Quantum Mechanics - I
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Lecture - 40
Time-dependent Hamiltonians

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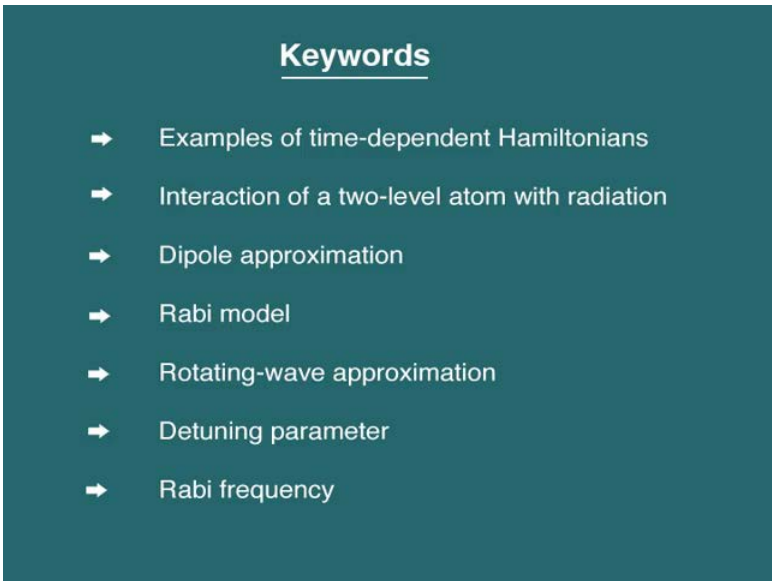

In the last few lectures, we have discussed stationary perturbation theory, also called the Rayleigh Schrodinger perturbation theory.

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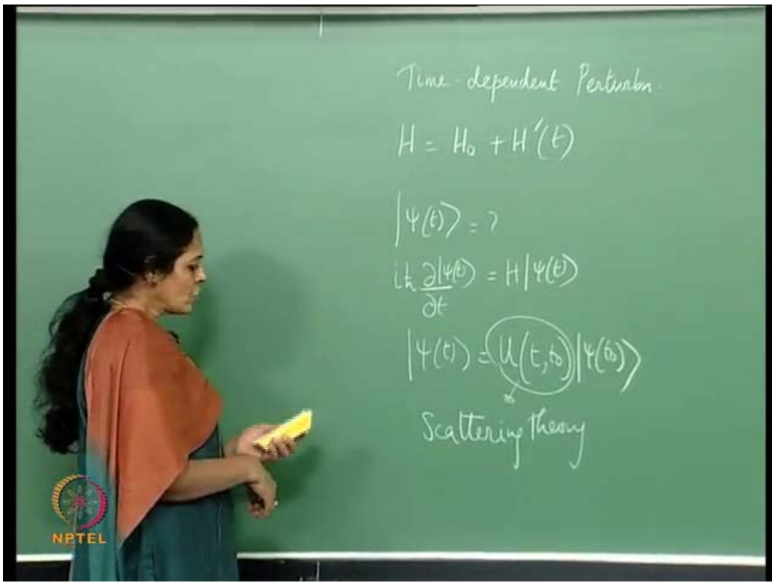

And there, we saw that the Hamiltonian had two parts. One was the free Hamiltonian H naught and then there was a perturbing Hamiltonian, H prime and H prime itself was not a function of time. And, that is why we referred to that as time independent perturbation theory, or stationary perturbation theory. Now, today I would like to discuss, the salient features of time dependent perturbation theory. That simply means, that H prime is a function of time.

This is one case where, time dependence is broughten because, the perturbation has a time dependence in it. Of course, there are other situations in reality. Where, you really do not think of it as a free Hamiltonian H naught, plus perturbing Hamiltonian H prime which is a function of time. Instead, you could think of a situation where, the total Hamiltonian is essentially the same. Except for a period of time, when it changes to something else.

So, I will not be discussing a situation of that kind. I would instead be looking at a situation like this, where the free Hamiltonian is not a function of time and then there is a perturbation, which is a function of time. This is clearly a straight forward generalization. Well, as straight forward as it can get, of what we studied in the Rayleigh Schrodinger perturbation theory. Where we saw the linear stark effect and the Zeeman Effect; now, you would also recall that in one of the earlier lectures, in a different context. We looked at the shell model of the nucleus and there we discovered that if you put in the spin orbit coupling l dot s.

As an extra piece in the Hamiltonian, degeneracies are lifted and in fact, you find that the total angular momentum j is a good quantum number. And the magic numbers in the shell model are explained pretty reasonably. Provided you do not go to higher magic numbers. The lower one’s like: 2, 8, 20 etcetera, are described very well, explained very well and justified. Provided we introduce the spin orbit coupling term, in the Hamiltonian.

Now there to, there was no time dependence. But, you should now be able to connect up. That shell model of the nucleus, with what we did in the Rayleigh Schrodinger perturbation theory. Because, the term in the potential which lifted the degeneracy, was the time independence spin orbit coupling term. So, in fact you have seen three examples of stationary perturbation theory. One is the linear stark effect, which came because we had an external electric field and then the Zeeman Effect; you could have used a magnetic field and of course, the shell model of the nucleus. Where the spin orbit coupling helps lift a degeneracy.

Now, in contrast we have this, where H prime is a function of time. The aim of course, is to find psi any state, which is expanded in the relevant Hilbert space of the problem. The aim is to find psi of t and if you look at the Schrodinger equation, you simply have i h cross delta delta t of psi of t, is equal to H psi of t. The general solution of course, would be u of t, t naught psi of t naught, where u is a unitary operator. I will call it the time evolution operator and psi of t naught was the initial wave function. That was given, the initial state of the system that was given to us.

So, of course, you need to find out u. In order to find out psi of t, given psi of t nought and then you know the wave function at time t. So, that is one aspect to time dependent perturbation theory, where you could expand this, in powers and look at the time dependence. There is something else which can happen. Where I really do not use perturbation theory at all, where in contrast to this, I have a situation say a two level atom. This atom interacts with say laser light, the radiation field, the coherent state, which is a laser light. And suppose; the frequency omega of the radiation field, exactly matches the frequency difference between the ground state and the excited state. Then it is resonance and it is possible, that all the atomic population in the ground state, initially in the ground state, just move up to the excited state.

Clearly, this is not a situation where, I will use perturbation theory. Instead, I simply have a model of a two level atom if you wish. Where a population, an atomic population in the ground state, just moves up to the excited state, at some instant of time and then comes back and this could be a process which keeps on happening in cycles. Now, such a situation is also a time dependent effect. As time changes the population, atomic population in the ground state moves to the excited state and then comes down and so on.

So, there are very many situations, where one has to deal with a time dependent problem. So, the first thing that I wrote was this. Where I have a unitary operator and I wish to find what the unitary operator is. The second thing is scattering theory, where the Hamiltonian itself changes over a small period of time and then the third thing, is where I do not use perturbation. But, I talk of an effective two level atom model, where the atom gets excited and goes to the excited state and then comes back to the ground state. Because, of interaction with the laser light. So, these are all various examples of time dependence dynamics.

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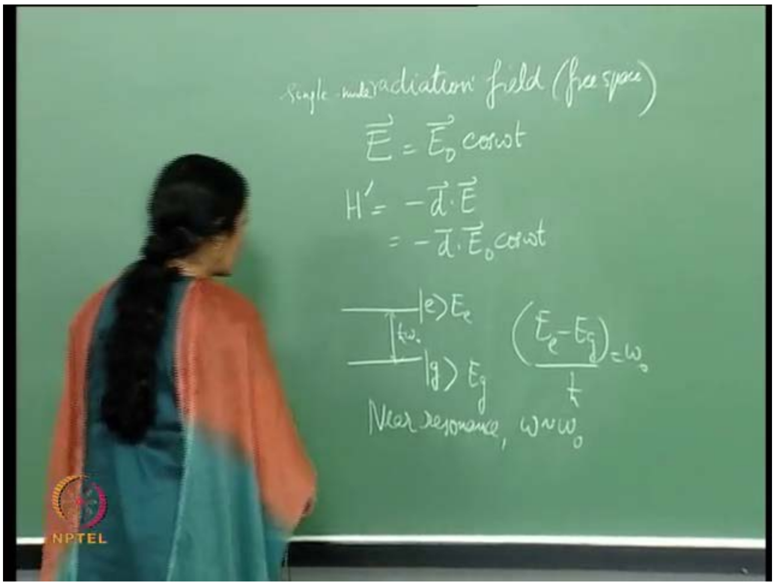

So today, what I would like to look at is the laser atom interaction model, which I spoke to you about last. So, I would be looking at a situation. First where I have a radiation field and free space and it is a single mode radiation field. So, I model it by an electric field, which is E naught cos omega t and omega is the frequency that I talk about. Now, this is in interaction, with the atomic dipole operator. I am using the dipole approximation and therefore, H prime is minus d dot E, which I can write as minus d dot E naught cos omega t. So, this is what I have.

I have an atom, which can be treated essentially as a two level atom, for the reason that I told you a little while ago. Because: if this energy is E e and this is E g, E e minus E g by h cross is omega naught. So, this energy difference is h cross omega naught and near resonance, that means, omega approximately equal to omega naught. The population in the ground state, just moves up only to this level and the other levels can all be forgotten.

This is not a perturbation. This is simply a situation where there is population transfer between two levels. And above all, there could be an instant of time, possibly an instant of time, or a set of times. Where the population which was initially only in the ground state completely moves up to the excited state. In contrast to the kind of perturbation theories that I have spoken about till now, they are essentially, the state is the original state of the system. With some perturbations which could be the, 1st order correction, 2nd order correction, 3rd order correction and so on.

Let me emphasize, in contrast to what we learnt earlier, where the state at any instant of time is largely dominated by what was initially the state before perturbation. And, perturbation simply makes some corrections to the state, in contrast to that. Here we have a situation, where the initial state can completely change. In the sense, that if initially the atomic population is in the ground state, it moves up to the excited state at some instance of time. And therefore, it is majorly populated only in the excited state. So, it is not a correction to the ground state. I will first look at this model.

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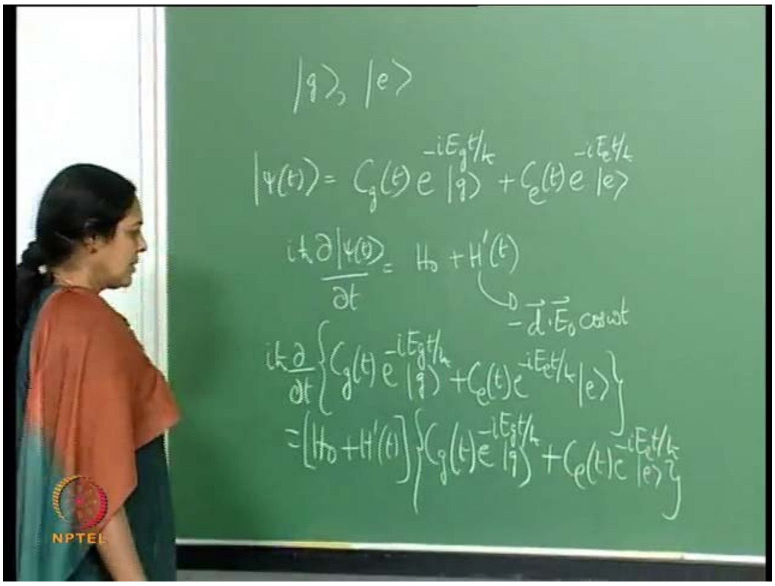

It is called; the Rabi model and we have the initial basis states g and e of the atom. The two level atom and of course, they evolve as Eigen states of the original Hamiltonian H naught. So, that at any instant of times the state of the system, the state of the two level atom. Would be some, coefficient e to the minus i E g t by h cross, ket g plus C e of t e to the minus i E e t by h cross ket e.

This is understandable, because at that instant of time that we are considering ket g would have evolved in this manner. Because, it is a stationary state, ket e would have evolved in this manner. But, you should be able to write the wave function, as a superposition of the evolved states at that instant. There is no reason to believe that these coefficients cannot depend on time. Indeed, in general they will and that is why the state has been chosen in this manner.

Now but, we also know that this is true. H naught plus H prime of t. Remember that H prime of t is minus d dot E naught, cos omega t. Now, if this is the Schrodinger equation and we plug this in, what do we have? We have i h cross delta by delta t, C g of t e to the minus i E g t by h cross ket g, plus C e of t e to the minus i E e t by h cross ket e. That is a left hand side and that is equal to H naught plus H prime of t, C g of t e to the minus i E g t by h cross ket g, plus C e of t e to the minus i E e t by h cross ket e. So, this is what I have.

Now, if I differentiate here for instance, there are two terms that depend on time. C g itself is a function of time and therefore, I will get a C g dot, where the dot stands for the derivative with respect to time. And of course, this also gets differentiated. Remember that ket g itself is not a function of time. It is the initial basis state.

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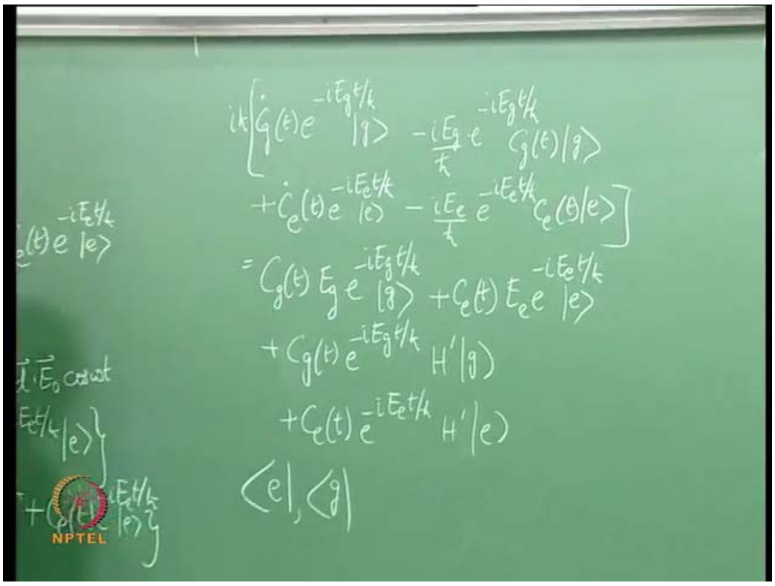

So, if we did that, we just have i h cross C g dot of t e to the minus i E g t by h cross ket g. Let us keep the (Refer Slide Time: 11:55) i h cross out of this. The next term is minus i E g by h cross, e to the minus i E g t by h cross, C g of t ket g. Now similarly, I have to differentiate this and I get plus C e dot of t, e to the minus i E e t by h cross, ket e minus i E e by h cross e to the minus i E e t by h cross C e of t ket e.

Now this, first of all g (Refer Slide Time: 11:55) H naught, acting on ket pulls out an E g, C g of t E g, e to the minus i E g t by h cross ket g. Similarly, H naught acting on this (Refer Slide Time: 11:55) term pulls out an E e. Ok and then I have H prime. So, H prime, remember is an operator, it is the, it has the atomic dipole moment d in it, plus H prime. So, I am going to write this as plus C g of t, e to the minus i E g t by h cross H, prime ket g. Plus C e t e to the minus i E e t by h cross, H prime ket e. So, this is what I have.

Now, one thing is clear. I can take this equation and flank it with ket e on this side and also ket g on this side, on the left hand side. Now, that should give me something. So, let us see what happens.

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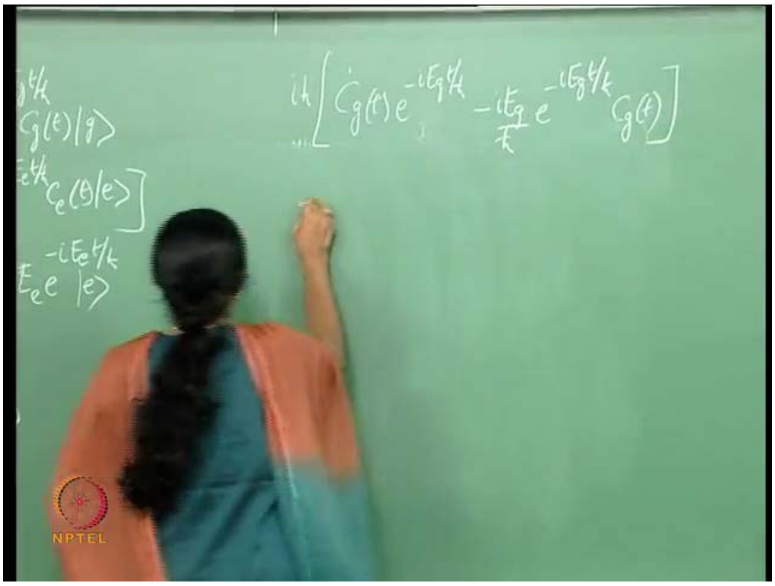

So, if I started with ket g, certainly the 1st term survives. Because, (Refer Slide Time: 15:02) ket g is normalized to 1. Similarly, this term also survives. But, these two terms disappear.

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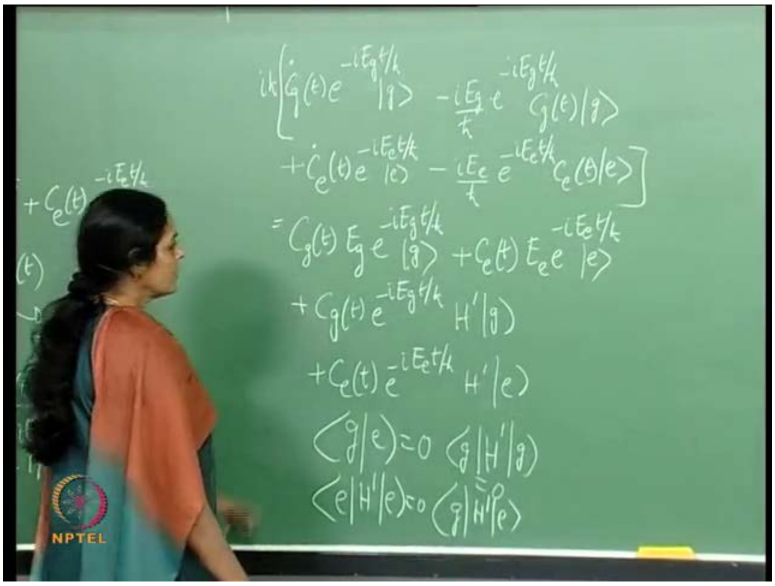

Because, ket g and ket e are orthogonal states and therefore, these two terms do not contribute. Here, it is a matrix element of H prime, but this will be 0. Because, you will recall from our discussion, even in the case of the stark effect, that the dipole operator cannot connect states of the same parity. Similarly, e H prime e is 0, but what survives is g H prime e and therefore, it is complex conjugate.

So, we were at this point and here you can see that this term will survive, this term will survive. Here g, H prime g is 0 but g H prime e will survive.

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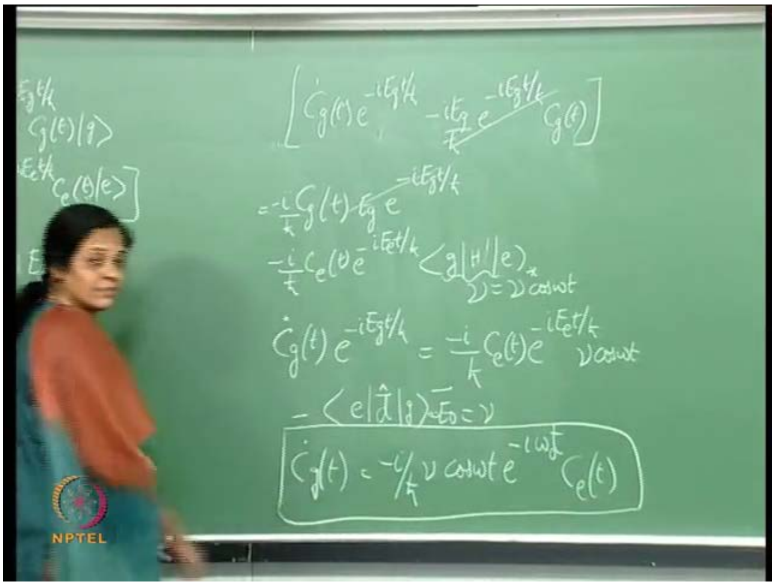

So, if you look at these terms, on the right hand side. You have C g of t, E g e to the minus i E g t by h cross. That term survives, (Refer Slide Time: 18:10) this goes. This goes but, this survives. This is what I have. Of course, I can bring the i h cross to this side. Look at what I have here. I have minus i E g by h cross, C g of t e to the minus i E g t by h cross. So, that goes and I am left with C g dot of t. Of course, I can write this as C g dot of t e to the minus i E g t by h cross, equals this object here. Minus i by h cross C e of t e to the minus i E e t by h cross and this object, let me call it nu and let me say that, that is also equal to its complex conjugate.

Because, generically this is real as you have seen from our discussion of the stark effect earlier. So, there is a nu. It is not just nu. There is a nu cos omega t, where I have used the fact, that the dipole operator between g and e, this dipole operator actually minus, the dipole operator between e and g dot E naught. I call that nu. What is left behind is simply a cos omega t, which I have put out there.

So now, if you look at this C g dot of t, equals minus i by h cross, nu cos omega t e to the minus i. That gives me if I take this to that side, it gives me a minus i E e minus E g by h cross. And therefore, I just have an e to the minus i omega naught t C e of t. So, this is my 1st equation. This is what I have. Similarly, I can get C g like I got C g dot of t. I can get C e dot of t and how would I do that?

Instead of using a bra g out there (Refer Slide Time: 18:26) I will put a bra e. Use the same facts that these quantities are 0 and that g and e are orthogonal kets. And, then I will be left with another analogous equation. This is the equation C g dot of t. Now, I will have another equation for C e dot of t, which I merely write down and which you can easily get by doing precisely what I did earlier.

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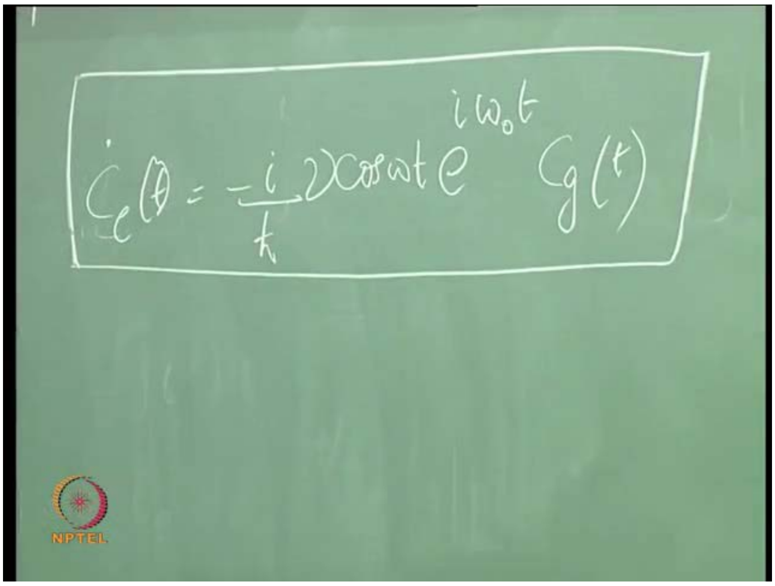

So, I will have C e dot of t is minus i by h cross nu cos omega t, e to the i omega naught t (Refer Slide Time: 19:21) that is one difference, C g of t. So, these are the two equations that I have. These are coupled equations. I can decouple them and get an equation just for C e and another just for C g by taking the differential once more. Now, if I did that I get the following thing.

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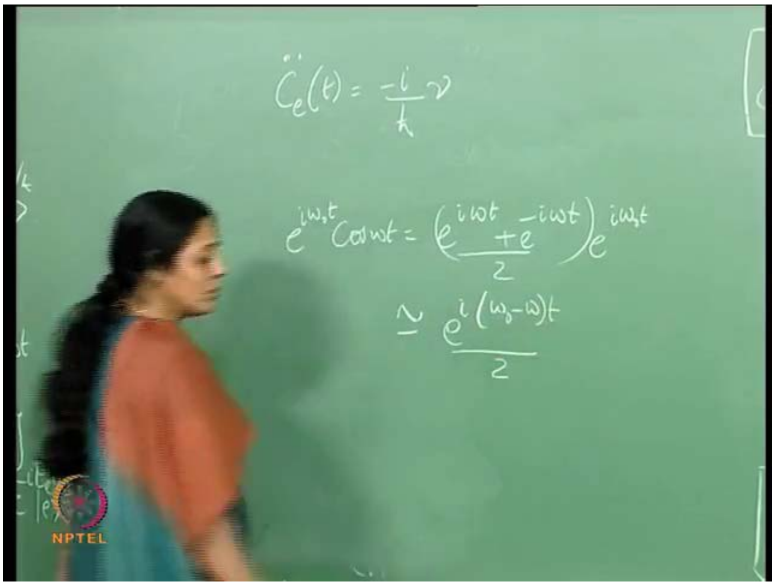

So, let us look at C e of t. So, C e double dot of t is minus i by h cross, nu cos omega t. But now, one should be careful. There is a C g of t as well. I want to use an approximation. Now, this approximation is the following: I write cos omega t, as e to the i omega t plus e to the minus i omega t by 2. Club it with e to the i omega naught t. And, as I said, the physics of the situation is this. That you are working with omega very close to omega naught and there are only two levels involved.

There is therefore, a priory no reason to retain terms of the form omega plus omega naught in the exponent. The relevant frequency, the contributing frequency is really omega minus omega naught. So, this object is going to be approximated, to e to the i omega naught minus omega t by 2.

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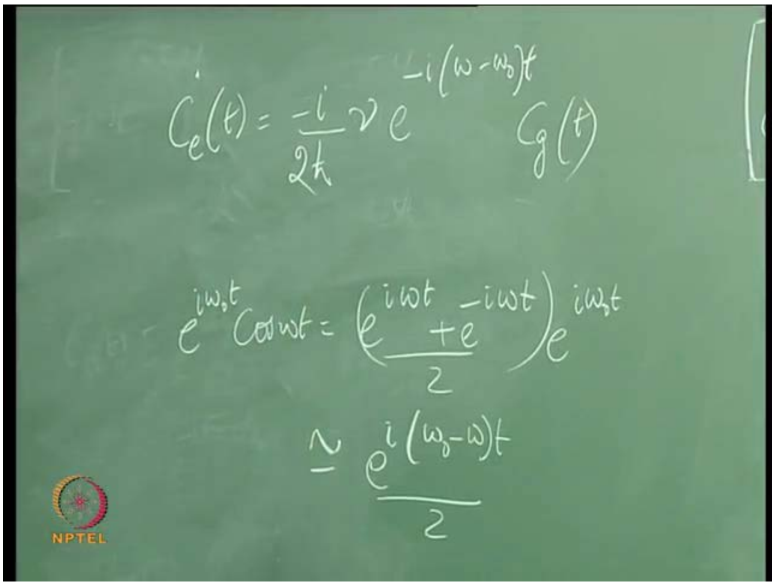

Now, if I did that C e double dot of t is minus i h cross, or let me first write C e dot of t. It is minus i h cross nu, there is a 2 h cross from here, e to the minus i omega minus omega naught t C g of t. So, this is an approximation, also called the rotating wave approximation. But, I will not get into those details here. However, the reason why this approximation is made is, precisely that the original state of the system is not what is preserved.

There are two levels that are involved and the population can jump between these two levels and, the laser is tuned. So, that omega is very close to omega naught. In fact you define something called a detuning parameter delta, which is essentially this difference omega minus omega naught and if delta is 0, you are at resonance and omega approximately equal to omega naught, your near resonance. So, it is the near resonance situation that we are considering here.

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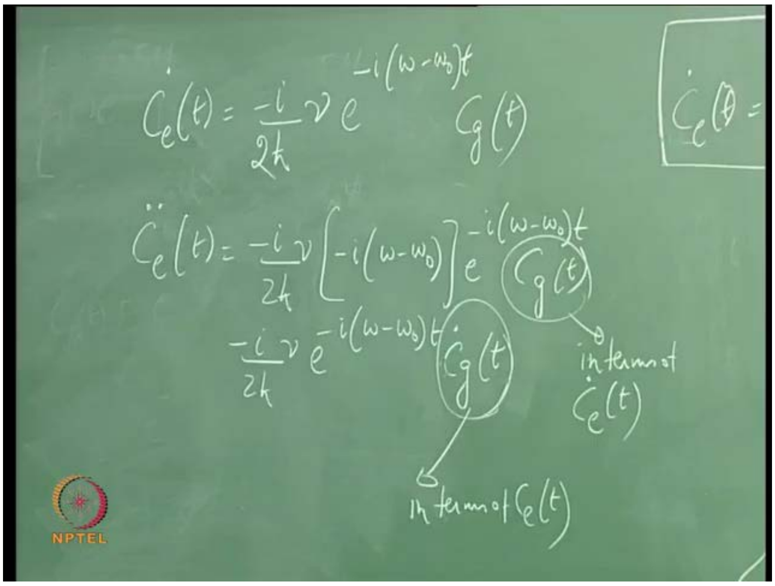

So, look at this equation. I will now take the 2nd differential. I get a minus i omega minus omega naught, e to the minus i omega minus omega naught t. C g of t and then there is another term, which is e to the minus i omega minus omega naught t, C g dot of t. Once more, here (Refer Slide Time: 19:21) I will use the rotating wave approximation and I will only retain the exponential term, which has omega minus omega naught and not omega plus omega naught.

So, this C g of t, is substituted for in terms of C e dot of t. Then, I have a C g dot of t here, which gets written in terms of C e of t. And therefore, I get a decoupled 2nd order equation, C e double dot of t related to C e dot of t and C e of t.

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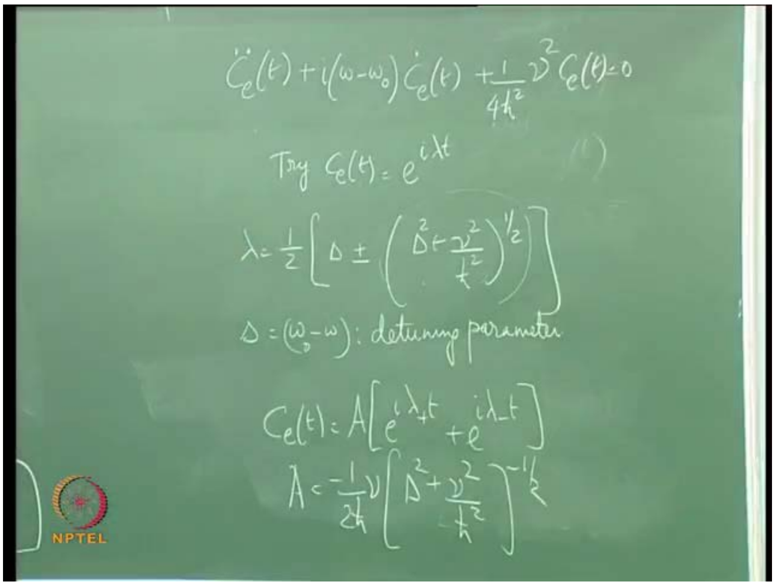

So, here is the 2nd order differential equation that I have for C e. I have C e double dot of t, plus i omega minus omega naught C e dot of t. Plus 1 by 4 h cross squared nu squared C e of t is 0. You see when I plugged in for (Refer Slide Time: 26:11) C g dot of t. I had an e to the minus i omega naught t and that I would have put in here. And then, I would have used C g of t in terms of C e dot of t out here and I would have put that in here. So, it is in this manner that I eliminate C g of t and I get an equation like this. Want to start with a trial solution for C e of t.

This seems, like a reasonable trial solution, for the following reason that you will recall that we wrote (Refer Slide Time: 11:55) psi of t as C g of t times this, plus C e of t times this and since the external electromagnetic field varies, like a cosine function. I would expect a sinusoidal variation in this as well. And therefore, it is a reasonable thing to start off with a trial solution, where C g of t and C e of t vary like e to the i lambda t and that is why this solution for C e of t. Now, if it does not work it will show up as a contradiction somewhere else.

But, at this level it is nearly commonsense, that has guided this choice of trail solution. Because, the field vary sinusoidally and therefore, I will expect the response to vary sinusoidally. Now, once you put in for C e of t here, that is a C e double dot and a C e dot. This 2nd differential will bring down a lambda squared. The 1st differential brings down a lambda and then of course, there is an e to the i lambda t. But, the e to the i lambda t will cancel out all over and you just have a quadratic for lambda. So, there is a lambda squared. It is an equation, which involves lambda squared, lambda and a lambda independent term. And therefore, it has two roots. I will leave this to you as a simple exercise. Just plug in C e of t to be that and obtain the value of lambda.

As I have already mentioned delta is an important parameter in atom optics. It is the detuning parameter and that is what appears here; nu you will recall is simply the matrix element of the dipole operator. Between the two states e and g, times an e naught. The two roots of lambda are these. There is an overall half and there Is a delta plus minus, this object. Look at this, delta squared has the dimensions of frequency and so does nu squared by h cross squared. And therefore, to the power of half has the, this delta square has a dimension of frequency squared and so does nu squared by h cross squared. Then delta is 0, if you are at resonance, this drops out. What is left behind is nu by h cross and that is an important thing to remember.

The matrix element plays a role there, the matrix element of the dipole operator between the ground state and the excited state. But, in general of course, these are the two solutions. What would I expect for lambda? A general solution, it could be either of these and I will call the two roots lambda plus and lambda minus.

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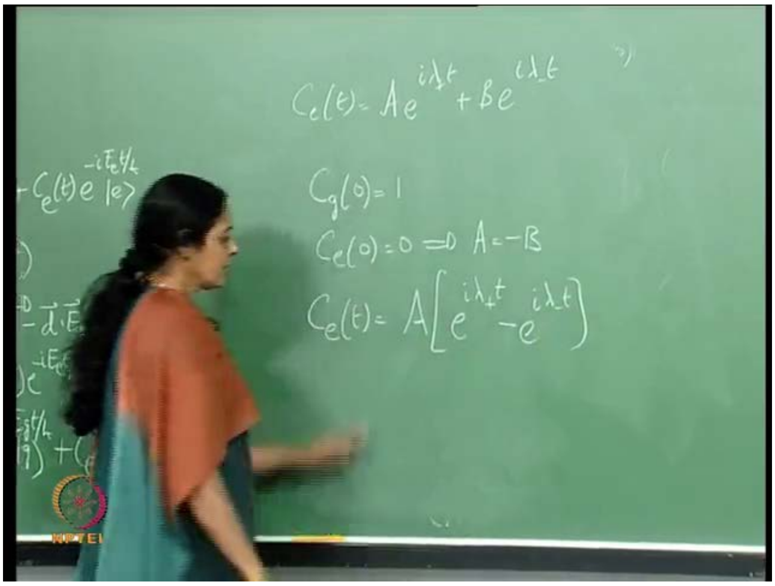

And then of course, the general solution for C e of t, can be written down. A e to the i lambda plus t, plus B e to the i lambda minus t. This is where I impose my initial conditions. I start by saying that the ground state is populated. The excited state is not populated. This is a time t equals 0. So, when I said time t equal to 0 it is clear, that A equals minus B .And therefore, the solution C e of t is A e to the i lambda plus t, plus minus e to the i lambda minus t. So, this is what I have for C e of t.

I can use the initial conditions (Refer Slide Time: 27:27) and fix A and A turns out to be minus 1 by 2 h cross nu, delta squared plus nu squared by h cross squared to the power of minus half. Now, given C e of t one can always find C g of t, up to a phase. Because, modulus C e squared plus C g squared should be (Refer Slide Time: 11:55) equal to 1, as the wave function is normalized to 1. So, I can substitute for A, substitute for the lambda's and I have an expression for C e of t and therefore for C g of t and therefore, for psi of t. Now, this is what I have for a solution for psi of t.

What does this tell me? It tells me a very important thing. First of all I identify something called Rabi frequency. The Rabi frequency itself comes from here. (Refer Slide Time: 27:27) It is delta squared plus nu squared by h cross squared to the power of half. So, in terms of the Rabi frequency, I can ask, what is the probability of the system being in the excited state? What is the probability for the system being in the ground state and so on?

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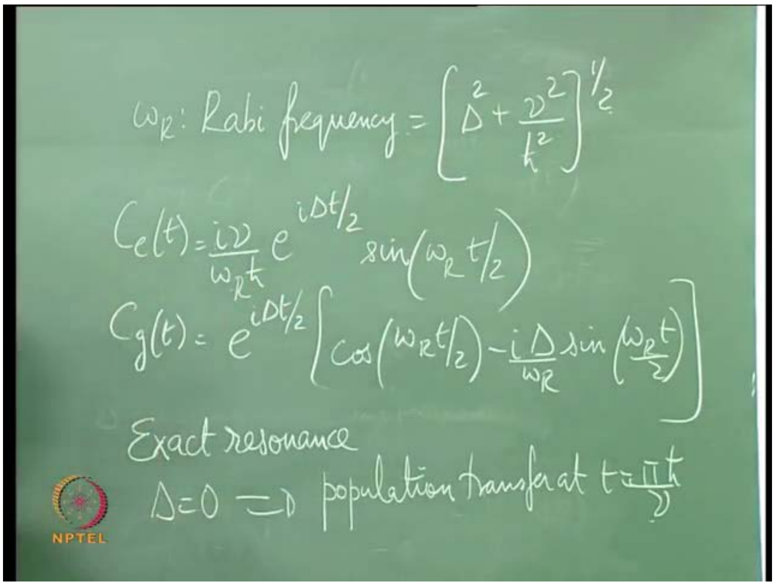

I identify the Rabi frequency, omega some R. The nomenclature itself comes from historically from nuclear magnetic resonance ideas. I identify it to be delta squared plus nu squared by h cross squared to the half. And then, C of t and C g C e of t and C g of t, take on these simplified expressions. This is just a bit of algebra and I leave it to you to verify this, just straight forward piece of algebra.

The important thing to note here is this. Apart from an e to the i delta t by 2. C e of t depends on time as sin omega t by 2 and C g of t has a cos term and a sin term. Now, suppose, the detuning parameter is 0, that means omega exactly matches omega naught.

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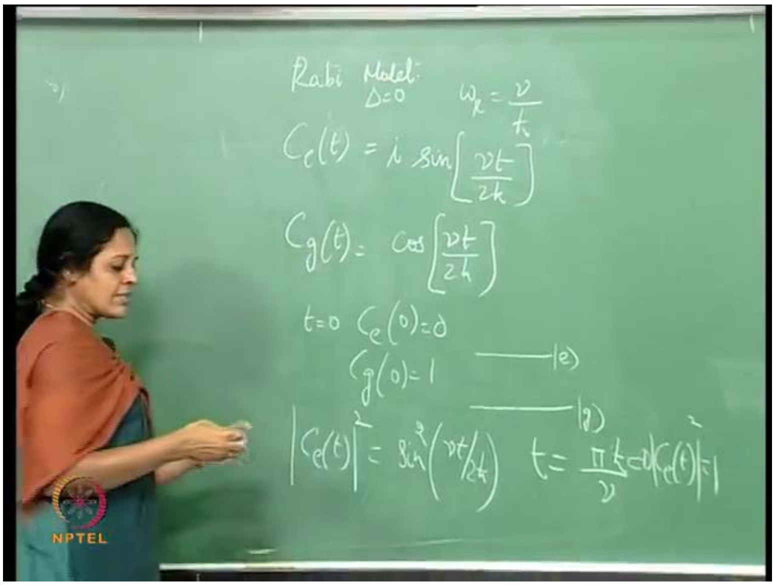

Then you can see, if delta equals 0. C e of t is i nu by omega R h cross, where omega r is now given by nu by h cross. You see, it is a delta equals 0 and that gives me a nu by h cross. So, I can that 2 cancels out and I just have an i. Since, delta is 0, this does not contribute. This is just unity, sin omega R t by 2 h cross. So, that is nu t by 2 h cross out there. And what about C g of t? C g of t, is cos nu t by 2 h cross. (Refer Slide Time: 34:01) This is 1 and that is a 0 there. So, this is what I have.

I have some very simple evolution dynamics for this system. So, what is the probability that, well at time t equals 0 it is clear. That C e of 0 is 0 and C g of 0 is 1, which is as it should be, those were our initial conditions. So, the population was completely in the ground state at time t equals 0 and then it oscillates sinusoidally. Because, I can now ask what is the probability that the system is in the excited state, at any instant of time. Well, that is sin squared nu t by 2 h cross.

And, this is interesting. Because, surely when sin squared nu t by 2 h cross takes the value unity, which it does for t equals pi h cross by nu. Then C e of t mod square equals 1. In other words, the probability of being in the excited state is 1 and the probability of being in the ground state at that instant of time is 0. So, there is a complete population transfer that is the nomenclature that is used. There is a population transfer, from the ground state to the excited state. But, it does not stay there because, again later on when sin squared of the argument is 0.The probability of being in the excited state becomes 0 and the entire population is in the ground state. So, what do we see?

We see this happening. When the detuning is 0, we see that it moves from the ground state to the excited state sinusoidally. Then back from the excited state to the ground state then from the ground state to the excited state and so on. So, if you look at the dynamics, it is a complete sinusoidal variation. And, this is the Rabi model and you can talk of a Rabi frequency of oscillation. Because, all ((Refer Time : 38:25)) done, you have a Rabi time, pi h cross by nu and corresponding to that. There is the Rabi frequency of oscillation. This is the Rabi frequency nu by h cross.

So, this is a simple model if you wish, where the time dependence is studied not using perturbation theory. But, saying that the system moves from the initial state to just one other state and all the dynamics is between these two states. In such a manner, that at certain instance of time, the system could have a probability 1 of being in the excited state and 0 in the ground state. This is called the Rabi model and it deals with a single mode radiation field, in free space interacting with essentially a two level atom.

Now, instead of working with free space, you could talk of the radiation field confined to a cavity. The cavity could be an optical cavity, a micro wave cavity. Whatever it is, the radiation field is confined to a cavity and it interacts with atoms, in that cavity. Normally, these would be multi mode radiation fields. But, as a simple example: I would like to take a single mode radiation field in a cavity and therefore, I will not use e’s e naught cos omega t. I will be more particular about what I write for the electric field inside the cavity and inside this cavity there can be photon creation and destruction.

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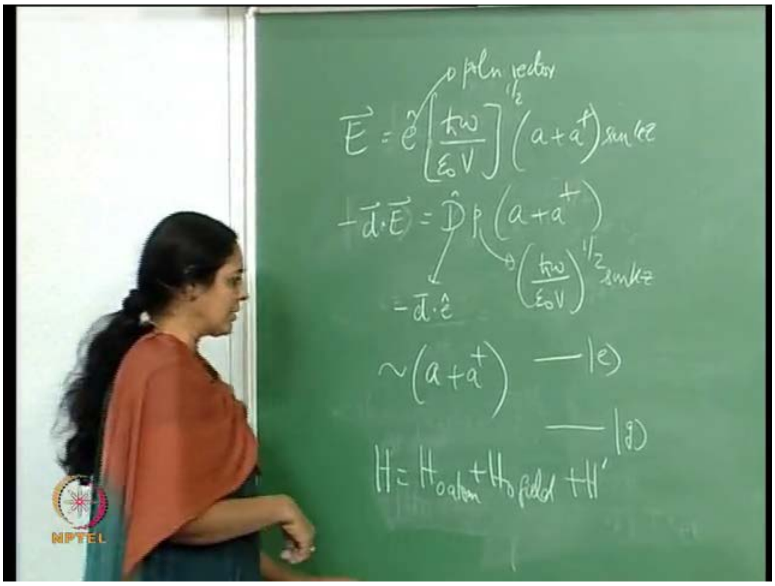

So, I move on to the next model of interaction of the radiation field: with matter, with atom and this would be the Jaynes Cummings model. So, let me move on to this model. As I said the electric field now is written in this form, a and a dagger are the photon destruction and photon creation operators respectively. And, in one of the earlier lectures, where we looked at the electric field in the cavity, we did arrive at this expression. So, I would urge you to go back to the back lecture and see how exactly we got this.

This is an the polarization vector and this is in some arbitrary direction e and therefore, if you look at H prime which is minus d dot E. I write this as minus d dot e out here, this e, that is a scalar product. And, then I have grouped all these constants h cross omega by epsilon naught V to the power of half sin k z as P. This set of constants: V is the volume of the cavity, epsilon naught h cross is the plank's constant, omega is the frequency and epsilon naught is the permittivity. And so sin k z because, I started with this electric field and then there is an a and an a dagger. This is the expression which is familiar to you, which we have derived earlier.

And now, if you look at this, you have this electric field, which is in terms of operators. It is essentially a plus a dagger and this is supposed to interact, with the two level atom operators. And, what are the two level atom operators? First of all, the Hamiltonian itself would now have three parts, which is H naught of the atom, plus H naught of the field, this is the free field Hamiltonian, that is the free atom Hamiltonian, plus H prime, which involves a interaction between the field and the atom.

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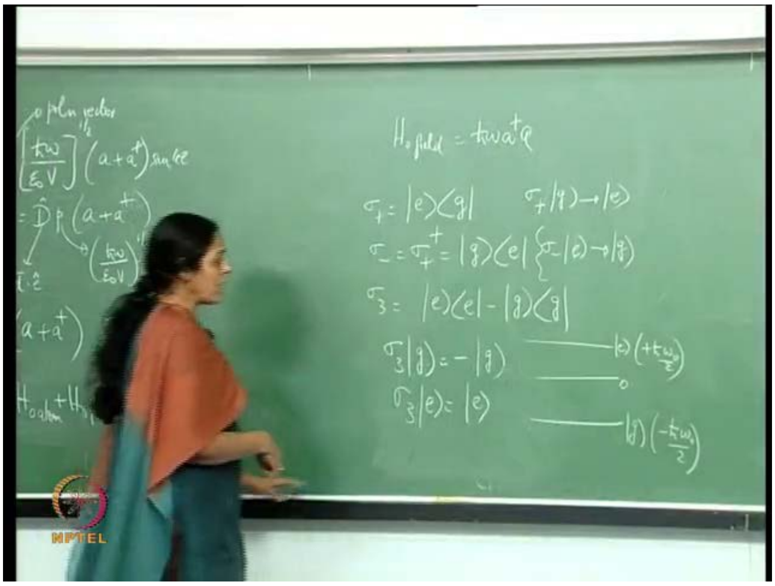

So, let us write down these terms one by one. H naught field, is simply h cross omega a dagger a. That is what you will have, if the photon creation operator (Refer Slide Time: 40:36) is a dagger and the destruction operator is a and the frequency is omega. This is H naught field.

Now, what about the atom? As far as the atom is concerned, look at the various operators. Again from one of my earlier lectures, I defined, sigma plus as this operator. Sigma minus as sigma, plus dagger and therefore: as this operator and sigma 3, as this difference. Now, we checked out that this satisfied the s u to e algebra. Now, one thing is for sure, if sigma plus acts on g it takes it to e. Similarly, if sigma minus acts on e, it takes it to g and so on. This is what I have. Now, what happens if sigma 3 acts on g? Sigma 3 acting on g, just gives me a minus g and sigma 3 acting on e, gives me a plus e.

So, suppose I have the reference level 0 here; in energy and suppose the ground state had energy minus h cross omega naught by 2 and the excited state had energy plus h cross omega naught by 2. Surely, I can write the free atom Hamiltonian very simply in the following manner.

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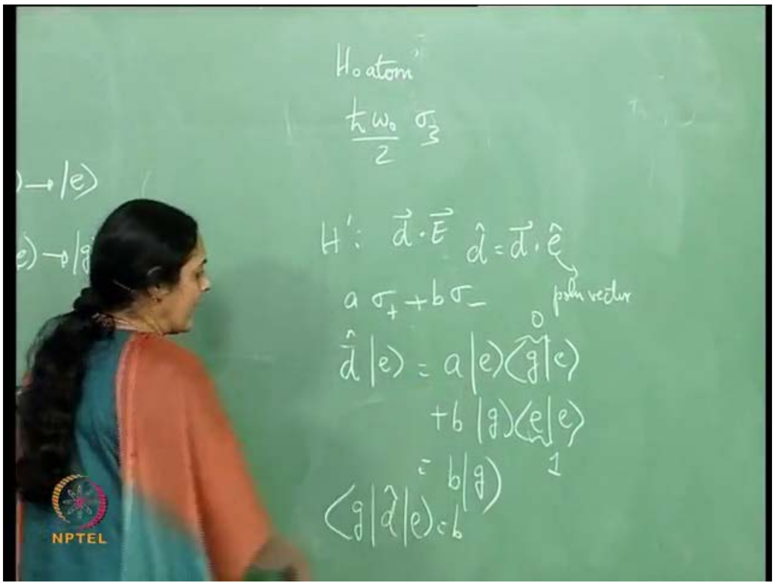

As h cross omega naught by 2, sigma 3, because, what happens. (Refer Slide Time: 42:53) Sigma 3 acting on g gives me a minus sign and ket g and therefore, picks up an energy minus h cross omega naught by 2. And, sigma 3 acting on e, picks up a plus sign in front of ket e and therefore, gives me energy h cross omega naught by 2 sigma 3. And therefore, this is H naught atom and then of course, I have minus d dot e. So, look at H prime, H prime essentially has this atomic operator, dotted with the electric field.

How do I model the atomic operator? What does it do? First of all, d should not connect states of the same parity. So, suppose d were of the form, a times sigma plus. Plus b times sigma minus, where a and b are some constants. What would it do? d acting on the state e, this operator acting on the state e, let us forget the vector sign. The operator acting on the state e, by d by this d, I mean d dot e polarization vector. And therefore, I have made a scalar of it. So, this is a sigma plus, plus b sigma minus. This is 0, this is 1.

And therefore, I get b g and therefore, g d e is b. It is now vanishing. On the other hand, if d acts on g, it will take it to E and this term will be 0 and I get an a. But, one is the Hermitian conjugate of the other.

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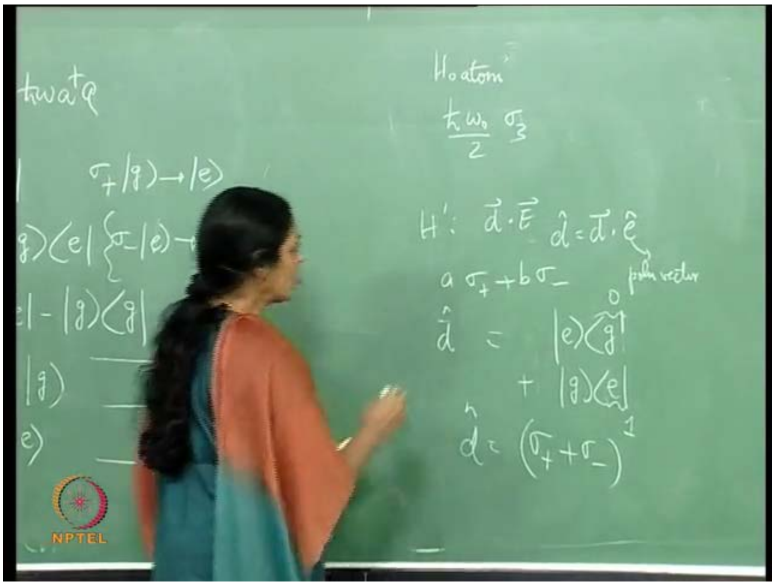

And therefore, I will remove this and I will just say it is e g plus g e. So, that would be my argument for writing the operator d, in terms of sigma plus and sigma minus. So, d is essentially sigma plus, plus sigma minus and then I need to worry about the interaction. So, I need to do a d dot e and the operator in E, was a plus a dagger. And therefore, the interaction Hamiltonian itself can be very trivially written now.

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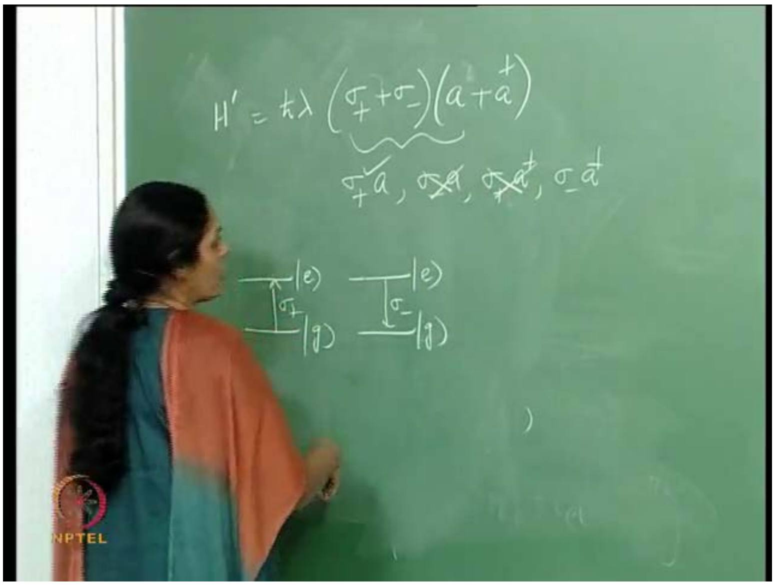

H prime is essentially, I can write it as some h cross lambda for dimensions, sigma plus, plus sigma minus, times a plus a dagger. But, if you now look at the combination, what are the terms? This gives me terms sigma plus a, sigma minus a, sigma plus a dagger, sigma minus a dagger. Sigma plus takes the atom from the ground state to the excited state. In the process, a photon has been absorbed. Sigma minus, takes the atom from the excited state, brings down the atom from the excited state to the ground state, releasing a photon. But, this is the photon destruction operator. Now, that cannot be, a photon has been created. And therefore, this term is not on physical grounds. This term cannot be put in.

Now, look at this term. Sigma plus absorbs the photon, so that g goes to e. Then the photon is not created. Again, on physical grounds this term cannot be there. So, what survives here is simply sigma plus a and sigma minus a dagger. I will continue from here next time. In fact, I have given you the various parts of the Jaynes Cummings Hamiltonian. There is a free radiation field; there is a free atom Hamiltonian and then the interaction term. We will proceed with the Jaynes Cummings Hamiltonian tomorrow and look at the various interesting features that appear using this Hamiltonian.