Physical Applications of Stochastic Processes

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Lecture-17
Level-Crossing Statistics of a Continuous Random Process

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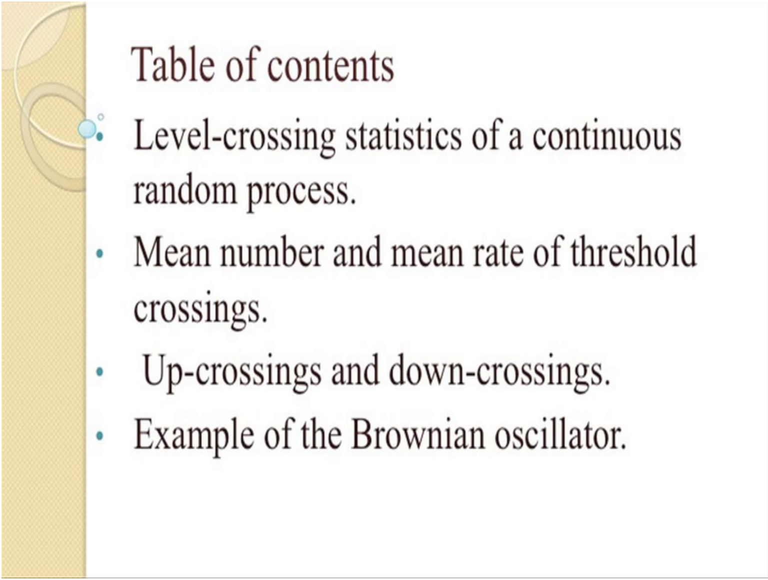

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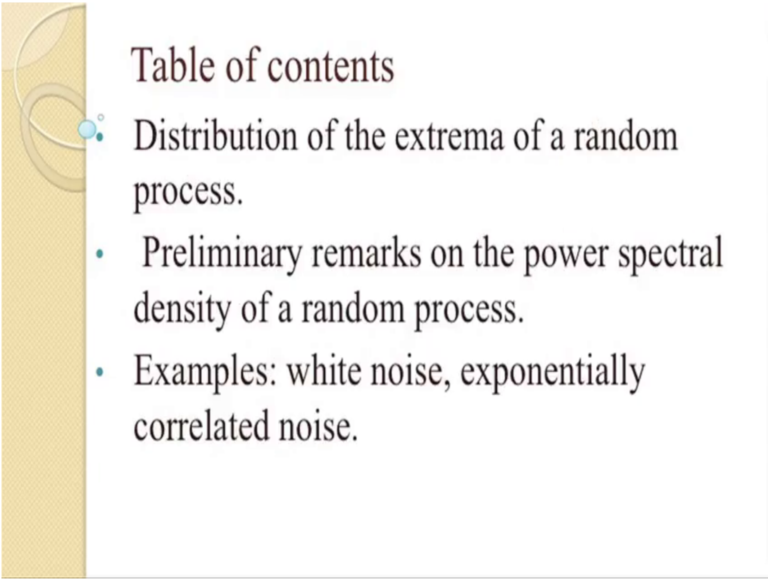

I wanted to talk about little bit about the Ito calculus today but then on looking over it I realized that it is a little formal and it is not very clear to me that really fits into physical applications of stochastic processes. But I will try to get back to it I am trying to write it out I am trying to say it in as simple a form as possible and I have not quite succeeded. So, since I do not want to get into a whole lot of formalism and in this class.

We will take a rain check on it I will come back to it a little later because I had like to see what is the simplest way of saying it explicitly saying it and so on? I mentioned a few things yesterday about the fact that it makes a difference when you have multiplicative noise as opposed to additive noise and it is a question of how you interpret how you handle this quantity dw which is the increment in a wiener process.

You certainly cannot write dw as dw over dt times dt because it is not differentiable. So, that is where the root of the problem is I am still playing with the idea of playing with how to present this in a simple enough form. So, we will take a rain check on it meanwhile there is one topic which is of practical importance in the study of noise and stochastic processes occurs all the time and it is a simple enough thing once you make a sufficient number of assumptions and it is got; as I said a lot of practical applications.

And this has to do with the following question given a random process in time and we are going to assume for this purpose sufficiently smooth random process in time one could ask what is the rate at which or one could ask how often does this process cross some prescribed threshold so you could say there is some level in this process and you would like to know the statistics of the level crossings of this process.

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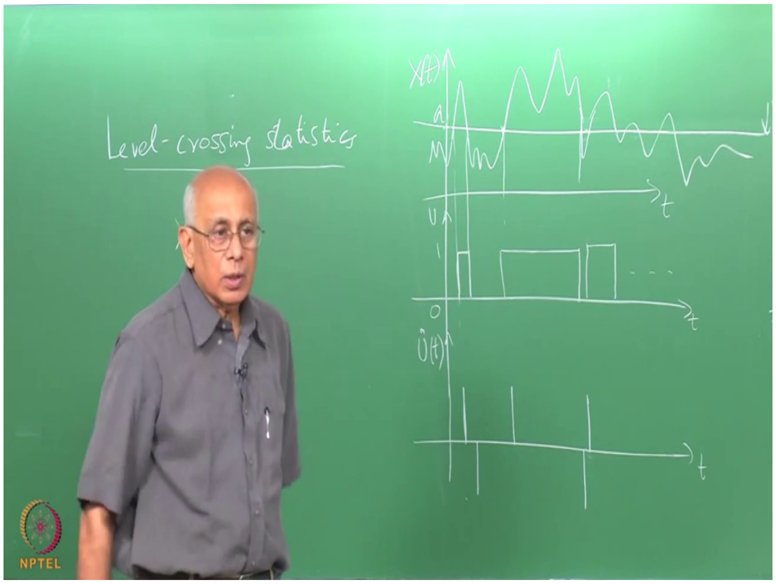

So, that is a very, very general statement and we will try to make it specific so level crossing statistics. So, let us look again at a process x of t and I am going to assume that this is continuous process this number one and secondly it is not as irregular as Brownian motion which to which this formalism is not applicable for reasons which will become clear. But if you plot this typical realization of this process the function of time we will assume it is an ongoing process we do not have to assume that it is a stationary process that is very important.

Because in practice what can happen is the following you might for instance record the rainfall continuously as a function of time the precipitation as a function of time and this is going to be irregular it is going to fluctuate etcetera and you would like to know when the precipitation exceeds per unit per hour or something like that exceeds a certain threshold value. I would like to see the statistics of this thing happened.

Now this could change with the seasons very slowly so it is not a stationary process definitely. So, whatever we are going to write down will be applicable even to non stationary processes with some provisos but if it is stationary then things will become simpler as you will see okay. So, it is some ongoing random process is a function of time x of t and it goes up and down in this fashion there is some kind of a regular process of this kind.

And then you ask I had like to know the statistics of when it crosses some prescribed threshold it could even be the origin it does not matter so some threshold like this where we put in a value a and this is a prescribed threshold and I had like to know about the statistics of these points where it crosses this crosses over and the other side okay for that purpose I realize that the immediate thing to do is to define another random process.

So, let us call this process U of t which = 0, if x is below a and = 1 if x is above a so weight of x of t - a and I plot this then schematically what happens if I plot to you on the same graph is that this point here in this interval U is unity it is up there and then it remains 0 till over this range well it is not a bad let us do it this way, so over this range it is again positive, so it remains goes like that and then there is a piece here and so on so this is what U of t does and this is unity.

So, the problem has become much simpler we do not care about the variation of x itself we only want to know is it above the threshold or not in other words when is you 0 when is it one that is it okay. Now what we want what we are getting at is the statistics of these instants of time. So, it is convenient to plot not U but U dot of t can imagine formally differentiating this process of course these are all singular quantities.

But you can differentiate a step function to get a delta function right then U dot of t = a delta function of x of t - a but it is got an x dot of t also because you have to differentiate this quantity. Now what does U dot actually looked like it is going to fire whenever x of t hits a namely at these points at all these points. But suppose it hits a at one particular value at some time T then this Delta function if you convert it to a delta function in time is going to have a divided by it is going to be t - whatever it is at that instant t is – ti.

So, a typical 0 would look like this thing would look like Delta of t - T I but it would be divided by the modulus of x dot at this point at this ti because that is what this Delta function does when you convert it to a delta function in t. But there is an x dot sitting on top okay so x dot divided by mod x dot is +1 if x naught is positive and - 1 of x dot is negative, so what we have is a delta function here of unit strength out here and one at this point of unit strength in the opposite direction.

And similarly there is an up crossing here so it goes up and there is a down crossing here so it goes down etc. So, this process you dot is just a sequence of pulse it is a pulse sequence in which you have +1 if it goes up and -1 if it goes down okay. So, the matter is now very straightforward what we got to do is to find for instance the number of crossings in a given interval of time you would like to know the number of threshold crossings in some time interval say t1 to t2.

Let us call it N a, t1,t2 and what is this = it is just the count of all these Delta functions you just have to integrate over T this quantity and that is it this U dot and that counts it automatically making sure that you have the modulus of course. So, this is = integral t1 to t2 dt and then you need mod x star of t that counts one for each of these guys because I put a modulus there and that is it.

Now this is a random variable because these points t sub I are at random because x itself is varying randomly with time so now we can proceed to look at the statistics of this number N what is the average number etcetera. So, we could ask over all realizations of this x of t different realizations in the given time interval t1 to t2 what is the average number of crossings level crossings okay.

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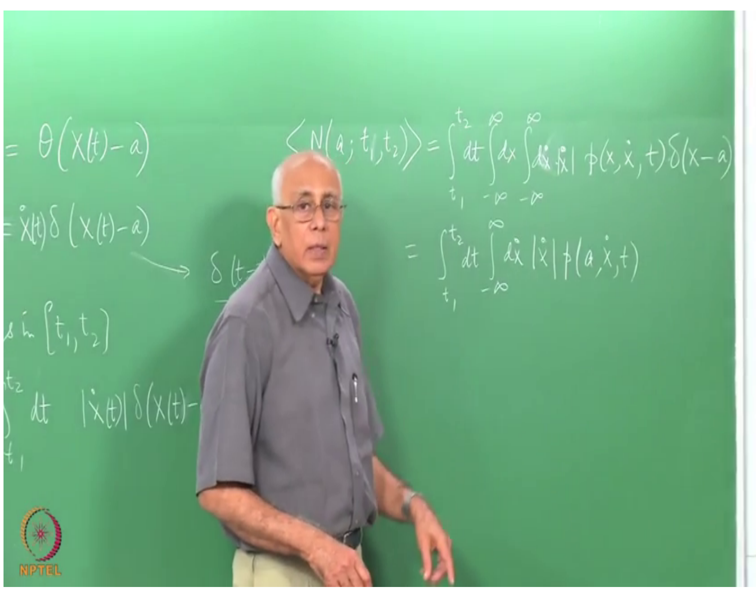

So, that will immediately be expectation N this quantity will be = what it say first there is an integral over this do you want t1 to t2 dt and then these are random variables these guys are random variables so now you can see that it depends not only on x but also on the on the random process x dot. We have not assumed stationarity or anything like that so these probabilities will be time dependent and what you need here is the joint distribution or density function not only of x but also x dot together.

We have looked at earlier we have looked at cases for a diffusing particle of the Joint Distribution of the position and velocity. So, you need a thing like that in this case in this general situation so you need some P of x x dot and t because this guy could be time dependent this probability density and you have to integrate over all possible - infinity to infinity dx integral - infinity to infinity dx dot but you also have modulus x dot.

So, that is the formal expression for the expected or average number of crossings of this threshold A. If you set a = 0 or sorry times this we need this guy also dealt of size times Delta function of x - a that is sitting here, so this is = integral t1 t2 dt - infinity to infinity dx dot that remains x dot and then this density function at the value a of x dot and t because there is a delta function here I can get rid of this integration over x.

If you put a = 0 you get the number of 0 crossings the average number of 0 crossings. So, the matter is not quite trivial because you need this joint density and then in that density you have to set a particular value whatever threshold value it is and then you have to do this integral. What would the variance of this number be like okay that gets harder and harder? So, the variance formula would require the mean square.

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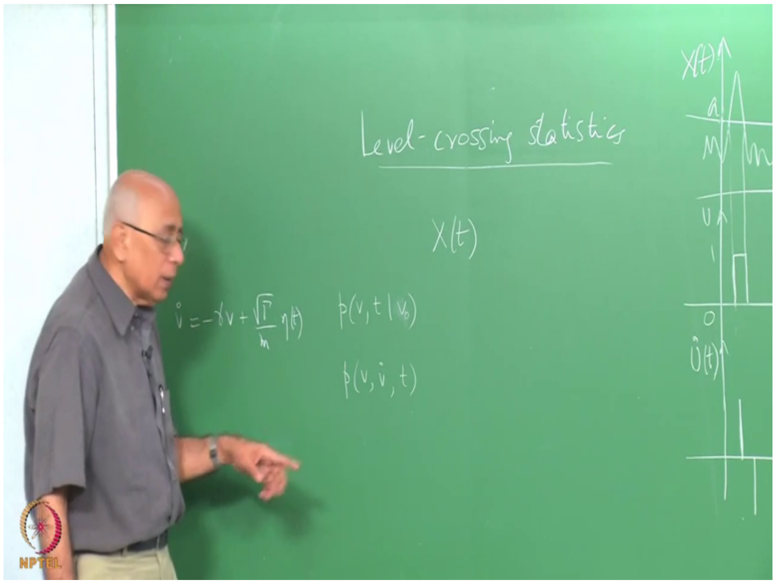

So, you require N squared of a t1 t2 this would be = at this formal level it would be = integral t1 to t2 dt integral t1 to t2 dt prime integral - infinity to infinity dx1 dot say dx2 dot - infinity mod x1 dot moment x2 dot these guys would be there and then a P of now it is a joint density so it is a x1 dot t a x2 dot t prime this two time joint density is also required probability or density is also required and formally you have to do this and then of course in general you require the moment generating function of this which would become fairly complicated but that is the formal expression.

As it stands can we tell what is going to happen can we very often you would like to know what are the up crossings and what are the down crossings like. So, you would like to look at the distribution of these points where it is crossing upwards like this and then the distribution of the down crossings right, so if you call that N and - for instance, so if I call N + up crossings number of up crossings of a t1 t2 what would this be = it would be = integrate t1 to t2 let us find the average number that is what we are interested in general this is = what should I do.

Here is the number of crossings actual crossings what should I do if I want just the up crossings. The integral over x is trivial that is gone it is just you replace it by a pardon me I mass integrate dx dot from 0 to infinity because it is going up that means x is increasing. So, the integral runs dt integral 0 to infinity dx dot and then there is no need for the modulus it is x dot P of a x dot and what about down crossings.

N- okay whatever yes that is right so a t1 t2 again integral dt t1 to t2 and what should I do, for down crossings I want to count the number of times it goes downwards from above a crosses to below a, well clearly x dot is negative because x is decreasing right. So, what do I integrate over - infinity to 0, but I should put as mod x1 dot mod x dot clearly right you can so that is the same as - it is quite right the x dot x dot t p of a x dot t.

Now this quantity here this guy here this is the expected or average number of crossings in this time interval here it is = the mean rate of crossings multiplied by the time interval integrated over the time interval and the rate may be dependent on time that is the reason why you have a t here explicitly the process is not stationary the rate itself might change with time right. So, this integral = mean rate of level crossings at time t.

So, it may change from time to time and we could call it = some part of a and of course there is obvious definition for the mean rate of up crossings mainly tore down crossings and so on okay. So, if you give me the joint density in x and x dot then whether it is stationary or not I can formally write down assuming of course the process is differentiable and so on I can write down a formula for the average number of crossings it is very straight forward here.

Now if the process should turn out to be stationary if x is stationary the next or is of course stationary differentiation makes it does not change anything then this joint densities and so on would be the steady state densities. Then if you know that joint density one can actually compute what this rate is. So, let us do it in the simplest case that we have available Gaussian process okay for which we actually know what the density looks like but we need it for both x and v.

So, we need a process in which x is stationary and x dot is also stationary the normal diffusion of a free particle we know that x dot is stationary if you use the logical model but x is not stationary. On the other hand can you think of an example again involving one dimensional motion where both x and v are stationary. We need to look for a process where there is no long range diffusion because then of course the variance of x becomes infinite and then it and it diverges with time and it is not stationary. So, what is the problem which is a standard problem which has got pardon me yeah the Beyonce Nolan Beck.

Now there is a little problem with Beyonce Nolan Beck thing as we applied to the velocity that is an exercise for you we are going to do that. We have the joint then we have the conditional probability density conditional probability density we have for the velocity okay. We do not have it with the acceleration included we have not put that in at all for the velocity process we have not done that yet.

So, it is a little tricky we are not talking here about conditional densities at all what we have our probability densities joint probability densities in a variable and its time derivative. Beyonce Nolan Beck for the velocity actually gave you P of vt given a v naught this is not what we are interested in at all, what we are interested in is a probability density that gives v v dot and possibly as a function of t.

Now let us go back and since you mentioned this example let us look at the particle which obeys the free Langevin equation and it looks like this v dot = - gamma v + this square root of gamma over m Eta of t we can in principle compute this density here v and v dot but remember what is going to happen this v dot is going to involve white noise here which is not differentiable it is not smooth so there are singularities here in this problem. So, we cannot talk about things where you have a white noise a bare white noise.

Somehow it is got to be this white noise has to be ameliorated in say it is got to be made mild it is got to be made harmless and you must have a stationary distribution for both a variable and its time derivative and even this is not enough when we want to find for instance the square and so on we need the joint velocity distribution and v1 at t1 v2 at time t2 and so on that is not so trivial. So, what is the other example again I am not limit process but not connected with a velocity.

What was the other problem we did where you had a steady state in both a stationary distribution in both position and velocity yeah we put an external potential what sort of potential the harmonic oscillator potential? The harmonically bound particle we found this object had a stationary distribution right in fact we found the stationary distribution.

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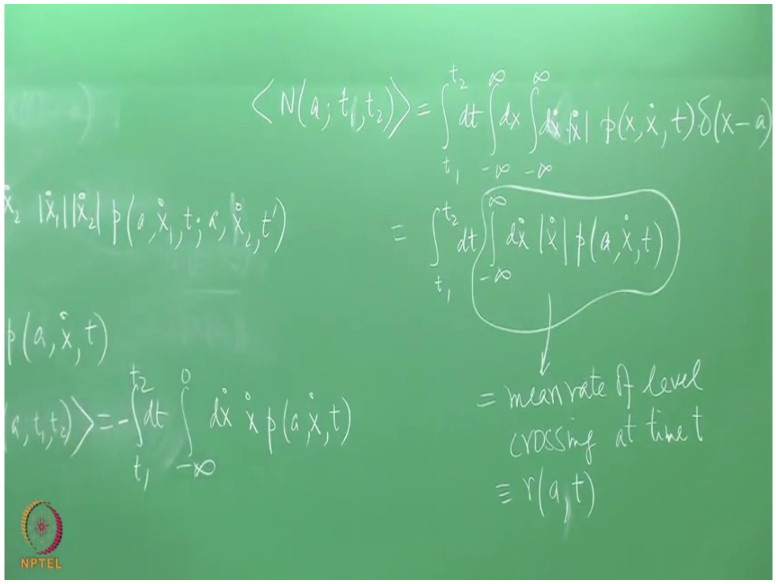

So, we found in this case for harmonically bound particle we are not talking about the conditional density at all we are just talking about the probability density anymore. So, we have P of xv and t or x dot and t in this case but this was stationary this distribution was stationary because the system is in equilibrium both the position and the velocity have Gaussian distributions right. So, we know that this is not there at all it is stationary in this problem and what is this = what is the Joint Distribution of the position and the velocity for a harmonically bound particle.

Each of them is a Gaussian and you know this from equilibrium statistical mechanics it is just the Maxwell Boltzmann distribution for each of them e to the - energy over k t normalized right. So, what is it for what is this guy here for the velocity yeah, it is m over 2 pi k Boltzmann T to the half e to the - mv squared over 2 k Boltzmann T and it is multiplied by the distribution in the position that is it right. So, this is m Omega naught squared over 2 pi k Boltzmann T e to the -m Omega naught squared x squared that is all we need that is all we need here.

And you have to plug that in instead of x dot I called it v here you have to plug it in and that is the end of the matter right. So, let us see what this rate looks like for such a process let us write down in general what the rate looks like in this Gaussian case.

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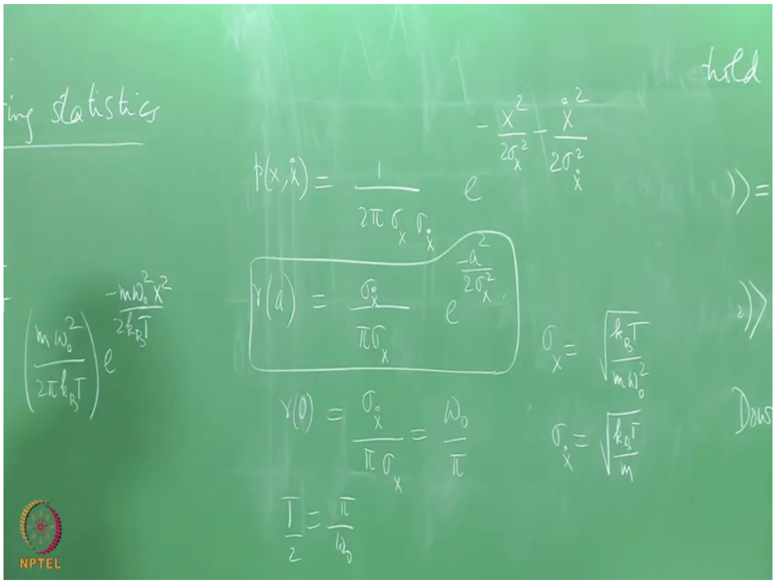

So, we have P of x x dot = 1 over 2 pi and then Sigma x square root of Sigma x squared and then there is a variance in x dot that is Sigma x dot in this fashion e to the - x squared over 2 Sigma x squared - x dot squared over x dot squared that is what the P of x x dot is and now let us ask what is the level crossing threshold in x between some number a specify some number a then what we need is the following r of a at any time t.

But it is exactly the same at all times so it is stationary. So, the rate at which the mean rate at which this oscillator crosses some point a on the x axis both upwards and downwards this is 1 over 2 pi Sigma x Sigma x dot and then what do we get e to the - so we have to do this we have to do this integral, so e to the - a squared over 2 Sigma x square because all I have to do is to set x= a in there.

And then an integral - infinity to infinity dx dot mod x dot okay e to the - x dot squared or 2 Sigma x dot squared I have to do this integral but that is not difficult because there is an x dot sitting here. So, what does that give us this is = first of all let us write this as twice the integral from 0 to infinity and get rid of the mod. So, this 2 goes away and you are stuck with this. So, let us put let us put x dot squared over 2 Sigma x dot squared = some u, so it says x dot dx dot the 2 cancels over Sigma x dot squared = du.

So, let us move the Sigma x square of this Sigma x dot square, so now that is going to give me a 0 to infinity du e to the - u and that is 1, but there is a Sigma x dot squared multiplying it so there is a Sigma x dot. So, this is the general formula when you have a Gaussian process and both x and x naught are stationary Gaussian processes. Of course we could have computed the up crossing down crossing etcetera in this case it would be half.

If you put a = 0 then it would be completely symmetric about that point in particular notice that r of 0 the 0 crossings = Sigma x dot or Pi Sigma x and that is it this factor becomes unity. Now let us see whether this is physically reasonable or not in the case that we know in this problem in the oscillator problem Sigma x= square root of kBT over m Omega naught squared and Sigma the velocity Sigma x dot = square root of kB T over m right.

So, what does this ratio become Omega naught by pi they are a factor of two missing somewhere very careful here that is fine okay. What is the time period of this oscillator the unperturbed oscillator? 2 pi over Omega naught right, so t over 2, every half period on the average it crosses the axis x = 0 of course that is exactly what it does okay. So, this big rigmarole here it is actually in this trivial case is given as the right answer so it checks this answer out.

But you now have an explicit formula for when it crosses any value a, how come this oscillator does not have any fixed amplitude it is going it is good there is a probability of finding it arbitrarily far from the origin but the variance is finite the variance of this oscillator in the steady state is finite stationary state but there is a probability of finding it anywhere why is that yeah it is in contact with the heat bath at temperature t, so it is energy is not fixed.

And as you know the energy determines the amplitude right the energy can be arbitrarily large of course it becomes less and less probable that it is moved far away in one direction or in the other direction it is going to stay near the origin most of the time it is Gaussian after all peaked about the origin but the fact is there is a non 0 probability of it being arbitrarily far from the origin and this is the rate at which that threshold is crossed.

It gets much smaller as they become significant you can see for a given temperature of course the temperature is increased then Sigma x squared is also increased and then a can become larger as exactly as you would expect. So, even the 0 crossings are taken care of in this problem now the difficulty of course is that you need this variance to be finite you need this value to be finite you could now ask a slightly more complicated question.

What about the maxima and minima of this random variable we have said it is got a nice realizations the sample the sample paths are nice and smooth and so on what about maxima and minima so let us see what happens there and it is related to this problem.

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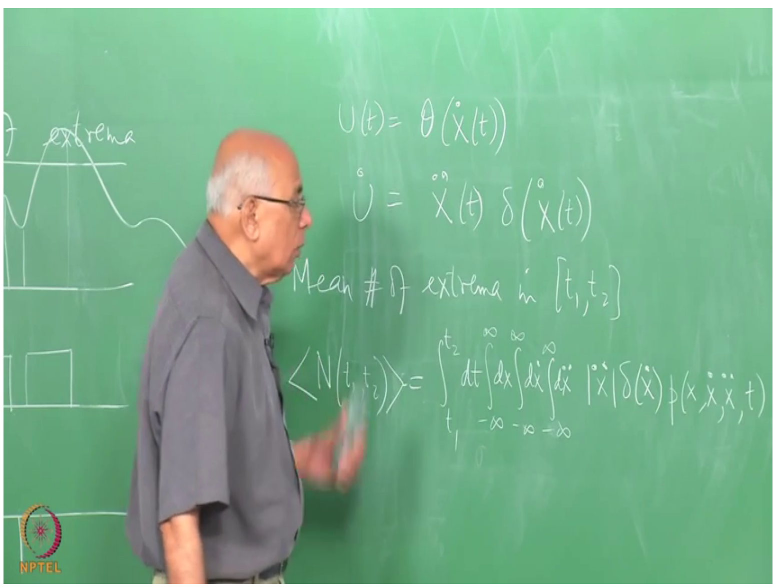

So, distribution or crossing of so they should really say the distribution of extrema. Now we have in mind a sample path which looks like this and I want the statistics of these points. So, I do the same thing as before I say look from a minimum to a maximum the slope is positive, the slope is 0 at extrema. So, I now construct U of t to be = theta function of x dot of t, so I am trying to find out the statistics of all these segments here.

So, in this segment this guy is positive then again in this segment the guy is positive and so on and 0 in the segments where theta dot is where x dot is negative. So, between a maximum and a minimum this process is 0 and between a minimum and a maximum it remains 1, +1 and as before if I compute so this is U if I compute if I calculate U dot the up crossings will correspond to unit Delta function with +1 weight and the down crossings would correspond to - 1 weight.

So, the Maxima would correspond to -1 weight and the minima would correspond to +1 weight, so U dot = x double dot of t delta function of x of t, so now I could ask okay what about the mean and then the mean number of these extrema in some time interval. So, the number of maximal means of extrema in t1and t2 go to n this is = an integral from t1 to t2 dt and then I have to compute this quantity here.

So, the number would say take modulus here x double dot of t Delta formally what it is. What I am interested in is over all realizations I want to know the mean number. So, I already wrote mean them but this is the mean number is = expectation of this = but I also have to integrate here. So, let us do this integral what should, I integrate over so there is going to be definitely an x double dot and then a delta of x dot that is going to sit there.

So, there is definitely an integral over these guys so integral - infinity to infinity dx dx dot dx double dot all are there because the process itself is x of t and I have to integrate over its density times the joint density of x, x dot, x double dot perhaps at time t if it is not stationary but in any case I need the joint density of not only the variable but its derivative and its second derivative so this makes sense only the function is twice differentiable in some sense some precise sense okay.

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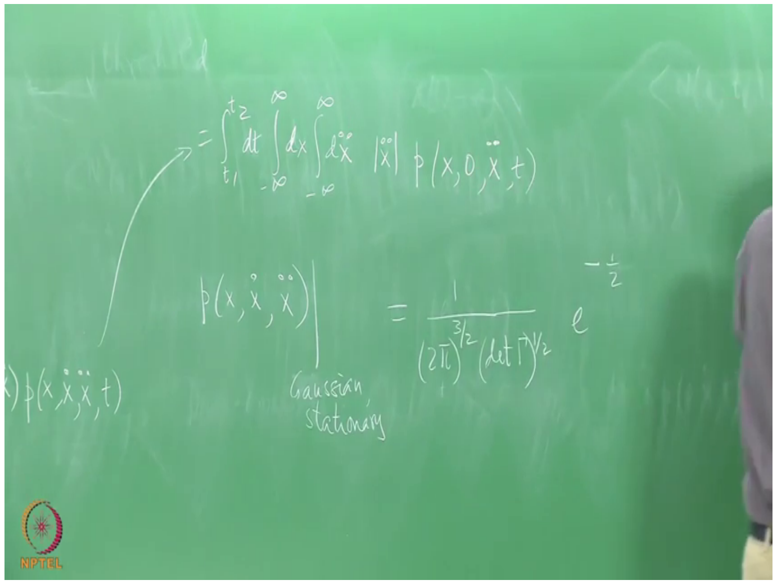

Generally you call them mean square differentiable that is the way the stochastic process would have differentiability put into it but you need sufficient smoothness to do this and of course as usual you can get rid of the x dot integral using the Delta function. So, this thing here becomes = integral t1 to t2 dt integral dx into dx double dot times mod x double dot and this is the overall values infinity but then a joint density which is x0 x double dot.

And maybe so this quantity now this full integral gives you the rate at which the extrema occur and as before if you assign a sign to x double dot we would be able to tell the maxima from the minima all you have to do is to make sure ask when is this positive or negative and then you can separate the positive from the maxima from the minima. What if I say look I am not interested in these maxima and minima I am only interested in maxima and minima which are above some threshold.

So, I am interested in this, this and any minimum above this what should I do then put in a so do not consider this put in a theta function put in another theta function which says x is bigger than a and go through the same process as before so that is not a that is a small extension of these formulas. So, you can always change to any arbitrary threshold and say I am not interested in fluctuations in extrema and there is a small values I am interested in it beyond a certain threshold okay.

Now what can we say about this in a Gaussian process suppose I tell you that you have a Gaussian process in which x, x dot and x double dot all of them are Gaussians and it is a stationary process all these are stationary random variables then what does this P look like in that case we wrote it down for the case of two Gaussians what we need to do is to do a generalization of that to three but there is a little wrinkle that appears here which you have to be careful about.

I have not discussed multivariate Gaussians in great detail or in any detail in this class but what happens then is the following. This fellow here let me write the general formula down so P of x, x dot, x double dot in the case when they are all stationary and when they are all Gaussian and stationary this thing looks like the following. You need to define a covariance matrix the moment you have a multivariate Gaussian.

You need to define a covariance matrix and it looks like this it is 1 over there is this 2pi square root for each of these factors so there is a three halves and then there is a determinant of this covariance matrix let us call it gamma to the power half e to the power – 1 half the usual standard half who sits there and then I need a notation you need a notation.

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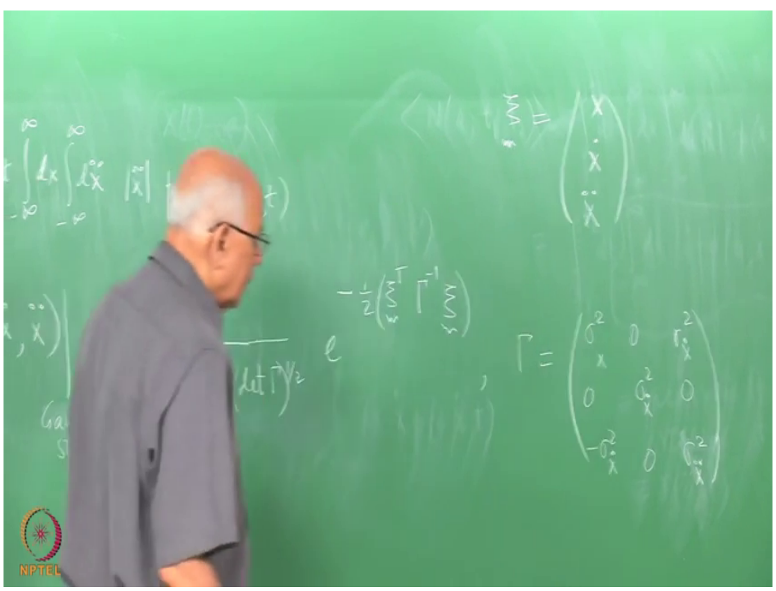

Now so let me I should not call it x, let me call it let me call it is Zi some vector this stands for x, x dot, x double dot you put this thing in a column vector and then it is e to the half Zi transpose gamma inverse Zi okay with this guy where this covariance matrix gamma looks like this it is got Sigma x squared, Sigma x dot squared, Sigma x double dot squared in the diagonal elements and then there is a 0 but there is a Sigma x dot squared here - Sigma x dot squared that is what this matrix looks like.

And you need to find its inverse and plug it in here crucial thing is this guy sits here in general any multivariate Gaussian. You know that so you plug this in into this expression do this integral and you know you know have in the stationary case this true integration is trivial its t1 - t2 times the rate out here and you can compute the rate of crossings. Now the real interest comes when you have a noise of some kind about which you do not know much.

Perhaps you can make a Gaussian assumption but you do not know very much more about this we would like to interpret what this whole thing is like what are these things really trying to tell you and so on. For this we need the concept of the power spectrum of the noise I have not talked about this at all yet but this is now the time to introduce it because we are now going to talk about noise processes. So, various kinds not necessarily Markovian not necessarily described by master equations but we look at various kinds of noise for a while now.

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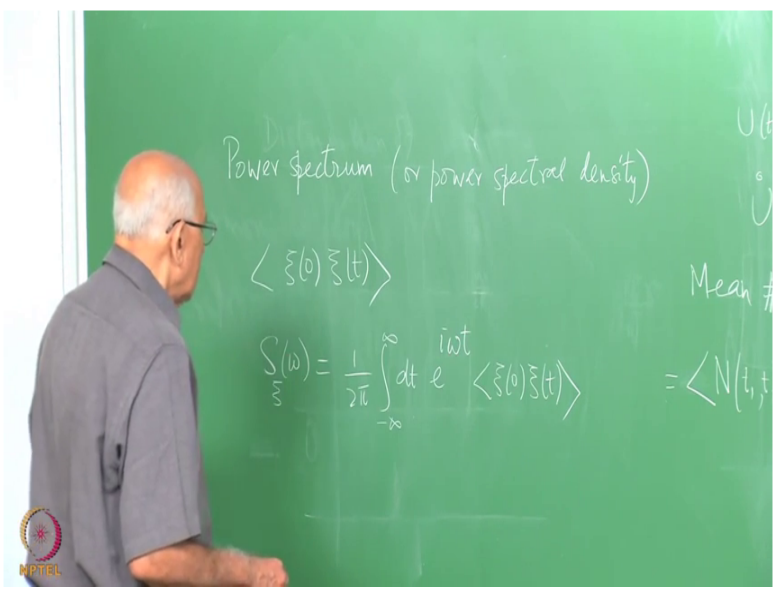

So, let me define first a power spectrum or power spectral density or we will do this in the simplest case first a 1 dimensional process first and let us look at a process which is stationary. So, we have a lot if you if the process is I you know that considerable amount of information is carried by this autocorrelation function yeah you can write down various properties of it and things like that we kind of expect that as t becomes very large this will decay to 0 from some finite mean square value etcetera.

Now this power spectral density is defined as s it is very simple as Zi of Omega it is just a Fourier transform of this okay it is a function of t, so nice smooth function of t in general so its Fourier transform is it there and I will tell you what the use of this definition is what it is going to do or what it actually is measuring. So, - infinity to infinity dt we need to choose the Fourier transform convention so I have chosen one.

Now in physical terms what it does is once you measure this and Zi is some kind of noise or random process it measures the strength of this of these fluctuations when you Fourier transform this it tells you what is the intensity in some frequency window between Omega and Omega + t Omega that is what this guy does here. And by the way all the others are matters of convention the + sign here the one over two pi here etceter.

Engineers normally define this as twice the Fourier transform without this 2pi factor and so on but these are maybe we will stick to one convention we will stick to this thing here now what is the import of this whole business it will turn out and we will see this explicitly that when you have some noise driving another variable which also becomes noisy as a consequence then the power spectra of the input and output variables are related to each other okay.

In fact the response of the system is measured by what is called a transfer function between these two which is dependent on the power spectra of the input and the output variables and there is a theorem called the Wiener Khinchin Theorem which I am going to talk about and which will exploit which will quantify this relationship between the input and output. So, this is basically what this does.

Now in the cases we looked at in the simplest cases we have looked at when we can write down what this guy is and then we will come back and look at its significance in greater detail first this Gaussian white noise that we had.

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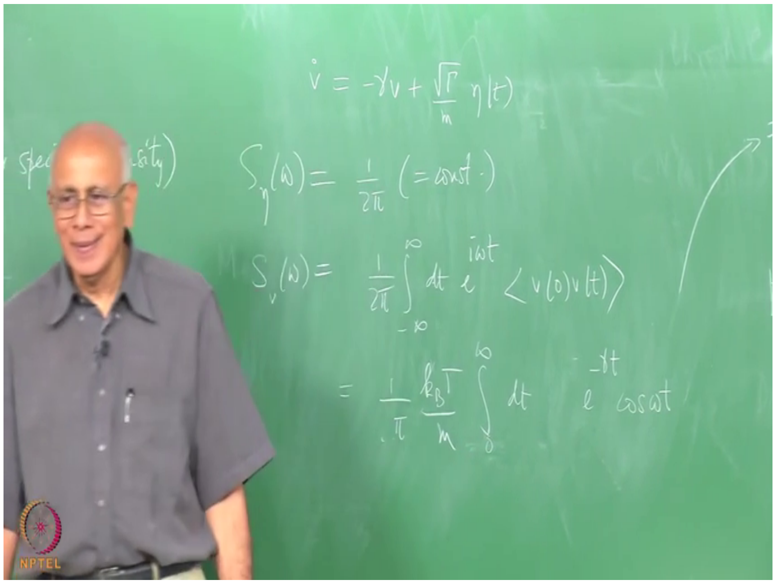

We had a noise Eta which essentially said it of t it of t prime is Delta of t - t prime stationary process so this is just a delta function. So, S Eta of Omega is = that was a 0 mean Gaussian process by the way I am assuming that the mean is 0 here otherwise a correlation is Delta x0 deltas Zi here the different deviation from the mean the correlation of the deviation from the mean. So, when this is just a delta function this just gives you one so is it 1 or two apply constant okay.

So, that is another way of defining white noise it says that when you do a Fourier transform there is equal intensity at all frequencies of course it is unphysical because certainly some energy is involved in producing this noise it is clear that you cannot have arbitrarily high frequencies the same intensity okay. So, that is a mathematical idealization by the way what was Sv of Omega in the case when you had Langevin amount particle.

So, remember that this process was defined by v dot s - gamma v + square root of gamma over m Eta of t what is this = what we need to do here is to put in the value for the correlation function right so this is = 1 over 2 Pi integral - infinity to infinity dt e of the i Omega t and then what is the correlation function of this velocity process the stationary velocity process it is e to the - gamma t that is the whole idea the velocity correlation time was gamma inverse and died down with one correlation time it was a Markov process exponentially correlated right, And then the strength was 1 over so it was 1 over 2 pi kBT over m that is the mean square value of this guy times integral - infinity to infinity dt e to the i Omega t, e to the - gamma modulus t and died down on both sides. So, of course that is a trivial integral to do first thing to do is to remove this thing and make it twice 0 to infinity e to the - gamma t and what is that going to be sorry before we do that.

Let us leave it like this - infinity to infinity with a 2 Pi e to the i Omega t but I break that up into cosine and sine and the sine vanishes because the rest of the integrant is an even function mod t here so I get rid of this 0 to infinity and then there is a cos Omega t only the real part survives and what is this guy what is this integral while going man that is it that is it is it is it gamma over gamma squared + Omega squared or Omega over Omega gamma squared + Omega squared.

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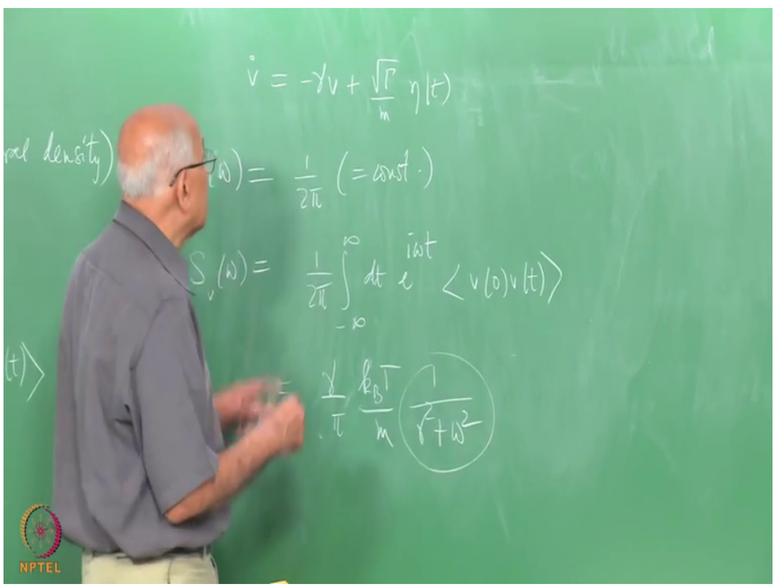

I am sure this is done by I do not know things have changed so I am sure you have done this e to the - ax cos Vx or sine vx I am sure you have done this integral right. So, elementary integrals what you have to do is to write e to the - ax + Ib x and that is trivial to do and then you take real and imaginary parts you get each of these integrals right. So, what is this answer for the cosine a over a squared + b squared or b over a squared + b squared how do you decide.

Even simpler way of how do you decide put b = 0 if you put b = 0 this just becomes e to the - ax for the cosine and becomes 0 for 9 so the sine must have b on top this is an integral right okay. So, this is gamma over gamma squared + Omega squared. So, it is gamma over Pi i 1 over gamma squared plus Omega squared crucial part is this what does it look like it is a Lorentzian shape.

So, now you see what is happening the effect of this the effect of the fact the fact that there is inertia in this problem tells you cannot shake this particle arbitrarily high frequencies with equal amplitude. So, what is the power spectrum do as a function of Omega when Omega becomes large it dies down like 1 over Omega squared. Whereas the noise that drives it has equal power everywhere at all frequencies.

But the response because there is inertia on the problem and damping in the problem this response does not follow the stimulus it is sluggish and this dies down as Omega squared for large values. We will see more about this and the more interesting cases are where this dies down the power spectrum dies down like a power of Omega which lies between 0.8 and 1.2 or something it is called 1 over F noise we will say a little bit about that okay.

So, let me stop here today but we will take up the idea of the power spectrum and what it does in greater detail.