expression A of 0 B of tau or if there is a slight base, yes. Okay A of 0 B of tau in equilibrium was equal to, we wrote an expression for it explicitly which was equal to an integral from minus infinity to infinity d omega e to the minus i omega tau phi AB tilde of omega divided by 1 minus e to the beta h cross omega, was there a factor here?

Was there a factor outside? There was an ih cross because we brought this ih cross across anything there is also 2pi, right? Divided by 2pi, right. So if I leave out factors of 2pi it will keep me honest because we do not want through another of these things, okay. So now suppose you said tau equal to 0 then you get an expression for this quantity.

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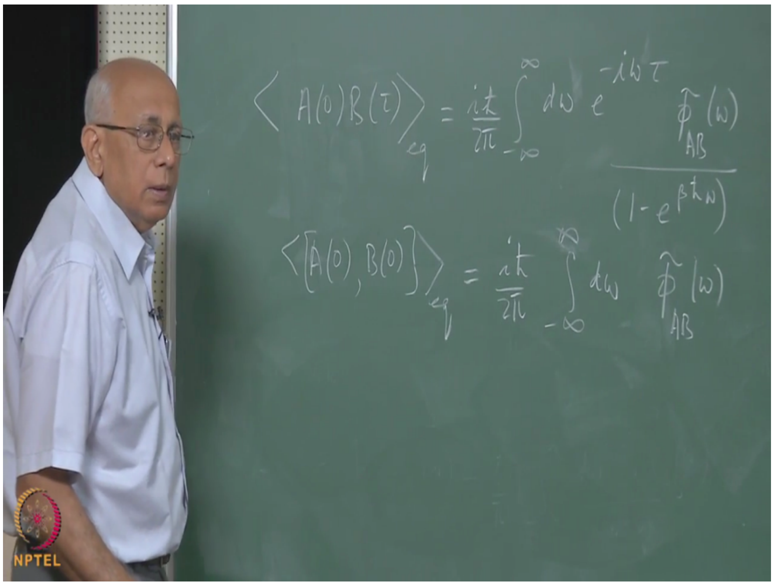

First of all if you look at the commutator itself A of 0 B of 0 equal time commutator equilibrium equal to ih cross over 2pi in that case this factor was not there, you simply had this and nothing else. So it is equal to just the integral minus infinity to infinity d omega e to the minus i, oh! This is gone pi AB tilde of omega. So the equilibrium value of the commutator is something you might be able to compute, if you are given these operators at equal times you can compute what the commutators are using canonical commutation relations very often.

So this is some function of x or P and so is this, you can compute this commutator and you are guaranteed that its equilibrium value is equal to the zeroth moment of the spectral function, you can write this in terms of all the matrix elements, so you are essentially getting some kind of some rules for the time.

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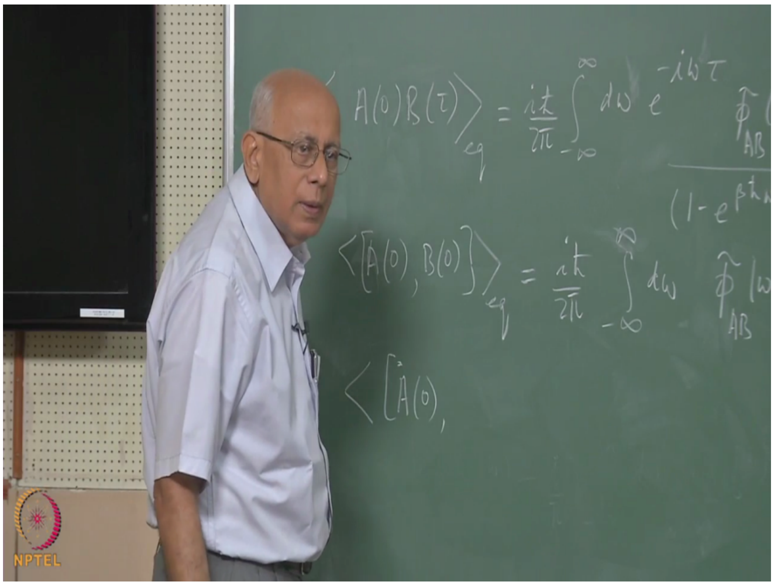

Now what would this be? That was for the anti-commutator, this is the commutator just the commutator, just the commutator, yes. I am just doing the commutator part we will see what is the use of anti-commutator is in a minute. You can actually guess what we are going to do or the anti-commutator, what the use is going to be? But look at what happens with the commutator. So what is A dot of 0 B of 0? What would this be?

Again there is an ih cross over 2pi and an integral minus infinity to infinity d omega, what you have to do to produce at dot? You have to differentiate with respect to t prime, right? So to do is to start with, this by the way is the same as A of t prime let us write it here. This is the same as A of t prime B of t and this fellow stood for t minus t prime.

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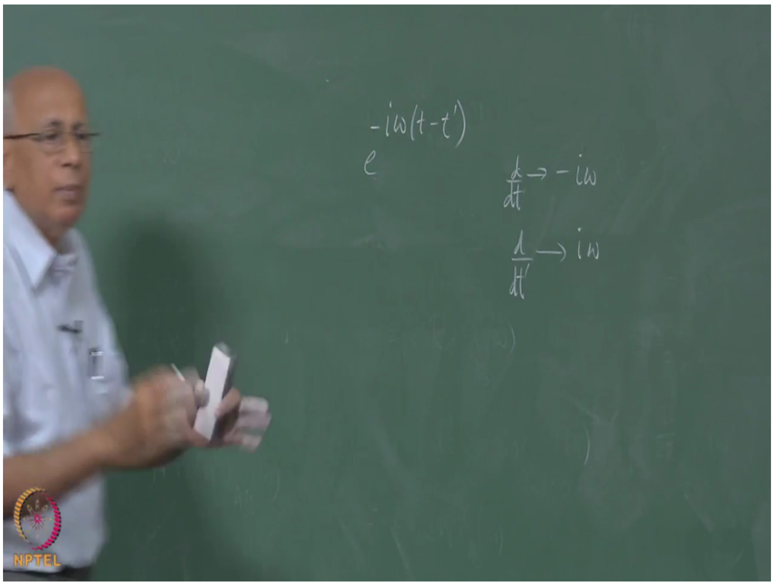

So if I differentiate, so the factor I have is e to the minus i omega t minus t prime, if I differentiate with respect to t I get d over dt brings down a factor minus i omega and after that I will put d equal to 0 if I differentiate with respect to t prime this brings down a factor i omega and we need to keep track of that.

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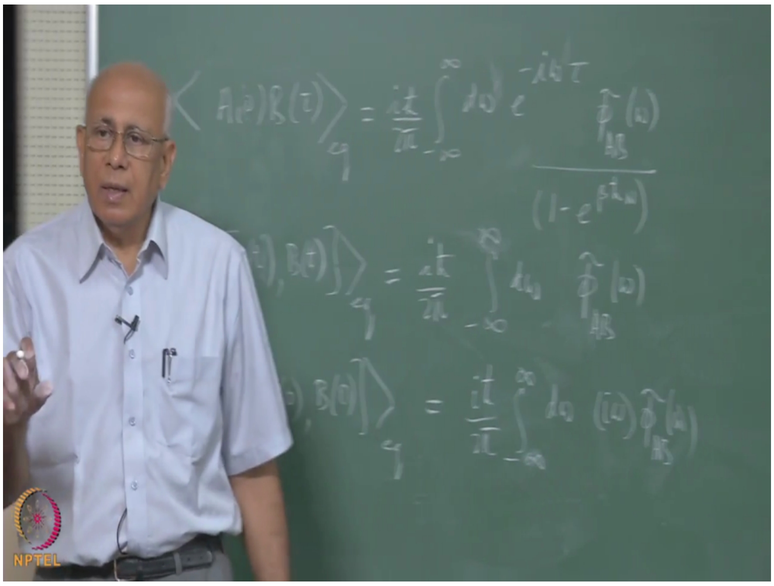

So I differentiate with respect either t or t prime and then set t equal to t prime and by stationarity this is same as 0 out here. So what is this equal to? This is going to bring down a factor i omega outside. So it is i omega times phi tilde of omega. So apart from this i factor it's the first moment of this spectral function and if you recall what the representation of the spectral function was in terms of all those omega mn Delta functions you can carry out this integral and you get omega times those matrix elements must be equal to whatever this point is.

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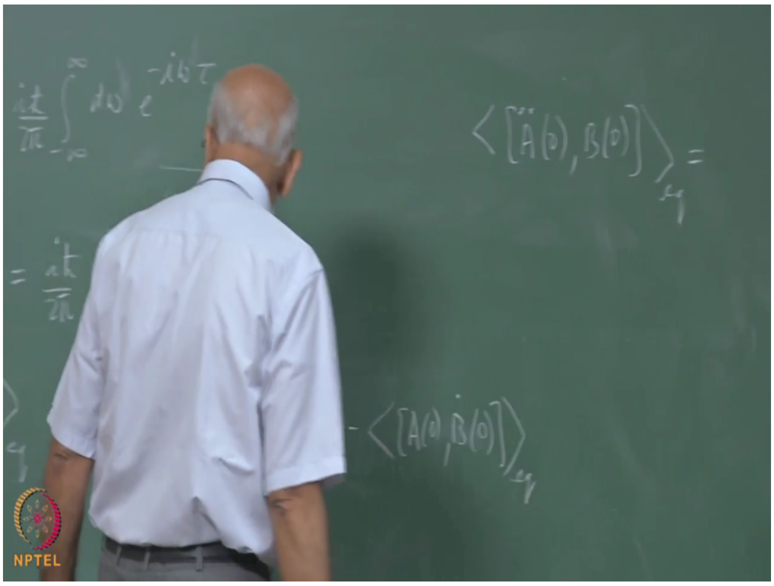

And you can easily see that this is also equal to minus A0 with B dot of equilibrium. Now this game can be played many times over, so what is expectation value of A double dot of 0, B of 0? Remember that these are all operators.

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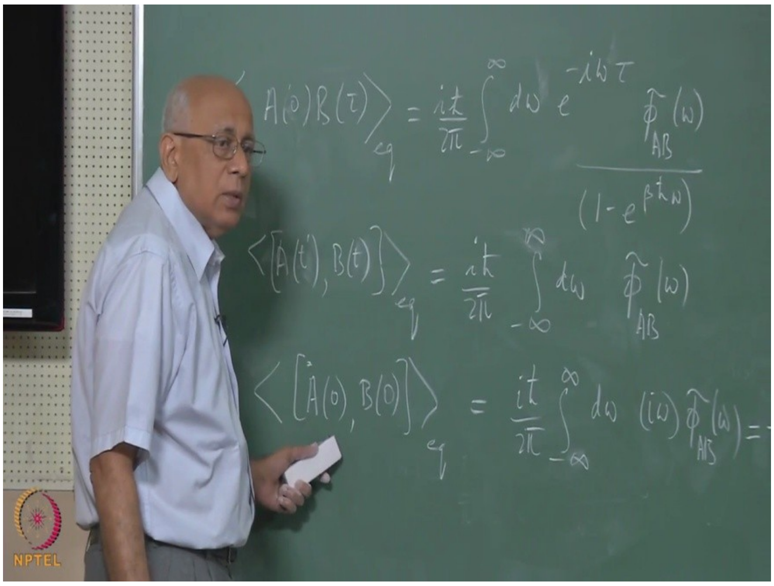

What is this equal to? I differentiate twice with respect to this, so I bring down an i omega twice.

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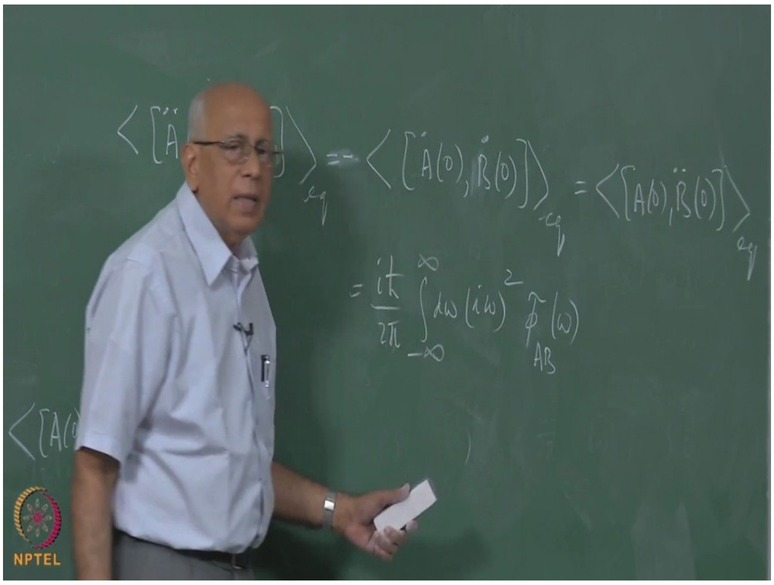

So this quantity is equal to A dot of 0, I could have B of 0 equilibrium with a minus sign and that is also equal to without a minus sign A of 0 B double dot of 0 A dot, so you do once this once that the sign changes we do both there it is minus i omega whole square and the minus goes away and that is equal to ih cross over 2pi an integral over d omega minus infinity to infinity i omega whole squared times phi AB.

So apart from the i factors it second moment of the spectral function, so you can see you can knock off the whole lot of moment relations now and each time you do a dot you have got a different dynamical variable but now you can write that dot as a commutator with the Hamiltonian h not. So you are really creating whole algebra of all the operators and the system and you are finding they are equilibrium expectation values of various commutators here.

And if you know the left hand side then you have constraints on the spectral function on the right side, okay. So the famous f sum rule the thomas go and write some rule these are all special cases at finer temperature of this thing here. So in the exercises I will give some of these examples what I mean to say is, this quantity for instance could also be written as follows.

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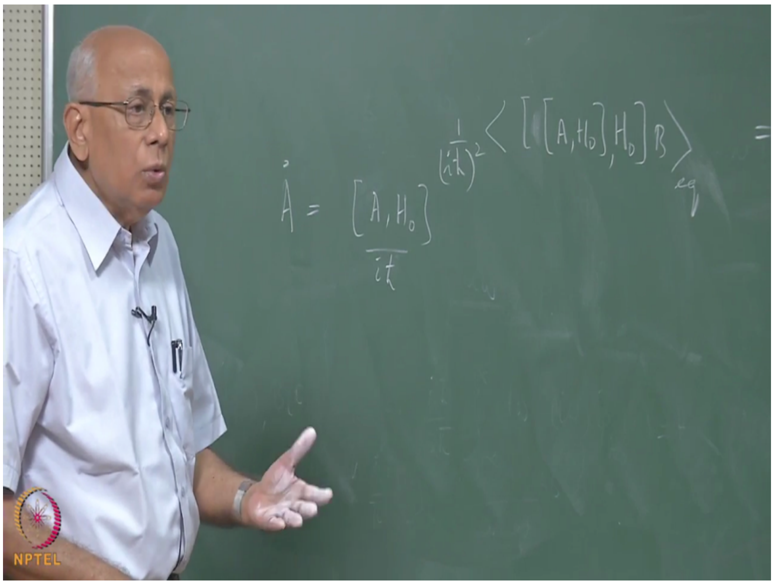

And so that the dot, so A dot remember is equal to commutator of A with H not divided by ih cross, so this quantity here is also equal to the expectation value 1 over ih cross whole square of the commutator of commutator of A with H not with H not B all at the same time. So all the multiple (()) (40:45) and commutators can all be discovered once you have this and therefore you have a lot of constraints. The various kinds of sum rules are obtained.

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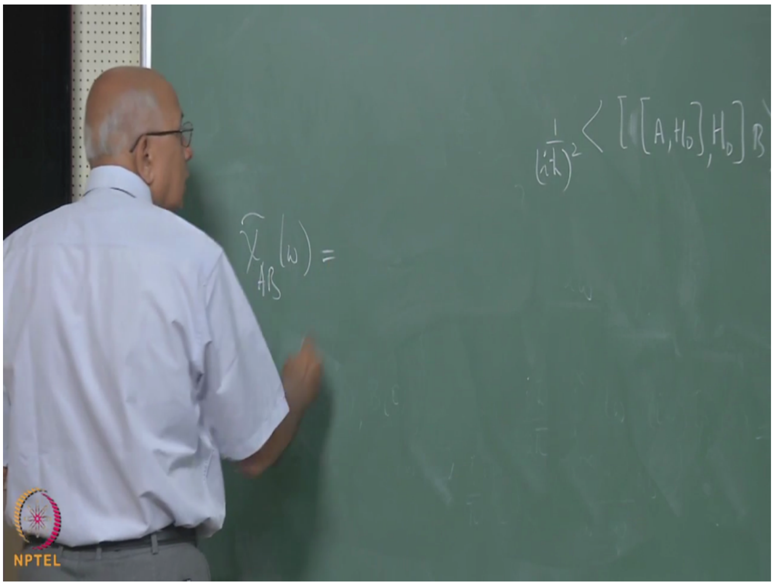

Now does that, is it of any use? Well, yes because look at what the representation of this quantity was; look at what we said about phi Kai AB tilde of omega this was equal to 1 over 2pi i, so I got this i right now and integral minus infinity to infinity d omega prime phi AB tilde of omega prime divided by Omega prime minus Omega. No, so let us write this as i over 2pi and observe the minus sign here.

So let us write this as omega minus Omega prime plus i Epsilon, yes. Oh! 1 here, right, okay. That is the advantage of latech if you did that it shows you an error immediately that you have missed something. Of course if you say go to error it goes to some random place, the previous error or maybe it introduces an error because latech can be self correct, AutoCorrect, latech they are all very impertinent.

So nowadays I always see any errors which remain are entirely due to the impertinence of AutoCorrect not due to me. So if you now ask what can we say about this you see, this quantity here has Delta functions at all the transitional frequencies. Now you could say, alright let us look at very high frequencies, significantly above all the transitional frequencies of the system.

Then the contribution here Omega prime will be, this will fire at all the transition frequencies, so omega is much bigger than all of them you get an approximation to this susceptibility at very high frequencies. So formally you can write this as i over 2pi you pull out an omega, so this is equal to summation, n equal to 0 to infinity, oh! That is even absorb this, some constants over omega to the n plus 1 sorry Cn to the n plus 1 where Cn equal to this i over

2pi integral minus infinity to infinity d omega prime phi AB tilde of omega prime times omega prime why that? Okay, alright.

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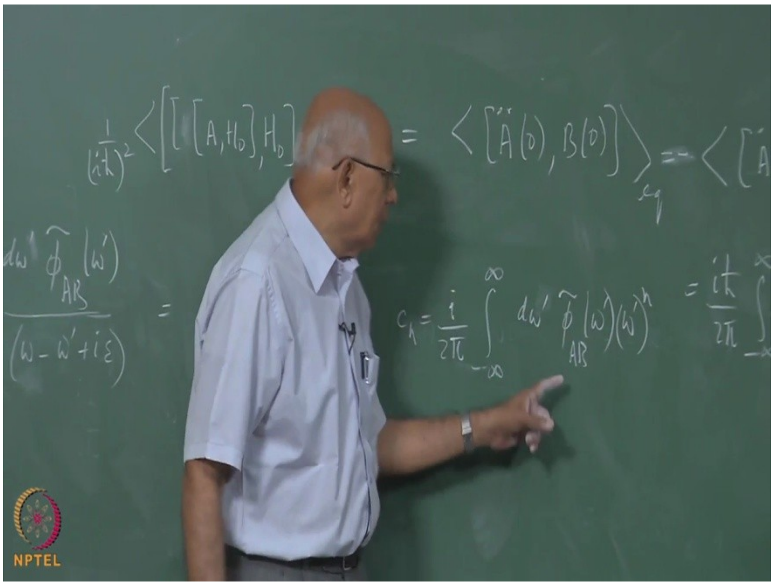

Omega prime, omega prime to the power, okay. I did a binomial expansion of 1 over omega minus omega prime pulled out in extra omega and then these are the coefficients, does not have to be but if you look at in cases where the matrix elements corresponding to those frequencies if they are small then you can throw those out because remember these Delta functions are all weighted.

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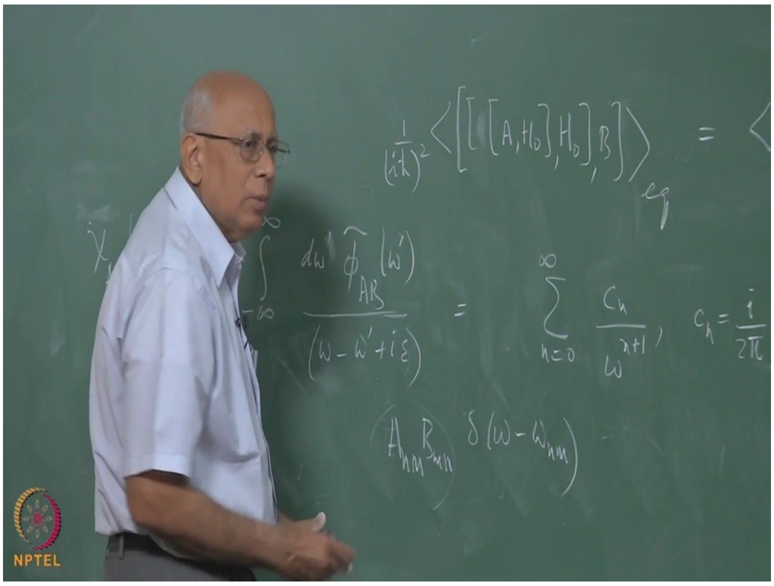

Remember that what happens in this spectral function is a series like this omega nm but this is multiplied by Anm Bmn, right. You will need this to be non-0 for that frequency to contribute but if you see alright these things die down as at very large values or whatever these quantum numbers are then you can get the satisfactory high frequency expansion of the susceptibility where these quantities these moments are known in terms of equal time commutators explicitly, okay.

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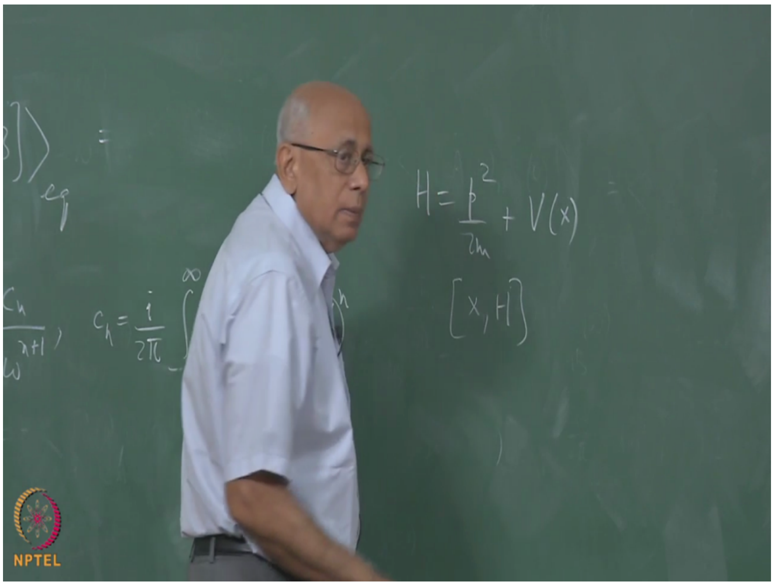

For instance the most famous of these is this first moment sum rule which you come across in quantum mechanics but what we have now is the values of these quantities at finite temperature that is important. For instance we know that if H is equal to P squared over 2m plus some potential V of x and you ask what is x with H, this of course is going to give you a commutator with this which is P over m but now if you ask what is x with x over H this is equal to 1 over m apart from some ih cross factors times some constant.

Because x commutes with any function of itself the only thing is this and you have this quantity here. Now this x with H is like x dot is like a velocity. So you got a position, the commutator of position with momentum and that is a constant we can therefore find out what this is? This is going to be a number here but on the other hand this is going to give you a nontrivial quantity on the left hand side.

So there is a big, you can play this game various systems you can find out what the high frequency expansions. Now what is the use of anti-commutator I will put some of these in the exercises but what is the use of the anti-commutator? Well, because you have A of 0 B of tau with a plus sign if you put A equal to B at equal times then you get mean square values of physical quantities.

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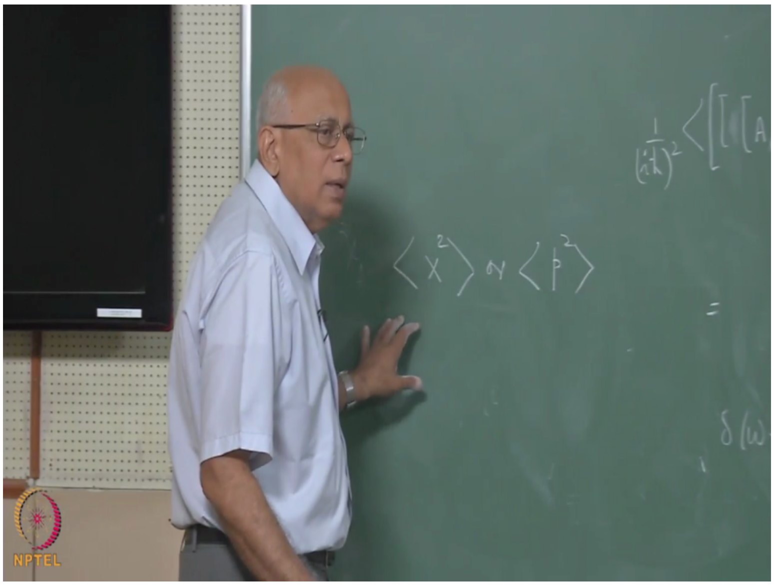

So you would for example get what is this quantity for an oscillator? Or what is P squared etc? Okay in equilibrium, you can therefore compute at finite temperature now you need to use this formula with that cot hyperbolic etc. Of course you know what this is for harmonic oscillator we know that at the high enough temperatures half m omega squared expectation of this goes to half kat and at 0 temperature it is going to go to half the ground state energy if you like but now in between what does it do? At finite temperature this is going to give you an exact answer, okay.

So in a nutshell the spectral function has this formal representation in terms of the transitional frequencies of the system but it also provides a great deal of information about various, due to some rules it is highly constrained and you get results for mean square values expectation values of commutators, nested commutators and so on. So in a given practical example one will have to of course make approximations but the formalism is correct here.

What remains for us to do in this business is to relate this quantity, relate formulas we have derived to the rate of dissipation we have got to make connection with the Fermi golden rule into a shot of perturbation theory I will do that and then we will take it from there.