Select / Special Topics in Classical Mechanics

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Module No # 10

Lecture No # 33

Classical Electrodynamics (iii)

Greetings. We discussed several features of the laws of classical electrodynamics in our previous class; we will continue the discussion. And in this class and in the next one, the focus will be on demonstrating the connections between the laws of electrodynamics and the special theory relativity.

As I mentioned earlier, electrodynamics is very huge topic; this needs a full course, which should be like a 40 hours lecture course or may be 45 and followed by another 45 lecture hours course. So, this is something that one would study over two full courses and here in just about three to four classes, we are trying to summarize some essence of the laws of classical electrodynamics and discuss the connections with this special theory of relativity.

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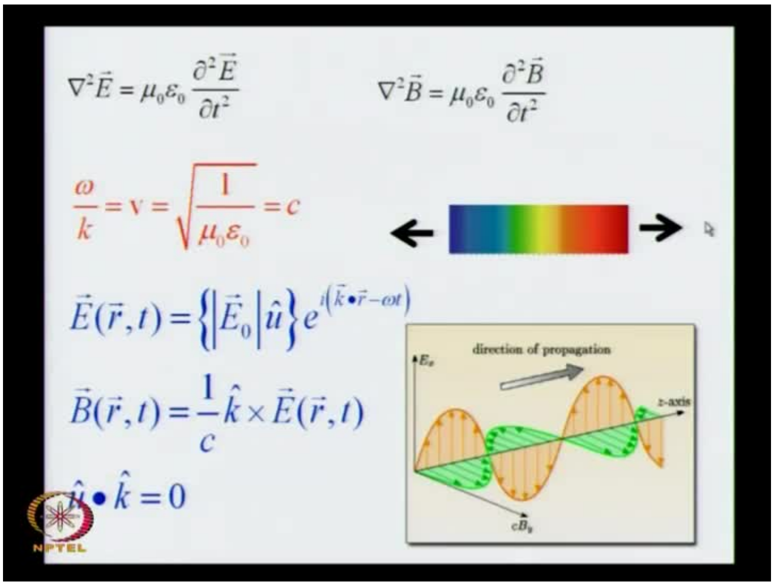

So, what we did in the previous class was to arrive at the Maxwell's equations from the empirical laws of electromagnetic formalism and how they were sort of augmented by Maxwell, and a lot of things have to come together to discuss this. So, what I will do is, I will spend some time recapitulating on some of the important ideas that we brought together and then we shall proceed just to refresh our minds. And once we get to the Maxwell's equations, we can perform some straight forward applications of vector calculus. In particular, take the curl of the, curl of E and the curl of B and from that what we got were these two wave equations - one for the electric field and other for the magnetic field. And we found that the solution to these equations are provided by the relation that you find over here, which immediately tells you that the electromagnetic phenomenon propagates as a transverse wave in space at the speed of light, which is determined completely by properties of vacuum.

And then, the visible part of this electromagnetic spectrum is what you see over here; the spectrum of course extends to shorter wavelengths on this sides and to longer wavelengths on the other side, and here is a picture of the transverse electromagnetic wave that the entire optical phenomenon are connected with.

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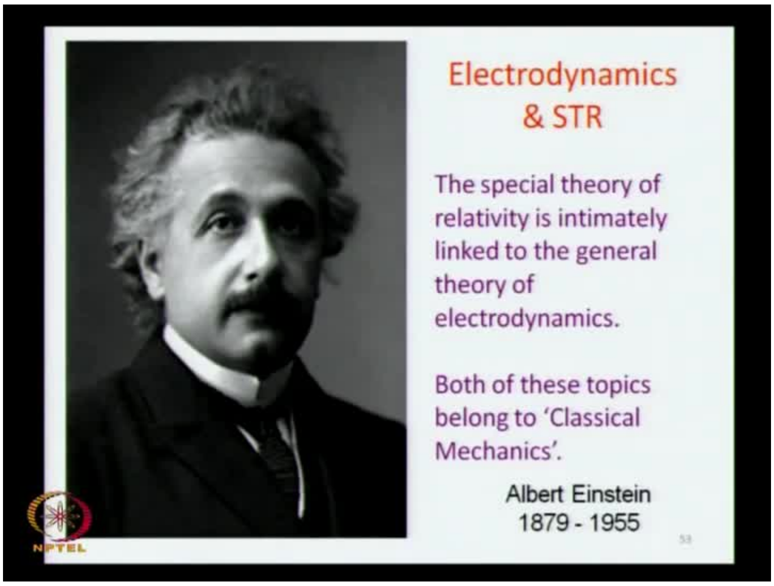

Now, as I emphasize this phenomenon has the special theory of relativity, which is intimately built into its formalism and we will see these connections. And therefore, it is natural to accommodate the laws of electrodynamics as well as the special theory of relativity in any course on classical mechanics.

So, classical mechanics, we normally develop the classical mechanics of particles, develop Newtonian formalism in terms of the cause effect relationship, then we also discuss the alternative formulation of mechanics, which is in terms of the principle of variation - the Lagrange's principle and the Hamilton's principle. Then, we also develop the machinery to do fluids, which is essentially the Newtonian kind of formalism, but applied to fluids and the main feature over there is this emergence of this convective derivative that one has to make use of when you take derivatives with respect to time.

So, we do a little bit of fluid mechanics and fluid dynamics and then what we did was to develop the tools of vector calculus, because what is of extreme importance in developing the equation of continuity in fluid mechanics and also the Bernoulli's theorem and the Gauss's divergence theorem of vector calculus and the stokes theorem of the vector calculus; so, we do that simultaneously. And we find that the tools that we have developed have got powerful applications in electrodynamics; so they come in handy, because we have already developed them in earlier units.

So, all of this belongs to the realm of classical mechanics, which naturally would include a discussion on the special theory of relativity, which was our topic for unit 6. So, you will remember that we discussed the special theory of relativity, the Lorentz of transformations; the Lorentz transformations, which we discussed, we also discussed the twin paradox in that context, just to get a little handle on time dilation and length contraction and how these are important. And integral parts of all the consequences, which follow from the fact that the speed of light is the same in every same inertial frame of reference. So, this something that we discussed at some length in unit 6 and we are going to need all of these put together in today's discussion, which is why I will spend a little bit of time recapitulating some of the essential features.

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So, let me remind you; we have discussed this that Galilean relativity; that if you have a game of cricket and you have got big spectacle in front of you, Brett Lee bowling to Sachin Tendulkar and the world is watching with anxiety is to what is going to happen next? And the idea here is that if this game was to be played on a huge flat truck, which is moving at a certain velocity and if that velocity is a constant velocity, then the game could be played just as well; the laws of mechanics are essentially the same it really makes no difference.

As a matter of fact in a certain sense, the earth is already some sort of a truck on which this game is being played; we often do not think about it, but the earth of course is rotating it is going round the sun, etcetera so it is not a static piece of, you know playground, which has no motion of its own in an inertial frame that we are referring this game to. And if this game was to be played on a big truck in other words, if it were played in another frame of reference, which is moving at a constant velocity with respect to the previous game, then the laws of mechanics would be exactly the same and you will see the same dynamics of the cricket ball; you will see exactly the same game with no difference and this is Galilean relativity.

Here the speeds that we are talking about that, if Brett Lee were to bowling at a speed of something like a 152 kilometer per hour, at a speed of 152 kilometers; right so that that is a typical speed at which these fast bowlers bowl.

And this is the speed which will be measured in the truck frame; if this game was being played on the truck. So, you have got a speedometer, which records the speed of the ball to be 152 kilometers per hour, which is quite fast. But if this speed was to be measured in a frame of reference, which is outside the truck and in this frame of reference the truck itself is moving at a certain speed and if that speed is exactly 152 kilometers per hour, which is opposite to this, then in this frame which is outside the truck, the speed of the ball would be 0 right and the ball is never going to get to Tendulkar; but it is Tendulkar who will get to him; right the ball will never move in this frame of reference right and this is Galilean relativity.

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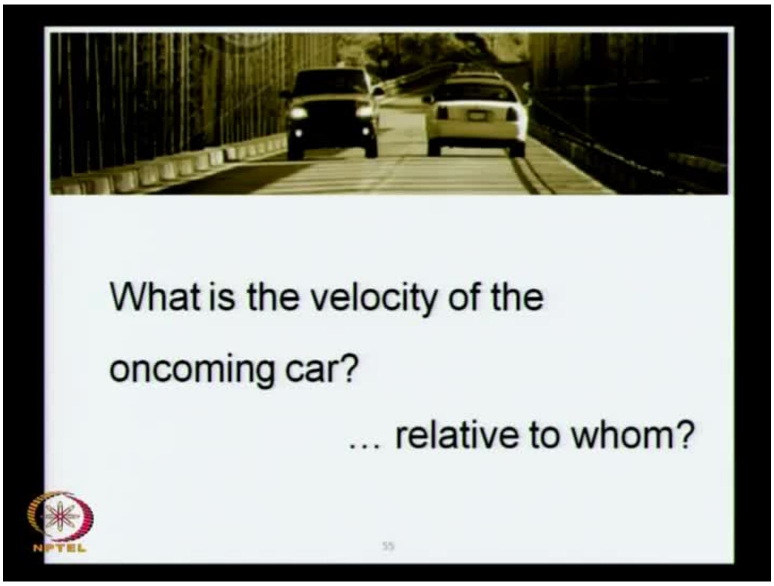

Now, the reason it works is, because we always have to talk about…; whenever we talk about velocity, we always ask velocity with respect to what? What is the velocity of the oncoming car? So, there is a car which is coming from one side and it is going on the other; or the cars are being driven on the right side I think, I took this picture from some American website; this is not, if you drive like that, you will get a ticket in India because both are on the wrong side.

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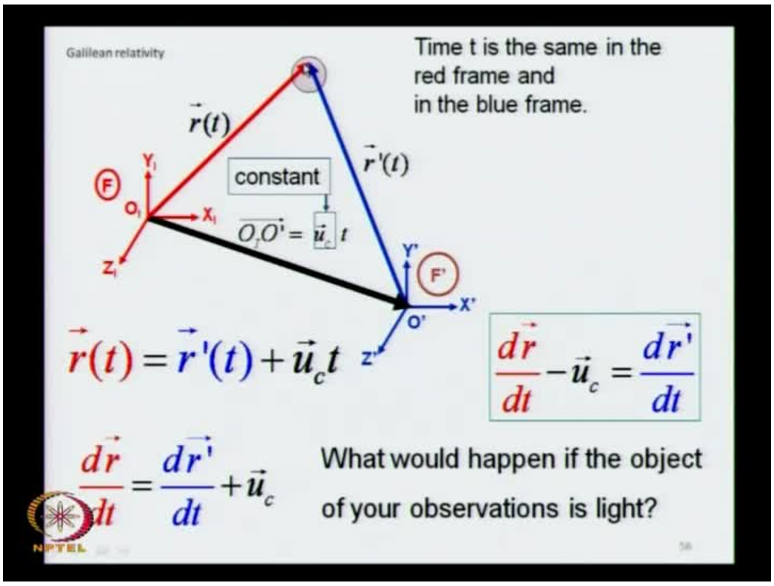

So, the velocity of either car really depends on who is the observer? With an observer who is on the road, one car is approaching toward him, the other car is moving away from him; if the observer is one of the two cars, the relative speeds are going to be different. And this is a fundamental question that we discussed, and you will remember that we discussed this very systematically by considering an observer in an inertial frame of reference, which is this red frame of reference. The index I has a subscripts, reminds us that this is an inertial frame of reference and if there is another frame of reference, which is moving at a constant velocity with respect to the inertial frame, then the distance between the origins, if this start out at t equal to 0 would increase linearly with time, where u c is the velocity of this blue frame with respect to the red frame. So, this is the displacement vector, which will have a length, which increases linearly with time.

Now, if you carry out the observations on a certain target, so this is the object you are looking at which has got a position vector r with respect to the red frame; and with respect to the blue frame, it has got a position vector, which is r prime t and here you have a triangle law of addition, so that r of t in the position vector of this object in the red frame is equal to this blue arrow, which is r prime t plus this displacement vector of the two frames of references, which is u c t.

Now, if you take the time differential of this, you get the velocity in the red frame to be not equal to the velocity in the blue frame, rather the velocity in the blue frame must be added by this u c to get the corresponding velocity in the red frame. So, this is the connection between the two velocities and this is what I meant by saying that, whenever you talk about Galilean relativity, you talk about the velocity of an object; you have to add or subtract the relative velocity of the two frames of references, it is as simple as that.

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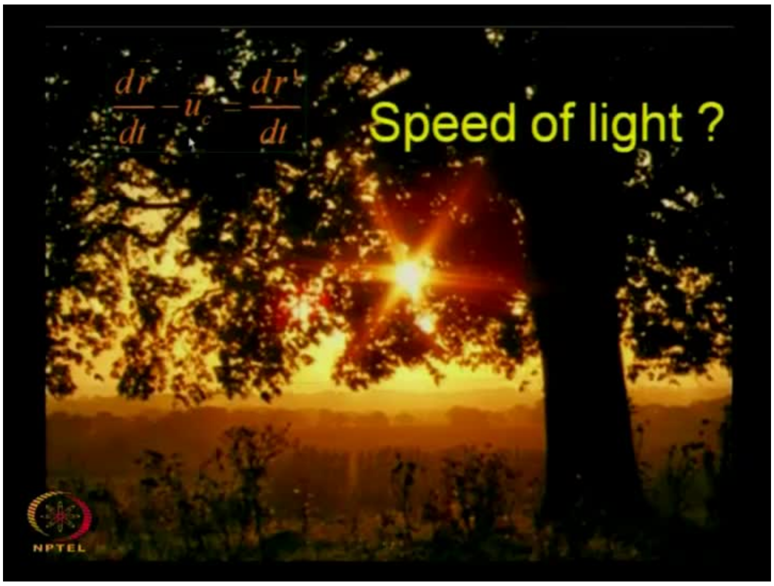

(Refer Slide Time: 12:22) With the difference that if the object you are looking at is light itself; this is the object of our observation. If you are observing light, then do these relations still hold or they do not, and what happens is really very amazing. Because here, this is the what we have is the connection between the velocities of an object; they must differ by the relative velocity of the two observers, but if you are looking at light itself, then the phenomenology has to be interpreted in completely different terms and this is a huge consequence of the special theory of relativity; it needed Einstein's genius to recognize this.

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So, this brings us to new set of transformation laws. So, the Galilean laws of transformation do not work when light is involved, when you are carrying out observations on light or when you are dealing with motions, which are quite high at high speeds and compared to normal speeds that you and I normally encounter; I know some of you try very fast, but even then it does not touch the relativistic fields, because the speed of light is huge.

So, then you have to make use of a different set of transformations; the Galilean transformations do not work; you need to make use of Lorentz transformation. So, we have discussed this all these features at great length in unit 6; so this is a very quick summary and recapitulation of some of the essential ideas.

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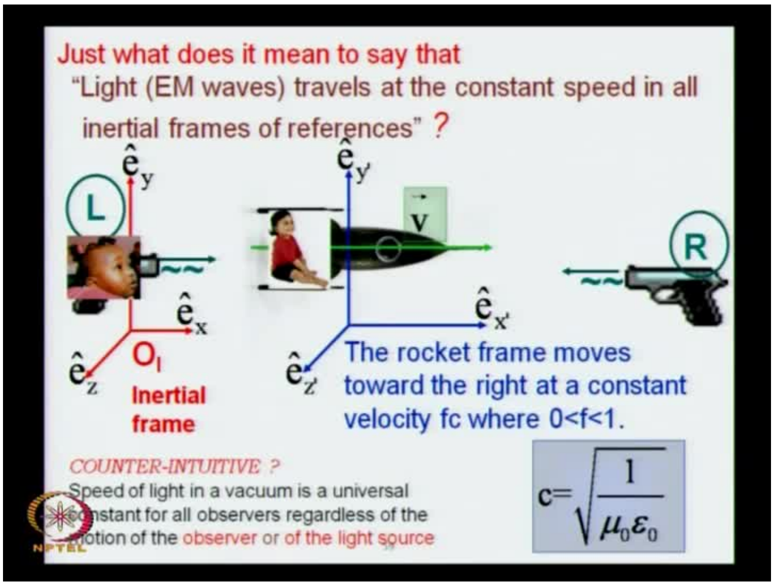

And let me remind you that, when we say that light travels at a constant speed in all inertial frames of references; now, this is a completely non Galilean idea. Because in a Galilean relativity, the relative velocities of the inertial frames of references must be added or subtracted to get the corresponding velocity of light. Now, that is something that you do not invoke when you are dealing with a statement like this that "light travels at constant speed in all inertial frames of references". So, let us see what is it that we mean by this, that if you have got a light gun, which shoots out a ray of light and you observe it in an inertial frame of reference and then we use this picture of this little observer, because physicists are becoming younger and smarter at a much early age.

So, we have got a young scientist over there, who is carrying out these observations in the in the inertial frame of reference and this little observer measures the speed of light; but then there is another observer, who is also young observer and this observer is moving at a constant velocity with respect to the previous observer and this observer could also measure the speed of light. And this observer is in fact in a rocket, which is going at extremely high speeds and no matter what the speed is, this is a constant velocity of the rocket with respect to the red frame and both the observers end up recording exactly the same speed of light regardless of the relative motion.

This is how the laws of nature work; not only that, if there is another light gun on this side, which emits light in the opposite direction and the speed of this light is measured by this moving observer or the stationary observer it does not matter. The two beams in frames of references, which have only a relative constant velocity; both of them are in inertial frames of references and they end up recording exactly the same speed of light.

Now, this is counter intuitive; it is not Galilean. Counter intuitive, as long as you do not polish your intuition to accommodate hard facts and this we learnt from Maxwell's equations that the speed of light is determined by properties of vacuum alone and by nothing else; it is the square root of 1 over mu 0 epsilon 0 and the Maxwell's equations hold good in all inertial frames of references, and Maxwell's waves travel at the speed determined by mu 0 epsilon 0. So, the reality of the constancy of the speed of light, which Einstein recognized as a characteristic feature of the special theory of relativity, which is which he formulated in 1905 was in a certain sense already sitting in Maxwell's equation, it needed somebody to detect that and bring it to the surface and build a formalism of this special theory of relativity based on that.

So, the manner in which this emerges is something that we recognize by considering this particular experiment; that you have a rocket, which is moving from the left to the right and at the frontend and the tail end of the rocket we have got two sources of light.

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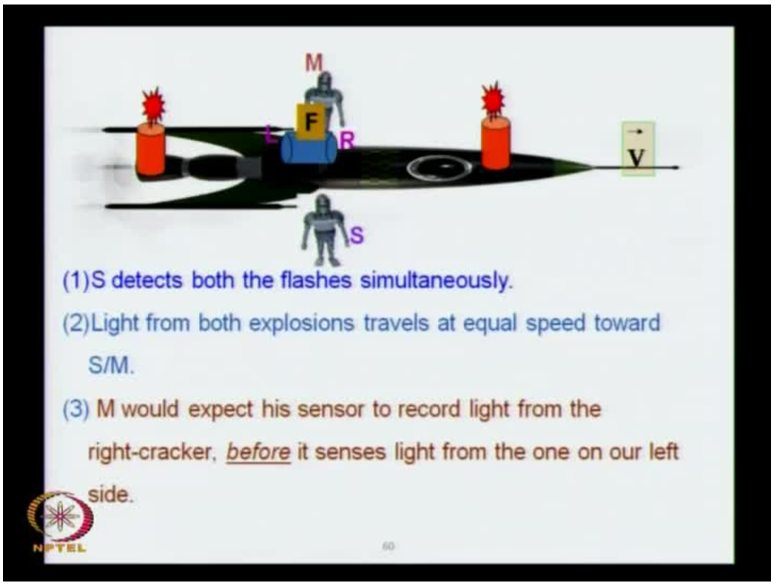

So, we discuss light coming from these two candles seen by two observers; one is in a frame of reference outside the rocket and the other is this observer, who I label as m, who is moving with the rocket and he is in the middle of the rocket, and these two candles light up and the light is seen simultaneously by the stationary observer, who is outside the rocket. But what happens is the observer, who is moving on the rocket, since light is moving at equal speeds from both the directions with respect to both observers, because both are in an inertial frames of references.

So, the speed of light is the same in every inertial frame of reference. So, the consequence of this is the fact that the observer m would think that his detector… he also has got a sensor and he finds his sensor to the right would record the light somewhat earlier than the sensor on the left because his sensors are moving from left to right.

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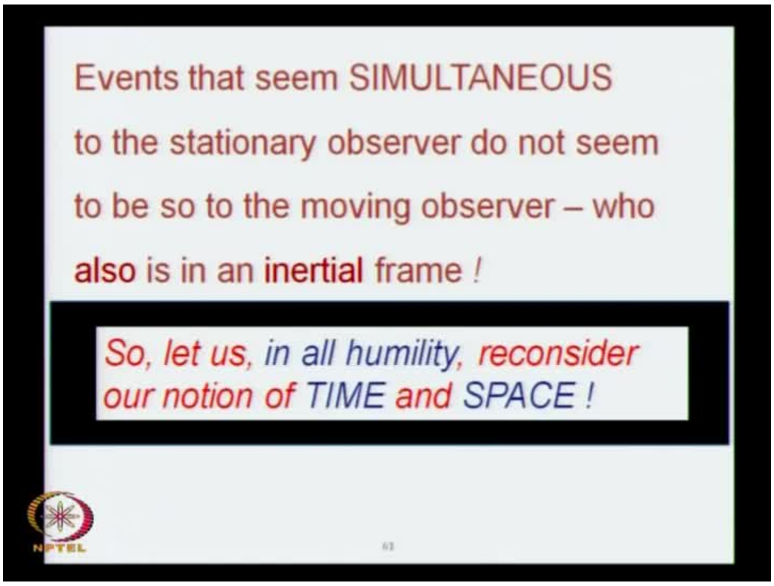

The consequence of fact that, what is simultaneous for one observer is not simultaneous for the other; now, this is not a new shock as I keep reminding. The shock is already in the observation that the speed of light is the same in every inertial frame of reference; this is an automatic consequence of that. But this is not the only consequence, because what happens is that, if you have to reconcile with the fact that what is simultaneous for one observer is not simultaneous for another observer, then we have to you know, reconsider our notion of time and space. So, what is time to one observer and what is space to the other observer?

When you compare space interval and time interval for these two observers, then the Galilean idea that these are invariance and a Galilean transformation does not hold good anymore, because simultaneity has to be redefined as a consequence of the fact that the speed of light is the same in every inertial frame of reference.

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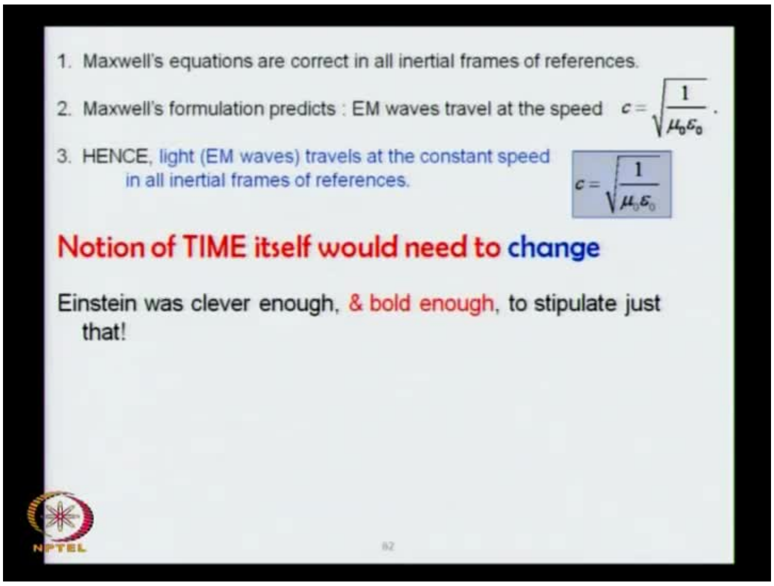

So, now, let us summarize what we have got. Maxwell's equations are correct in all inertial frames of references. Maxwell's formulation predicts the electromagnetic waves to be propagating at a speed determined by properties of vacuum; the electric permittivity in the magnetic permeability of free space and therefore, light travels at a constant speed in all inertial frames of references. Let us accept this as a ground reality and if this is the same if this is the case, then it means that the notion of time will have to change, and Einstein very boldly accepted this, that what is time for one observer is not the same for the other observer. Speed being distance over time; if time is not the same and speed or distance also cannot be the same. So this leads us to notion of time dilation and length contraction; so we discussed these elements in unit 6.

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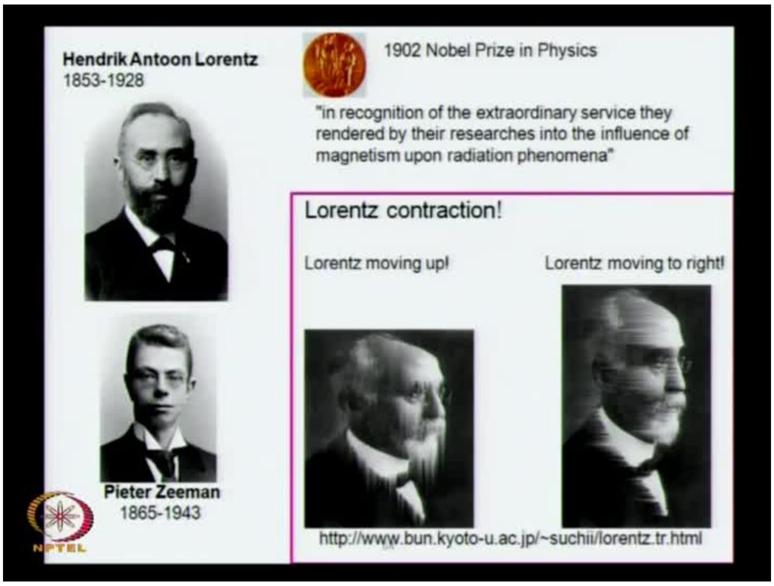

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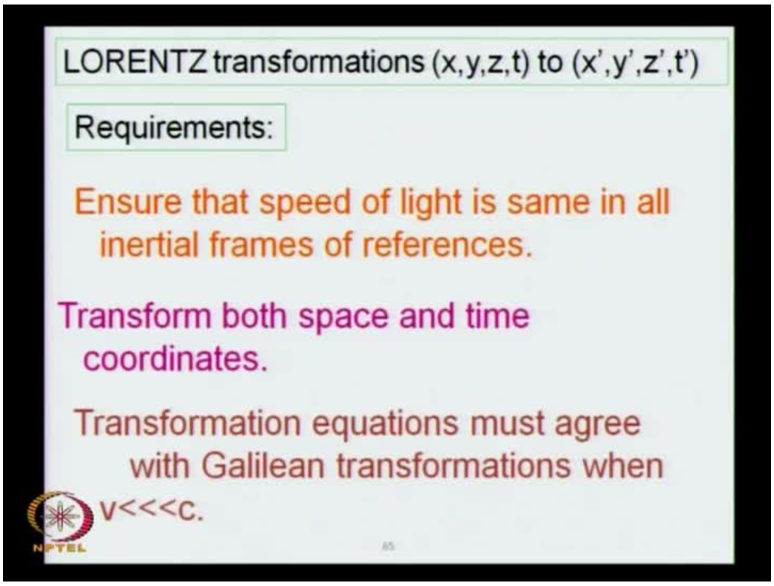

So, Lorentz contraction is an important idea over here; it goes together with time dilation. Now, there is nothing miraculous about neither Lorentz contraction nor about time dilation, because both are automatic consequences of the constancy of the speed of light in every inertial frame of reference. So, there is only one shock namely, the fact that the speed of light is the same in every inertial frame of reference; everything else comes as an automatic and essential consequence of the constancy of the speed of light in every inertial frame of reference. Lorentz got the Nobel Prize, which he shared with Pieter

Zeeman in 1902 and then, we must reformulate the laws of transformations; when you compare events in one frame of reference with another frame of reference, which is moving with respect to the first frame of reference.

So, now, these transformation laws will no longer be governed by Galilean relativity, but they it will be governed by the Lorentz Einstein relativity. So, the requirements of the Lorentz transformations are that they must ensure that the speed of light is the same in all inertial frame of reference; this is where it departs fundamentally from Galilean transformations. They must therefore, transform both space and time coordinates. And Galilean transformations of course are not quite absurd; so they must agree with Galilean transformations when the relative speeds are not very high compared to the speed of light.

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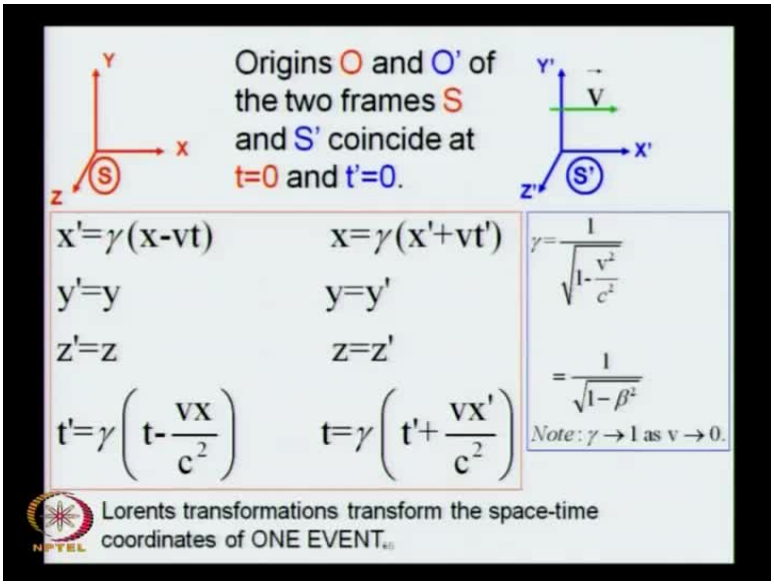

So, the laws of transformations, which give you these features, are known as the Lorentz transformations. And these are the relations that we have discussed in unit 6 that, if you have one frame of reference and another frame of reference, which is moving at a constant velocity with respect to the frame S, then the coordinates x, y, z and t in one frame of reference are connected to the x, y, z and t at the other frame, which I denote with these primes, and this x prime, y prime, z prime and t prime are then mixtures of x and t, and t prime is a mixture of t and x. So, space and time gets scramble they lose their individual identity they actually get superposed, they get completely scrambled in this process.

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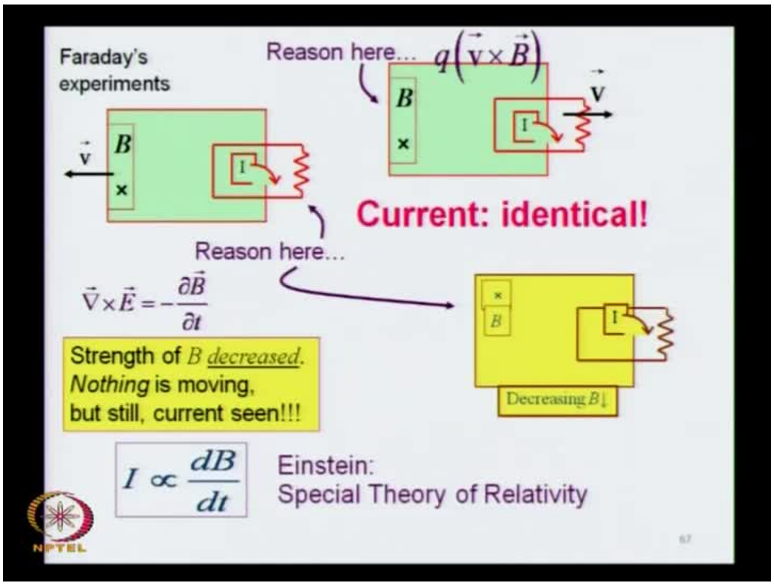

Now, we also discuss the Faraday-Lenz experiment. Now, you will see how all of these pieces come together-that if you have a magnetic field in a region, which is perpendicular to the plane of the screen. So, you see an arrow which is going into the plane of the screen and you have got a loop and you drag this loop from the left to the right, you find that a current is set up in this loop and the reason for this current to be set up is the Lorentz force v cross B. But there is a charge over here, which would then get dragged to the right and it will undergo acceleration in the direction of q times v cross B which in this figure in this geometry will be from bottom to the top on this leg, as you can see from the direction of the cross product and then it will end up setting up a current in the clock wise direction. But we also found that if you do not move the loop, but you drag the field from right to the left, you get an identical current; even if the charge on this conductor is not moving at all, because this loop is not dragged this time, but it undergoes the same acceleration.

So, you cannot invoke the Lorentz force conduction to explain this result, which is amazing and this is something which Einstein analyzed in great details. In fact, an analysis of phenomena - electromagnetic phenomena, such as these which were the primary considerations which led Einstein to discover the symmetry between electric and magnetic processes, which are very neatly packed in Maxwell's equations and come to terms with the constancy of the speed of the light as a consequence of this.

Now, let us see how it happens. So, the reason that a current is set up over here is not this q v plus B, but it is a fact, that the magnetic field is decreasing in this region and the same thing happens if you move neither, but just decrease the magnetic field. So, the current is set up not because the charge is given a certain velocity and therefore, it undergoes acceleration in the direction of q times v cross B; but it is because the magnetic field is changing, it is reducing; it is a completely different region, but it generates exactly the same effect.

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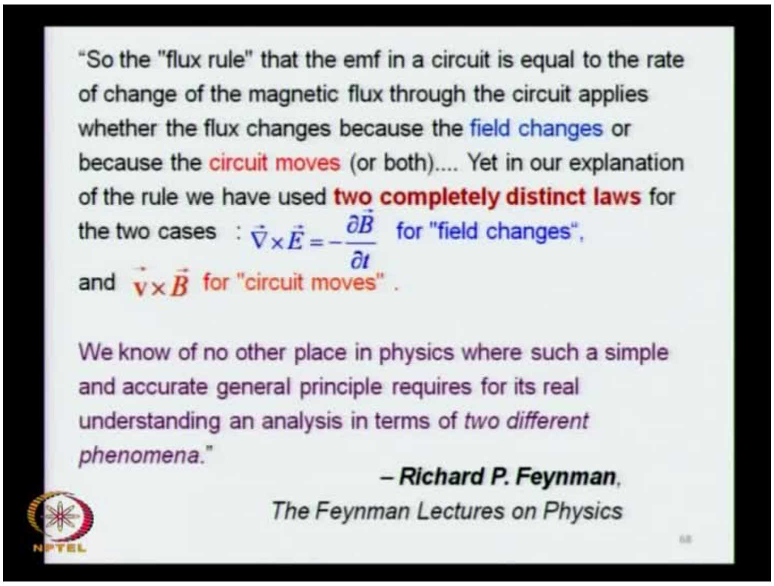

Now, this relationship is at the heart of the special theory of relativity. Now, here, I am going to read out this; it is a very famous quotation; so it is here in quotes and I will let you guess, who said this, because some of you will recognize it. So, let me read this statement for you. So, the "flux rule" that the emf in a circuit is equal to the rate of change of the magnetic flux through the circuit applies whether the flux changes because the field changes or because circuit moves; these are the two conditions that we consider. One was the motion of the circuit; the circuit moves that is one; the other, the field changes del b by del t these are two different things. So, let us read it again, that the emf in a circuit is equal to the rate of change of magnetic flux through the circuit applies whether the flux changes because the field changes or because the circuit moves or both.

Yet in our explanation of the rule we have used two completely distinct laws for the two cases. One that the curl of E is given by the negative rate of change with respect to time of the magnetic field when the field is changing, and the other comes from the v cross B term, when the circuit moves these are two completely different ideas.

Now, I do not know if you have recognized the person who has written this. So, let me give you one more hint, because in the same context he goes on to say that, we know of no other place in physics where such a simple and accurate general principle requires for its real understanding and analysis in terms of two different phenomena.

The effect is the same, but it is understood in terms of the circuit moves in one case and in the other case that the magnetic changes with time. Obviously, the motion of an object and the magnetic field changing are completely two different phenomena, but they are both generating exactly an identical result. And here, this a very dramatic statement here, that we know of no other place in physics where such a simple and accurate general principle requires for its real understanding and analysis in terms of two different phenomena, and who other than the grandmaster, who teaches physics to the whole world for generations could have said it other than Richard Feynman, this is straight out of Feynman lectures.

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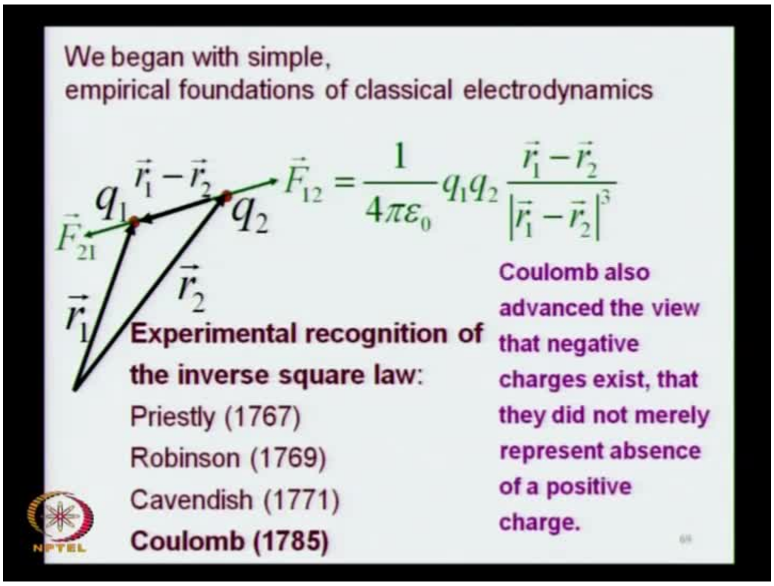

So, let us take this forward; what we did was to begin with some very simple consideration; we began with the Coulomb's law, which we discussed in our previous class. This is the one over distance squared law; this was recognized as a law on the basis of experimental observations carried out by Priestly, Robinson, Cavendish and Coulomb, but it is called as the Coulomb's law, because Coulomb did something more than what the earlier experimentalists did. And that was that he also advanced the view that negative charges exist, they did not merely represent absence of positive charges. The electron of course was discovered much later, in was it in 1996 are there about, in Thomson's experiments and so and so it was much later toward the end of the 20th of the century that the electron was discovered in radio activity, Becquerel and so and so you know this was a much later discovery. So, at the time of Coulomb, of course, the electron was not identified; so this is the Coulomb's law and we began with this, we accept this as a one over distance squared law.

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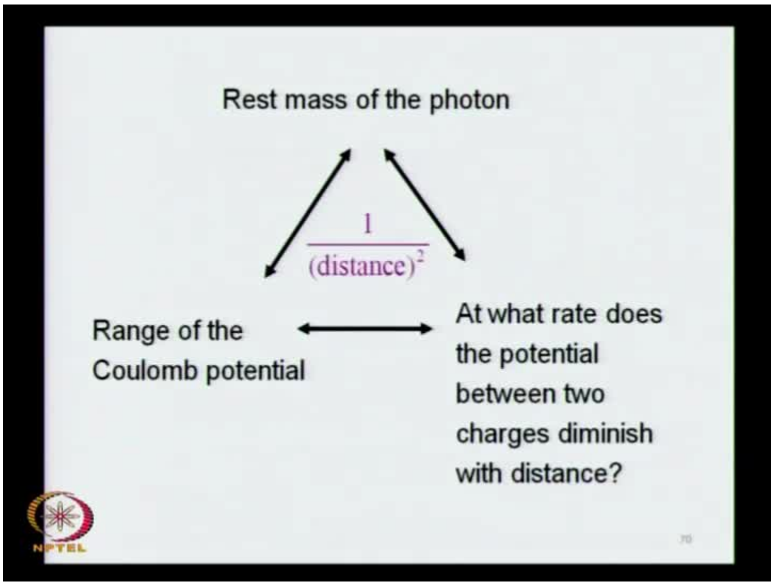

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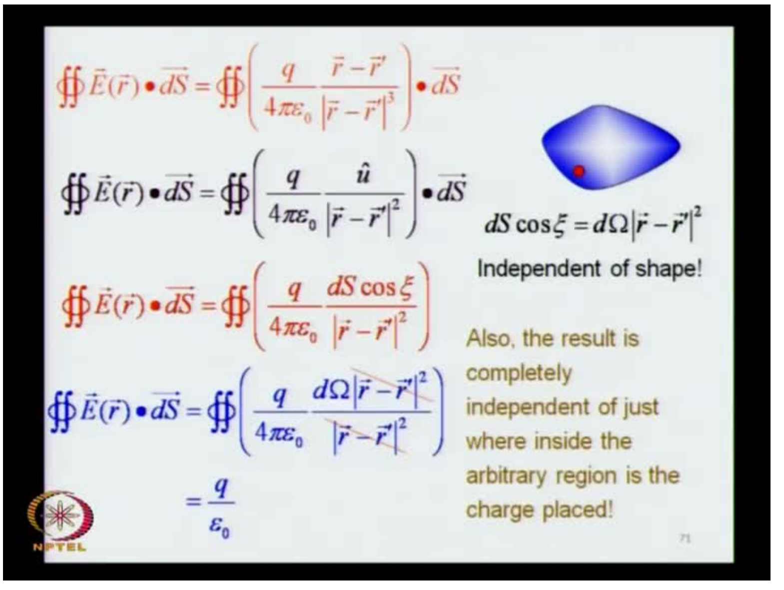

We found that the accuracy of this law as a one over distance square is intimately connected with the rest mass of the photon, which is intern connected with the range of the Coulomb interaction itself; that the Coulomb interaction has got an infinite range is also connected with the rest mass of the photon and all of these interconnections we discussed in an earlier class. And then what we did was to plug in the Coulomb's law over here and consider the electric intensity at any point which is on a surface of arbitrary shape around that source point, no matter what that shape is, did not have to be any regular shape rectangular, parallelepiped, spheres, cylinders these are all regular shapes, but you could take any arbitrary shape and all you do to that is little bit of geometry and nothing else.

And by simply constructing the surface integral and recognizing the fact, that the surface element dS cosine xi is d omega distance square independent of the shape which we did in the previous class. And this is a quick recapitulation of this fact, that this relation enables us to strike out this distance square in the denominator with that in the numerator and then you are left with a relation, which is the surface integral over a closed surface of E dot dS is equal to q over epsilon 0 and this is really the Gauss's law. Because with Gauss's divergence theorem, you can write the left hand side as the volume integral of the divergence of E.

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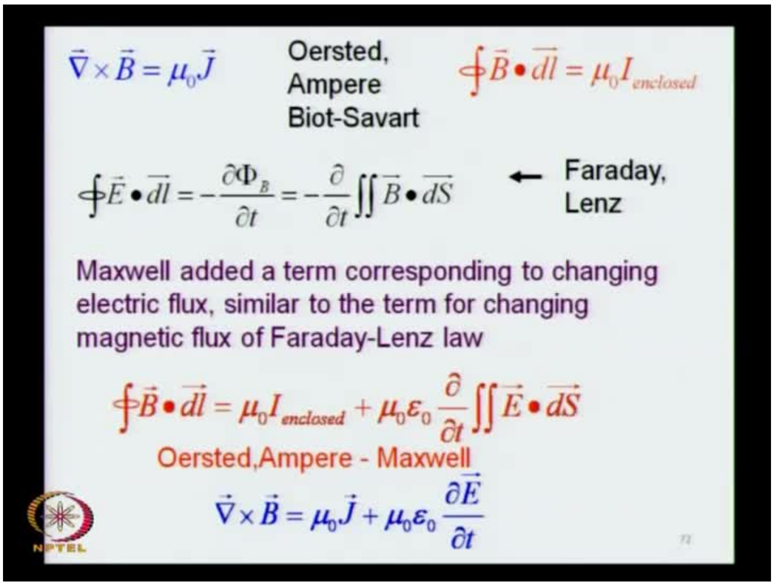

The other things we did was to pick the thread from the Biot-Savart law, get the amperes law out of it through direct integration and explain the Faraday-Lenz observations in terms of the rate of change of the magnetic flux, because we found that if there is a rate of change of magnetic flux, which is what I discussed just a little while ago, then also you can generate a rotational electric field.

So, what Maxwell did was to add a term corresponding to change in the electric flux, very similar to the change in the magnetic flux. So, here you have in the Faraday-Lenz the magnetic flux and what to have on the right hand side is a change in the magnetic flux. So, Maxwell added a term very similar to this by exploiting symmetry and similar to this b dot Ds, he added this E dot dS to this relation, which came from the Ampere Biot Savart law and what you have is really the Ampere - Maxwell law. Now, this is how Maxwell augmented the law, which was formulated by Oersted and Ampere on the basis of experiments; this came out of Maxwell’s intuition, he recognized that there has to be a symmetry between electricity and magnetism.

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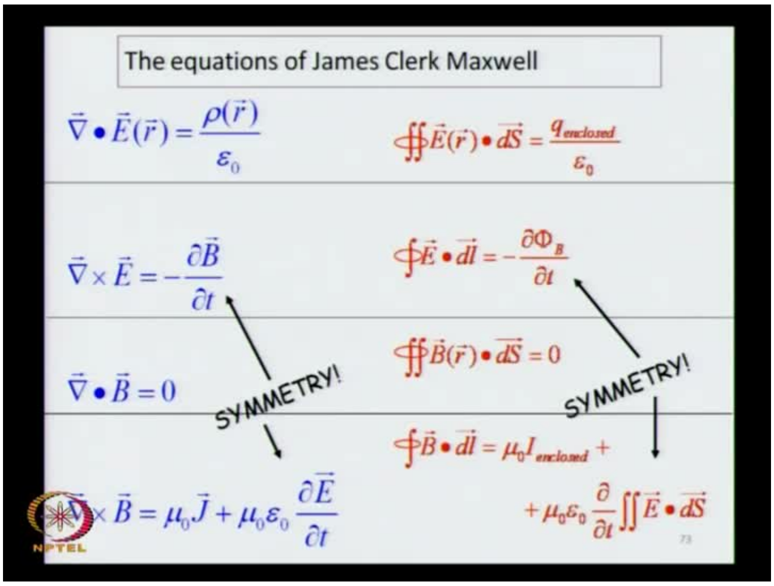

So, now, you have the complete set of equations modified by Maxwell, which came from empirical observations of Coulomb, Gauss, Ampere and it was augmented by Maxwell to include this term over here, which comes from the rate of change of the eclectic flux. And then, we consolidated these together in which we formulate both the divergence and the curl of E, and also both the divergence and the curl of B in the famous Maxwell's equations.

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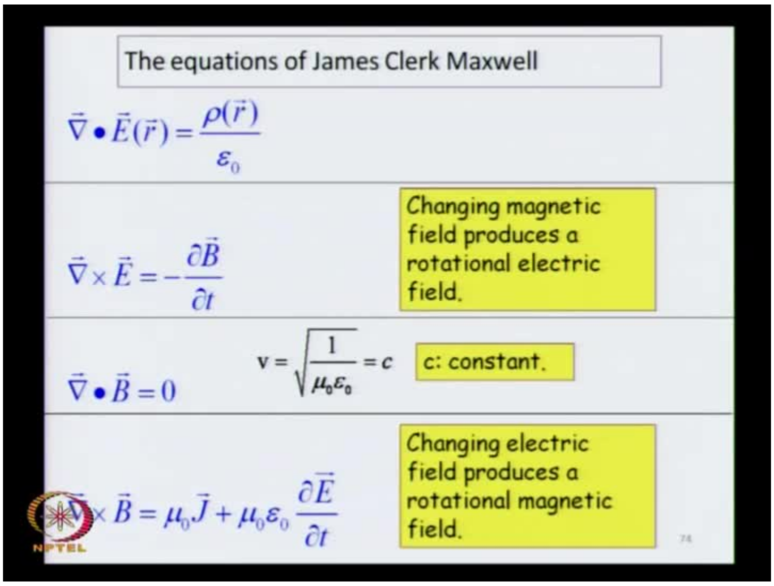

And from Helmholtz theorem, we do know that both the curl and the divergence are needed to specify the vector field, apart from which you of course need the boundary conditions. So, Maxwell was inspired by this symmetry element; you can write these in terms of integral equations also, these are completely equivalent to the equations in the point form. And our essential conclusion is that there is a change in magnetic field, which produces a rotational electric field; likewise, there is changing electric field produces a rotational magnetic field. So, this was Maxwell's contribution to what the experimentalist knew before Maxwell.

And then, by carrying out further vector analysis we find that the electromagnetic wave propagates at the speed of light and this is how Maxwell connected the electromagnetic phenomena with optics; and if optics corresponds to the electromagnetic radiation in the visible range, then of course this is the part of the entire electromagnetic spectrum at wavelengths beyond the red and also to wavelengths, which is shorter than the violet. So, VIBGYOR is the visible region, but at both wavelengths which are shorter and longer than the visible region of the electromagnetic spectrum; every electromagnetic wave satisfies these relations, which is why they are so powerful.

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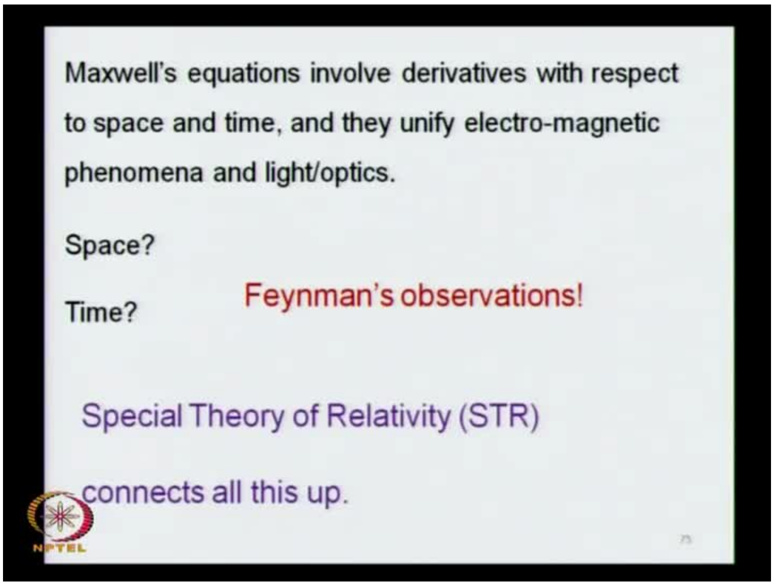

Now, what do the Maxwell's equations have? They involve derivatives with respect to space and time, they unify electromagnetic phenomena with light and optics, but space and time are not completely independent of each other, they are scrambled in the Lorentz transformations.

So, this will have a major bearing on our understanding of the electromagnetic phenomenon. So, you remember, Feynman's observations that you have completely different reasons that del b by del t to be the explanation for the current being set up when you are dragging the magnetic field and the v cross B term being invoked, when you are dragging the loop and you have completely different phenomena different explanations for the same effect.

So, this comes because of these connections; so this comes essentially from Einsten's theory of relativity. This is known as the special theory of relativity, which connects all this, puts it all together. It is called as the special theory, because subsequently about 10 or 11 years later, Einstein formulated the general theory of relativity of which this is the special case.

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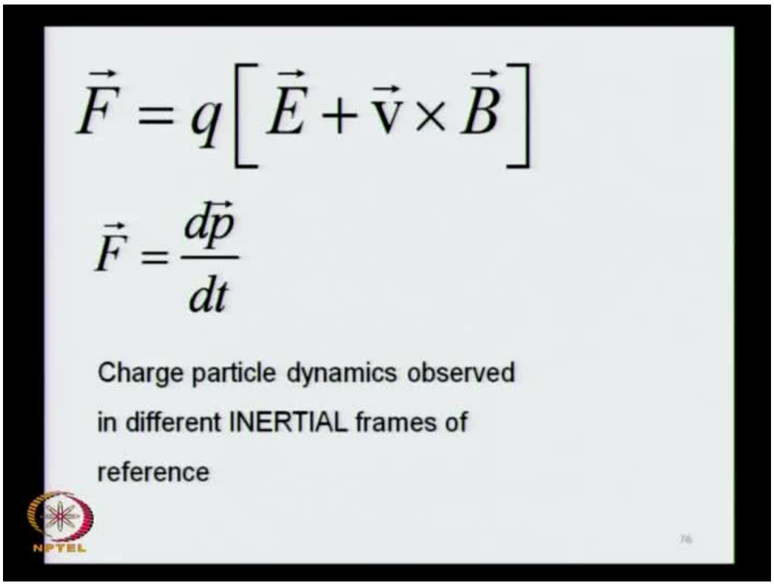

Of course, what you need a little more than Maxwell's equation to solve problems and electrodynamics is the Lorentz force; and this force would generate a change in momentum at as a rate to which this force is equal. So, dp by dt is equal to the Lorentz force and what we need to do is to study, how the charged particles respond to electromagnetic fields, but you observe them from different inertial frames of reference;

from one frame of reference and then from another frame of reference, which is moving with respect to the first frame of reference, but at the constant velocity.

When you carry out these comparisons, there are features that you are going to worry about. Because if you were to apply Galilean relativity, then what is this v? This is the velocity of the charged particle and this velocity of the charged particle to one observer is not the same to another observer even if he is in an inertial frame.

So, there are no Pseudo forces; both the frames of reference is that we are talking about are essentially inertial frames of references. You invoke Pseudo forces, which we studied at great length in unit 5 was it? Yes, it was in unit 5 that we discuss the Pseudo forces and we had to plug-in the Pseudo forces, whenever we had to analyze motion in one frame of reference, which is accelerated with respect to an inertial frame of reference.

Now, we are comparing observations by two observers, neither is accelerated with respect to the other, they are moving at a constant velocity with respect to the other; there is no acceleration of the two observers relative to each other; both are therefore in inertial frames of references. But what is velocity to one observer cannot be the same to the other and then it will be tempting to ask that this velocity will be different. And therefore, if you take a naive view of this force, one would think that the force would be different and then would you guess that the dynamics will be different.

Now, the consequence of this would be the two observers; both of whom are in inertial frames of references we will see different physics; you do not think that should happen. And Einstein incorporated this as an integral element of his formalism of the special theory of relativity; this comes as the second postulate, if you might call it that the laws of physics are the same in all inertial frames of references even when you are dealing with electrodynamic phenomena.

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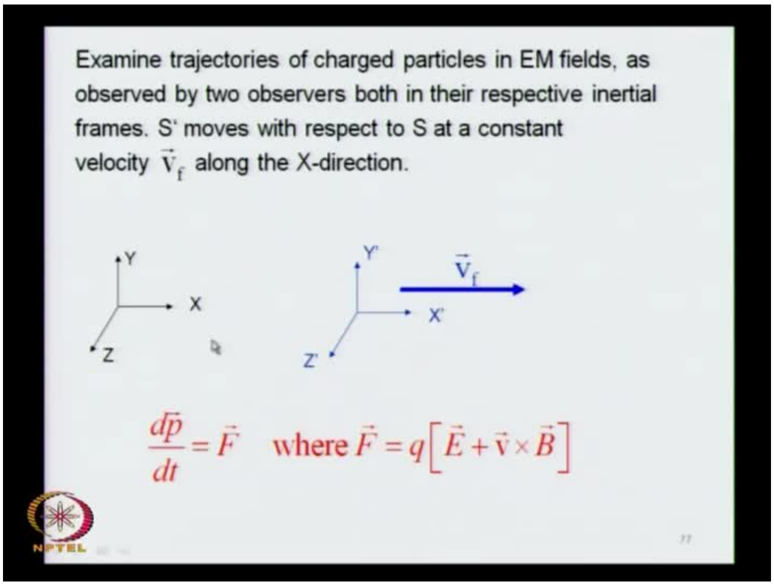

So, what we will do is examine the trajectories of charged particles in electromagnetic fields; these are the two frames of references. We consider there is one frame of reference, which is the black frame; another frame of reference, which is the blue frame, which is moving with respect to this black frame, at a velocity V f and then we must understand the consequences of the electromagnetic field E and B on the dynamics of a charged particle of charge q.

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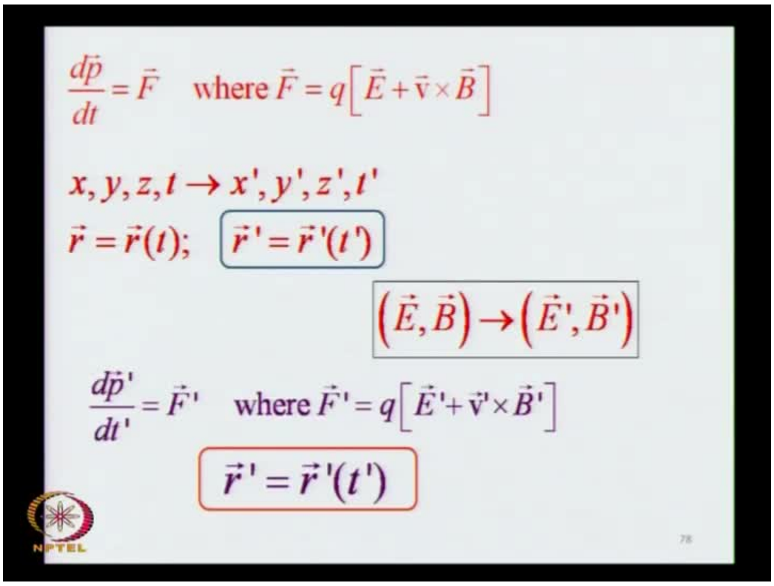

What is the physics that will be seen by these two observers? So, this is the question that we will examine. Now, we have accepted that Galilean relativity will not work; it cannot, because the speed of light is the same in every inertial frame of reference, it is a completely non Galilean view point.

As a result of the Lorentz transformations, which must now replace the Galilean laws of transformation. The parameters x, y, z and t of one frame would transform to x prime, y prime, z prime and t prime in another frame of reference, because t is no more absolute. And the trajectory seen in the unprimed frame of reference will be given by how the position of the charge particle changes as the function of times. So, this functional dependence and time will provide you the trajectory of the charged particle in the unprimed frame of reference, whereas in the primed frame of reference the trajectory will be given by how the transformed position vector is given as a function of t prime, which is not the same as t of the previous observer.

So, the trajectory in the primed frame will be given by how r prime is given as a function of t prime, which is different from t. Now, this can be reconciled by admitting that what is electromagnetic field for one observer cannot be the same E and B to the other observer, because if it were to be the same, you will end up arriving with the conclusion that the two observers in the inertial frames of references will see different physics.

Now, that is not something that you except, and the reason is what is electric field for one observer is not the same electric field for the other observer. Likewise, what is the magnetic field B for the first observer is not the magnetic field, not the same magnetic field for the other observer; what the other observer sees is E prime and B prime, which are both different from E and B.

So, what the second observer should do is to set up an equation of motion, which is the rate of change of momentum - will be given by the force. But his force will be given by charged times E prime plus v prime cross B prime and all of these quantities are different in the primed frame of reference including the electromagnetic field and then he must solve this equation of motion and how do you solve the equation of motion? You integrate it; put in the initial conditions, pops out the trajectory something that you do in high school physics.

So, it is the same thing; it is all classical idea, there is no quantum theory here, which is why the special theory of relativity and laws of classical electrodynamics. Maxwell's theory is an integral part of classical mechanics, which is why we chose to include at least, the essence of this formalism in our discussion on this course on classical mechanics.

So, the observer in the primed frame must then integrate this equation over here; put in the initial conditions and get its trajectory r prime as a function of t prime. So, notice that this r prime as a function of t prime can be obtained over here by carrying out these Lorentz transformations this is one method that you can get.

The other method is to set up a completely new equation of motion with the primed electromagnetic field; integrate this equation of motion and not this. Now, these two are different equations - one is dp by dt equal to f, the other is dp prime by dt prime is equal to F prime; this has E and B, this equation has got E prime and B prime, this has got the velocity v, this has got the velocity v prime .

And the solutions are given in such a way that they describe the trajectory and you see the trajectory in the prime frame to be given by r prime as a function of t prime in the primed frame of reference coming out from the Lorentz transformations or alternatively, by setting up a completely different equation of motion in terms of the new electromagnetic field seen by the second observer, and solving this new equation of motion and then integrating it.

So, there are two alternative methods of getting the solution. Now, if all this is mambo jumbo these two relations - these two trajectories will not agree with each other. So, do we see an agreement between this solution, which has come from a Lorentz transformation of x, y, z, t to x prime, y prime, z prime, t prime, and this r prime as a function of t prime, which has come from a completely different technique by solving a different equation of motion with different E prime, v prime and B prime. If these two solutions agree, then it would mean that the laws of transformations for x, y, z and also for E and B that we are that we are discussing, they must be correct and that would be the test.

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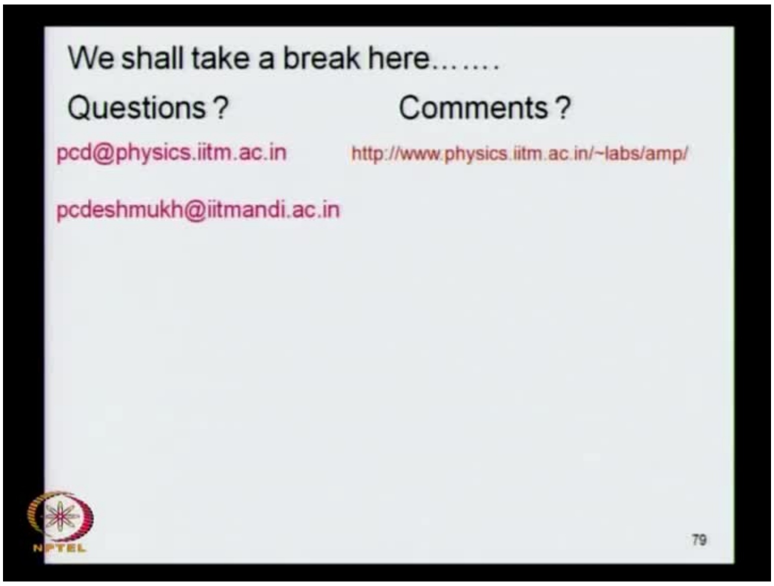

So, we will take a short break over here and then in the next class, we will actually take some special cases. We will take an electron, put to it in an electromagnetic field and see how it will respond to the electromagnetic field subject to, different initial conditions, to different values of the electric and the magnetic field and then find out how the solutions compare with each other when you view the solutions from one frame of reference and carryout the Lorentz transformations of the trajectory. All carry out the transformations of the electromagnetic field E and B to E prime and B prime and solve a new the new equation of motion and do they agree.

So this is something that we will discuss in the next class. So, if there is any question or comment, I will be happy to take otherwise, we will take a break here and then discuss this application in the next class. Yes

In non inertial frame the velocity of light will be defined

See, non inertial frame of reference is not the issue over here; non inertial frame of reference comes into play, it has absolutely nothing to do with what we are discussing; we are discussing the constancy of the speed of light in every inertial frame; that the two observers that we talked about.

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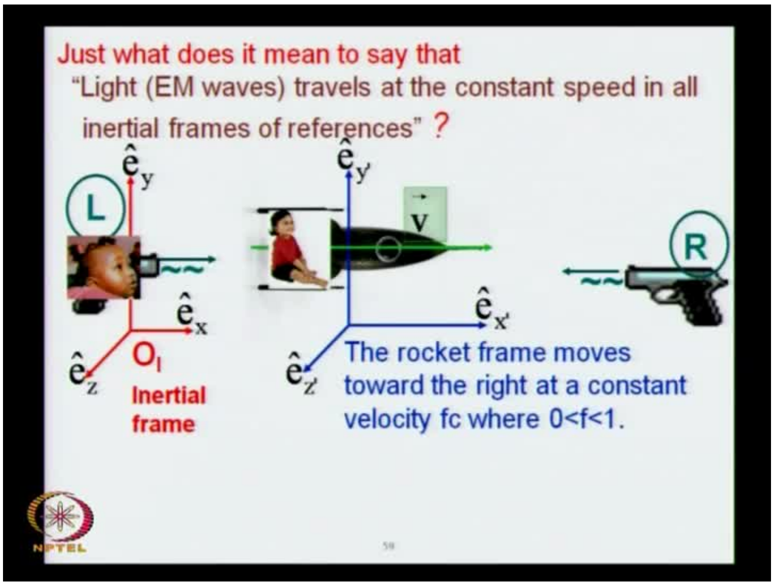

Let me go back to this picture here; these were the two tiny observers who are measuring the speed of light, they are both in inertial frames of references - one observer is in the red frame, the second observer, who is also very smart, is moving with respect to the first observer. But his motion with respect to the first observer is essentially at a constant velocity and therefore, both of them are in inertial frames of references; when one is not moving at a constant velocity with respect to each other, only then do you consider what we call as motion in an accelerated frame of reference.

So, we are not talking about Pseudo forces at all, the only force we are invoking is a real physical interaction. Mind you, the dynamics of a charge particle in an electromagnetic field must be governed only by the real physical interaction and physical interactions in nature are very few; you have either gravitational interaction or electromagnetic interaction or nuclear strong or nuclear weak or some integration thereof.

So, there is nothing like getting into the Pseudo forces; if you compare observations by two different observers, one of who is in an accelerated frame of reference and then if you still want to make the Newtonian cause effect relationship work, then he will have to plug in a Pseudo force.

He will not be able to explain it on the basis of the Lorentz force alone that is the idea; the only physical interaction that he would invoke is the Lorentz force, but that requires the fact that the two observers are moving at a constant velocity with respect to each other; if they are not, then the observer in the accelerated frame of reference will have to invoke a Pseudo force, he cannot explain this dynamics in terms of the Lorentz interaction alone, he will have to invoke the Pseudo force yes your question.

So, here, observing the velocity of light is constant in all in (( )) My question is what will be if I am in in a rotating frame or in non inertial frame, then what is the velocity? then will it be c or something like that something (( ))

In a rotating frame of reference whatever see there are the trajectories will obviously be different; the dynamics will be different and this is part of the reason that we discussed this great length in unit 5. We analyzed the motion in a rotating frame of reference also, and whatever dynamics that you see will have to invoke Pseudo forces, which is in fact to a certain extent a continuation of my previous answer, because the rotating frame of reference is also an accelerated frame of reference.

So, the moment you observe physical phenomena in an accelerated frame of reference, then you will have to invoke Pseudo forces, because if a ray of light is taking some time to reach you from the source and in the mean time you have turned.

Then this is the given

Then the trajectory of the ray of light is obviously going to be different, because light does not reach you at infinite speed; if it were to reach you at finite speed, then before you could turn, you would already see it, but since it is approaching you at a finite speed, before you see it, you would have turned and the only way you could explain this will be by invoking Pseudo forces.

So that is a different question altogether; it has nothing to do with the fundamental reality that the speed of light is constant in every inertial frame of reference. Now, this is the completely non intuitive idea and Einstein had the genius to reconcile with it, any other question? So, we will when we come back, we will discuss the rest of it.