Select/Special Topics in Classical Mechanics

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Module No. # 04

Lecture No. # 13

Dynamical Symmetry in the Kepler Problem (ii)

We will continue our discussion. What we did in the previous class was we began with the equation of motion, played with it and discovered various conservation laws.

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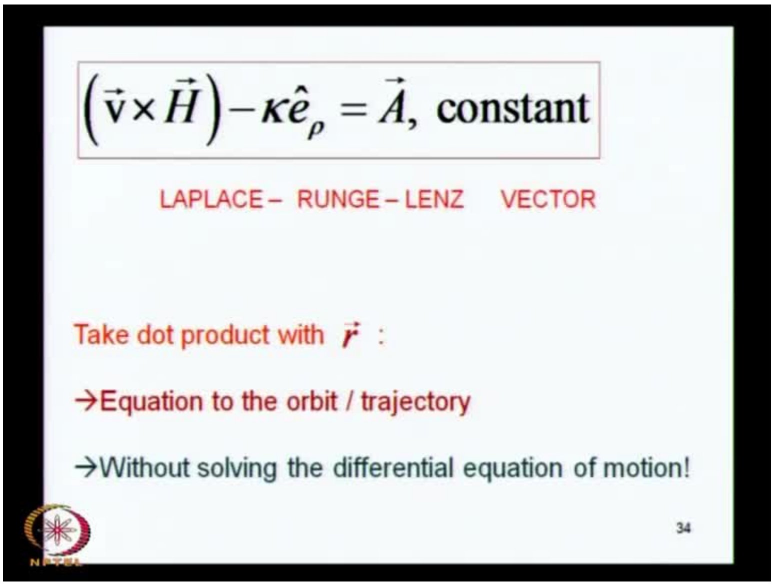

One of the quantities that were conserved was Laplace-Runge-Lenz vector. What we will do now is use this to get the equation to the orbit. Again, there is a novelty in this process because whenever you solve an equation of motion, which is the purpose of the goal of a problem in classical mechanics. It is to find what characterizes the mechanical state of a system and how does the system evolve with time. When you solve this problem, you get the trajectory of the particle or the system in the phase space or in the configuration space, you get the trajectory or you get the orbit. It means a closed trajectory, which is an orbit, but essentially you are looking for the solution. You are looking for the trajectory and you have to find that the problem in mechanics is to find this trajectory. The method to get this trajectory is by solving differential equations, either the Newton's equation or the Lagrange's equation or the Hamilton's equation.

What we will do is just play with this Laplace-Runge-Lenz vector. We will not solve the differential equation and we will not solve the equation of motion, but still we get the trajectory. It is not that fun and let us see how we do that. We will get it by simply taking the scalar product of the Laplace-Runge-Lenz vector with the position vector and that is all you do. By taking this scalar product, we will get the orbit equation, which is completely different process and solving the differential equation to get a trajectory.

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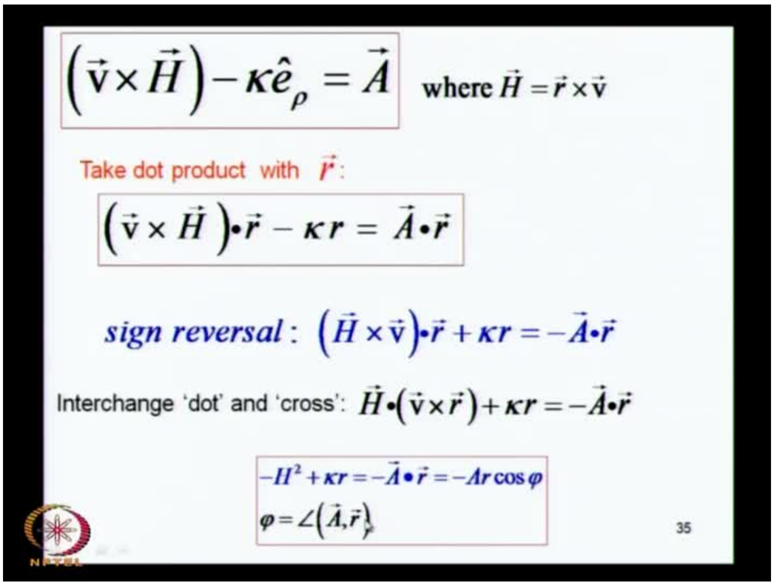

Let us take the dot product with the vector r and you have two terms. Here, r is the radial vector. So, the dot product of kappa times e rho with r gives you kappa r. On right hand side, you got the dot product of the Laplace-Runge-Lenz vector A with the position vector.

Now, let us reverse the signs in this equation. Instead of v cross H, I have H cross v in the first term and instead of this minus sign, I have got a plus sign over here. On the right hand side, I get a minus sign over here. So, I just do a sign reversal and have got triple product. I can always interchange the cross and the dot and let us do that. It is H dot v cross r plus kappa r. We have met this quantity earlier as v cross r, it is the negative of the specific angular momentum. Here, r cross v is the specific angular momentum and v cross r is the negative of the specific angular momentum.

Now, we have the dot product of H with the negative of H, so that gives us minus H square and you have kappa r. Thus, this is equal to minus A dot r, which is A r cosine phi, where phi is the angle between the Laplace-Runge-Lenz vector and the position vector.

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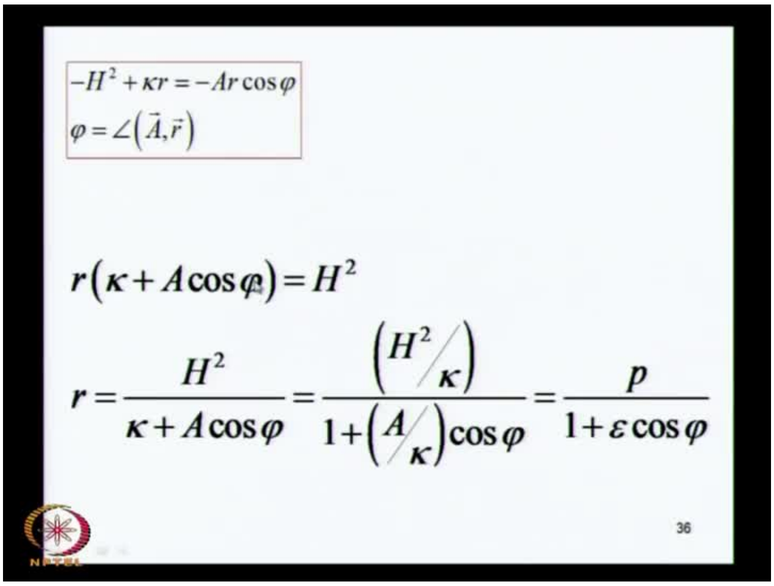

If we look at this equation, this already gives us the orbit equation. It tells you the relationship between r and phi, which is what an orbit equation is. Some of you will recognize what this equation is, but we will rewrite this in a form, which is more familiar to us. So, we will just carry out some transformations.

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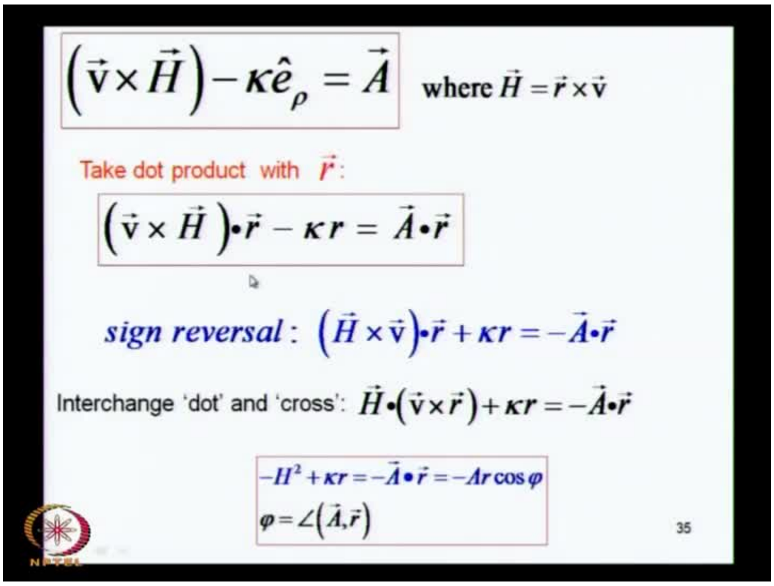

I take r as common between this term and this term and bring H square to the other side of the equation. It is r times kappa plus a cos phi equal to H square, which essentially means that r is equal to p over 1 plus epsilon cosine phi, which is an equation for the ellipse. This is in a very familiar form in polar coordinates. The equation to the ellipse is essentially p over 1 plus epsilon cos phi, epsilon being the eccentricity. So, this equation and this equation is no different. It is just a rearrangement of that equation. We got it simply by constructing the scalar product of the Laplace-Runge-Lenz vector with the position vector. We did not solve any differential equation and we did not integrate the equation of motion, but we got the trajectory. Is it wonderful or not?

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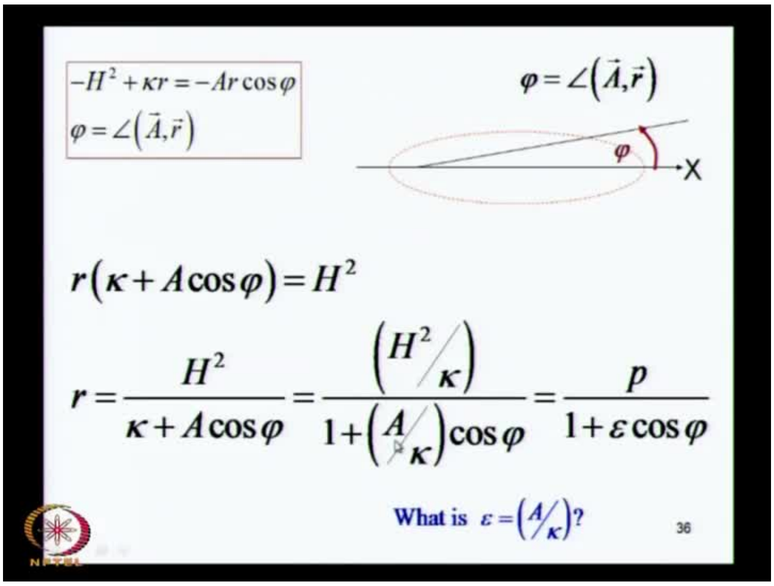

Here, we got this as the equation to the ellipse and this is the kind of geometry we are working with. We need to find what this eccentricity is and this eccentricity e or epsilon is coming as a ratio of the magnitude of the Laplace-Runge-Lenz vector with kappa, but this is in our own units. In some other units, some of the constants will be cancelled, when you take this ratio.

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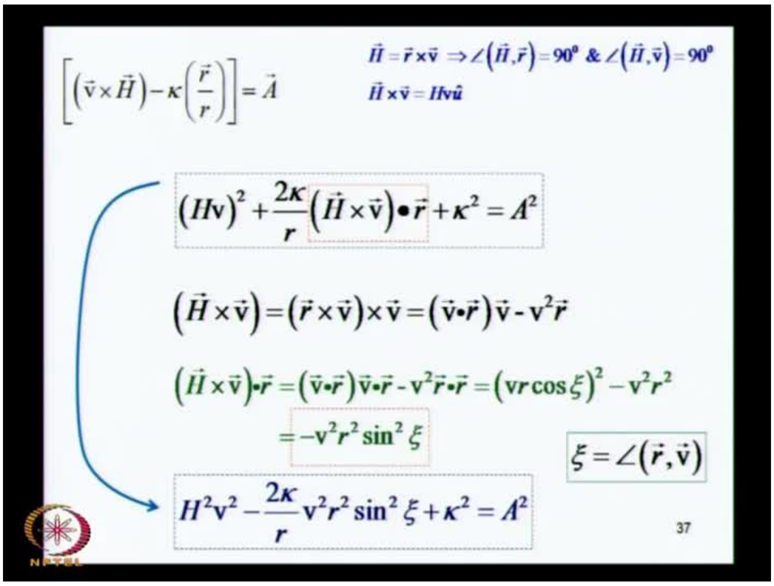

We need to get that eccentricity and to get that what we do is to take the scalar product of the Laplace-Runge-Lenz vector with itself. So, we take the scalar product on the left hand side with itself and the scalar product of the right hand side with itself. So, A dot A will give us a square on the right hand side. On the left hand side, we have got v cross H, which is nothing but H v times the unit vector because H is obviously orthogonal to v . The angle between H and v will have to be 90 degrees because H is defined r cross v, so it has to be orthogonal to v.

There is no angle between H and v, other than 90 degrees. It is always phi by 2 and this dot product of v cross H will give us the square of H v. The dot product of this quantity with itself will give us kappa square. This is just the unit vector dotted with it, which give you the number 1. So, the dot product of this quantity with itself give us kappa square and you have got the cross terms between these. Since the scalar product is commutative, you get a factor of 2 over here. You should have a minus sign, but I have written it with the plus sign because instead of v cross H, I have written an H cross v over here.

This is the result that we get and we now have this H cross v sticking over here. We expand this because H itself is the specific angular momentum as r cross v. We will carry out this expansion, so this is the dot product of this outer vector with v times; the middle vector v minus the dot product of this vector v with the adjacent vector v gives us v square times the remote vector, which is r. This is again the bac cab rule, which I have used. We have got this H cross v and now what we need over here is H cross v dot r. We got H cross v and we take its scalar product with the position vector r. We take the scalar product of this right hand side with the position vector r. So, you are already have a v dot r and from the projection of v on r, we get another v dot r over here. From the second term, we have minus v square and then the dot product or the scalar product of this position vector with r. So, this is minus v square r dot r and if the angle between v and r is xi, then v dot r is v r cosine xi. So, you get the square of this because there are two of these terms.

It is v r cosine xi square and you have a v square over here. So, you can take the v square r square as common and recognize that 1 minus sin square theta. You get the minus sign common and the result is v square r square sin square xi. This is what we get for this term over here. So, let us simplify this and this equation, which has H v square in the first term. In the second term, you have 2 kappa over r; there is a plus sign, but this box H cross v dot r has got a minus sign over here. So, the result is minus 2 kappa over r, which is this coefficient here and then the v square r square sine square xi. This kappa square comes here and you have the square on the right hand side. So, there is no new analysis, it is just a rearrangement of terms. Now, this is a very nice form because you can recognize this v r sin xi to be the magnitude of the specific angular momentum because that is r cross v.

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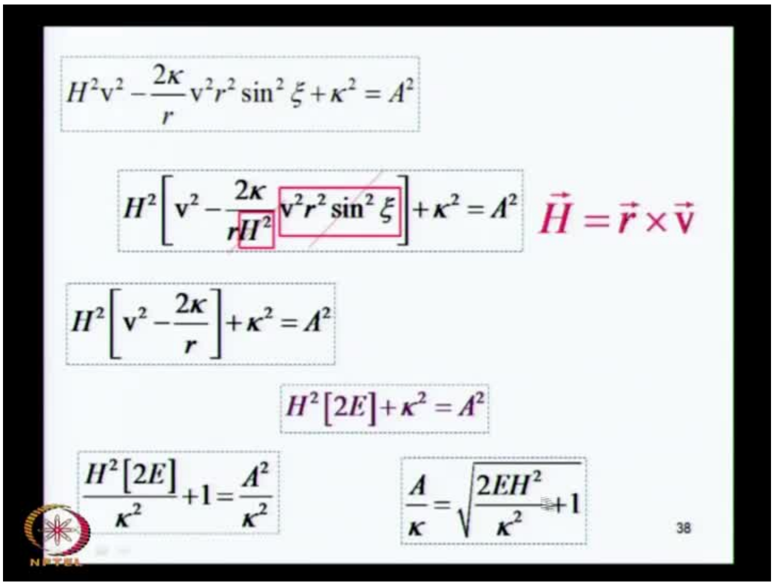

I take H cross square and H square as common in the first two terms. It is already sitting there explicitly in the first term. Since I have taken it as common, I must divide the second term by H square. I leave alone the third term as kappa square. I can cancel this denominator, H square with this v square r square sine square xi because H is r cross v. So, it is r v sin xi and that is the magnitude of H. The square of it, in the numerator cancels the square of it at the denominator.

Now, you have a rather simpler form, in which the sin of the angle disappears; the sin xi disappears, v square minus twice kappa r is twice the energy. This is in units of mass because this is the specific mechanical energy - m v square by 2. This is what we will interpret as the kinetic energy and this is v square, so it is twice the kinetic energy and this is twice the potential energy, twice the specific kinetic energy plus twice the specific potential energy because that how we have defined kappa. So, this is nothing but twice the energy. So, H square twice the energy or specific mechanical energy plus kappa square equal to A square. Rearrange these terms, in which you can divide this entire equation by kappa square. It gives us A over kappa and we needed that A over kappa to get the eccentricity. If you remember, we got the equation to the ellipse. We had the eccentricity, but the eccentricity was in terms of A over kappa. Now, we have in terms of known quantities, which are the energy and the angular momentum. A over kappa is this quantity, it is the square root of the left hand side. So, it is the square root of 2 EH square over kappa square plus 1.

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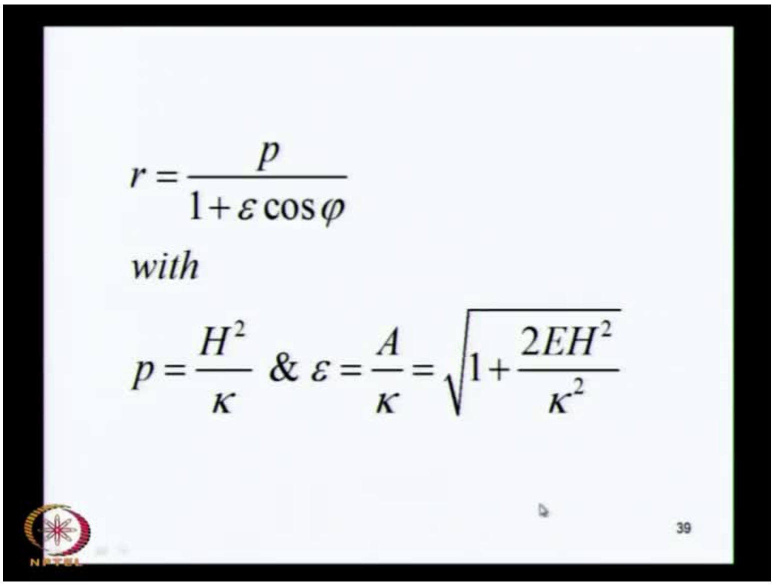

This is the complete orbit equation in polar coordinate. So, this is the equation to the ellipse. We have achieved a lot by playing with the equation of motion. We discovered various conserved quantities.

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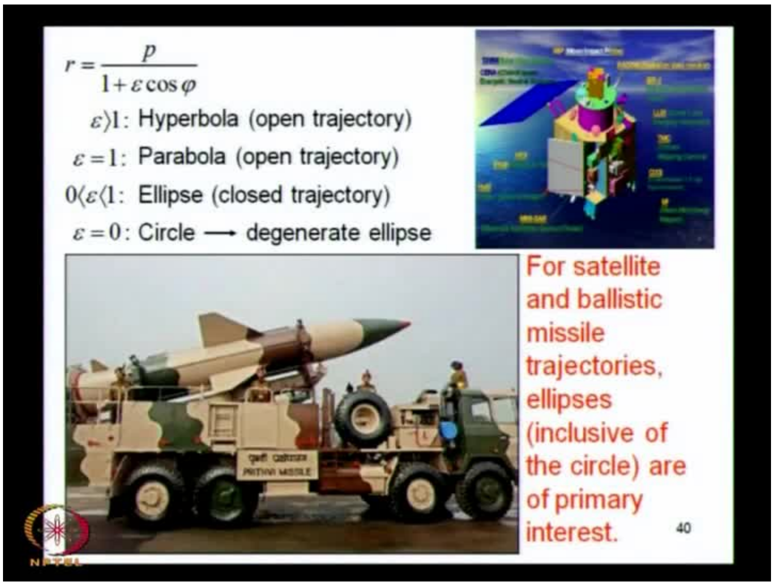

We connected these conserved quantities with symmetries. These are obviously extremely important relations in rocket technology, in satellite trajectories and so on. Depending on the eccentricity, you will get different kinds of orbits. So, you may have an open trajectory, you may have an hyperbola. If the eccentricity is greater than one, you may have a parabola. If the eccentricity is exactly equal to 1, you may have closed orbits. If the eccentricity is less than 1, it is the ellipse. It is degenerate with the major axis and the minor axis. If they are equal, you get a circle. So that is what will happen, if the eccentricity is 0. These are of course very important in rocket technology, satellite shaping, orbit shaping and so on.

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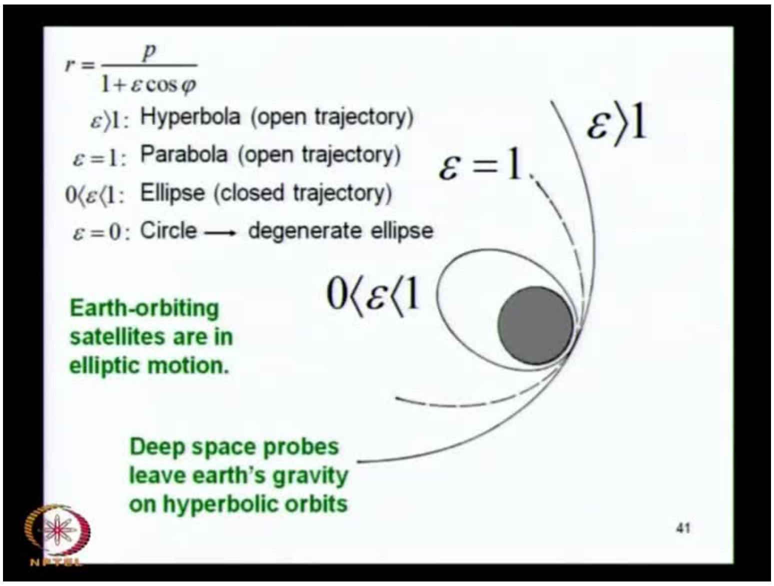

You have to really maneuver these eccentricities. If you have a deep space probe, for example, you need to launch the object and let it go outside the earth's gravitational pull in closed orbits. You might have to send it in hyperbolic orbit and this is what is called as orbit shaping. One really has to adjust these parameters to get decided trajectories.

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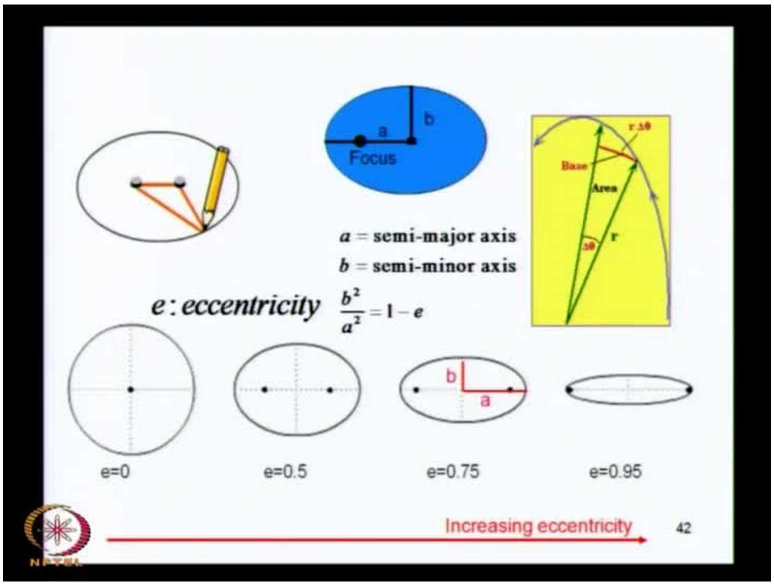

Here you can see that these eccentricities can be easily adjusted by changing the ratio of the major axis to the minor axis or vice versa.

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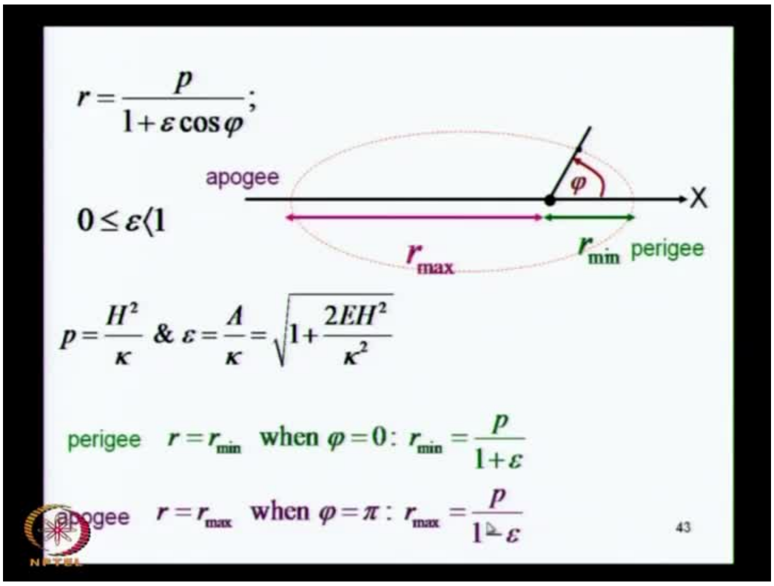

These are the essential parameter and this distance is the minimum distance. Let us deal with the ellipses because these are the closed orbits that we have to work with, for satellite trajectories or for orbits of planets in the solar system. The closest approach is what is called as the perigee and this is the point of perigee. The minimum distance will be given by this term, when the denominator is the largest. This will happen, when the cosine is the largest and this will happen, when the angle is 0 because for phi equal to 0 cosine phi becomes plus 1, which is the largest value that it can take.

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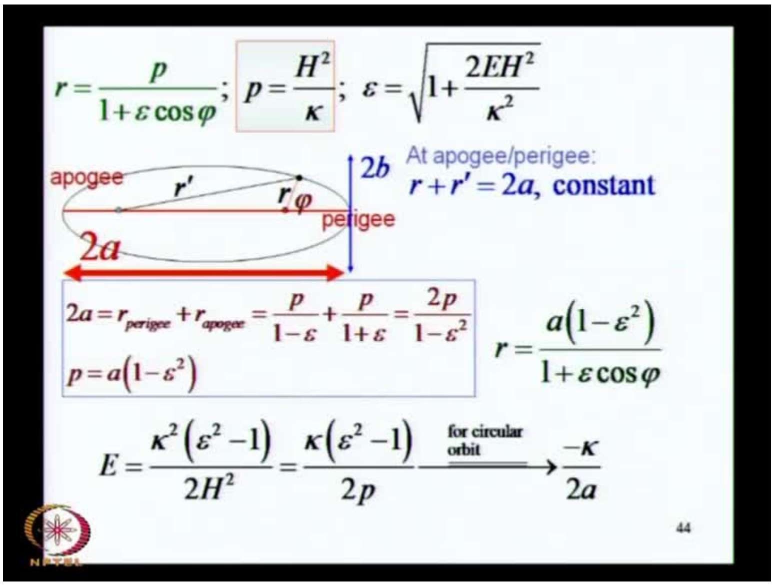

The minimum distance to the perigee is p over 1 plus epsilon. The apogee is the maximum distance, which is p over 1 minus epsilon. You can see that if you just add these distances r and r prime from the 2a to foci, then this 2a is a constant. You can get it simply by adding the minimum distance and the maximum distance. This allows us to write the numerator p, in terms of the eccentricity and half the major axis. So, this is a familiar form, in which you write the orbit equation. For a degenerate ellipse, eccentricity is 0 and so this will be minus kappa over 2a.

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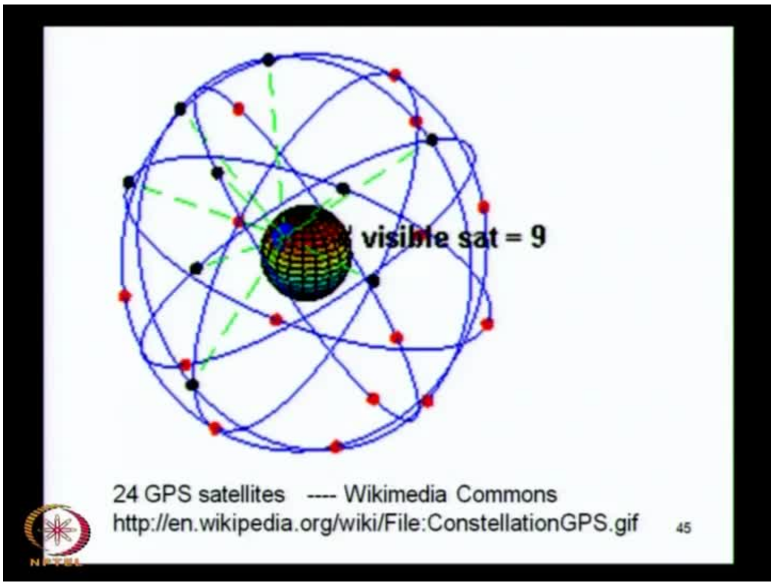

These are just rewriting these relations in known forms and in more familiar forms. Of course, these are important for various technology applications. It means I do not know if you are listening to my lecture or under the desk, you are using your cell phone to send sms or joke to your friends. These cell phones require GPS system and all of these GPS systems are monitored by various satellites.

At any given time, there have to be a certain minimum number of satellites, which are visible from a point in space. You can see that all of these orbit equations and all the properties, which govern the orbit equations are the conserved quantities. These are changed is by firing rockets, if you want to do some orbit shaping. these are essential elements of modern technology.

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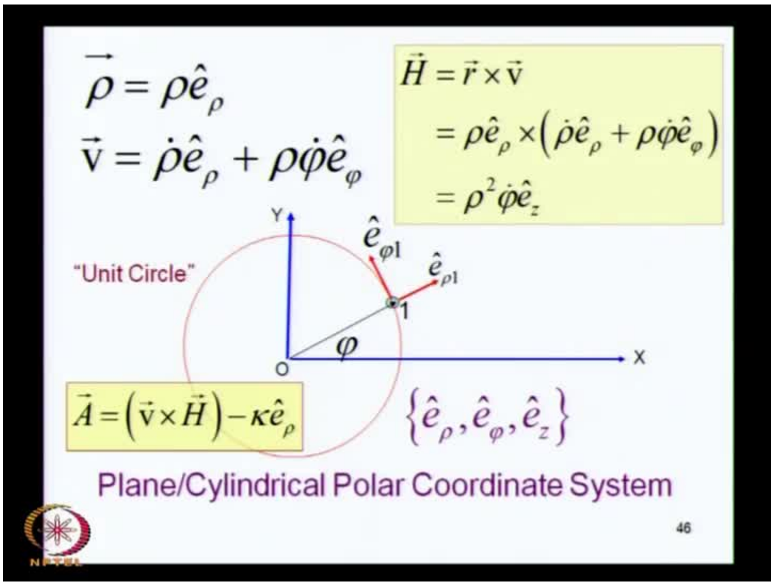

Now, what I will be doing is to use the polar coordinate systems and we have discussed this unit 3. So, I will not spend any time in developing the polar coordinate system. I will use it and I will just remind you that in the polar coordinate systems, we could do with the plain polar coordinate system, but we also have dealt with the angular momentum, which is perpendicular to the plane of the orbit. If you want, you can think of the cylindrical polar coordinate system.

Motion is confined to a plane because angular momentum is conserved. This is the specific angular momentum rho square phi dot. The angular momentum is in the direction e z, which is perpendicular to the plane of the orbit. In this coordinate system, we will work with the Laplace-Runge-Lenz vector, which is v cross H minus kappa times e rho and e rho being the polar unit vector along the radius in the plane.

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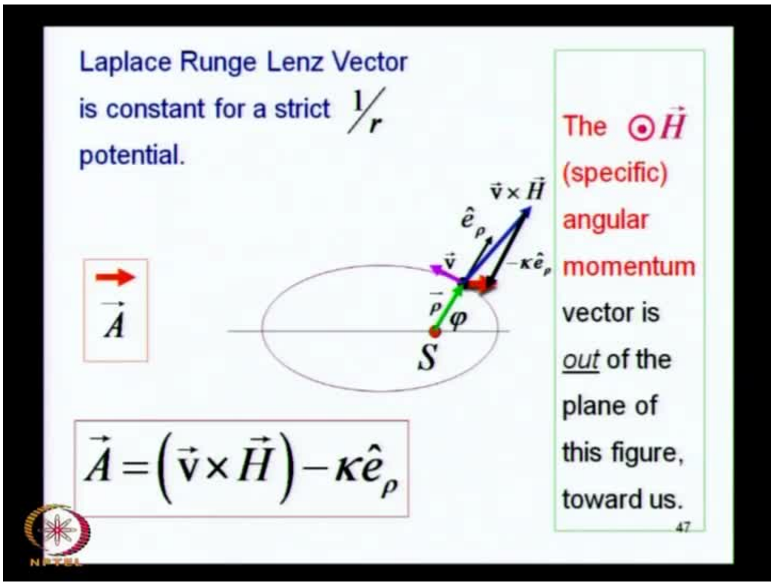

Now, let us have a look at the direction of the Laplace-Runge-Lenz vector. This direction is given by v cross H minus kappa e rho. So, this is the direction of e rho, this is the direction of the velocity of this planet going round the sun. The angular momentum is out of the plane in this figure and it is towards us. This is the manner, in which this ellipse is going that is the manner, in which this planet is going in this plane. So, this is the direction of the angular momentum, which is seen by an arrow that is perpendicular to the plane of the screen pointed towards us. So that is the direction of angular momentum. Therefore, you can determine the direction of v cross H.

You need the direction of v cross H and from this v cross H, you must subtract kappa e rho to get the Laplace-Runge-Lenz vector A. Now, we have got v, we have got v cross H and from this, you must subtract kappa times e rho. Here, e rho is from the sun to the planet in the direction of the position vector. So, this is the direction of e rho. This black arrow over here is minus e rho. It has a direction opposite to it, but parallel to it and scaled by the factor kappa.

So, minus kappa e rho will be anti-parallel to e rho. So, this is minus kappa e rho, if you take v cross H minus kappa e rho. You have to take this vector as v cross H minus kappa e rho. So, the Laplace-Runge-Lenz vector will be this red vector and this is the direction of Laplace-Runge-Lenz vector.

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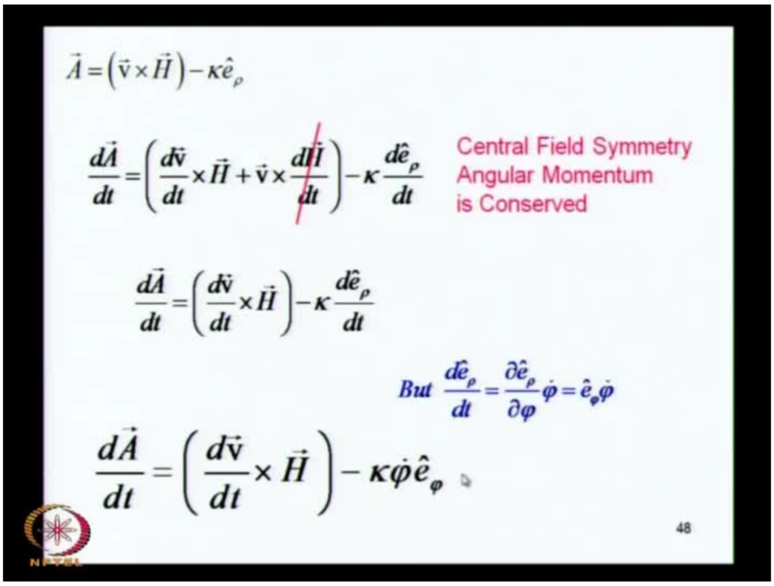

Let us have a look at the time derivative. We already know it is a conserved quantity and so its time derivative must vanish. Let us confirm that its time derivative actually vanishes because we have not done that. We got the conserved quantity by playing with the equation of motion. Now, we reassure ourselves that its time derivative vanishes. So, let us take the time derivative of the Laplace-Runge-Lenz vector.

The time derivative of the Laplace-Runge-Lenz vector will be dv by dt cross H plus v cross dH by dt from the time derivative of this term. Kappa is a constant and then you have the time derivative of the unit vector. These unit vectors change from point to point and from time to time though the time derivative of the angular momentum. Of course, they vanish because angular momentum is conserved in the central field. So, dH by dt will go to 0. Now, you are left with these two terms - dv by dt cross H minus kappa time d by dt of e rho, but this we know from our knowledge of the plane polar coordinate systems.

We had done this is in unit 3 that the time derivative of the unit vector e rho is equal to phi dot times e phi. The change in any unit vector is always orthogonal to that. So, the change in unit vector e rho will be along e phi. Its magnitude is something that we determined explicitly in the previous unit 3. We found that de rho by dt is equal to e phi times phi dot. This is the term, which will go here. So, de rho by dt will be replaced by phi dot times e phi, so let us do that.

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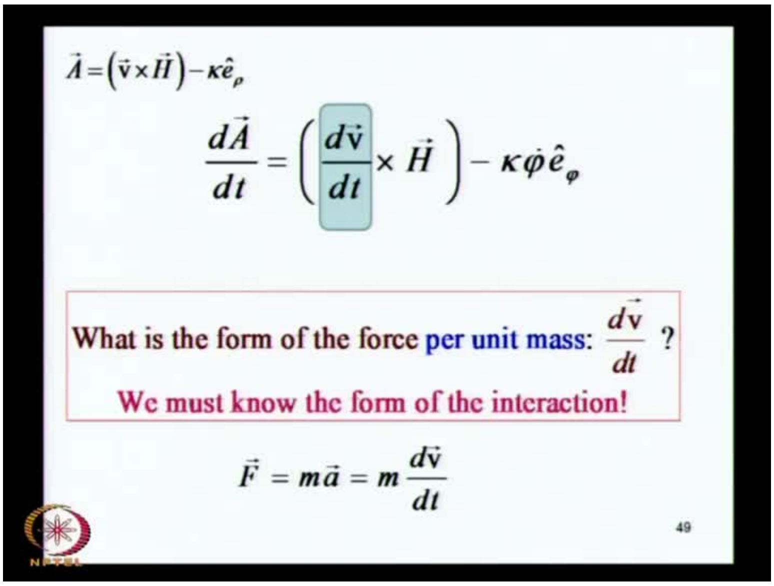

We have dA by dt, which is the rate of change of the Laplace-Runge-Lenz vector equal to dv by dt cross H minus kappa times this quantity, which is phi dot e phi. That is the relation that we have on the screen as dA by dt is dv by dt cross H minus kappa phi dot e phi.

Now, what do we need we need? It is dv by dt and that is acceleration. Acceleration is the force per unit mass, so it is the specific force. We are defining; we are developing this entire analysis for these quantities, which are qualified by the term specific. We dealt with the specific mechanical energy, we dealt with the specific angular momentum and now, we deal with the specific force, which is the force per unit mass. It is acceleration because F is equal to ma, so per unit mass and it is dv by dt.

What we now need is dv by dt or in other words, we need the force and we need the interaction. What is this interaction? This interaction must be plugged to find what dA by dt is. Unless we know the interaction, we cannot proceed. We must write the explicit value for dv by dt, so we need the force.

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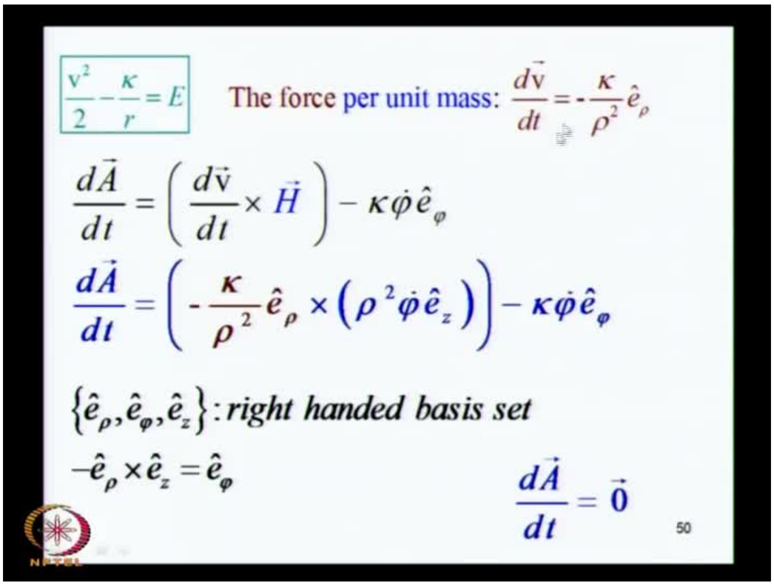

Now, this force is the potential energy minus kappa over r. The force is the negative gradient of the potential. So, we know that this force is minus kappa over rho square. It is the negative gradient of the potential. So, we put dv by dt equal to minus kappa over rho square times e rho. Instead of this quantity - d dv by dt, we have this minus kappa over rho square e rho. Now, everything is written in terms of these polar quantities rho, phi and the constant kappa. You already know that in the cylindrical polar coordinate system, e rho e phi e z constitute right hand side of unit vectors. So, the cross product of e rho and e z is equal to minus e phi. So, minus e rho cross e z is what you have over here. This minus sign e rho cross e z is equal to e phi itself.

You find that the first term is exactly equal to the second term. You got a phi kappa phi dot scaling the unit vector. The first term and the second are exactly equal and they cancel each other. The time derivative of the Laplace-Runge-Lenz vector vanishes. We have reassured ourselves that the Laplace-Runge-Lenz vector is a constant, not surprisingly because we got it as a conserved quantity.

In ensuring that it is a conserved quantity, we did make use of the fact that the force is one over r square. This one over r square has been used in assuring that the Laplace-Runge-Lenz vector is a constant. If we had not used this, we would not have been able to convince ourselves that A is a constant of motion.

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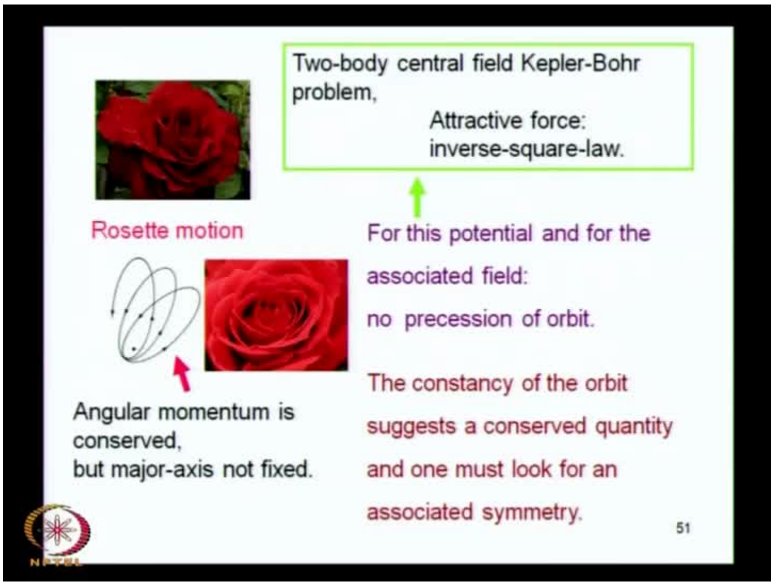

If you have this ellipse and there is a certain quantity, which is conserved quantity. In the previous figure, you saw that it was from the focus to the perigee and the constancy of this vector is what that really prevents this ellipse from précising that you have an ellipse like this. The reason that it is not precise and goes to another orbit is the conservation of energy or conservation of angular momentum. They does not guaranty that the ellipse will not precese what guaranties that this direction from the focus to the perigee is fixed. If that direction is fixed, the ellipse itself is fixed. So, this is the reason that there is no Rosette motion because if you look at these precesion from a distance, it would look like the petals of a lovely rose, The motion is still beautiful, even if it is not Rosette.

In this Rosette motion, the angular momentum is conserved; the energy is conserved, but the orbit is not fixed. So, the fact that the two body Kepler problem is the same thing with the Bohr problem in quantum mechanics. If you look at the hydrogen atom, you have an electron, which is going around the proton and the potential is 1 over r. The force is 1 over r square is the coulomb force in this case. It is a great wonder of nature that gravitational force and the coulomb force are 1 over r square forces.

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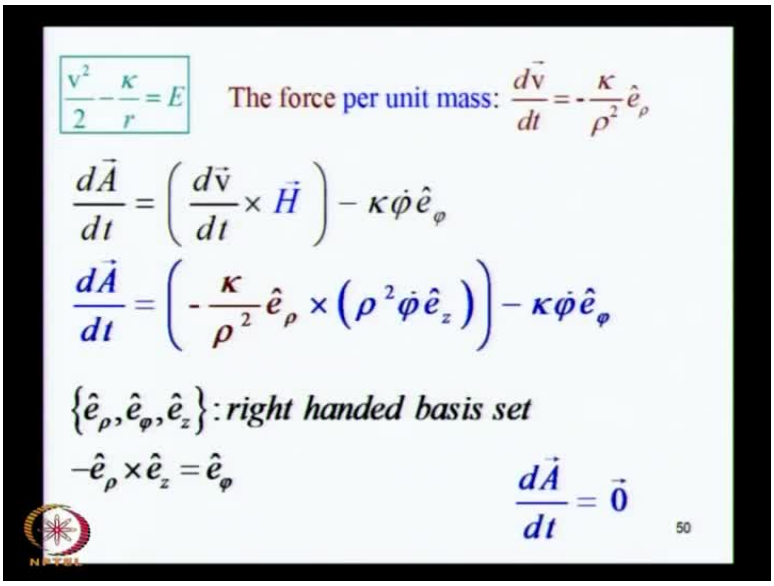

Now, this potential for the associated orbit, we can find that the orbit remains fixed. The fact that the force is 1 over r square was an integral factor in proving in our previous slide. As you see here, this dv by dt is the force equal to 1 over r square. Force is an essential part of the proof that the Laplace-Runge-Lenz vector is a constant. If this were not there, the first term would not have cancelled the second term to give 0. These two terms - the first term and the second term, they had to kill each other exactly and then only Laplace-Runge-Lenz vector will be a constant.

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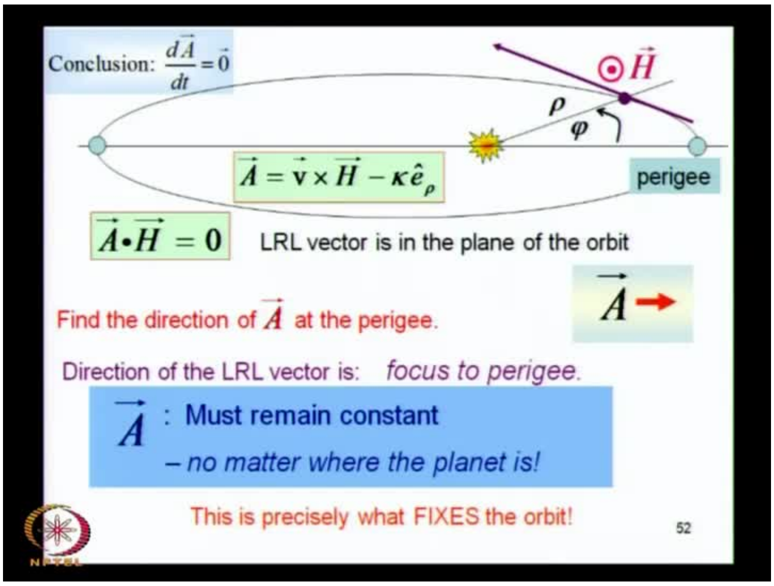

We need to know the force and let us have a look at the geometry once again. You can immediately see that the Laplace-Runge-Lenz vector must be orthogonal to the angular momentum. If you take the dot product of the Laplace-Runge-Lenz vector with H, the first term is a triple product with H appearing twice, so it would vanish. The angular momentum is orthogonal to the position vector, so this scalar product goes to 0. That tells us the angular momentum, the Laplace-Runge-Lenz vector is on the plane of the orbit, but in which direction is in the plane.

You can draw a number of vectors in this plane, so let us draw it. We know that it is in the plane of the orbit. Let us find what its direction will be and let us do it for just one point. Let us determine it at the perigee and this is the perigee. What is the direction of v cross H at this point? V is from bottom to top, H is from the plane of the paper to (.) So, what is the direction of v cross H? V cross H minus kappa e rho. Here, e rho is from the focus to the perigee minus e rho. It is like this. You can find that the direction of the Laplace-Runge-Lenz vector is from the focus to the perigee. What is going to happen, if this is the direction of the Laplace-Runge-Lenz vector for the point at the perigee? It will have to be the same, no matter where the planet is because it is a constant of motion and it is a conserved quantity.

Throughout the orbit, the planet is in the direction of the Laplace-Runge-Lenz vector. We will always be this and we now know that if the direction is always from the focus to the perigee, then the direction of the major axis itself becomes fixed. The ellipse gets fixed and the ellipse cannot precese. So, there will be no Rosette motion and the fact that there is no Rosette motion is because of the conserved quantity that the Laplace-Runge-Lenz vector is the constant of motion and it is a conserved quantity. Now, we ask the question that we know what fixes the orbit; it is the constancy of the Laplace-Runge-Lenz vector.

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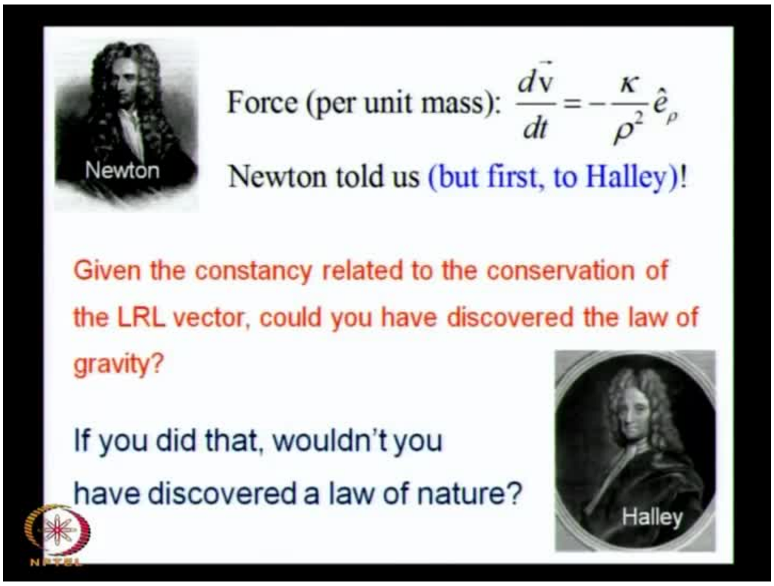

We have used the fact that the force is 1 over r square. Now, we know 1 over r square is the force between two masses because Newton told us. You and I did not discover the law of gravitation, I certainly did not and I do not know about you. We got it from Newton and in fact, the first person to know about it was Halley. After whom, Halley's comet is famous and this is very interesting because you know that we discussed earlier, how Kepler deduced that the orbits are ellipses.

Kepler said that the orbits are ellipses because he did curve fitting to his experimental data. Halley later asked this question to Newton, what would be the orbits? What will be the equations to the orbit? What will be the trajectory like? Newton said it would be an ellipse, but Newton's basis was different. He did not say it on the basis of Kepler's laws nor on the basis Tycho Brahe of data. Newton had invented calculus at the age of 22 or 23. There was no idea of function limit etc. So, he invented calculus and set up the differential equations. He deduced and he discovered the law of gravity. So, he was able to plug in that force in F equal to ma and then integrate it to get the ellipse. It was a totally different ball game.

Now, we again invert the question that given the constancy related to conservation of Laplace-Runge-Lenz vector, could you have discovered the law of gravity? You know that the law of gravity is essential to the demonstration of the fact that the Laplace-Runge-Lenz vector is a constant and 1 over r square was essential. Now, if you did not know that 1 over r square is the nature of gravitation force, could you have discovered it, if you had known that the Laplace-Runge-Lenz vector is constant. So, this is the moral of the story that you can invert these questions and discover the laws of nature by looking at these constant quantities. You have a conservation principle and from this conservation principle, you can deduce what the law of nature is.

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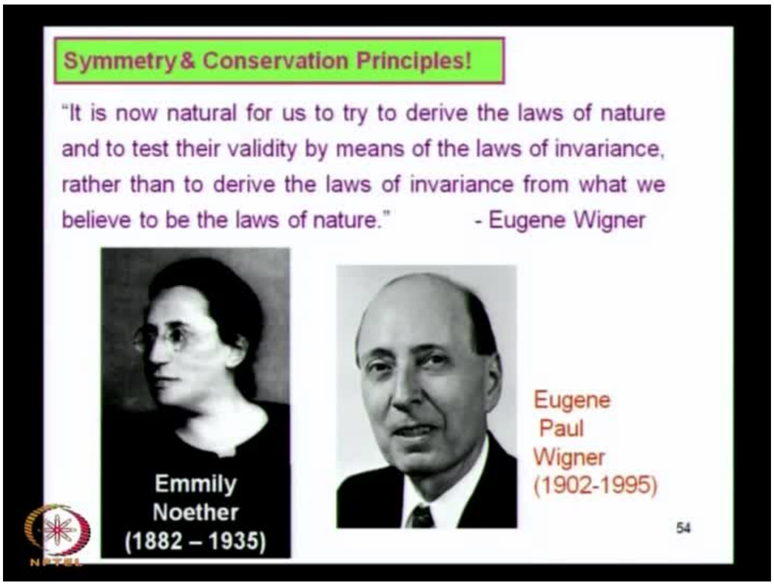

There is one question that remains to be answered, what is the nature of the symmetry? You got a conserved quantity, which is the Laplace-Runge-Lenz vector. What is the symmetry associated with this? When we encountered the conservation of energy, we discovered the symmetry, which is symmetry with respect to changes in time evolution with respect to time. When we met the conservation of angular momentum, we connected with the symmetry with respect to rotations.

As I pointed out, I want to repeat this point that Eugene Wigner elucidated to us that it is natural for us to try to derive laws of nature. Test their validity by means of the law of invariance rather than derive the laws of invariance that we believe to be the laws of nature. This is invert thinking, which can lead us to new physics. We can discover new laws of physics using this technique.

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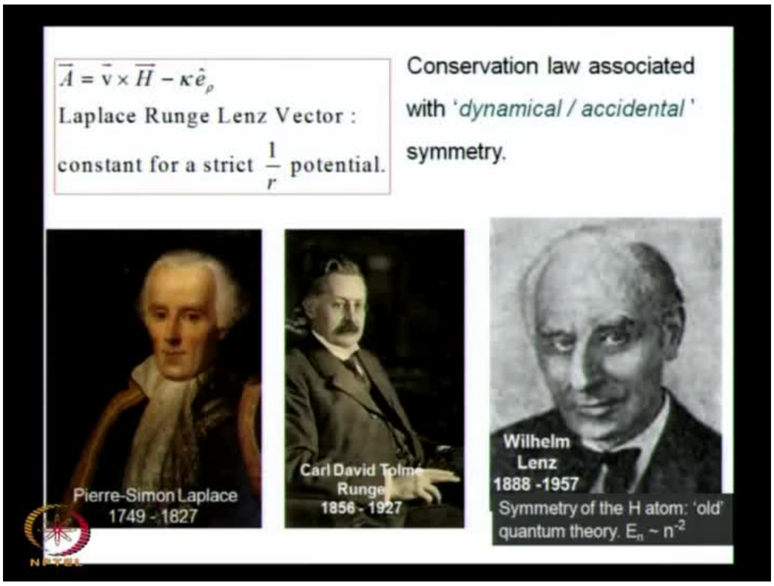

The symmetry associated with the constancy of the Laplace-Runge-Lenz vector is the fact that the force must be 1 over r square that the potential must be 1 over r. It is not a geometrical symmetry, it is not like something like rotation and it does not changes. It is not like translational symmetry, like taking an object from here to here in homogeneous space or it is not like taking an object and rotating it in isotropic space, in which the properties of space are the same in all directions. So, this is the different kind of symmetry. You can call it as an accidental symmetry, if you like. In fact, this was called as accidental symmetry, but a more appropriate term for this is dynamical symmetry. The reason it is called as dynamical symmetry because it comes from dynamics.

It comes from the cause effect relationships; it comes from the nature of the interaction. The nature of the interaction between these two masses is given by the 1 over r potential gm and m 2 by r. The force is given by the corresponding 1 over r square law and it is this dynamics; it is this nature of the physical interaction, which goes in to the heart of the constancy of the Laplace-Runge-Lenz vector. This is connected with the conservation principle and this conservation principle has a corresponding symmetry as one would get from Noether's theorem. This symmetry is therefore called as dynamical symmetry and this is known as the Laplace-Runge-Lenz vector. Lenz name is also associated with this and this is not the same Lenz, who we meet in the Faraday Lenz law. This is the Lenz Wilhelm, who dealt with the quantum mechanics of the hydrogen atom.

You also have 1 over r potential and there is a similar feature, which was studied by Wilhelm Lenz.

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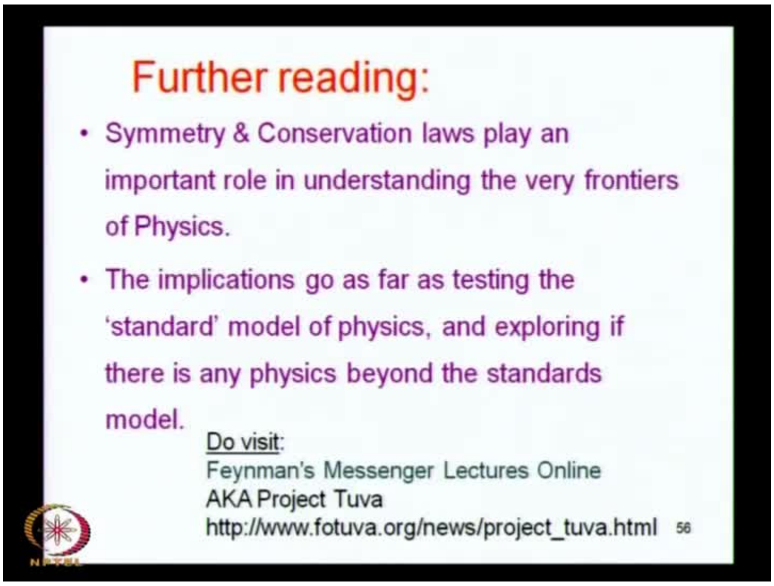

We basically conclude this topic over here. I will suggest a little bit of further reading because the ideas have very deep and profound consequences in physics. Symmetry and conservation principles govern fundamental laws in physics. These connections and the implications are used to test the standard model of physics to explore, if there is physics beyond the standard model. I will strongly encourage you to listen to Feynman's lecture. It is available on the web; you do not have to write down this link. You just Google as Feynman's lectures online or Feynman's messenger lectures online or this Tuva project Feynman's has a wonderful lecture on these topics. I will strongly encourage you to read it and listen to that lecture.

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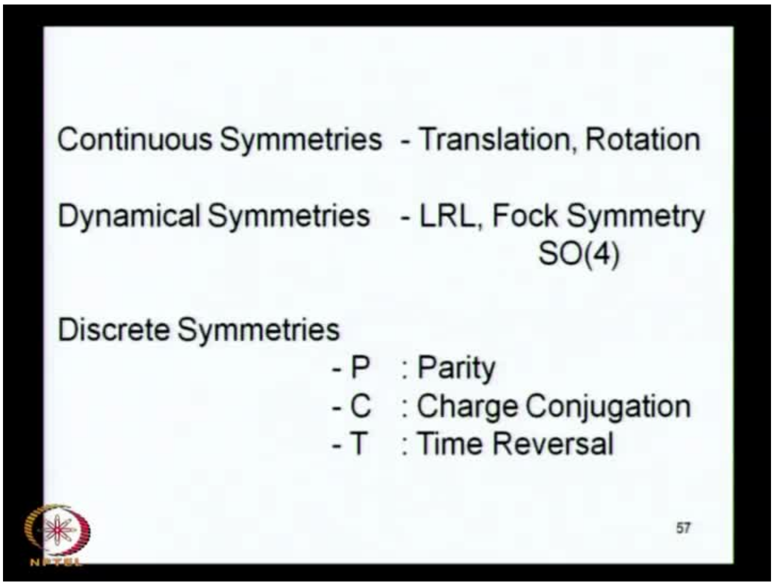

There are other symmetries in physics, which are important. You have continuous symmetries, the translation and the rotation that we have already discussed. We have discussed these dynamical symmetries. The dynamical symmetry is the Laplace-Runge-Lenz vector in the context of the quantum mechanics of the hydrogen atom. It is known as the Fock symmetry or the SO 4 symmetry. There are discrete symmetries like parity, charge conjugation and time reversal.

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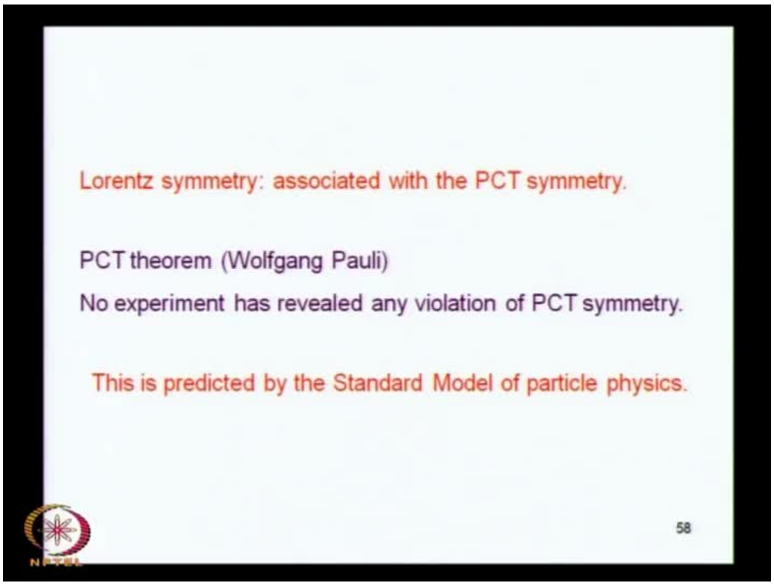

There is a very famous Lorentz symmetry, which is connected with the PCT theorem through the… This is known as the PCT theorems. It is proved by Pauli and there is no experiment in physics that has ever been done, in which the PCT symmetry has been violated. So, this becomes a prediction of the standard model of the particle physics. Any departure from this will be a test on the standard model. It will be a suggestion of whether or not there is any physics beyond the standard model.

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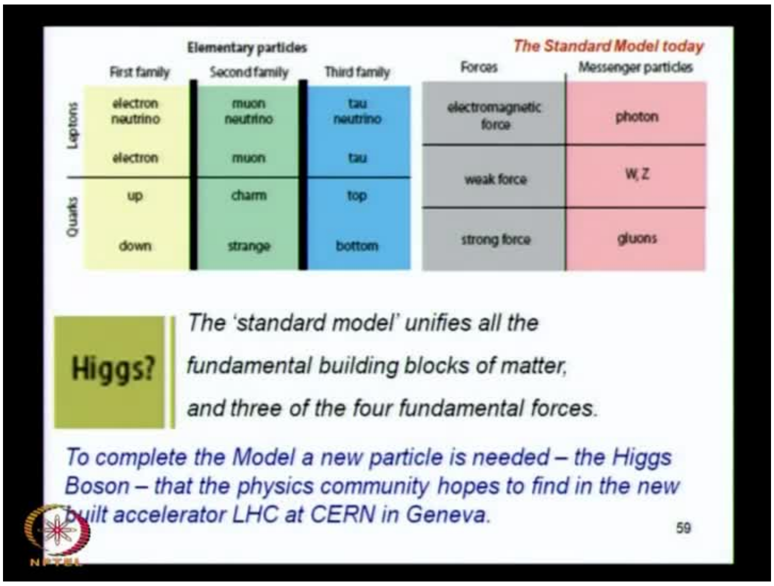

These are some of the issues, which people are working with. These are the essential elements of the standard model of physics, but there are certain number of elementary particles like what are the forces? What are the known forces at least? This standard model requires the Higgs Boson for its completion, which is one of the main objectives of the experiment at Large Hadron Collider. These are very exciting issues, which come out of these consideration symmetries. Conservation laws discrete symmetries that is Lorentz symmetry, if there is any physics beyond it and so on.

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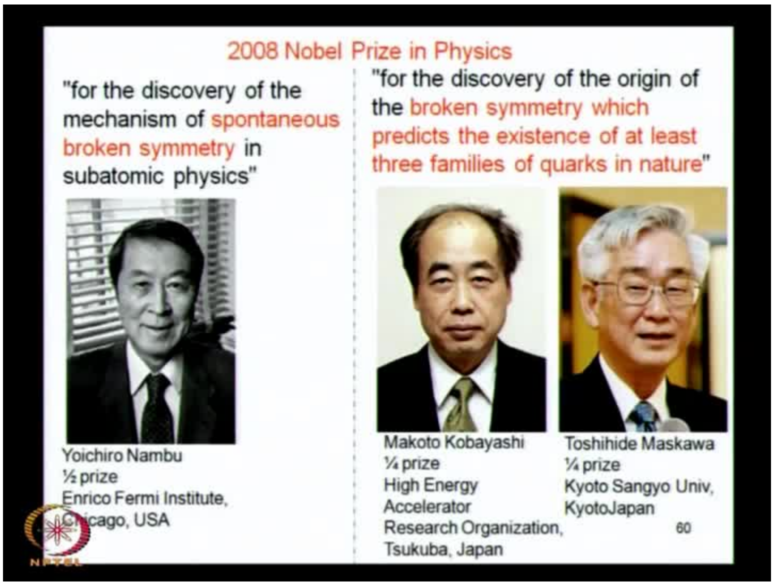

In fact, quite recently, just the year before last, the Nobel Prize went for the spontaneous breakdown of symmetry. There is another kind of symmetry that you normally expect, which is the symmetry between matter and anti-matter. If the symmetry is seen to be broken, then what are the reasons for it? How does one understand it? The Nobel Prize for 2008 went to Nambu Kobayashi and Maskawa for throwing some light on these issues. These are clearly beyond the scope of this course, but these are some suggestions for further reading for some of you.

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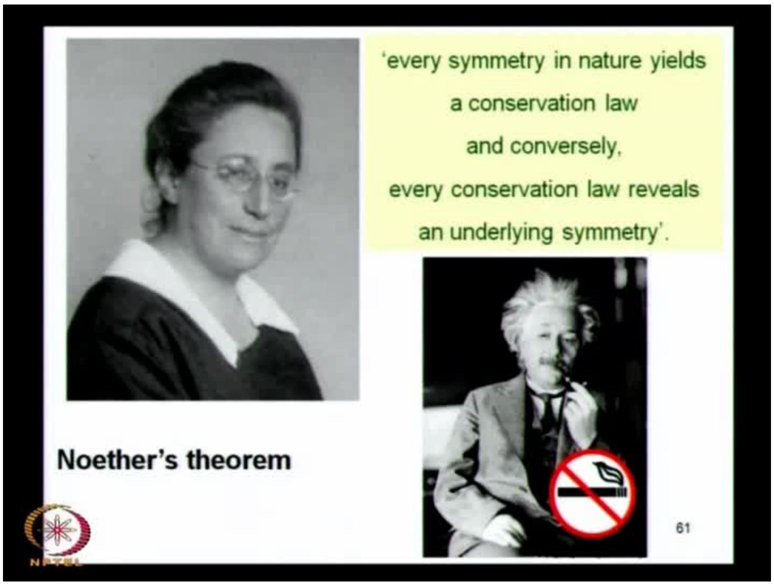

They all come from the basic idea that we have discussed. We can understand even in terms of two body Kepler problem. For every symmetry principle, there is a conservation law and for every conservation law, there is an underlying symmetry. This whole thinking of going to the physical law from the symmetry principles, it is something that we began with Einstein. We will keep showing you this picture of Einstein, smoking and this will keep on reminding you, not to smoke. I think at the end of this course, if I can discourage smoking, it will be an achievement and I would think that is the real message.

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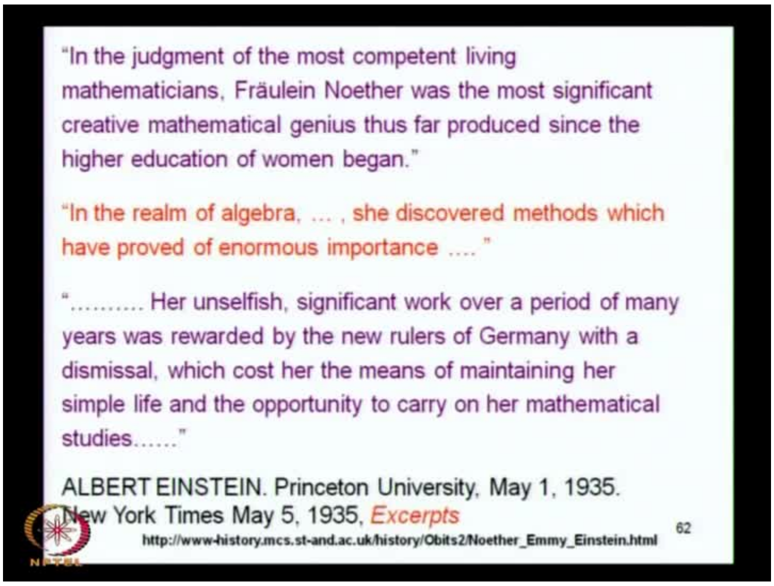

I would like to conclude by quoting from Einstein's tribute to Noether, who very sadly died rather young. Einstein wrote in the obituary, which was published in new york times in 1935. What Einstein says about Noether is that in the judgment of most competent living mathematicians, Fraulein Noether was the most significant creative mathematical genius, thus far produced, since the higher education of women began.

In the realm of algebra, she discovered methods, which have proved to be of enormous importance. Her unselfish, significant work over a period of many years was rewarded by the new rulers of Germany with a dismissal, which cost her means of maintaining her simple life and the opportunity to carry on her mathematical studies. She went through a lot of injustice in Germany at her time. With that I conclude this particular unit.

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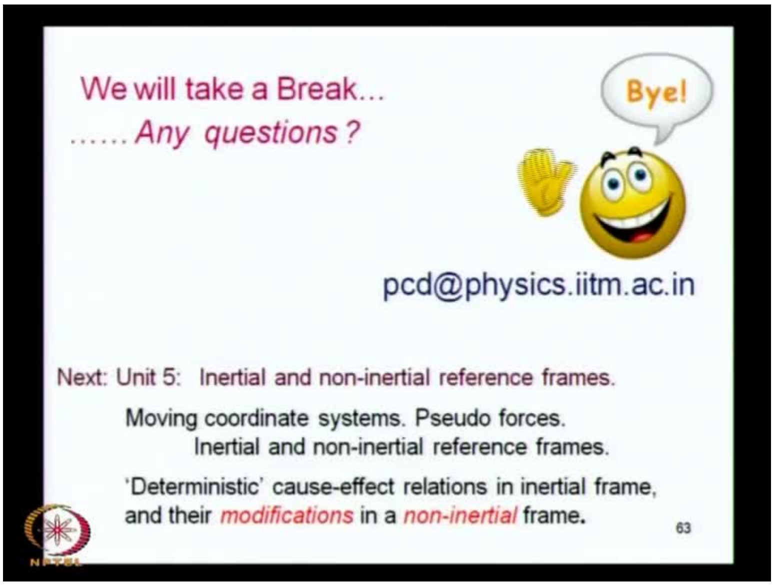

If there are any questions, I will be happy to take. In the next unit, we will discuss inertial and non-inertial reference frames. We will try to understand causality and if there are, what is behind the principle of causality? What kind of effects do these generate? Do all effects that we see come from known causes? Are they results of causes, which do not exist? This is a fictitious cause, but that is for unit 5, so that is what we will discuss in the next unit. If there are any questions, I will be happy to take

Since, we have discussed the merit of Lagrangian mechanics, it is easy to identify the conserved quantities. So, even in the Newtonian regime also, I think now it is easy to recognize conserved quantities.

Well, so far, the underlying physics in concerned, it is not different for Newtonian mechanics or Lagrangian mechanics. The foundations of the two schemes are completely different. The foundation of Newtonian mechanics is in the principle of causality that there is a cause effect relationship that a stimulus, which is the physical interaction generates a response. The response is linearly proportional to the force to the stimulus F equal to ma and that is the heart of Newtonian mechanics. So, this stimulus response connection is the linear response of a system to a stimulus. It is the basis of Newtonian formulation and the cause effect relationship is not invoked at all in the Lagrangian or the Hamiltonian formulation.

The Lagrangian formulation and the Hamiltonian formulation make no reference to the cause. It does not talk about the effects; it only talks about how a system evolves as a function of time, which is the main goal of solving a mechanical problem. It tackles that problem by not invoking the stimulus response relationship. By saying that any system evolves in such a way that the action, which is the integral l d t. Here, l being the Lagrangian and that this action is an extremum. It is the principle of variation, which is at the foundation of Lagrangian mechanics.

The consequences or the results of these two schemes are not different at all. It is for this reason, the connections between symmetry and conservation laws can be seen in either scheme. Physics is not different, the foundations are different, the platforms are different and the Newtonian mechanics rest on the cause effect relationship on the principle of causality. The Lagrangian and the Hamiltonian schemes depend on stem from the principle of variation. These are the fundamental foundations and these are two different platforms to give you the trajectories.

Both tell you, how a system evolves as a function of time. The important thing to keep track of is that in solving any problem in physics. Essentially, what you are solving? How do you characterize the state of a system? How does the system change with time? If you answer this question, your problem is solved. You do it using either Newtonian mechanics or you know Lagrangian or Hamiltonian mechanics. When you cannot do it, you come to terms with the fact that the position and momentum cannot be known simultaneously. Accurately, you need quantum mechanics and then you say that the system is described by the state vector or the wave function. How does this evolve with time that is the Schrodinger equation.

Again the problem is the same that the system is characterized by the wave function and its time derivative. Its evolution with time is what will solve the problem for you. So, both the variational methods and the cause effect principle of causality or determinism give you the solutions to the mechanical problem, but the platforms are different. Physics is the same and details are different. Of course, in this case, you are able to deal with constraints and so on in a much easier way in Newtonian mechanics. If you keep track of every constraint, you can set up a detailed equation of motion. Taking into account, every constraint, we will still get the same connection between symmetry and conservation laws because that is where the physics is.

I heard that mercury has some problem with these equations, which was later solved by Einstein. Can you brief me about the problem with mercury and these equations?

Well, it is a very interesting question. Its answer has very far reaching consequences in physics. Now, we have seen that if the force is strictly 1 over r square, it describes the force and the orbit would be an ellipse. Its major axis will be fixed, so the orbit will not precese. Now, careful observations on mercury tell us that mercury orbit actually does precese. What is the reason that is preceses? There can be a variety of reasons - one is that the solar system is not just a two-body system. Only if you solve the two-body Kepler problem, you will have ellipse as the solution.

There are perturbations that are not just the earth and the sun, but there is a Jupiter, there is a Saturn, there is a Moon. All of these will influence certain perturbations, but these perturbations are not the reason why the orbit of mercury preceses. The orbit of mercury preceses because Newton’s law itself is an approximation and Newton’s law presumes certain things. Newton’s law is formulated in the domain of mechanics, which is essentially Galilean relativity, whereas the laws of mechanics are relativistic and Einstein discovered this.

It is these relativistic consequences, which are responsible for the precision of the orbit of mercury. So, you have to get into field equations, which comes from Einstein’s theory of relativity and those give you the correct orbit for mercury. It is rather remarkable that this was one of the first test of Einstein's theory. So, it comes from the non-constancy of the Laplace-Runge-Lenz vector and so on. The basic origin of that is the fact Newton’s law of gravity is only approximate. It is a non-relativistic approximation to the correct law, which has to come from the theory of relativity. So, we will stop here for this unit.