Quantum Mechanics- I
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Lecture - 24
Wave Mechanics of the Simple Harmonic Oscillator

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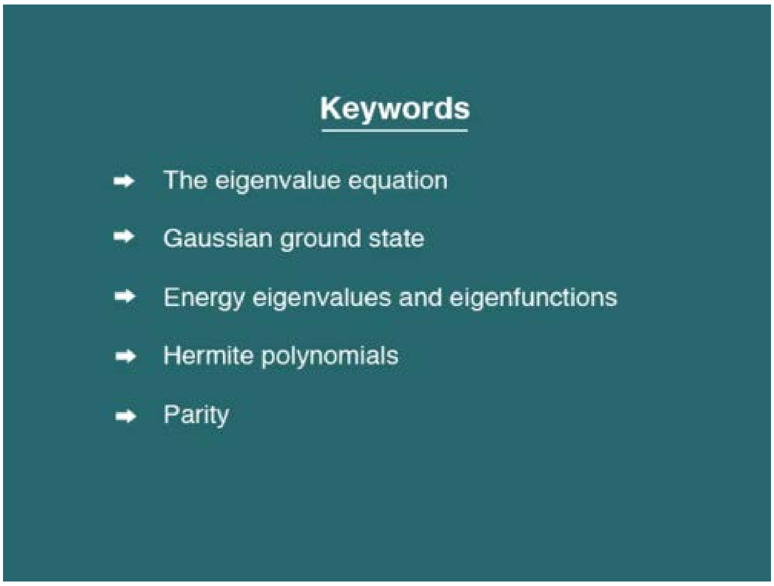

Now, in the last lecture we were looking at the simple harmonic oscillator and this was in the position representation.

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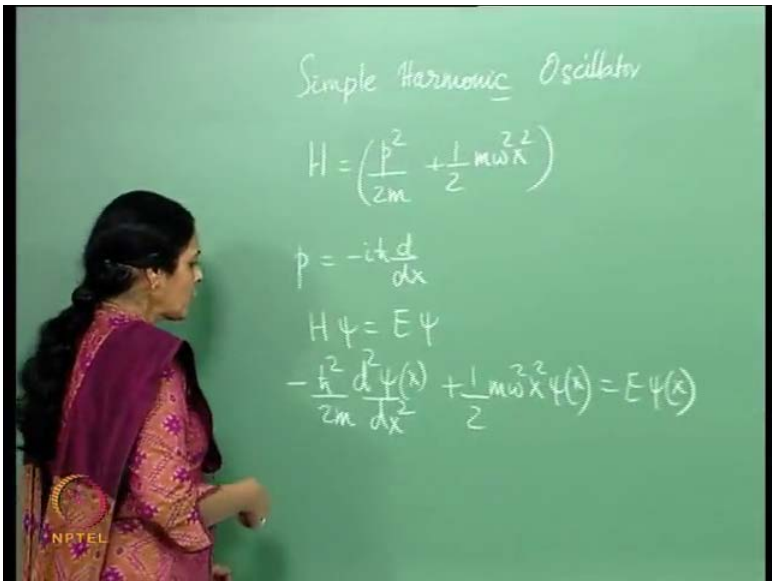

So, we will continue with the linear harmonic oscillator, the simple harmonic oscillator. And we realised that the equation to be solved given the Hamiltonian H is p square by 2 m plus half m omega square x square. This is the Hamiltonian and p is to be written as minus i h cross d by d x in the position representation. The equation itself is H psi is equal to E psi and that was the time independent Schrodinger equation. And therefore, we had minus H cross square by 2 m d 2 psi of x by d x square plus half m omega square x square psi of x is equal to E psi of x.

Of course, we realised that there was a part in general which was a function of time and that was not the time independent Schrodinger equation. We have taken these Schrodinger equation and written the wave function as some psi of x chi of t and then of course, this was the part that depends on x and chi of t itself had a solution E to the minus i e t by h cross and that is how you define a stationary state. Because, then the expectation values do not change in time and mod psi square again does not change in time and so on.

So, we are really looking at solutions to the stationary state wave function in the position representation. And, since we have already worked out the harmonic oscillator problem using the abstract operator method a and a dagger and the commutation algebra commutator a a dagger is identity. We can now do it in the position representation draw parallels between what we got in the abstract operator method and what we will be getting here in terms of the Eigen values and Eigen functions.

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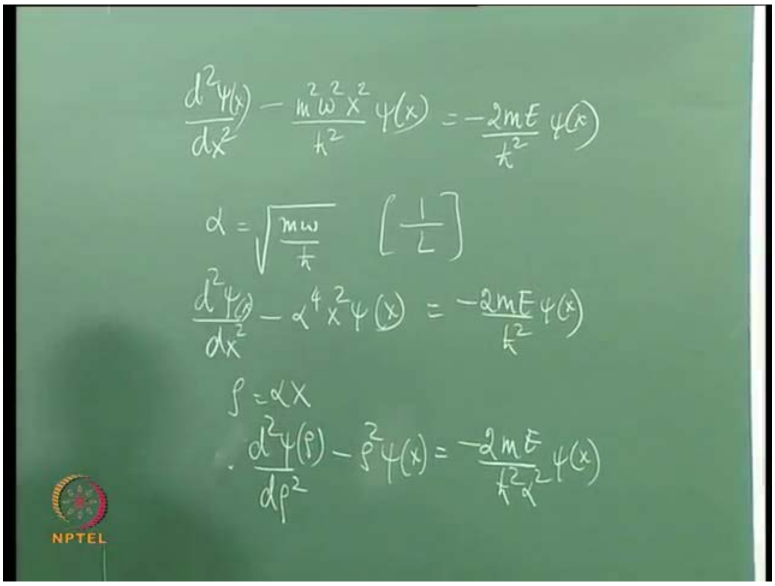

So, we got to a point where I wanted to write this in a more convenient form and therefore, we had d 2 psi by d x square if I suppress the index x, minus m square omega square x square by h cross square psi of x is minus 2 m e by h cross square psi of x. Then I said, let us define alpha as root of m omega by h cross and then indeed this becomes d 2 psi by d x square minus alpha to the 4 x square psi of x is minus 2 m e by h cross square psi of x. Now, we would like to work in terms of dimensionless quantities.

So, I can identify a length scale in this problem. You see this object has dimensions of inverse length that is the dimension. And therefore, if I define an object rho which is alpha x, rho is a dimensionless quantity and then we can recast this equation in terms of rho for d 2 psi by d x square would become alpha square d 2 psi of rho now by d rho square minus alpha to the 4 x square. But then you see if I bring down this alpha square I get a minus rho square here. Psi of x is minus 2 m e by h cross square alpha square psi of x. This object 2 m E by h cross square alpha square is again a dimensionless quantity.

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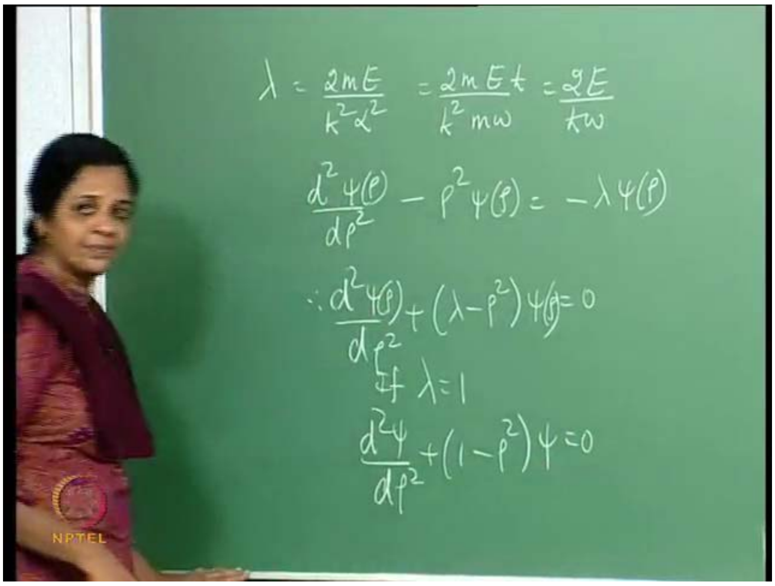

I define lambda as 2 m E by h cross square alpha square. And since alpha was root of m omega by h cross and therefore, I have an m omega by h cross here and so this quantity is 2 e by h cross omega. And that is very nice, because now I have an equation in terms of objects which do not have dimensions. I will comment on this shortly, but I now have d 2 psi of rho by d rho square minus rho square psi of rho is minus lambda psi of rho and therefore, I have d 2 psi by d rho square plus lambda minus rho square psi of rho equals 0. What is it that we did in the abstract operator formalism? Which was the parallel of what I have done now?

I have now written things in terms of dimensionless quantities. I have introduced a rho which is dimensionless because alpha had the dimensions of the inverse length and therefore, alpha x was dimensionless. The equation itself is cast in terms of dimensionless quantities like rho and lambda.

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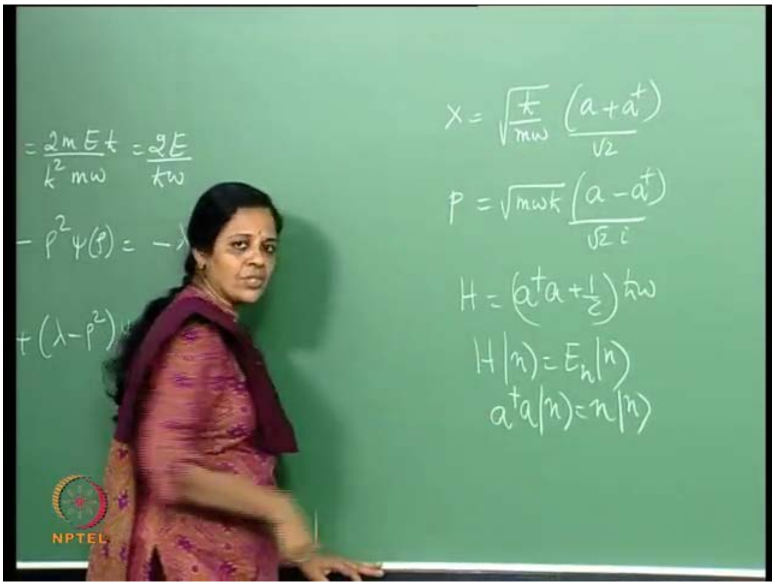

Now, you will recall that the when we did the abstract operator method in terms of a and a dagger, the ladder operators. We said that x was root of h cross by m omega a plus a dagger by root 2 and p was root of m omega H cross a minus a dagger by root 2 i. This has dimensions of length and this has dimensions of momentum. And therefore, a and a dagger themselves were dimensionless quantities.

So, when we worked in the abstract operator formalism. We worked with a and a dagger and wrote the Hamiltonian as a dagger a plus half h cross omega and then when I have an equation like H ket n is E sub n ket n, where n is the state of the oscillator and E sub n is the corresponding energy Eigen value. That is the same as saying a dagger a plus half h cross omega acting on the state n is E sub n ket n. That equation again is in terms of of objects which are dimensionless, a and a dagger which are dimensionless quantities. I have scaled out an h cross omega.

So, I might as well just pull that out and say a dagger a plus half ket n is E in units of h cross omega ket n and if I forget the half as well for the moment. I have a dagger a n is n ket n where I realise that E sub n is n plus half h cross omega where n is an integer and since I have removed the half h cross omega I have this. So, you see this equation that we wrote when we worked out the harmonic oscillator problem using the abstract operator method was an equation in terms of dimensionless quantities. Scaled out the h cross omega wrote a and a dagger in terms of x and p such that they were dimensionless and then you have this equation.

(Refer Slide Time: 04:48) The analogue of that is precisely this. Except, that I have written it in the position representation and therefore, I have d by d rho and so on occurring here and rho square and so on. But, the whole thing has been recast in terms of dimensionless quantities. So, it is like writing this equation in the position representation and that is what I have done. So, I have psi of rho here. Now, when I solve this equation, I will 1st try to find out if I have any solutions for specific values of lambda and that gives me a handle on solving this equation.

(Refer Slide Time: 04:48) So, for instance if lambda is 1 then d 2 psi by d rho square plus 1 minus rho square psi equals 0, but you see an equation of this form can be factored and written in a different manner all together.

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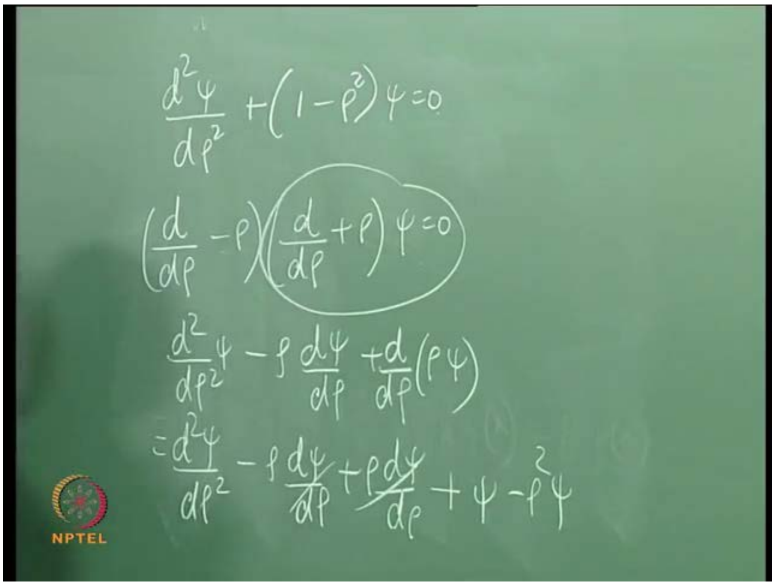

I have the equation d 2 psi by d rho square plus 1 minus rho square psi equals 0 and that can be written as d by d rho minus rho d by d rho plus rho psi equals 0. Because, this is just d 2 by d rho square psi, minus rho d psi by d rho plus d by d rho of anything that comes after that so, that is the same as d 2 psi by d rho square minus rho d psi by d rho plus rho d psi by d rho and that cancels out plus psi. And, then I have a minus rho square psi so, that gives me a plus1 minus rho square so, which is what I have here. Therefore, the solution of this equation could well be got from this d by d rho plus rho psi equals 0, but that solution is obvious if d by d rho plus rho psi equals 0 then that implies that psi is a Gaussian in rho.

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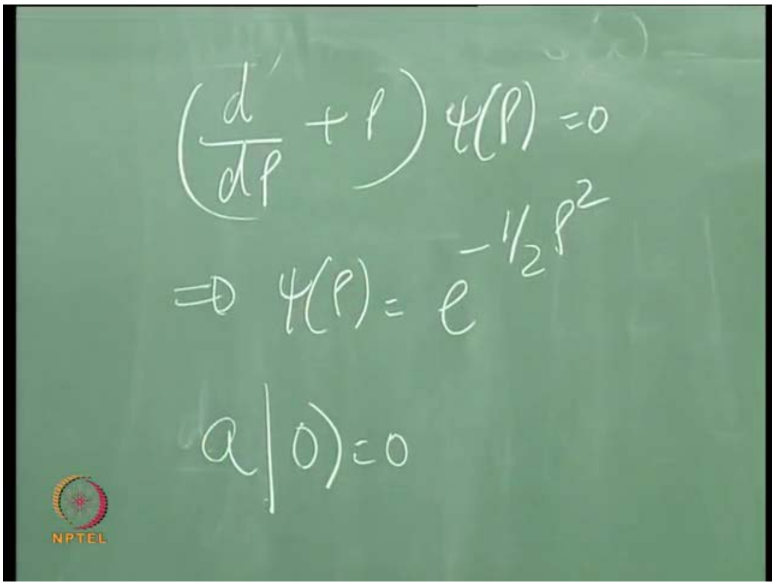

So, d by d rho plus rho psi of rho equals 0 implies psi of rho is e to the minus half rho square and therefore, that is also a solution of this equation. (Refer Slide Time: 09:28) Since, psi is a Gaussian and it satisfies this 1st order equation it also satisfies that equation. So, I have a definite solution for lambda equals 1, but I know from whatever I have learnt using the abstract operator method that there is a Gaussian solution for the equation a on ket 0 is 0. You see then we wrote a in terms of x and p, wrote p in the position representation as minus i h cross d by d x and then we got the Gaussian solution, (Refer Slide Time: 06:21) because if I had started there and I had substituted for a, here from this if I solve for a, I simply have root 2 x.

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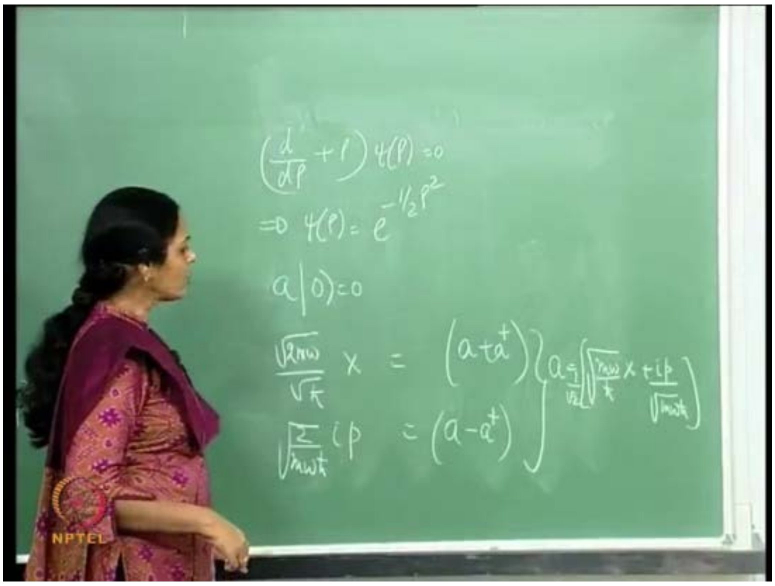

Let us write down a in terms of x and p. So, I have root 2 x equals root of h cross by m omega a plus a dagger and therefore, I have root of 2 m omega by root of h cross x is equal to a plus a dagger. Then, I have an i p again with the root 2 and on this side I had a root of m omega h cross and therefore, root of 2 by m omega h cross i p is a minus a dagger which gives me a. I can solve for a and I have a is equal to root of m omega by h cross x plus i p by root of m omega h cross. But that was 2 a and therefore, I have all this divided by 2. So, I have a is equal to1 by root 2 times this object and that is fine because I have again made it dimensionless, apart from the 1 by root 2 which I need to put in.

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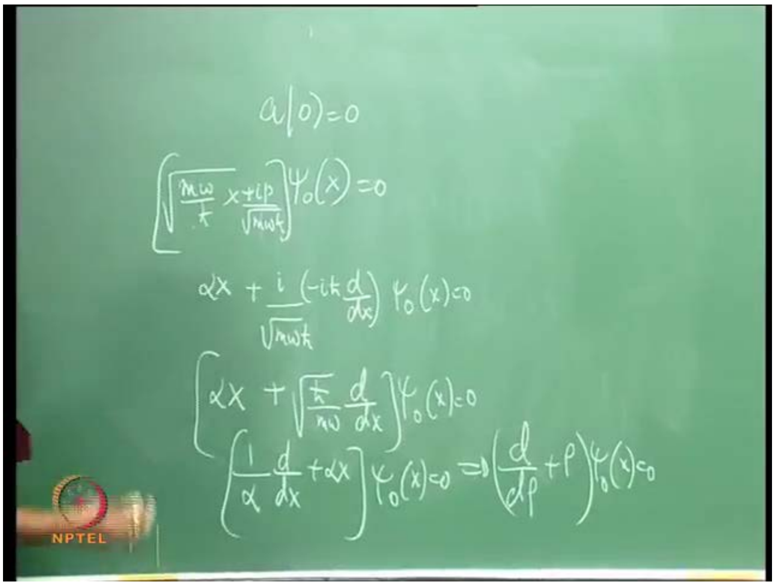

Now, in the position representation how does this equation look a on ket 0 is equal to 0 looks like this. In the position representation let me write this as psi 0 of x and a itself can be written in terms of x so, essentially I am trying to say that root of m omega by h cross x plus i p by root of m omega h cross acting on psi 0 of x is 0, but root of m omega by h cross is alpha. So, I have alpha x plus i by root of m omega h cross p is minus i h cross d by d x psi 0 of x equals 0. So, I have alpha x plus root of h cross by m omega d by d x psi 0 of x equals 0 that is the same as saying that gave me an alpha down stairs. So, 1 by alpha d by d x plus alpha x psi 0 of x equals 0 in terms of rho which is alpha x that is simply d by d rho plus rho psi 0 of x equals 0, which is exactly what I have here. (Refer Slide Time: 12:03)

So, it is good to draw a parallel and understand how exactly the abstract operator method when written in the position representation will give me this. I started with writing a ket 0 is 0, but if a is written in terms of x and p and p itself is minus i h cross d by d x. I simply get that equation where the solution which is a Gaussian solution. So, that is one thing that I have learnt that for a specific value of lambda, lambda is equal to 1 the solution is Gaussian. (Refer Slide Time: 09:05) And, then by inspection I see the following: Go back to the general equation. Let me look at the asymptotic form of psi of rho, because I know that it has to satisfy boundary conditions. The wave function has to vanish sufficiently fast, has to go sufficiently fast to 0 at spatial infinity that is as rho goes to plus minus infinity.

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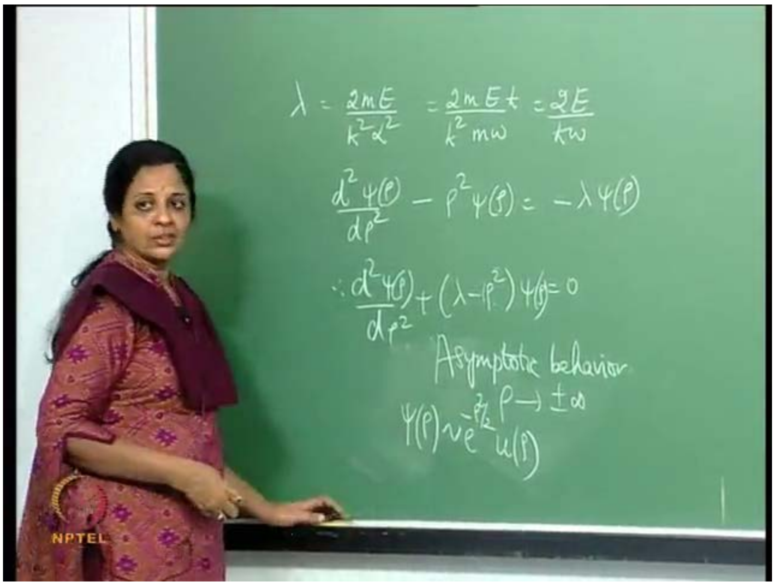

But when rho goes to plus minus infinity this object over powers this and really the value of lambda becomes immaterial. So, in order to study asymptotic behaviour that is as rho goes to plus minus infinity I might well take lambda to be 1 and if I did that I can see that psi of rho should go as e to the minus rho square by 2. Apart from other functions of rho, I will call it u of rho. The dominant behaviour for rho going to large values is E to the minus rho square by 2, goes as E to the minus rho square by 2 and that is all that I needed to know. Now, I can substitute this solution for psi. Recast the differential equation as a differential equation for u, find out u and then I know the full solution, the wave function for the simple harmonic oscillator in the position representation.

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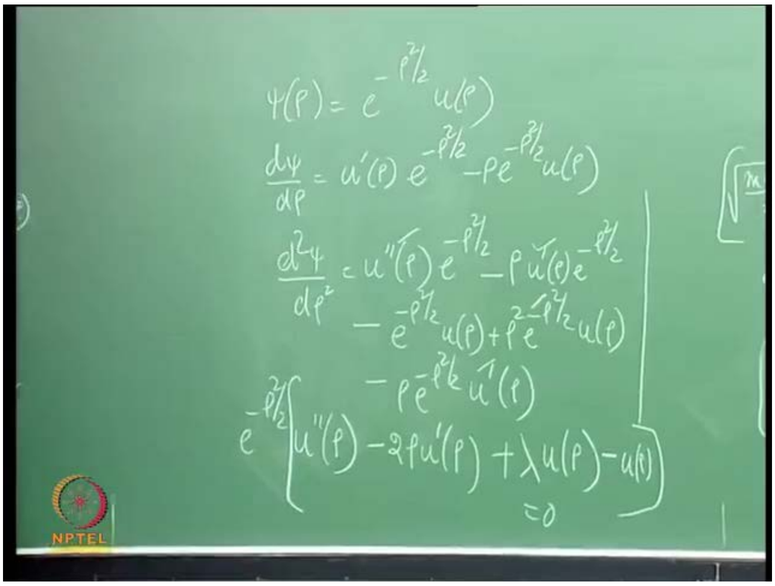

So, that means the following. Let me write psi of rho as e to the minus rho square by 2 u of rho. So, that means d psi by d rho is u prime of rho 1st derivative of u with respect to rho that is psi prime, but I need psi double prime of rho that is the u double prime of rho. I am now going to have a 2nd order equation in u that is what I have from the 1st term. So, this is what I have for d 2 psi by d rho square and then I substitute it back there.

(Refer Slide Time: 16:01) I am going to have an equation which involves both the u double prime and u prime. I have a minus rho e to the minus rho square by 2 u prime of rho and the same thing out here. So, that gives me a minus 2 rho and therefore, I have the equation itself becomes d 2 psi by d rho square gives me u double prime of rho. I am going to pull out an e to the minus rho square by 2. That is the common thing in all of them and the asymptotics are dictated by e to the minus rho square by 2. So, that is u double prime of rho minus 2 rho u prime of rho so, I have taken care of these terms.

And then I have plus rho square e to the minus rho square by 2 u of rho (Refer Slide Time: 16:01) and then I have minus rho square e to the minus rho square by 2 u of rho. So, there is a cancellation there and then of course, I have lambda so, I have plus lambda 5 of rho and I have pulled out an e to the minus rho square by 2 therefore, I have a lambda u of rho that is there. So, I have already taken care of all these terms except this one which is a minus u of rho. So, this object is equal to 0.

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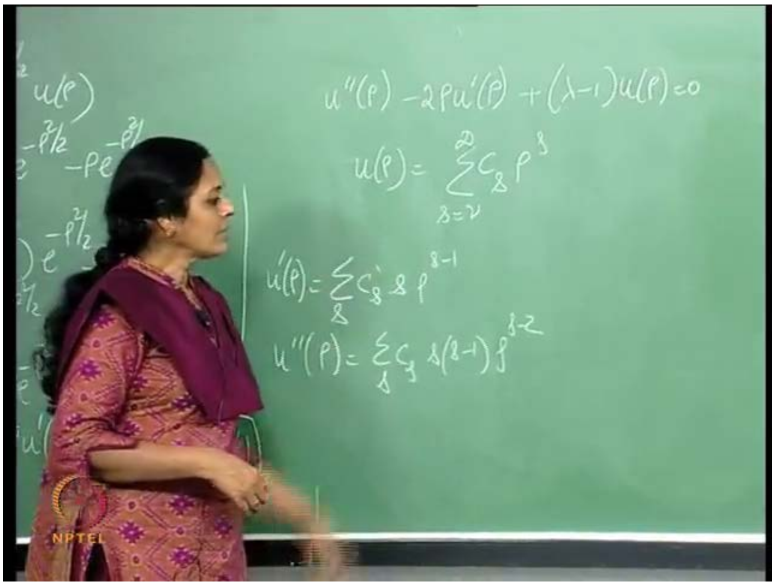

So, let me write this in a convenient fashion. I have u double prime of rho minus 2 rho u prime of rho plus lambda minus 1 u of rho equals 0. So, this is the equation for u 2nd order equation which needs to be solved. Once, I solve for u I can put that back in psi of rho just multiply it with the Gaussian e to the minus rho square by 2 normalise it suitably so, that the wave function obeys the probabilistic interpretation. It is normalised to unity and then I have my answer.

But to solve this equation, I will use the fact that u of rho. We are now working in function space I will use the fact that u of rho can be expanded in terms of polynomials of rho, I can write it as a polynomial of rho. Therefore, u of rho let me try this solution it is a summation over C s rho to the power of s. Again, I am interested in the asymptotic behaviour so, definitely I need to worry about s taking large values and s could perhaps take all values. So, let me say s starts from nu C s rho to the power of s some value nu does not matter the small values, I am worried about the asymptotic value. So, let me substitute that there and therefore, I have summation over s C s. So, u prime of rho is summation over s C s s rho to the s minus 1 which gives me u double prime of rho is equal to summation over s C s s times s minus 1 rho to the s minus 2.

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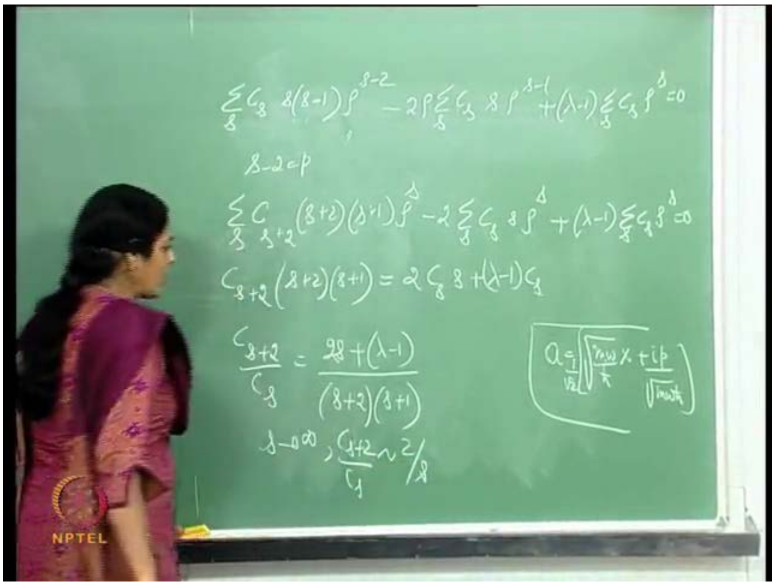

Now, substitute this I will keep this for later perhaps. So, when I substitute this back I have summation over s C s s times s minus 1 rho to the s minus 2. That is what I have for u double prime minus 2 rho u prime u prime is simply summation over s C s s times rho to the s minus 1 plus lambda minus 1 u that is equal to 0. Now, here if indeed this should give me 0 it is clear that the coefficients corresponding to every power of rho should vanish. So, let us compare let us write down the coefficient of rho to the power of s so, for that let me redo the 1st term. Call s minus 2 as p so it is summation over p suitable summation I am only interested in the asymptotic value so, when s goes to infinity p also goes all the way to infinity.

So, s is p plus 2 so s minus 1 is p plus 1 rho to the power of p that is the 1st term and I can go back to calling p as s because that summed over, that is just the dummy index. That is my 1st term minus 2 summation over s C s s rho to the power of s so, I am just trying to pullout coefficients of rho to the s plus lambda minus 1 summation over s C s rho to the s is 0. And therefore, I have C s plus 2 for any s, C s plus 2 times s plus 2 s plus1 is equal to 2 C s s plus lambda minus 1 C s. Therefore, C s plus 2 by C s is 2 s plus lambda minus 1 divided by s plus 2 times s plus 1. Now this is a good thing to know because we are interested in what happens for large values of s. The asymptotic behaviour is going to be dictated by this ratio and one wants to see if the series converges at all.

So, let us look at this ratio. For large s this ratio goes as 2 by s because I have a 2 s that is negligible compared to that for large s and here I have an s square that is a leading term and therefore, I get an s by s square so, 1 by s. So, for large s this goes as 2 by s, but I know the another series. I am now trying to find out the solution for u of rho as a function of rho. I know of a series which for large s has this asymptotic behaviour and that is unfortunately the series solution for e to the rho square.

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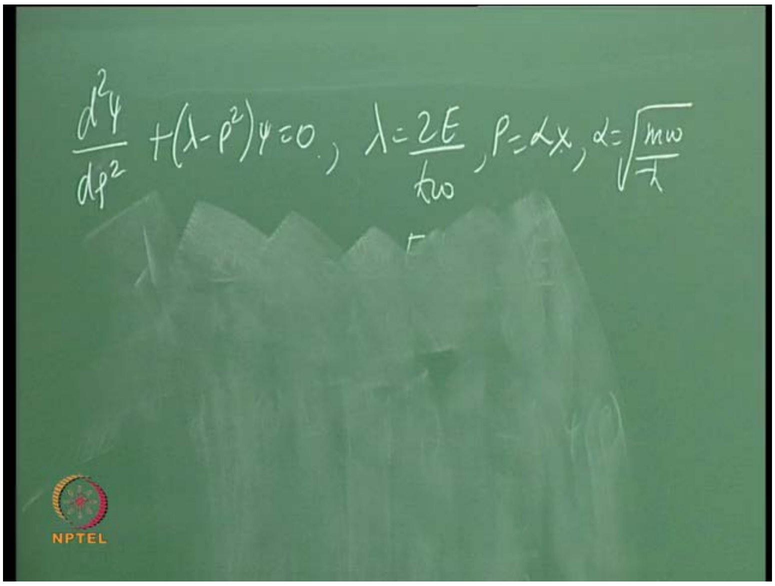

If I expand e to the rho square so, maybe it is good to put down this equation somewhere d 2 psi by d rho square plus lambda minus rho square psi equals 0 and lambda was 2 e by h cross omega and rho was alpha x. Alpha itself was root of m omega by h cross so, there we are.

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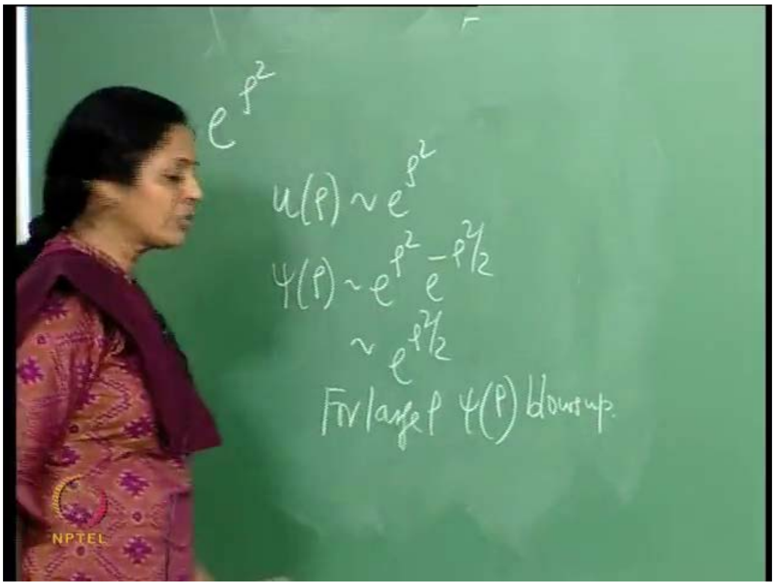

So, if you look at the series e to the rho square and, let us write this as summation over c as rho to the power of s. You find that for large s, it goes as1 by s and that is very unfortunate as it stands because that is like telling me that u of rho is essentially e to the rho square and therefore, psi of rho is e to the rho square times e to the minus rho square by 2, which I guessed from the asymptotic form when lambda was 1 and this is therefore, e to the rho square by 2 which is unfortunate. Because for large rho it blows up but I want psi of rho to go to 0 as rho goes to infinity.

(Refer Slide Time: 21:46) So, I cannot let terms for large s survive in the series if at all I hope to get a solution. So, the only way to handle this is if the series truncates at some point so, that the coefficients corresponding to large s simply do not contribute. (Refer Slide Time: 19:52) Some value of s where the series truncates and therefore, the summation over s does not go all the way to infinity, but get stops somewhere at a lower value, the finite value. And if that happens then clearly I do not have to worry about the asymptotic behaviour and predict that to be e to the rho square. So, let us see how exactly this series will truncate. I do not want that. So, one way of solving the problem is to look out if the series truncates somewhere.

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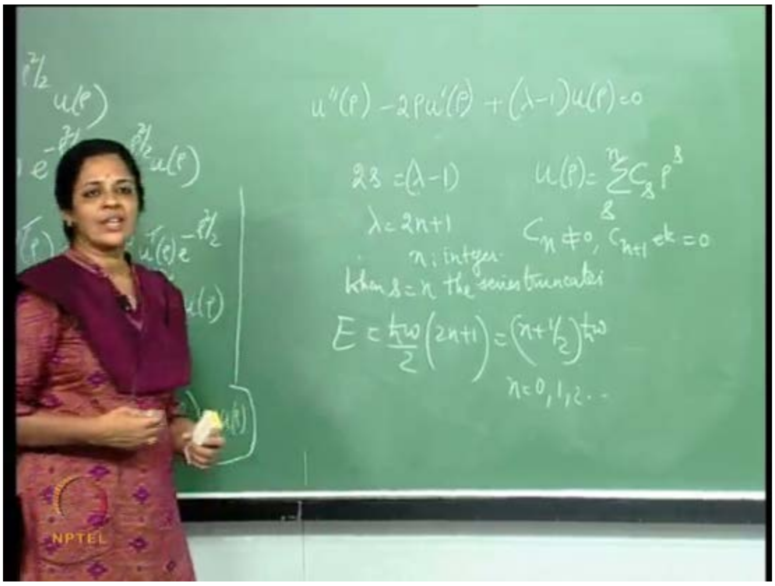

And that is possible because if 2 s is equal to lambda minus 1.

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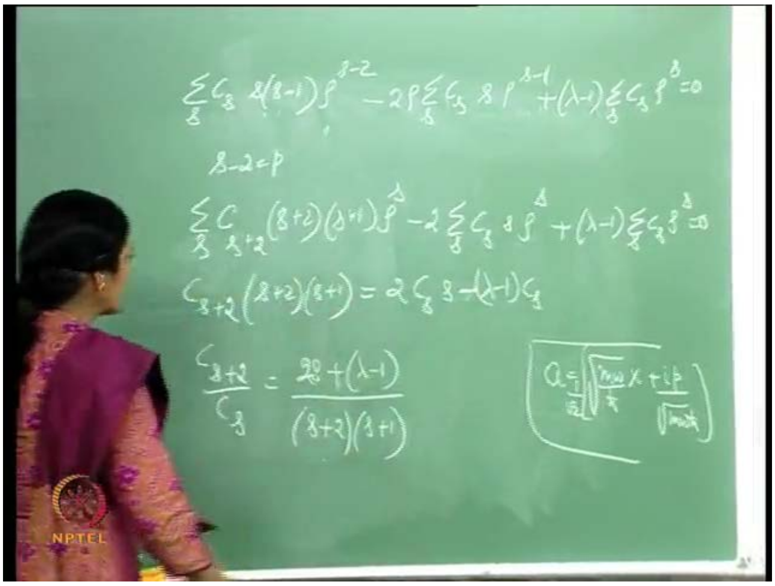

This is 2 s minus lambda minus1 here. If 2 s is equal to lambda minus 1 and then I know that the series truncates. But, since s is an integer lambda must be equal to 2 n plus 1 where n is an positive integer and then when s is equal to n the series truncates. So, the summation over s goes from the lower value whatever, it is all the way to n where lambda is 2 n plus1 where n is a positive integer.

So, that is a possible way of handling this problem and indeed that is a correct way of doing it, but if lambda is equal to 2 n plus 1 go back to the definition of E, E is h cross omega by 2 times lambda. So, E is n plus half h cross omega. So, you see this is very intimately connected with each other. The fact that the asymptotic behaviour of psi should be admissible, the fact that psi should go to 0 for large values of rho and therefore x, puts a constraint on lambda because the series has to truncate and that in turn tells me, what the Eigen values are.

So, the Eigen values are of the form n plus half h cross omega and in principle n can be zero 1 2 3 anything And when n is equal to 0 it tells me that lambda is equal to 1 and that is the lowest value that n can take, because s has to take positive values. So, lambda is equally to1 really corresponds to the ground state of the oscillator and indeed we saw that the ground state of the oscillator, the wave function in the position representation is a Gaussian wave function and that is how things get linked up with each other.

In any case going back here suppose, there is some value of n where this series truncates that is very nice because then u of rho is simply summation over s all the way to n C s rho to the power of s and that is it. So, C n is not equal to 0, but c n plus 1 etcetera are 0. It is a finite series, now look back at this and if I do that: What kind of equation do I have here? This equation is essentially the same as the equation satisfied by the Hermite polynomials because the equation for the Hermite polynomials is like this.

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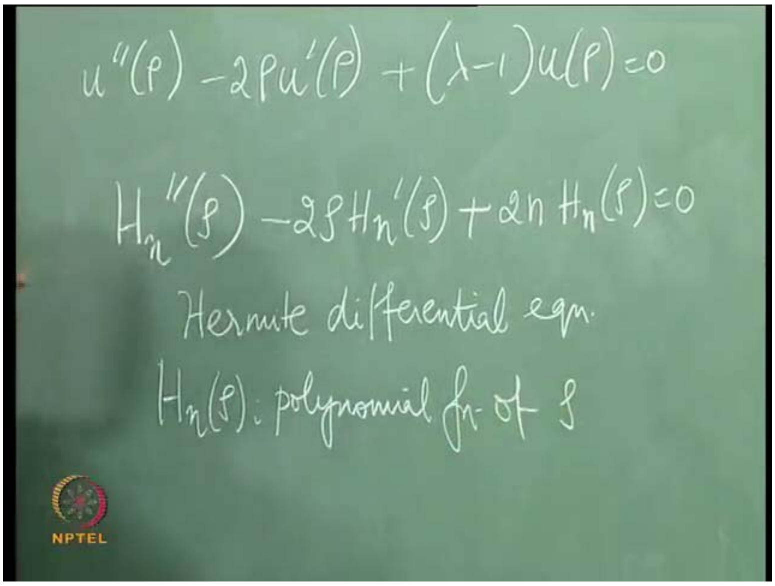

Suppose, H n of rho is a Hermite polynomials the solution of the equation H n double prime of rho minus 2 rho H n prime of rho plus 2 n H n of rho equals 0 it is the Hermite differential equation. These are polynomials in rho; H n of rho is a polynomial in rho. I briefly mentioned the properties of H n of rho in an earlier lecture said there in function space they could form a basis. The function space that we are considering here for the harmonic oscillator problem is clearly l 2 minus infinity infinity because rho takes values minus infinity infinity so, the solution of u of rho is essentially H n of rho.

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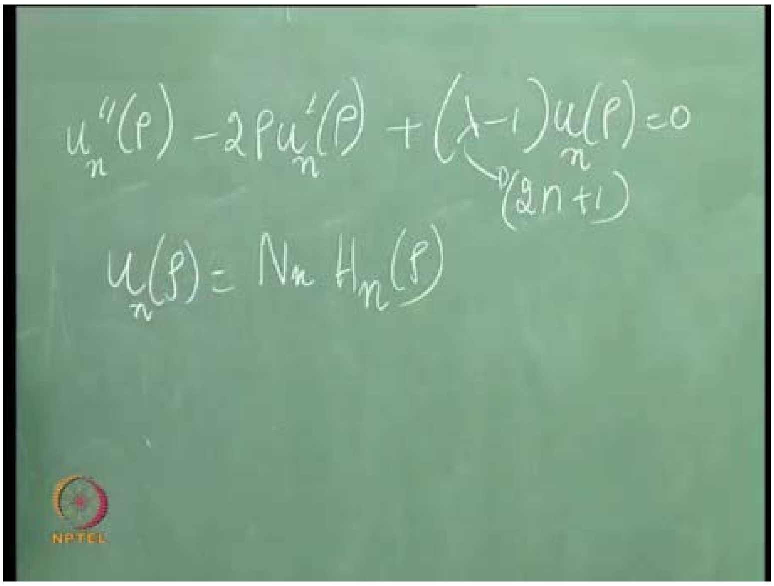

Apart from some normalisation constant and therefore, u of rho is some normalisation H n of rho and therefore, I would like to call this u n of rho. Lambda itself is 2 n plus1 and therefore, the n is brought in here.

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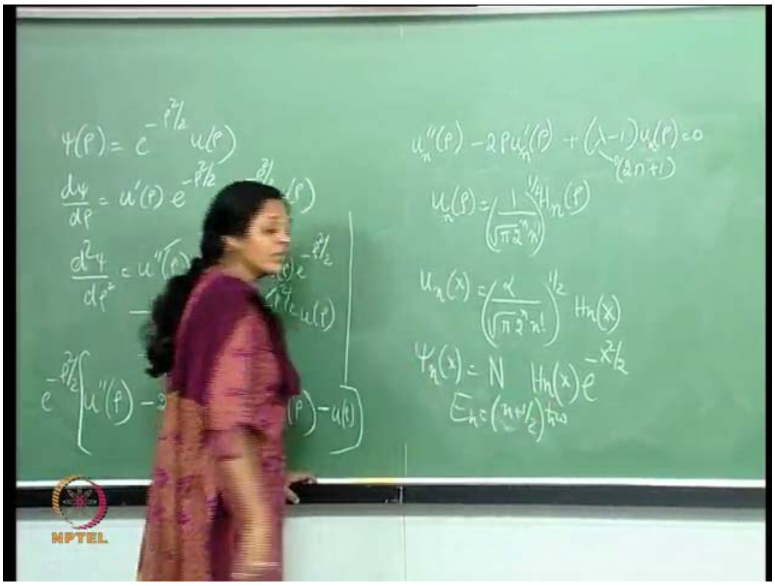

So, we could well write u n of rho minus 2 rho u n prime of rho and a u n of rho there, this lambda. So, that is what I have. So, this is some normalisation which I well call it N H n of rho and clearly the normalisation would have 1 by root pi 2 to the power n n factorial to the half. From the orthonormality property for H n which I just now wrote and therefore, this is what I should be having for u n of rho. I need u n of x and since rho is equal to alpha x, u n of x is alpha by root pi 2 to the power of n n factorial to the power of half H n of x so, this is what I have. And this is normalised to unity the u n's form an orthonormal basis set of functions, but now going back to psi I now write psi n of x because for a given value of n and therefore, lambda and therefore, e I have a certain wave function and that is given by this object u n of rho which is essentially H n.

So, let me not write the normalisation. It is some normalisation n, H n of x times the Gaussian e to the minus x square by 2. The Gaussian takes care of the fact that the wave function goes to 0 sufficiently fast at space infinity. The H n of x themselves arose, because the series had to be truncated in order that the wave function was an admissible wave function. Not every wave function which is square integrable can be a solution to our problem its only those wave functions which satisfy the boundary conditions imposed by the physical requirement of the probabilistic interpretation. That wave function must be normalised to 1 and therefore, the probability of seeing the system in the given region of space is 1. It is that, that is going to bring out an H n of x here and therefore, I have Eigen functions of the oscillator given by psi n of x with the energy Eigen values given by n plus half h cross omega.

So, in this method of working out Eigen values and Eigen functions there are some crucial points which are worth noting and which we will be using in subsequent problems that we will consider. Wherever, possible we will go to dimensionless variables. In this case we went to rho and lambda, where rho itself do not have the dimensions of length x ((Refer Time: 36:07)) and alpha has dimensions of 1 by length and therefore, rho was dimensionless. Similarly, lambda did not have dimensions because lambda was 2 e by h cross omega.

So, we will do that always and that as I said connects up with the abstract operator method, because we wrote that in terms of a’s and a daggers which were dimensionless objects. In fact I had explicitly illustrated that a on ket 0 is equal to 0 really turned out to be the differential equation with lambda set equal to1 here in the case of the position representation. These are the wave functions, but I know that these wave functions must have definite parity.

I know this because of an earlier argument that I had given that the Hamiltonian for the harmonic oscillator for the 1 dimensional oscillator commutes with the parity operator and from that I had earlier shown that the wave functions would have definite parity. The Eigen states of the Hamiltonian would have definite parity either positive parity or negative parity, and that there is a complete set of common Eigen states of the parity operator and the Hamiltonian.

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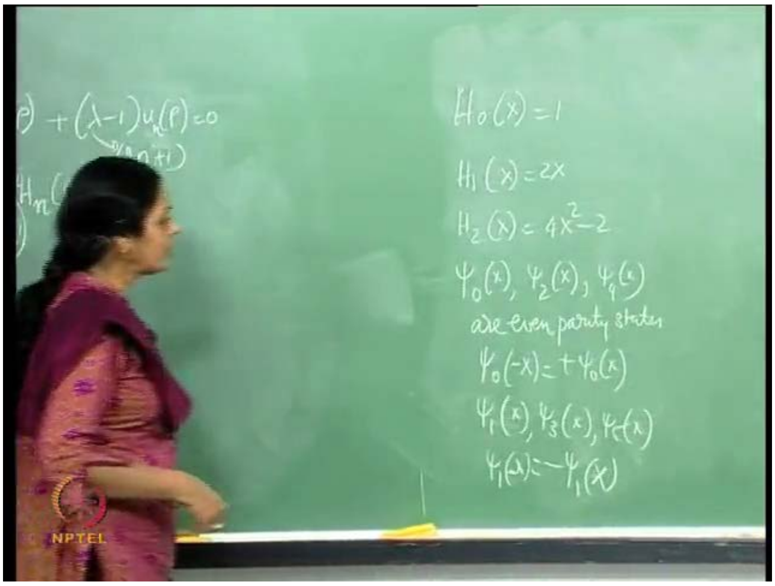

Now, that you can verify right here. Because the h ns can be written in the following fashion: H 0 of x is 1, H 1 of x is 2 x, H 2 of x is 4 x square minus 2. So, you see this has odd parity, this has even parity, that has even parity and so on e to the minus x square by 2 anyway is in even parity state when s goes to minus x e to the minus x square by 2 does not change sign and therefore, these wave functions have a definite parity. It is clear that psi 0 of x, psi 0 of x, psi 2 of x and so on are even parity states that means psi 0 of minus x is plus psi 0 of x and so on. And psi 1 of x, psi 3 of x, psi 5 of x are odd parity states that means psi 1 of minus x is minus psi 1 of x and so on.

(Refer Slide Time: 33:11) This property is reflected by the fact that the psi n of x is essentially H n of x and the H n's themselves have even and odd parity like I have shown here. Now, if you look at the wave function and have a schematic plot of these wave functions as a function of x you can schematically see the following.

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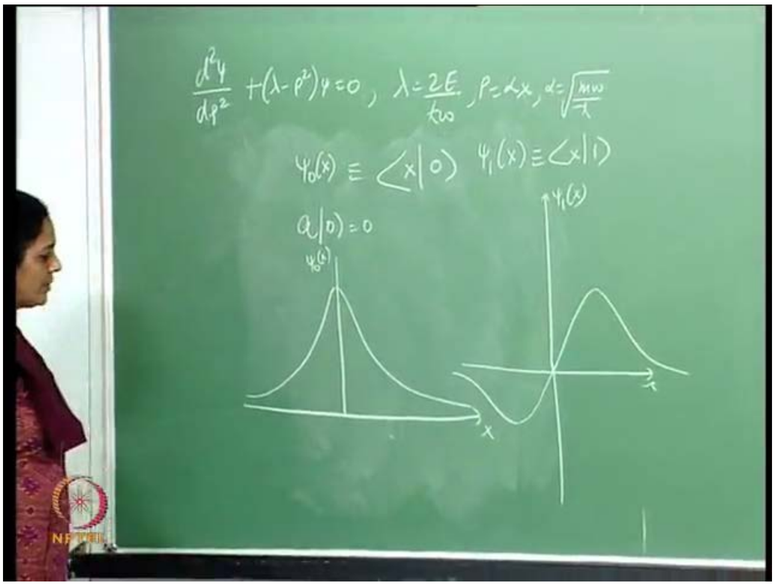

So, if you take psi 0 of x remember that this is the same in my notation as x 0, where a on ket 0 was 0. This written in the position representation is psi 0 of x and if I plot psi 0 of x versus x, get a Gaussian in x centred at x is equal to 0. Then, if you look at psi 1 of x and psi 1 of x is x 1 in my notation when n takes the value 1. Goes to 0 at plus infinity and minus infinity, but that is antisymmetric function of x and therefore, you can see that it has odd parity.

Look at the number of times it cuts the axis. Psi 0 of x does not cut the axis; it just goes all the way to plus minus infinity. There is no finite value of x where the wave function vanishes as far as psi 0 of x is concerned. As far as psi 1 of x is concerned it has one point where it vanishes it is a node. A node is a point where the wave function vanishes.

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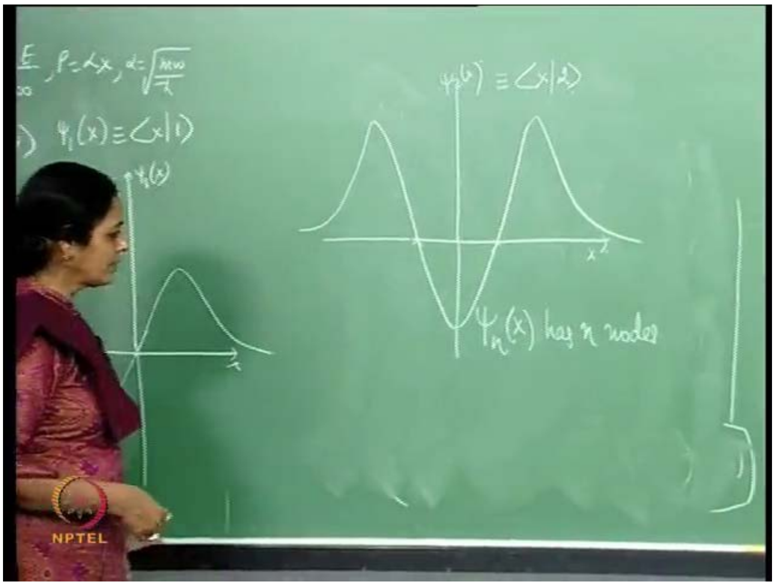

Now, look at psi 2 of x that is an even parity state. The wave function goes to 0 at space infinity. It is a symmetric wave function cuts the axis at 2 points. These are the nodes. This in my notation is x n is equal to 2, quantum optics I would call it the 2 photon state I have written the 2 photon state in the x representation if I were talking in the language of quantum optics. In the case of simple harmonic oscillator I would simply say that it is the 2nd excited state of the oscillator so, we have this.

(Refer Slide Time: 39:20) Now, the number of nodes is increasing the ground state had 0 nodes, the 1st excited state of the oscillator has 1 node, the 2nd excited state has 2 nodes and so on. And why does the number of nodes increased with energy? Now, that is something that we can see rather easily, because in general the number of nodes would increase with energy for the following reason.

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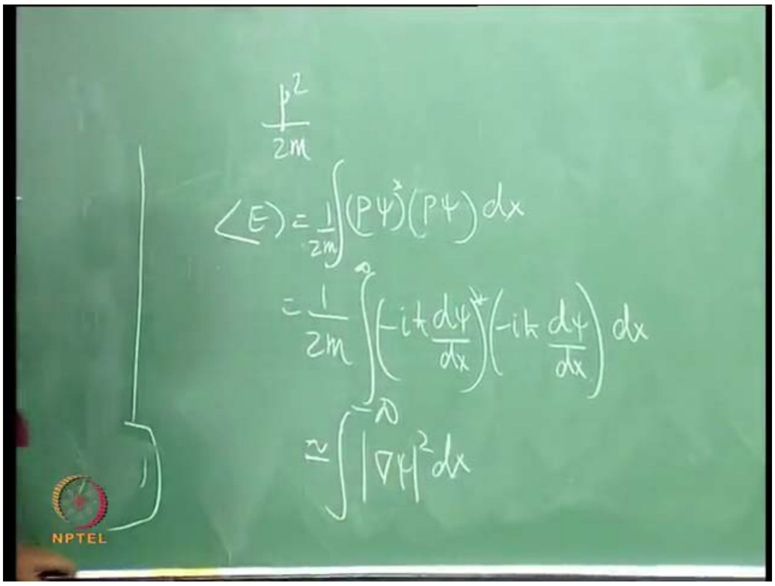

The contribution to the Hamiltonian comes from p square by 2 m and therefore, I have P psi, P psi star I am trying to find out the expectation value of the energy, the contribution from the kinetic energy. Since that had a p square by 2 m and I can pull out the 1 by 2 m, I would just have P psi star P psi and this is all a function of x and therefore, I have d x. Where, I have psi of x and p itself is minus i h cross d by d x. So, the expectation value of the energy is integral minus infinity to infinity minus i h cross d psi by d x star minus i h cross d psi by d x d x. But apart from constants involves a gradient operator so, that is essentially an integral over grad psi mod square d x.

And, since the gradient operator adds to the energy the more the gradients, the more the number of times you have to go up and down there is an increase in energy (Refer Slide Time: 40:57) and every time you cross a node you go down go up. Whereas, here (Refer Slide Time: 39:20) that does not happen. Here, you go up once here you go up twice and so on. Therefore, as the number of nodes increases there is no work done. The gradient operator brings in a larger contribution and therefore, and as the energy levels increase you find that the number of nodes increases. In the case of the oscillator problem the number of nodes increases by 1.

(Refer Slide Time: 40:57) The n-th excited state of the oscillator has n nodes. That means the wave function cuts the x axis n time and that is what you see here. There is a last point that I want to make before I close this lecture to connect up the abstract operator method that we had with whatever we have written here.

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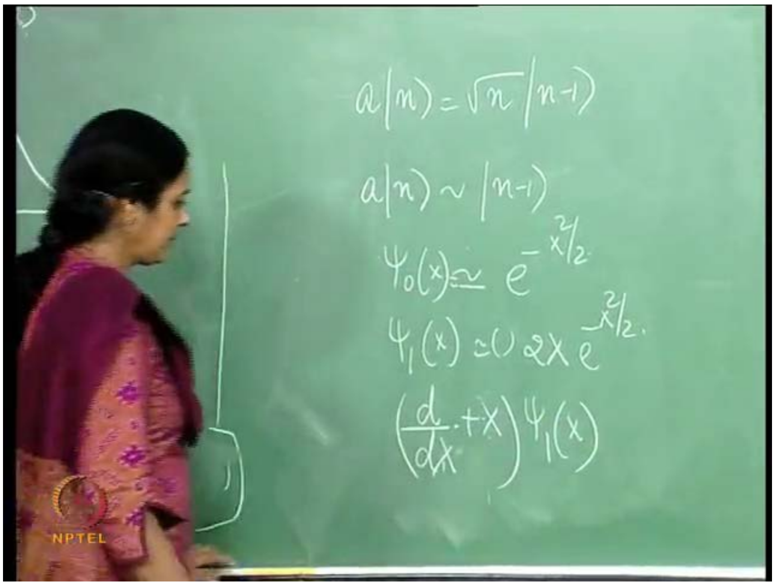

We know that in the abstract operator method we had shown this, barring constants it is clear that a on ket n is essentially ket n minus 1 so, let us see if this is really true. I know that psi 0 of x that corresponds to ket 0 is e to the minus x square by 2 it is a Gaussian and then there is a normalisation which I will forget for the moment but it is essentially a Gaussian. And then psi 1 of x is the Hermite polynomial H 1 of x which is 2 x e to the minus x square by 2. That is again apart from normalisations.

So, let us see if a in the position representation acting on psi 1 really gets it to psi 0 of x. So, a you will recall is d by d rho plus rho and this is supposed to act on this object so, let us worry about d by d x plus x acting on psi 1 of x. Let us evaluate this.

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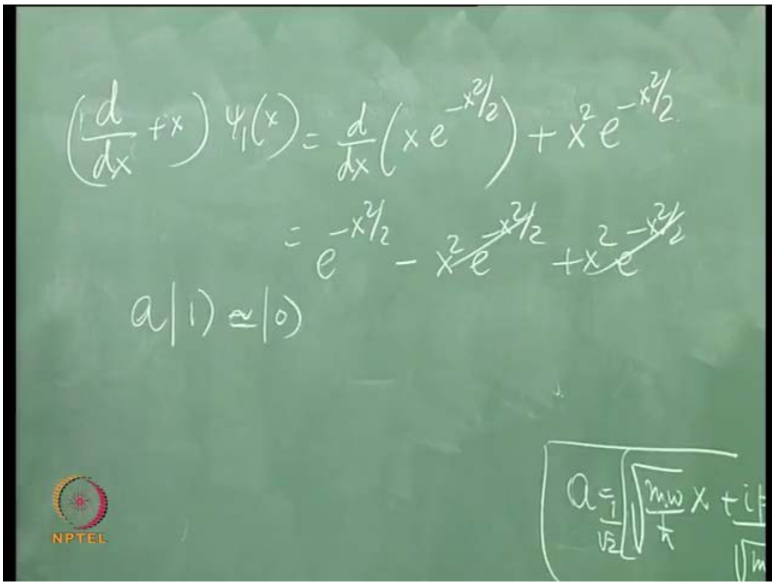

That is the analogue of a on ket 1. Of course, I am forgetting constants I am just trying to see the general pattern. So that is e to the minus x square by 2 plus minus x square e to the minus x square by 2 plus x square e to the minus x square by 2 and that gives me the Gaussian. So, you see a on ket 1 is essentially ket 0 the essentially because, I have not put down the constants. This is good enough for me to see it. Similarly, if you look at a dagger on ket 0 you can check that that gives me ket 1. In other words, a dagger acting in the position representation acting on e to the minus x square by 2 takes it to x e to the minus x square by 2 and that is what you have plotted here.

(Refer Slide Time: 39:20) This is e to the minus x square by 2 schematically. This is x e to the minus x square by 2 and so on. So, we have checked therefore, that the operator formalism exactly matches with the position representation formalism of the harmonic oscillator problem. Now, it is a matter of convenience. If all we need are the energy Eigen values and the energy Eigen functions we could well work with the operator method provided we are not interested in the position. Provided we are not interested in the manner in which the oscillator oscillates in time, what are the instantaneous values of position? And so on. If that is a matter of interest to us then it is more convenient to write things in terms of the position representation.

Otherwise, all these results could have been got perhaps with lesser work in some sense if we use the abstract operator formalism. And now, we have established that both of them are equivalent methods of solving the same problem. In the position representation the energy Eigen states of the oscillator. The Hilbert space is separable spaces say the numerable infinity of Eigen states and each one of them happens to be representable essentially as appropriate Hermite polynomials producted with a Gaussian, suitably normalised. That is a wave function in the position representation and the corresponding Eigen values of course, are n plus half h cross omega where n takes value 0, 1, 2, 3, and so on.