Quantum Mechanics- I
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Lecture - 10
An Interesting Quantum Superposition: The Coherent State

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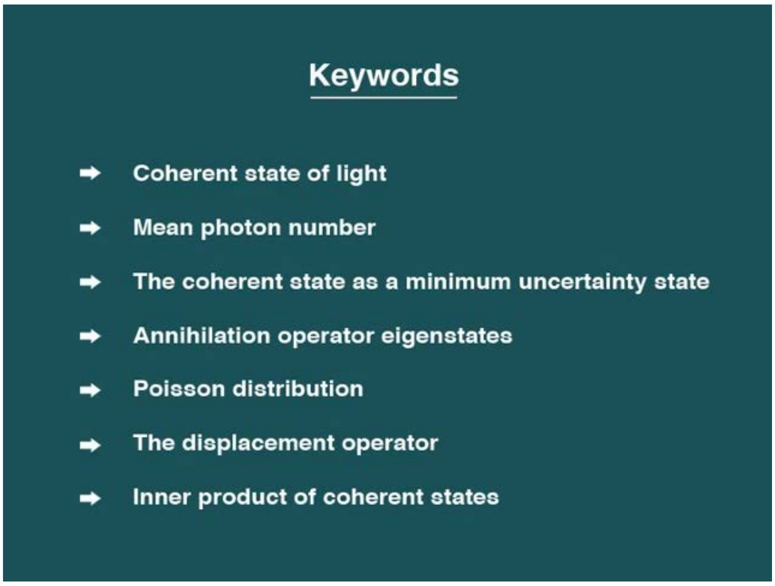

In my last lecture, I spoke about squeezed light, among other things. And I showed that squeezing is a very interesting property peculiar to the quantum world, where it is possible that the variance in x for instance of the observable x in a certain state which would be the squeeze state is less than half, if we work with units where the plank's constant is set equal to 1. I want to consider some more interesting quantum superpositions of the photon number states. For instance in today's lecture, in particular I want to talk about an infinite superposition of photon number states a very particular infinite superposition of photon number states that is called the coherent state of light, ideal laser light is an example of a coherent state.

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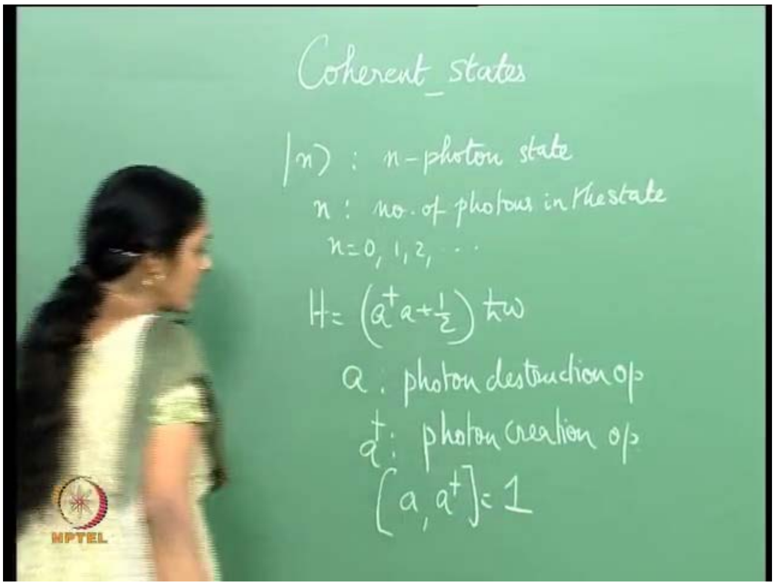

Therefore, today I will talk about coherent states just for recapitulation in quantum optics, n would be the n photon state, ket n would be the n photon state, which means n the label n is a number of photons in the state. So, that n can take values 0, 1, 2 and so on. The Hamiltonian under consideration is a dagger a plus half h cross omega half h cross omega being the 0 point energy and omega being the frequency of the radiation field. Radiation field is the quantized electromagnetic field a and a dagger are the photon destruction operators and creation operator respectively. And they satisfy the algebra, the commutator algebra, a dagger is equal to 1 now, one stands for the identity operator.

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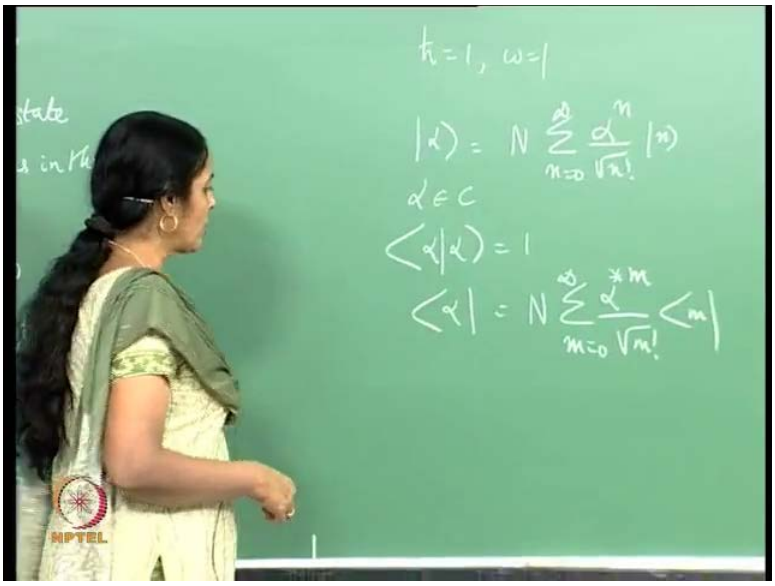

I work with units h cross equal to 1, possibly omega equal to 1. Just for convenience we can always put back the h cross and the omega using dimensional arguments. Now, I consider the superposition ket alpha, alpha is any complex number, given by some normalization constant n which we will find out. So, this is an infinite sum it has the 0 photon state, the 1 photon states, the 2 photon state and so on. So, we have an infinite superposition of photon number states and the weight is different it is alpha to the power of n by root n factorial corresponding to ket n. First of all let us normalize this state, I want this property alpha, alpha is equal to 1 the inner product is equal to 1. So, the bra is n summation m is equal to 0 to infinity alpha star to the power of m by root m factorial bra m.

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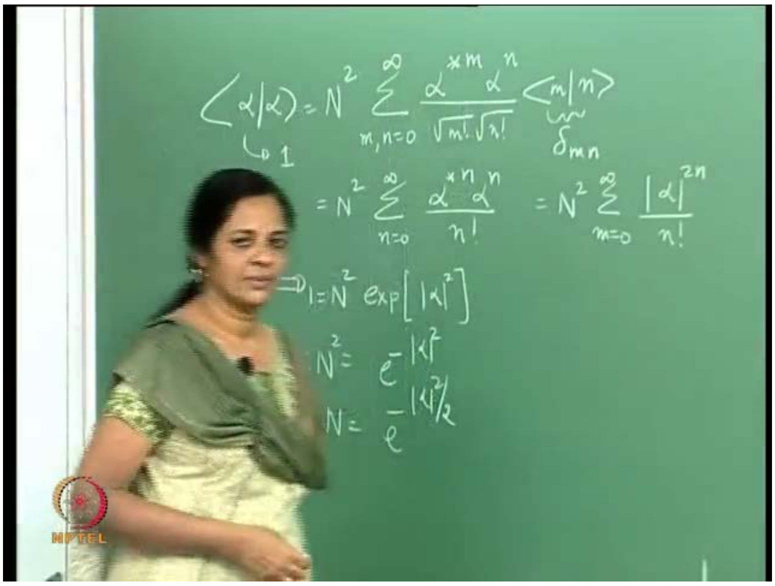

And therefore, if I try to find the inner product which I will work out, there is an N square summation m comma n equals 0 to infinity alpha star to the power of m alpha to the power of n by root of m factorial, root of n factorial the inner product of bra m with ket n which is what I have got from there. But, this is the fock basis, this is the natural basis and they are orthogonal states so I have a delta m n. Which means, I can remove one of these summations perhaps I can remove the label m replacing all m's by n. But, this quantity is simply N square summation m equal to 0 to infinity modulus of alpha to the power of two n by n factorial.

And if this object is normalized to 1, this implies that 1 is equal to N square and this is the familiar exponential series, exponential of mod alpha square. And therefore, N square is e to the minus mod alpha square and N is e to the minus mod alpha square by 2. So, I have fixed a normalization it is e to the minus mod alpha square by 2. And therefore, the normalized state this is the coherent state, it is called the coherent state because it is satisfies certain properties, there is spatial coherence and there is temporal coherence for the moment I would choose to call ket alpha as a coherent state. (Refer Slide Time: 01:14)

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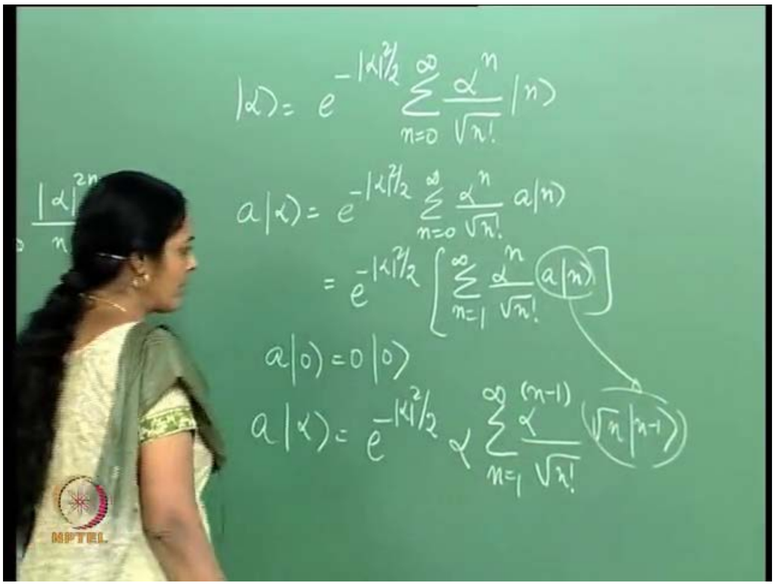

So, ket alpha normalized is e to the minus mod alpha square by two summation n equals 0 to infinity alpha to the n by root n factorial ket n. This infinite superposition has some very interesting properties. Let me find out the action of a on the state ket alpha. This is e to the minus mod alpha square by 2, I can well take the operator inside the summation. I would like to pull out the 1st term corresponding to n is equal to 0 because that is alpha to the power of 0 root of n factorial a on ket 0 and that is a 0, because a on ket 0 is 0. And therefore, I am left with this object, I have used the property that a annihilates the 0 photon state. In the case of the simple harmonic oscillator a acting on the vacuum state gave me 0.

So, the vacuum state, in the case of the simple harmonic oscillator the ground state of the simple harmonic oscillator. Equivalently in this problem the 0 photon state is an Eigen state of the annihilation operator with Eigen value 0. Because I can well write it in this manner it is a trivial Eigen state of the annihilation operator, trivial in the following sense that the Eigen value is simply 0. And therefore, I have a acting on ket alpha, is e to the minus mod alpha square by 2, I can pull out an alpha summation n is equal to 1 to infinity alpha to the n minus 1, because I have pulled out alpha by root of n factorial a acting on n is root n, ket n minus 1 this object a acting on n is root n ket n minus 1. It is easy to see what the answer is going to be on simplification.

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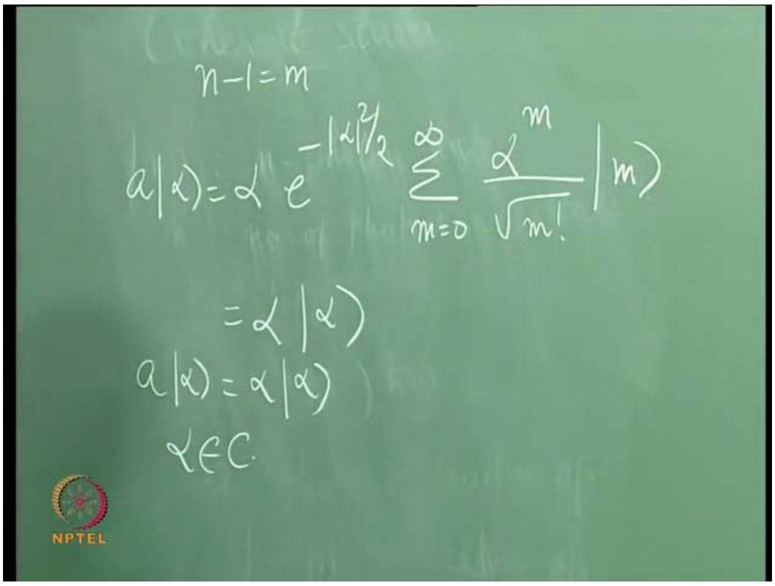

I can replace n minus 1 by m. So, that the summation goes from m is equal to 0 to infinity (Refer Slide Time: 06:52) and therefore, a acting on ket alpha is alpha e to the minus mod alpha squared by 2. Summation m equals 0 to infinity, alpha to the power of m, because there is a root n there and a root of n factorial here gives me a root of n minus 1 factorial in the denominator, which is the same as the root m factorial and that is the ket m. But, this is same as the coherent state ket alpha. And therefore, I have shown that alpha ket alpha is an Eigen state of the operator a, with Eigen value alpha where alpha can be any complex number, I have not imposed any conditions on the value of the number alpha.

So, you see I have an Eigen state another Eigen state of the annihilation operator apart from the (Refer Slide Time: 06:52) 0 photon state and this is an infinite superposition of photon number states. It is non trivial in the following sense that the Eigen value is a non

0 number in general. So this is an annihilation operator Eigen state, with further interesting properties.

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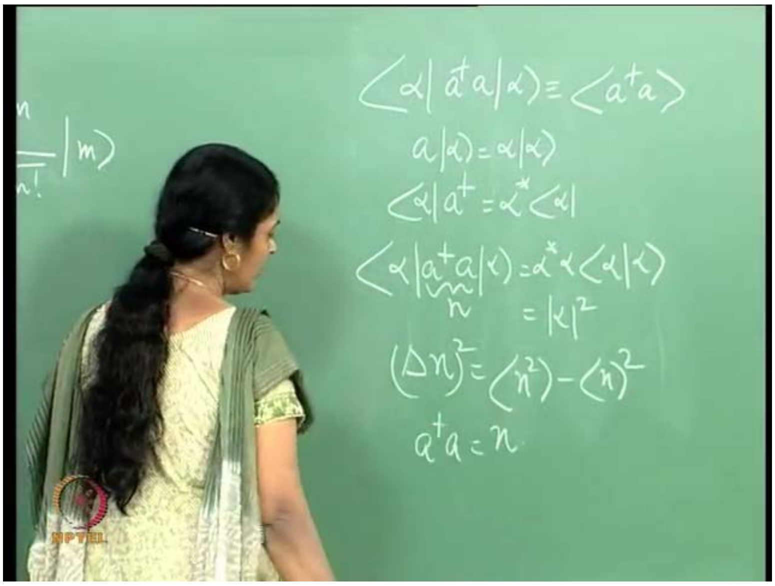

For instance, I could find out the main photon number in this state, this is identical in my notation to an object a dagger a, within this bracket. So, what is a mean photon number in this state alpha, since a acting on alpha is alpha alpha. It is clear, that if I take the Hermitian conjugate I would get this and sandwiching that therefore, I have alpha a dagger a alpha, is an alpha star alpha inner product of the ket with the bra and that is mod alpha square because that is normalized to unity. Therefore, the mean photon number in the coherent state, is equal to mod alpha square. But the mean photon number is the intensity of the light, therefore mod alpha square essentially, gives us the intensity of the light that is used.

I would like to find out the fluctuation. Representing this by n I would like to find out the fluctuation delta n in the number but, I know that delta n is expectations n square minus expectation n the whole square delta n square the variance in an is expectation n square minus expectation square and the whole square. Now where a dagger a, is simply given by n that is my notation so let me compute these objects.

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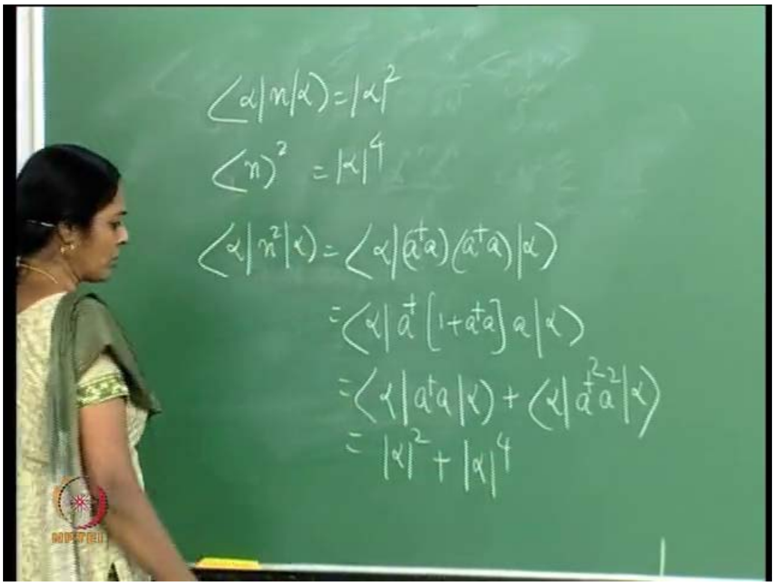

You see the expectation value of the operator n was mod alpha square and therefore, expectation value of n the whole square is mod alpha to the power of 4. I need to compute expectation n square, but that is the same as doing this: n is a dagger a, I should remember that there is a careful ordering that needs to be maintained here, this is quantum mechanics I cannot write this as a dagger square a square I cannot pull this across a. Because, I know that there is an nontrivial commutation relation between a and a dagger they do not commute with each other.

So, I could write this as alpha a dagger a, a dagger is 1 plus a dagger a and then there is an a from here. So this quantity is alpha a dagger a alpha that is my first term that is an a dagger a and an a there and I have an alpha a dagger square a square from the next term. But, this I know is mod alpha square and this I know a square on ket alpha gives me alpha square and a dagger square on bra alpha gives me alpha star square therefore, this gives me mod alpha to the 4. And therefore, I know the variance in the photon number.

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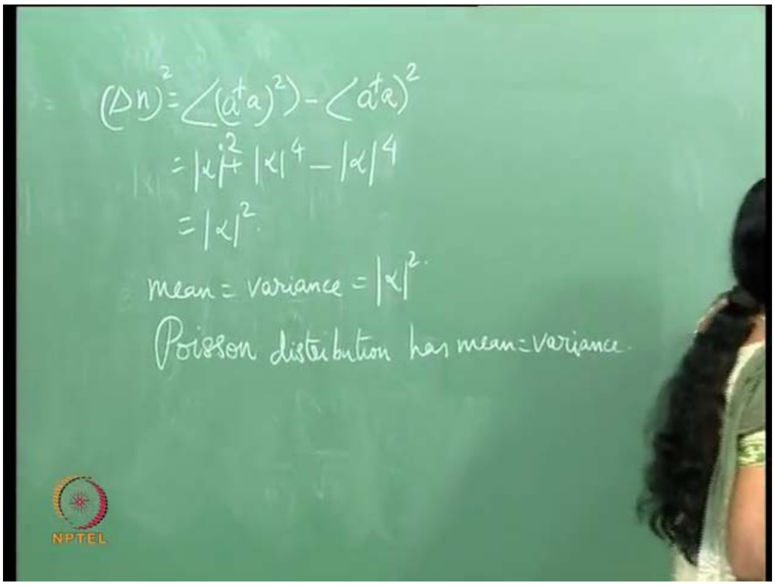

So, delta n square is expectation a dagger a square minus expectation a dagger a the whole square, which is mod alpha square plus mod alpha to the 4 minus mod alpha square minus mod alpha to the four and that is equal to mod alpha square. Now this is an interesting result, because the mean is equal to the variance equal to mod alpha square, which is typical of a Poisson distribution. This is a Poisson distribution has mean equal to variance. I could do better, I could find out the probability of detecting m photons in the state ket alpha.

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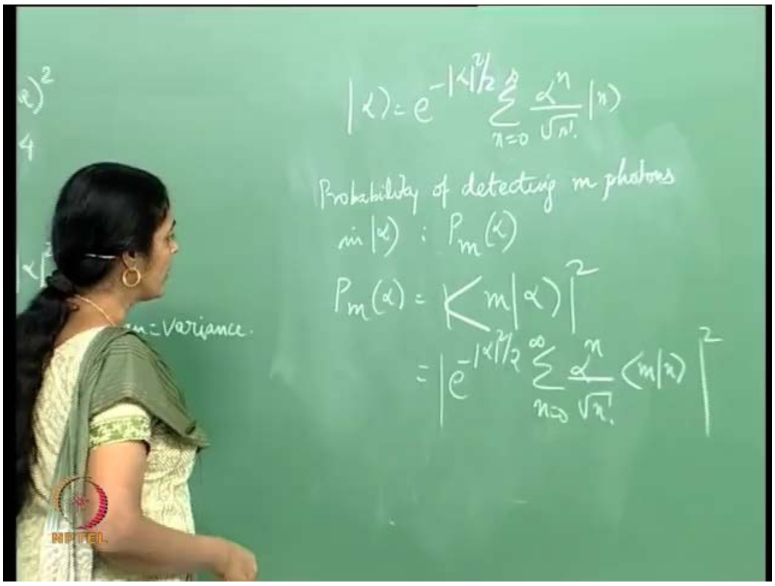

Since ket alpha is an infinite superposition of photon number states, the normalized sate is given by this. The probability of detecting m photons in the state alpha, is given by P m of alpha and it is clear that P m of alpha is simply m alpha mod square. You will recall, from the probabilistic interpretation of quantum mechanics, that indeed the probability of finding m photons in this state, would be to take the inner product of m, with alpha and then mod square it. That should pull out the weight age given to the sate ket m here and mod square it. So, this quantity is simply e to the minus mod alpha square by 2, summation n equals 0 to infinity, alpha to the n by root n factorial m n mod square. This is delta m n because of orthogonality.

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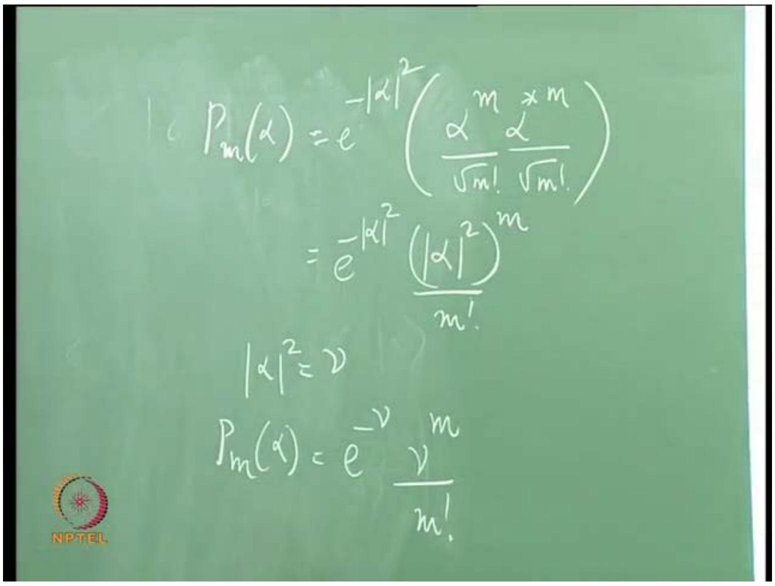

Therefore, I have P m of alpha equals e to the minus mod alpha square, I have already squared this, summation n equals 0 to infinity. The summation goes because of the delta m n and I just have alpha to the m by root n factorial mod square alpha star to the power of m by root m factorial and that is all that I have. (Refer Slide Time: 16:11) This quantity the delta m n changes all n to m replace all n by m remove the summation and mod square, the mod square means the complex conjugate has to multiply this. But, that is the same as e to the minus mod alpha square, mod alpha square to the power of m by m factorial.

So, if I just denote mod alpha square by nu, P m of alpha is e to the minus nu, nu to the power of m by m factorial, which is typical of a Poisson distribution. So, this a Poisson distribution in nu and this is the kind of result that can be tested in principle with the idle laser light which is represented by the coherent state ket alpha. When I am at this I also wish to find out, the uncertainty in x, x is a plus a dagger by root 2 and we should remember that x is not position in this contest x is really related to the electric field e.

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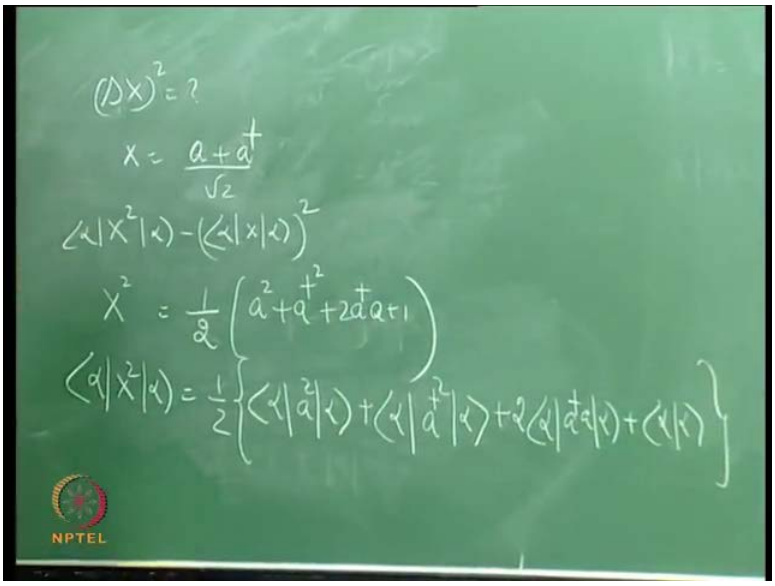

So, I want to find out the variance in X, where X is a plus a dagger by root 2 in the state ket alpha. So essentially, I need to find expectation X square minus expectation X the whole square. So, let me begin with expectation X square, X square given from the last lecture is simply a square plus a dagger square plus 2 a dagger a plus 1. Because in X square you would have had an a a dagger plus a dagger a and I use the commutation relation and get this. Therefore, alpha X square alpha is half alpha a square alpha is the 1st term plus alpha a dagger square alpha that is a next term, this is what I have.

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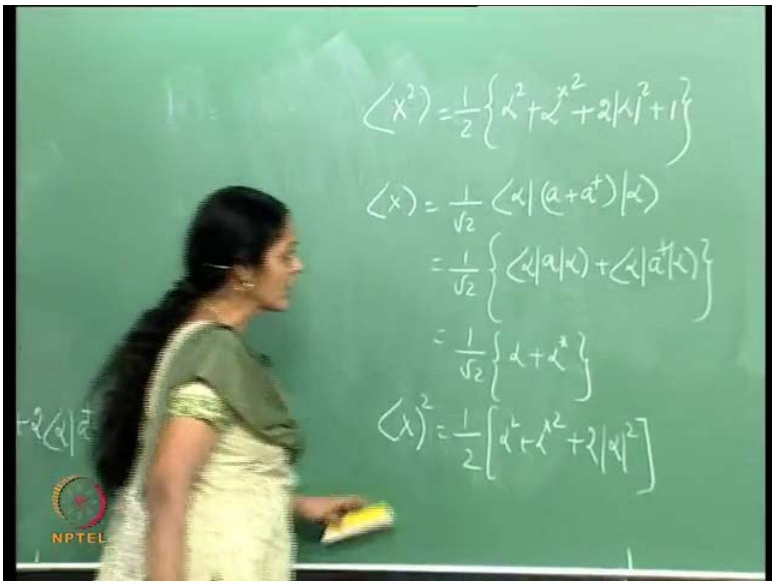

And therefore, expectation X square, in the state alpha (Refer Slide Time: 20:07) a acting on alpha pulls out alpha as the Eigen value and therefore, a square acting on alpha gives me alpha square ket alpha. But, the inner product of ket alpha with bra alpha is 1 and therefore I am left with number alpha squared. (Refer Slide Time: 20:07) The second term is a dagger square, I could work with a dagger acting on bra alpha, that pulls out an alpha star twice over gives me alpha star square and use the fact that the inner product of ket alpha, with bra alpha is 1 and therefore, I get alpha star square. Equally well you could have realized (Refer Slide Time: 20:07) that this object is the complex conjugate of this object.

And therefore, if the expectation value a square is alpha square, the expectation value of a dagger square is alpha star square. Plus a acting on ket alpha (Refer Slide Time: 20:07) is alpha ket alpha and a dagger on bra alpha is alpha star, there is a 2 out here, so that is gives me plus 2 more alpha square plus 1 (Refer Slide Time: 20:07) simply because this object is 1. So, this is expectation X square, expectation X is simply 1 by root 2 alpha a plus a dagger alpha which is 1 by root 2 the 1st term is the expectation value of a, the 2nd term is expectation value of a dagger. And this is clearly alpha plus alpha star, I need a expectation x the whole square, that is gives me a half when, I square this, I get alpha square plus alpha star square and since alpha and alpha star are just numbers I have 2 mod alpha square.

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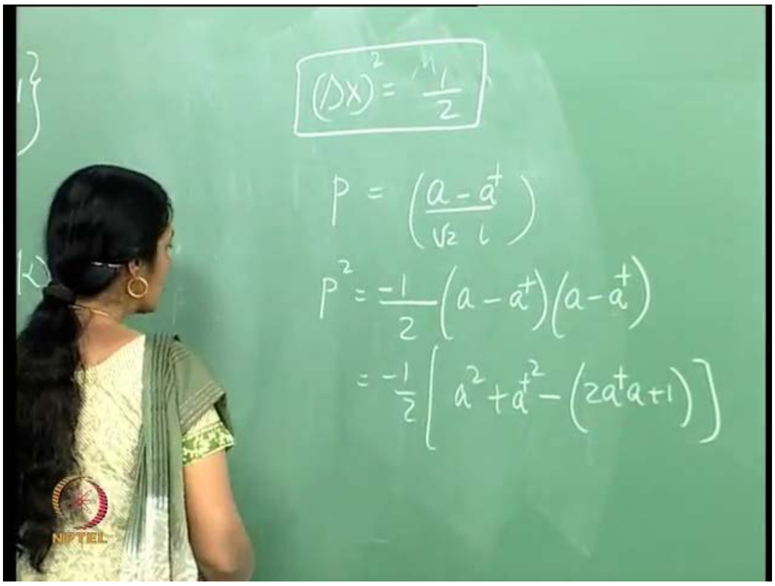

I need to subtract expectation X the whole square from expectation X square, which gives me the variance in X. In this object, (Refer Slide Time: 21:54) so when i subtract expectation X squared from expectation X square 1st term vanishes, so does the 2nd term and so does the 3rd. But, what survives is a half. And therefore, in units where h cross is equal to 1 delta X square is equal to half, the variance in X is half in the coherent state ket alpha. This is analogous to the ground state of the oscillator equivalently the 0 photon state, wherein units where the Plank's constant is set equal to 1 delta X squared is equal to half.

Now let me compute, the variance in p again p in this context is related to the magnetic field and is not be confused with linear momentum. I emphasize the fact that, while the model the framework that we used in terms of a and a dagger and the Hamiltonian the framework is the same. Be it the simple harmonic oscillator or the quantized electromagnetic field, the interpretations physical interpretations are different.

And p is defined as a plus a dagger a minus a dagger by root 2 i. I need to find P square and that is a minus half a minus a dagger the whole square and if you recall, this is a squared plus a dagger squared minus, there is an a dagger a minus a dagger so that is a minus 2 a dagger a plus 1 in the bracket. Because, a a dagger can be written as 1 plus a dagger a and that is how I got this. So, this is P square I need to find the expectation value of P square in the state alpha.

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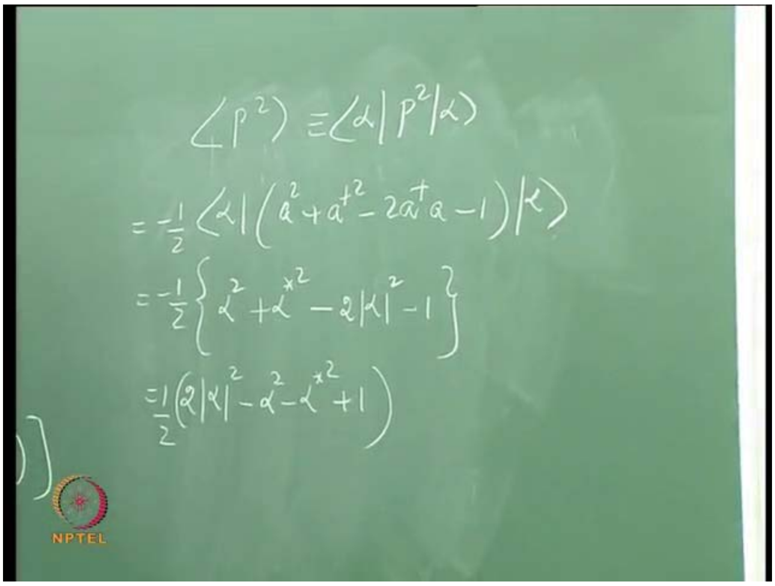

So, expectation value of P square in the state alpha that is the same as this object. This quantity is alpha there is an overall factor of minus half, so that is a minus half a square plus a dagger squared minus 2 a dagger a minus 1 alpha. Now, once more as before the 1st term gives me alpha squared, the 2nd term gives me alpha star square, in the expectation value the 3rd gives me minus 2 mod alpha square and the 4th gives me a minus 1. There is an overall minus sign and therefore, this just gives me 2 mod alpha squared minus alpha squared minus alpha star squared plus 1 half of that. So, this is expectation value P square need to find expectation of P and square it. Which is what I will do now.

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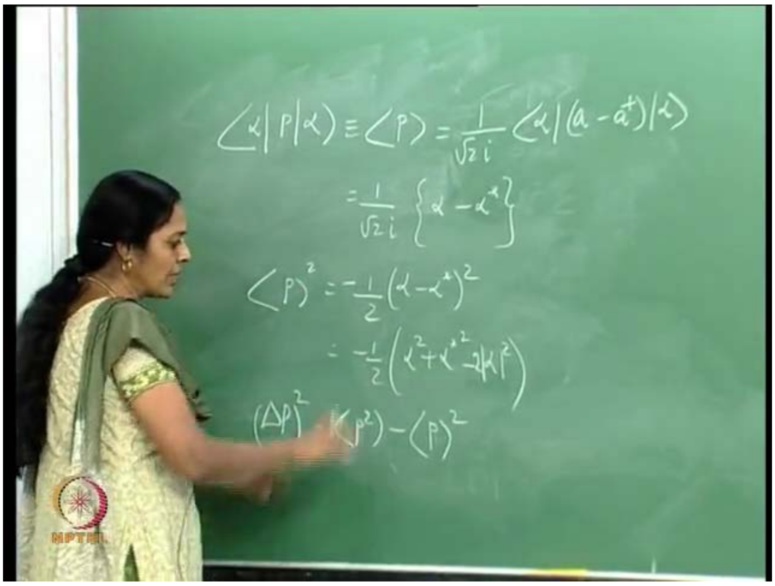

Now, expectation value of P in this state alpha, my notation is this: is 1 by root 2 i alpha a minus a dagger alpha this object is 1 by root 2 i the 1st term is alpha and the next term is alpha star with the relative negative sign between them out there. So expectation P the whole square is half with a minus sign alpha minus alpha star the whole square which is minus half alpha squared plus alpha star squared minus mod alpha squared with the 2. I need to find the variance in P delta P the whole square, that is a variance is expectation P squared minus expectation P the whole squared. (Refer Slide Time: 26:56) Look here for expectation P square, there is a minus alpha squared by 2 minus alpha star squared by 2, this gives me once I put in this minus sign it cancels out with an alpha squared by 2 here and an alpha star squared by 2.

Similarly, I have (Refer Slide Time: 26:56) a mod alpha squared out here with a plus sign and here, I have more alpha squared. But, now with the negative sign because I am subtracting expectation P squared from expectation P the whole squared and therefore, this object is simply the half, that I had when I found the expectation value of P square that was the last term with the half.

So, what we have established is that delta X squared delta P square they are both equal to each other and equal to half in units, where the Plank's constant is set equal to 1, which is precisely what happened with the ground state of the harmonic oscillator. It is precisely what happened with the 0 photon state, in other words the coherent state is also a minimum uncertainty state, in X and P, where X and P are not to be misunderstood in this contest as position and momentum. So, in the canonical variables X and P the coherent state also is a minimum uncertainty state. One starts wondering if there is any way in which, we can use a suitable operator act as suitable operator on ket 0, to get ket alpha. In other words I wonder if there is a unitary transformation, which takes ket 0 to ket alpha. And indeed that is true without making much ado about it, I will first give you the unitary operator and then we will see how exactly this unitary operator takes ket 0 to ket alpha.

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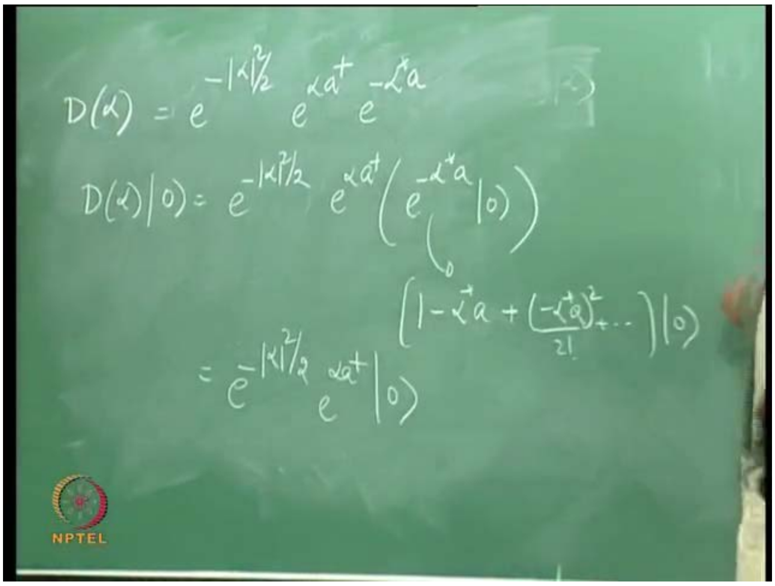

Consider the operator D of alpha, given by e to the minus mod alpha squared by 2 that is a number, e to the alpha a dagger e to the minus alpha star a. Consider, this operator the order is very important because a and a dagger do not commute with each other. Let me find out the action of D of alpha on ket 0 the overall number is the same it is e to the minus mod alpha squared by 2, e to the alpha a dagger e to the minus alpha star a acting on ket 0. I emphasized that, this exponential series is of the form 1 minus alpha star a plus minus alpha star a the whole square by 2 factorial plus so on. So, it is a sum of series of operators where the 1st term is the identity operator acting on ket 0. Well I know that a acting on ket 0 is 0 and here to a square on ket 0 is 0.

So, every term here acting on ket 0 simply destroys it what survives in e to the minus alpha star a acting on ket 0, simply the leading term the identity operator acting on ket

0.So, this object is e to the minus mod alpha squared by 2 e to the alpha a dagger ket 0 because the identity operator alone gives me a ket 0 and all other operators in that series act on ket 0 to give me 0 the number 0. Now this object can be evaluated.

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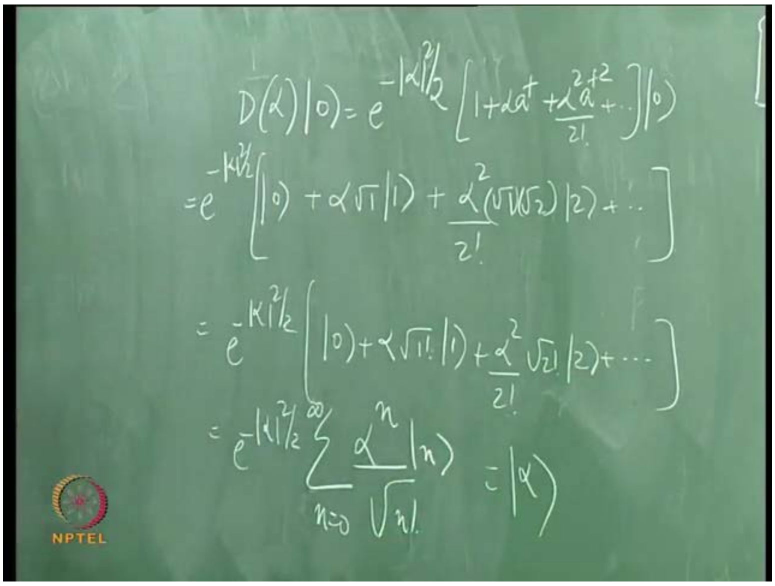

So D of alpha acting on ket 0, is e to the minus mod alpha squared by 2. I have 1 plus alpha a dagger plus alpha squared a dagger squared by 2 factorial plus so on acting on ket 0, as an infinite series of operators acting on ket 0, where the leading term is ket 0. I have an overall constant e to the minus mod alpha squared by 2 the leading term is ket 0 a dagger on ket 0, is root 1 ket 1, I would like to keep the root 1 there explicitly, so that a general formula emerges for this infinite set of operators acting on ket 0 plus alpha square by 2 factorial. The 1st time a dagger acts on ket 0, it pulls out a root 1 gives me ket 1 then a dagger acts on ket 1, to give me root 2 ket 2 plus so on.

And now the structure is clear this is e to the minus mod alpha squared by 2 ket 0 plus alpha root of 1 factorial ket 1 plus alpha squared by 2 factorial root of 2 factorial ket 2. and so on. This is really e to the minus mod alpha squared by 2 summation n is equal to 0 to infinity alpha to the power of n by root n factorial ket n. Because, that is gives me a root 2 factorial in the denominator, this could be written as a root 1 factorial in the denominator the next term will give me a root 1 into root 2 into root 3 in the numerator, the denominator will have a 3 factorial and therefore, I pull out root 3 factorial in the denominator and so on. So, this is what I have, but this is simply the coherent state ket alpha.

And therefore, I have an operator which acts on ket 0 to give me ket alpha, the operator is of this form, it is called the displacement operator. It is called the displacement operator because this displaces the ket 0 takes it to ket alpha, one sees this better in the position representation which I have not yet attempted to do. But, in subsequent lectures I will show that, if I write if I give a position space representation for ket 0, it would be a Gaussian and for ket alpha to be another Gaussian which is simply displaced from the original Gaussian.

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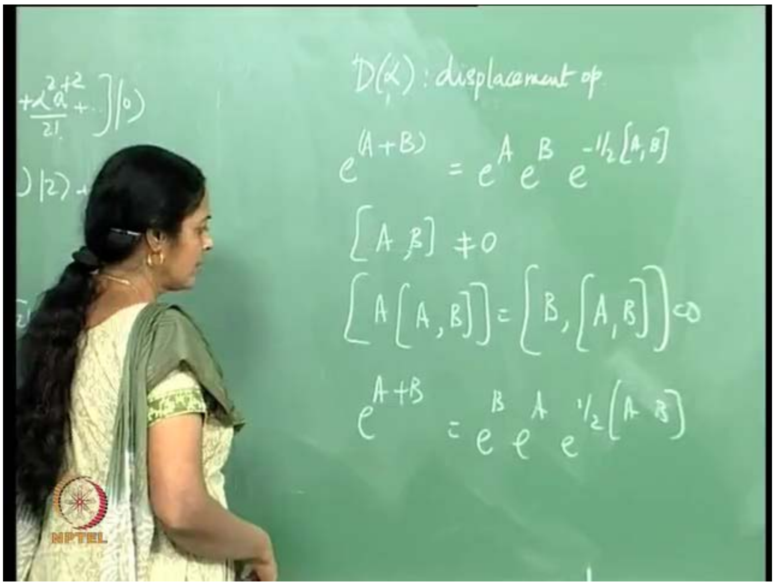

So D of alpha it is called the displacement operator. I have very carefully split (Refer Slide Time: 31:43) and written D of alpha. In general there is a disentanglement relation which says that if, you have 2 operators A and B and you have e to the A plus B you could write this as e to the A, e to the B, the order is important 1st e to the A then e to the B, e to the minus half, the commutator of A with B provided the commutator of A with B, is not equal to 0. However, the commutator of A with A with B is equal to the commutator of B with A with B is equal to 0.

Now if these conditions are satisfied, you could write e to the A plus B where, A and B are operators as e to the A, e to the B, e to the minus half commutator of A with B, you could equally write, e to the A plus B as e to the B, e to the A. e to the plus half commutator of A with B.

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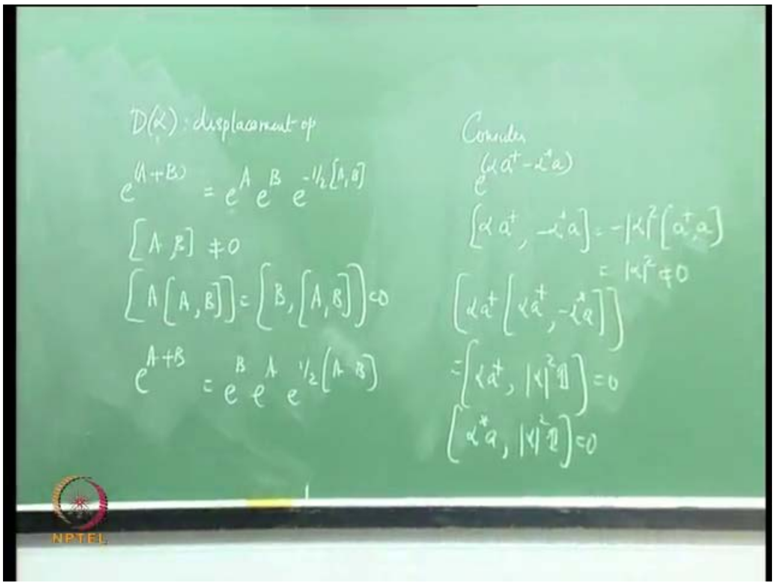

Let us look at this, example: Consider, e to the alpha a dagger minus alpha star a, consider this operator. This is like A in my example (Refer Slide Time: 36:49) and minus alpha star a is B, the commutator of alpha a dagger with minus alpha star a, is simply minus mod alpha squared a dagger with a commutator and that is just mod alpha square. Because, this gives me a minus 1 and therefore, with that minus sign it gives me a plus 1 and that is in general not equal to 0

Now, look at the commutator of a with a with b. Where this object, is simply the commutator of alpha a dagger, I have just evaluated this here and this is number, so this is mod alpha square identity operator. And since any operator commutates with the identity operator the commutator of (Refer Slide Time: 36:49) A with A with B in this example is 0. Similarly, you can see that the commutator of B with A with B is 0 because the commutator of alpha star a with mod alpha squared identity it is also equal to 0. And therefore, (Refer Slide Time: 36:49) I can split using this disentanglement relation, I can split this and write it as e to the alpha a dagger (Refer Slide Time: 36:49) e to the minus alpha star A, e to the minus half commutator of alpha A dagger with minus alpha star A. Which is, precisely what I have written here.

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So, this object here is really e to the alpha a dagger minus alpha star a. I could have written it in this manner or I could have used the disentanglement relation and written it in this manner. The disentanglement relation itself is one of a variety of formulae, which belong to the Baker-Campbell-Hausdorff formula and variance of the Baker-Campbell-Hausdorff formula. So, I have used one such specific formula in this case and I have written the displacement operator, (Refer Slide Time: 31:43) as e to the alpha a dagger minus alpha star a or I can pull out this e to the minus mod alpha squared by 2 and write it in this manner, which is what I have done. What I have shown, is that the displacement operator acting on ket 0, gives me ket alpha as the resultant state. It is obvious that the displacement operator is a unitary operator.

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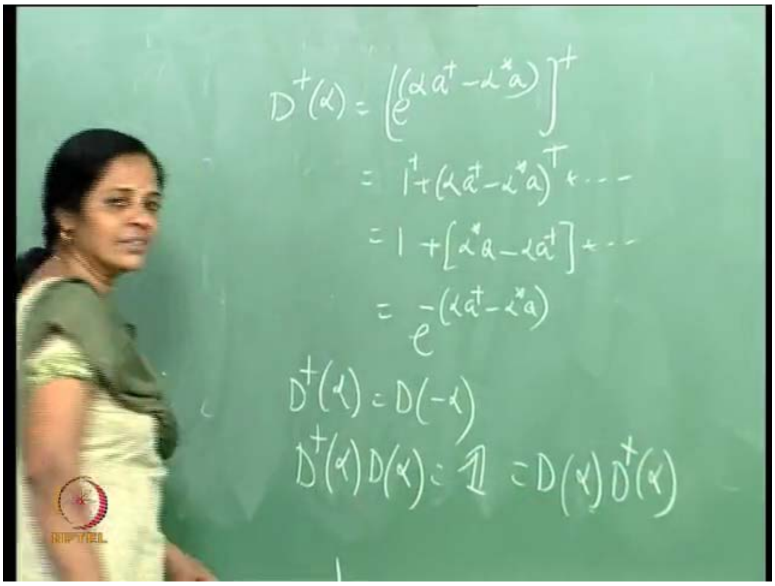

Because D dagger of alpha, is e to the alpha a dagger minus alpha star a dagger and I could have done this term by term, it is like taking the dagger of the identity operator plus alpha a dagger minus alpha star a dagger plus so on and that is the same as 1 plus alpha star a minus alpha a dagger plus, which is the same as e to the minus alpha a dagger minus alpha star a.

So, D dagger of alpha is the same as D of minus alpha. If here, I had simply changed the alpha to minus alpha, I would have got e to the minus alpha a dagger plus alpha star a. But, D dagger of alpha, times D of alpha is identity because it is exponential of minus an operator times exponential of plus the same operator. That is equally true that D of alpha D dagger of alpha, is the identity operator. And therefore, this rings a bell, I seem to have done a unitary transformation. The unitary operator in this case, is a displacement operator D of alpha the unitary transformation has taken ket 0 to ket alpha.

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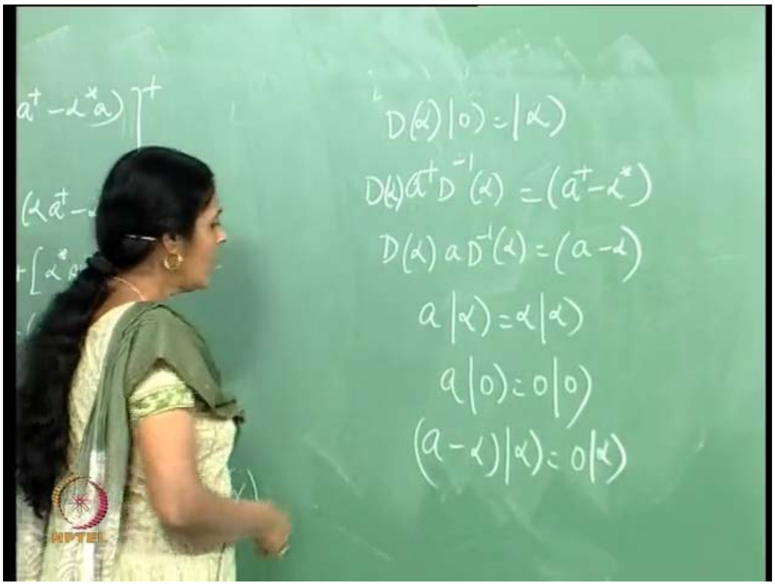

I could argue further and attain to see, if it is the same unitary operator that takes a dagger, to a dagger minus alpha star and a to a minus alpha. With the reason why I might expect this to happen is because I know that a on ket alpha, is alpha alpha a on ket 0 is 0 ket 0 and therefore, a minus alpha on ket alpha is 0 ket alpha. So, I wonder if the transformation the operator which transformed ket 0 to ket alpha also transforms a to a minus alpha and a dagger to a dagger minus alpha star and as I have discussed, in one of my earlier lectures when a state transforms in the following manner the operators, which act on that state would transform with D of alpha the same operator flunking it on this side and its, inverse on that side.

And since, this is a unitary operator D inverse is the same as D dagger and indeed this is true it is possible to show that the following identities are true and that is something that I will work out in detail at a subsequent in a subsequent lecture but, before that, I would like to look at the ortho normality properties of coherent states.

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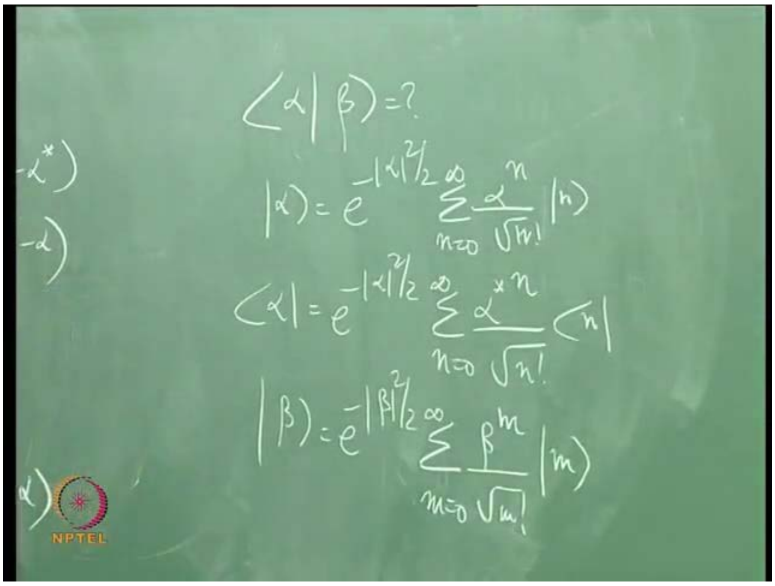

In particular, what is this object? Now, alpha is a label for a coherent state and alpha can be any complex number. So, if I take 2 coherent states one of them with the label alpha and the other with the label beta. Then, what is the inner product of alpha with beta? If they are orthogonal to each other the answer would be 0 and if they are not, then I will have a non 0 value for the answer. So, let me look at this ket alpha is e to the minus mod alpha squared by 2 summation n equals 0 to infinity, alpha to the n by root n factorial ket n. Therefore, the bra is this object and ket beta is e to the minus mod beta squared by 2 I use a different index for the summation, so as to avoid confusion.

It is a same Hilbert space that, I am looking at the fock basis or the number state basis or the natural basis, is given by the label ket m or ket n where the label takes value 0 to infinity staring from the 0 photon state upwards. The combination itself alpha and beta are different because mod beta squared and mod alpha squared are different. The mean photon number in this state, is mod alpha squared the mean photon number in that state, is mod beta squared. And now, I wish to find the overlap the inner product of alpha with beta.

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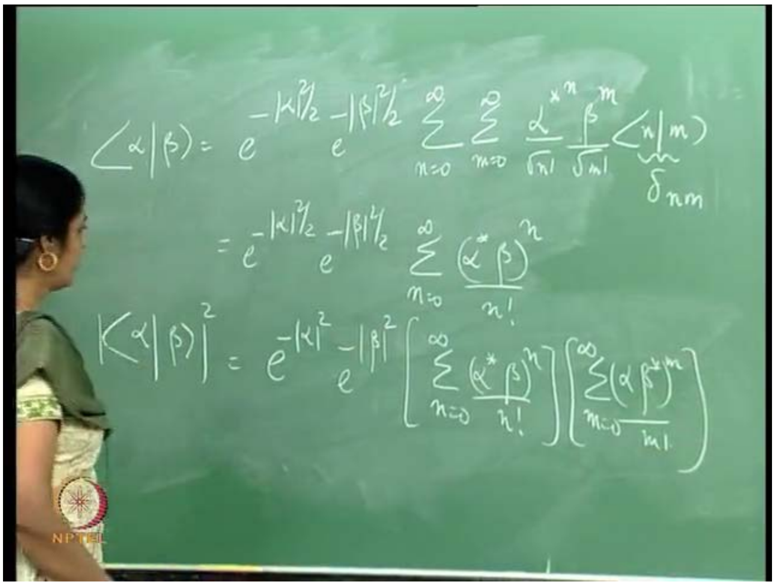

So, this has a mod alpha squared by 2 e to the minus mod beta squared by 2. There are 2 summations. (Refer Slide Time: 45:13) And since, there was a bra alpha, there is an alpha star to the n beta to the m by root n factorial root m factorial bra n ket m and as before this gives me a delta n m, which means I can remove one of the summations and perhaps replace all m's by n's. Alpha star beta, to the power of n by n factorial and that is all I have. I need to simply this. Now let me consider, alpha beta mod square, now that object is e to the minus mod alpha square e to the minus mod beta square and summation n is equal to 0 to infinity alpha star beta to the power of n by n factorial, times summation m equal to 0 to infinity alpha beta star to the power of m by m factorial.

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To understand what this could be? Let me, look at the expression, for e to the minus modulus of alpha minus beta the whole square. Now this object, is e to the minus alpha minus beta times alpha star minus beta star which gives me e to the minus mod alpha squared from the 1st term, mod beta squared from the 2nd term minus beta alpha star minus alpha beta star. So I can easily see that, I have e to the beta alpha star plus alpha beta star here, apart from e to the minus mod alpha square, e to the minus mod beta square, (Refer Slide Time: 47:08) that is an e to the minus mod alpha square, e to the minus mod beta square and I have e to the beta alpha star plus alpha beta star. And indeed this object turns out to be equal to that. And therefore, it is easy to verify that these this double summation out here summation over n, summation over m is precisely this.

Therefore, modulus of alpha beta the whole squared is e to the minus modulus of alpha minus beta the whole square. Now, one thing is clear; let us imagine for a moment that alpha and beta are real and beta equal to minus alpha and alpha large. Then, alpha beta mod square, is e to the minus that gives me an alpha minus minus alpha and therefore, that is a 4 mod alpha square and for alpha large it is 0, because that is like e to the minus infinity. And therefore, these 2 states ket alpha and ket beta which are not orthogonal to each other, can be approximated to being orthogonal to each other provided beta is taken to be minus alpha and alpha itself is large, I have already spoken to you about qubits in quantum computation.

We spoke about, spins and there we had qubits in terms of which quantum computation and quantum logic gates, could be constructed. This is another example: There have been attempts to use two coherent states, of this type such that they can be considered to be orthogonal to each other to a fairly good approximation and use them as the 2 qubit states instead of the column vectors 1 0 and 0 1. We have here, ket alpha and ket beta satisfying these conditions. So, that quantum logic gate operations can be implemented using these as qubits. Now, this is an area of a large amount of interest in current day research. This lecture is meant to give you a flavor for what an infinite superposition of states can do.