Thermal Physics
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Lecture - 59
Thermodynamics of Cavity Radiation

Hello and welcome back to another lecture of this NPTEL course on thermal physics. Now in the last lecture, we talked about cavity radiation or black body radiation and we talked about the properties of diffuse radiation, we talked about radiation pressure, its energy density and parameters like that. And also we have computed what is called the Stefan-Boltzmann law or black body emission.

And then we took one step further and we gave a statement that this law is in general applicable for any I mean any object, let it be a black body or let it be a nonblack body, which is of course, not an ideal emitter, but still with some correction factor. That correction factor is given by epsilon which is the average emissivity of that particular object. For black bodies of course, average emissivity should be maximum.

Now let us for this lecture let us focus on the thermodynamics of cavity radiation.

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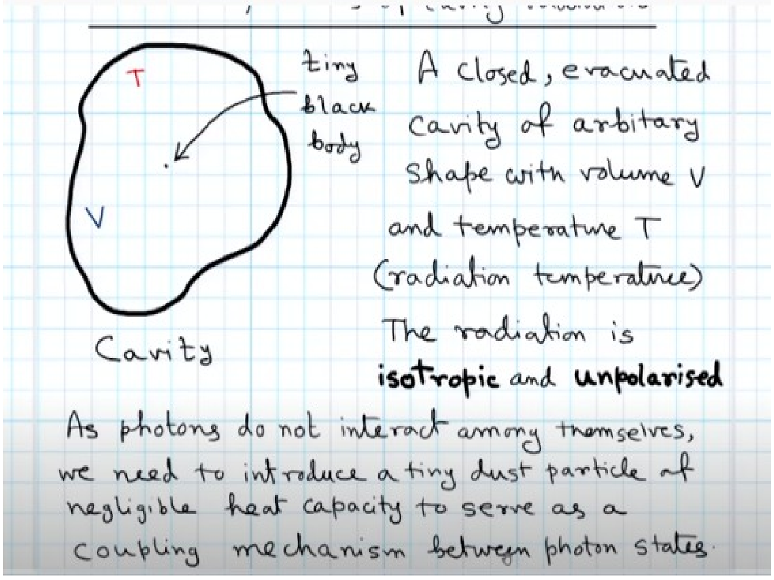

A closed, evacuated cavity of arbitary shape with volume V and temperature T (radiation temperature)

The radiation is **isotropic** and **unpolarised**

As photons do not interact among themselves, we need to introduce a tiny dust particle of negligible heat capacity to serve as a coupling mechanism between photon states.

Now in a we have already introduced cavity radiation and we have made a statement like this, cavity radiation is actually equivalent to a black body radiation. Now for today's lecture, we will be focusing on the thermodynamics associated with this cavity, which is of volume V of any arbitrary shape and size.

Only thing is it has a volume V and the temperatures of the wall, temperature of the wall of this cavity is maintained at a constant value T. Now so that means, the radiation inside is at uniform temperature. Also, we know that these are all diffused radiation inside. So the radiation is isotropic and unpolarized; isotropic, homogeneous and unpolarized at the same time.

Unpolarized is something that we have not discussed explicitly, but this is also a property of the diffuse radiation that it cannot hold any specific polarization by itself. Now there is a technical point here. We need to introduce a tiny black body, could be I mean this cavity is in general evacuated. So let us assume that there exist a small dust particle which is black in color and it has negligible heat capacity.

Now this we have to introduce inside this cavity in order to, order to enable the thermodynamics of the system. Now this is a technical requirement, because photons do not interact with themselves. Of course, photons can interact with the cavity wall, but in between them there is no direct mechanism, coupling mechanism.

But introducing this tiny black body actually enables the coupling mechanism between different photon states. But anyway, this black body will not, I mean this tiny black body will not affect our calculation in any sense, but without this none of our calculations are valid, okay.

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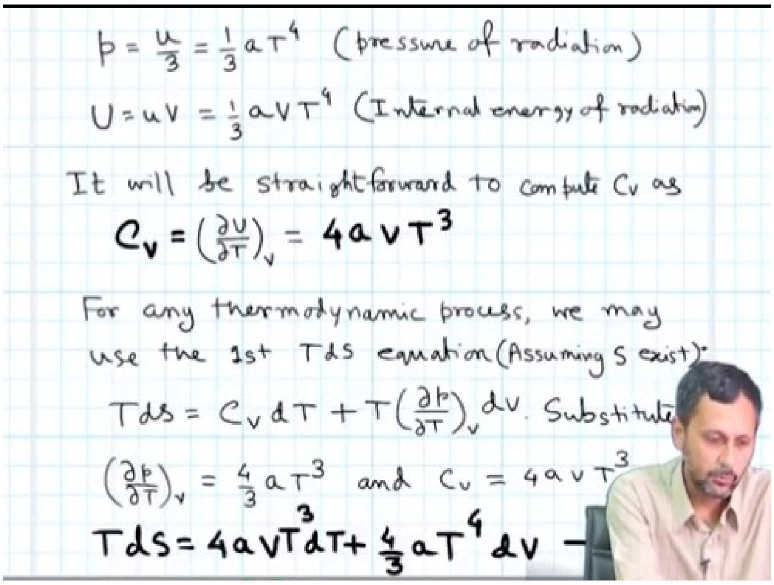

$p = \frac{u}{3} = \frac{1}{3} aT^4$ (pressure of radiation)

$U = uV = \frac{1}{3} aVT^4$ (Internal energy of radiation)

It will be straightforward to compute C_v as

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v = 4aVT^3$$

For any thermodynamic process, we may use the 1st TdS equation (Assuming S exist):

$TdS = C_v dT + T\left(\frac{\partial p}{\partial T}\right)_v dV$. Substitute

$\left(\frac{\partial p}{\partial T}\right)_v = \frac{4}{3} aT^3$ and $C_v = 4aVT^3$

$$TdS = 4aVT^3 dT + \frac{4}{3} aT^4 dV \;-$$

So we have already known that the pressure p is equal to u by 3 is equal to 1 by 3 a T to the power 4 which is called the pressure of radiation. Now a being a constant of proportionality which we know we it has a relation with the Stefan-Boltzmann constant sigma that we have used in the last lecture, while solving the problem. In this case for today’s lecture, we will be simply using this as a.

Now U is equal to, U is the, capital U is the internal energy of radiation. That also we have used in our previous discussion and that is nothing but the energy density u, small u, times the volume of the cavity. That means u times V which is 1/3 aVT to the power 4, right. Now once we have this and we start calculating other thermodynamic parameters of the system, it is straightforward to compute C V as C V is equal to del u del T V.

That means, if we just take a derivative of this expression with respect to temperature keeping volume constant, we get 4aVT cube, right. Now for any thermodynamic process, we may use the first TdS equation that we have seen previously. That it is a general form, I mean this is a general equation without making any assumption regarding this particular system.

Also here we have to assume that the entropy exist. I mean we have not proved that yet. We will come to that. But for now, let us assume that entropy for this system is existent. So then we can write this TdS equation as TdS is equal to C V dT plus T del

P del T V dV, right? So this is the first TdS equation. And then we have to substitute del p del T V as 4 by 3 a T cube.

This V is actually I mean keeping V constant is a redundant information here. Because anywhere for this particular system p is a function of temperature only. So there is no dependence on volume anywhere, right. And of course, we have seen in the previous line only that C V is equal to 4a V T cube. And then we can write this as TdS is equal to 4a V T cube dT plus 4 by 3 a T to the power 4 dV.

So this is we call it equation 1. And this equation is the equation we are going to use for next few minutes, okay.

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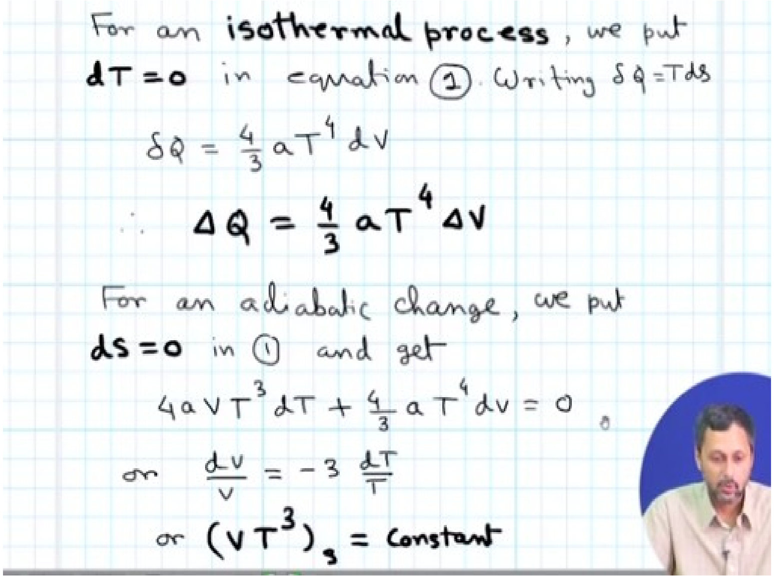

For an **isothermal process**, we put

$dT = 0$ in equation ①. Writing $\delta Q = TdS$

$$\delta Q = \frac{4}{3} a T^4 dV$$

$$\therefore \Delta Q = \frac{4}{3} a T^4 \Delta V$$

For an adiabatic change, we put

$dS = 0$ in ① and get

$$4aVT^3 dT + \frac{4}{3} a T^4 dV = 0$$

$$\text{or } \frac{dV}{V} = -3 \frac{dT}{T}$$

$$\text{or } (VT^3)_s = \text{Constant}$$

Now what happens in an isothermal process? In an isothermal process, we have temperature difference equal to 0. So we put dT is equal to 0 in equation 1. And we can also write dQ for TdS because dQ is the total heat requirement, infinitesimal heat change which is also represented by TdS given that the path that is joining the process is an isotropic process, sorry it is a quasistatic process.

Now of course isothermal process is a quasistatic process. So this relation holds and we can write dQ is equal to 4 by 3 a T to the power 4 dV and integration a simple straightforward integration and T becomes constant because it is an isothermal process. So we get delta Q is equal to 4 by 3 a T to the power 4 delta V.

So we see that in an isothermal process, if we change the volume in an isothermal process, please remember in this type of cavity rate irradiation, we have only two independent parameters. One is volume, the other one is temperature. The pressure happen to be a strong function of temperature only.

So that means, out of this three independent parameters that we typically get for a hydroscopic system or for equivalent three parameters for other systems, here we have only two of them totally independent of each other and one is already dependent on the other. Now that is also not very unusual, because typically we have a hydroscopic system, but also we have an equation of state that means, out of those three, only two can be chosen independently.

So that way we are not losing any generality, when we are taking V and T as independent parameter. But here the equation of state is the relation between p and T, right. Now for an adiabatic change, what do we have? We have ds is equal to 0. So that means, if we go back to the equation, first equation, we see the left hand side becomes 0. So we have 4aVT cube dT plus 4 by 3 aT to the power dV is equal to 0.

So rearranging gives you know many terms gets canceled and simple rearrangement gives us dV by V is equal to minus 3 dT by T, which means VT cube, s stands for an adiabatic process where entropy is conserved, is equal to constant. So we have this relation. This is the general relation for adiabatic change in a cavity radiation system, okay.

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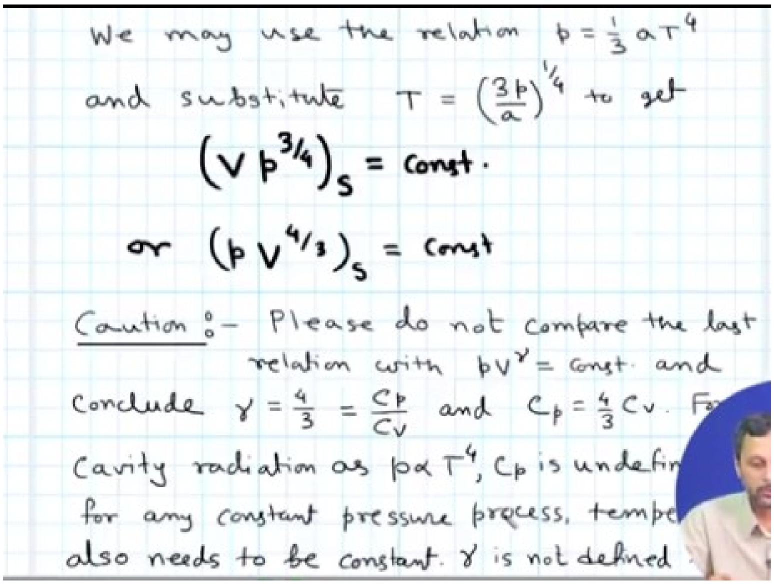

We may use the relation $p = \frac{1}{3} a T^4$

and substitute $T = \left(\frac{3p}{a}\right)^{1/4}$ to get

$$\left(V p^{3/4}\right)_S = \text{Const.}$$

or $\left(p V^{4/3}\right)_S = \text{Const}$

Caution :- Please do not compare the last relation with $pV^{\gamma} = \text{Const}$ and conclude $\gamma = \frac{4}{3} = \frac{C_p}{C_V}$ and $C_p = \frac{4}{3} C_V$. Fo

Cavity radiation as $p \propto T^4$, C_p is undefin

for any constant pressure process, tempe

also needs to be constant. γ is not defined

Also we can use the relation that p is equal to 1by 3 aT to the power 4. That means, we can substitute T is equal to 3p by a whole to the power 1 over 4 in this relation, which is V T cube is equal to constant. And finally, we can get pV to the power 4 by 3 is equal to constant. Alternatively V p to the power 3 by 4 is equal to constant. So either way these are all valid relations.

So similarly, we can try to and sorry in this case the relation between p and T will not work because p and T they already are, there is already one defined relation between them. So we can have a relation between V and p that involves p and p or a relation that involves p and T, right. So these are the adiabatic processes that we have.

Now if we, I mean if we somehow we are you know, provoked to compare this relation p V to the power 4 by 3 is equal to constant with the relation that p V to the power gamma is equal to constant for an ideal gas. So we might go one step forward and conclude that gamma is equal to 4/3, which is equal to C p by C V for this process. And C p is equal to 4 by 3 C V.

Now if we do that, that will be a big mistake. Why, because in this in the cavity radiation as p is directly proportional to T to the power 4 that means, basically they are correlated by this relation over here p is equal to 1 by 3 aT to the power 4. That means, any change in temperature that means, any delta T I mean what is C p actually?

C p is the amount of energy or amount of heat that will be absorbed by the material at constant pressure for an unit temperature rise. Now in this case, if we have constant pressure that means, if delta p is equal to 0, so delta T has to be equal to 0. So that means, in our system this particular system, C p is not even defined. C p is undefined quantity, C p does not exist.

So you have to be bit careful here because in some of the books you might find that it is mentioned that C p is infinity, okay. So it is not the case. C p is actually undefined, so that we cannot assign the value infinity to it arbitrary, right. Similarly, the other parameters like bulk modulus, elastic modulus, it can be proved that those parameters are also undefined, not infinity for this particular system, okay.

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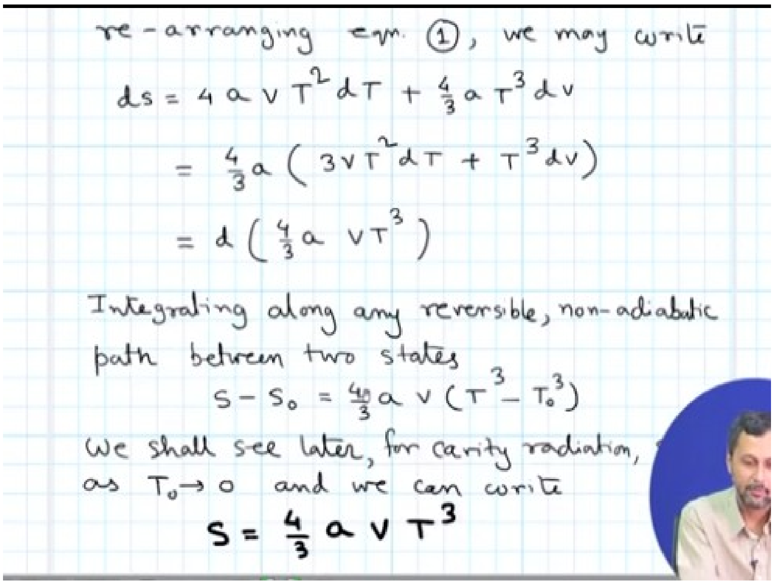

re-arranging eqn. ①, we may write

$$ds = 4 a V T^2 dT + \frac{4}{3} a T^3 dV$$

$$= \frac{4}{3} a \left(3 V T^2 dT + T^3 dV \right)$$

$$= d\left(\frac{4}{3} a V T^3 \right)$$

Integrating along any reversible, non-adiabatic path between two states

$$S - S_0 = \frac{4}{3} a V \left(T^3 - T_0^3 \right)$$

We shall see later, for cavity radiation, as $T_0 \to 0$ and we can write

$$S = \frac{4}{3} a V T^3$$

Now if we, now if we go back to this equation and if I write this, I mean if I just divide the right hand side by temperature and both sides by temperature, so we have ds in the left hand side and in the right hand side we have 4a V T square dT plus 4 by 3 a T cube dV, okay. So which is we can simplify and write d of 4 by 3 a V T cube. So ds is equal to d of something.

Now basically we can integrate this equation along the along any reversible non-adiabatic path between the initial and the final state and finally, we get S - S 0 will be equal to 4/3a V T cube minus T 0 cube where S 0 is the entropy corresponding to the temperature T 0. Now in the next lecture that will be the last lecture of this lecture series, we will see that as the temperature goes to 0, entropy for this particular system also goes to 0.

So that means, we can write s the absolute entropy at a given temperature T as 4 by 3 a V T cube. Now for now you have to accept this before we discuss the third law of thermodynamics and Nernst’s statement on the third law. We have to wait for some more time to learn little bit more about this S going to 0 as T going to 0 thing. But we have if we accept that we have s is equal to 4 by 3 a V T cube for the cavity radiation.

Now that means what? We have pressure temperature relation already. Volume is an independently defined parameter. We have now we have internal energy, we have entropy. And that means, we can take it forward and we can actually define the other free energy functions or energy functions for this particular system.

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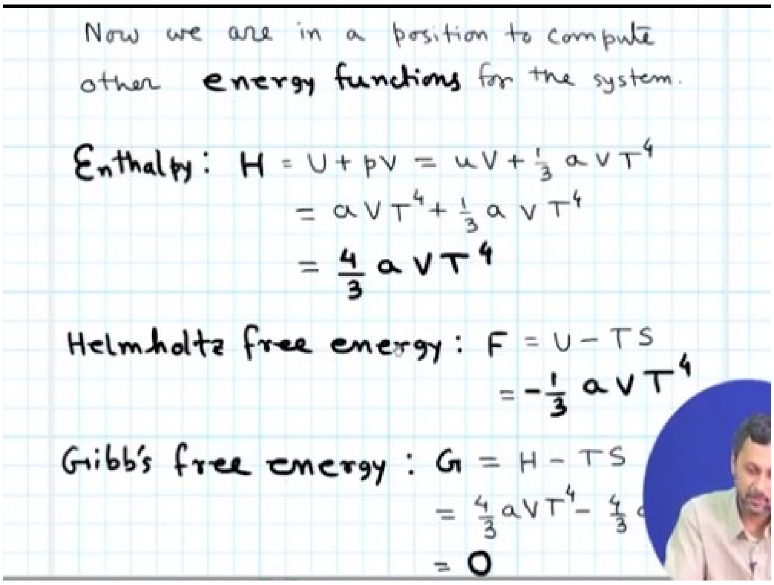

Now we are in a position to compute other **energy functions** for the system.

Enthalpy: $H = U + pV = uV + \frac{1}{3} a V T^4$

$= aVT^4 + \frac{1}{3} a V T^4$

$= \frac{4}{3} a V T^4$

Helmholtz free energy: $F = U - TS$

$= -\frac{1}{3} a V T^4$

Gibb's free energy: $G = H - TS$

$= \frac{4}{3} a V T^4 - \frac{4}{3}$

$= 0$

So and we do that we have enthalpy is equal to H, which is U plus pV. So that means, you small u V plus 1 by 3 a V T to the power 4. Once again we can substitute for small u which is a V T to the power 4 and we have 1 by 3 a V T to the power 4. So we get H is equal to 4 by 3 a V T to the power 4. Now Helmholtz free energy F is equal to U minus TS, simply it becomes minus 1 by 3 a V T to the power 4.

Gibbs free energy G, H minus TS. H is 4 by 3 a V T to the power 4 minus TS. What was S? S is this. So it will be 4 by 3 a V T to the power 4 S times T. And we get G is equal to 0. Now this G is equal to 0 has significance as we will see very soon.

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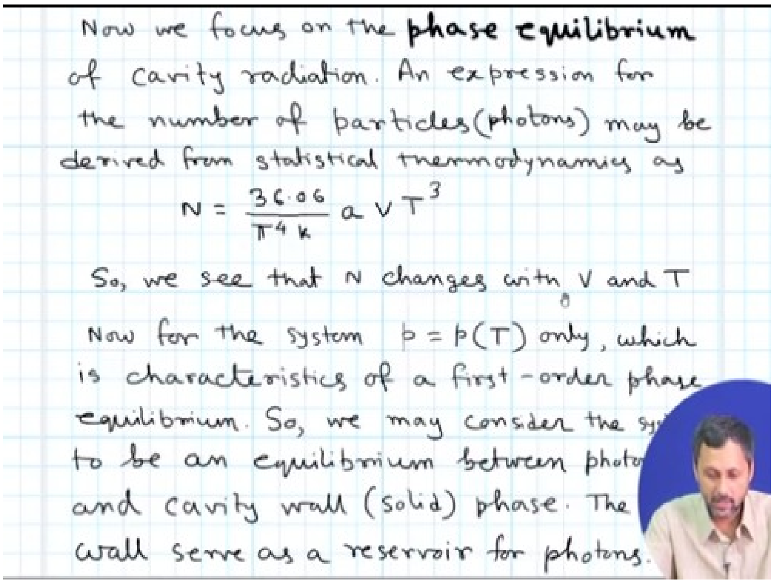

Now we focus on the **phase equilibrium** of cavity radiation. An expression for the number of particles (photons) may be derived from statistical thermodynamics as

$$N = \frac{36.06}{\pi^4 k} a V T^3$$

So, we see that N changes with V and T

Now for the system $p = p(T)$ only, which is characteristics of a first-order phase equilibrium. So, we may consider the sy to be an equilibrium between photo and cavity wall (solid) phase. The wall serve as a reservoir for photons.

Next, we can also define a phase equilibrium in this system. How come? Let us look at it this way. First of all let me tell you that from statistical mechanics, by application of proper statistical mechanics protocol, we can get the total number of photons, which is equivalent to particles in this particular case is equal to N is equal to 36.06 which is actually an approximation of the zeta function.

36.06 divided by pi to the power 4 K a V T cube. Now what do we see here? We see that N is a, I mean N changes with V and T. So if we keep the temperature same and we have an isothermal expansion of the cavity that means, N changes. If we keep the volume same and we increase the temperature of the cavity, let us say we have an adiabatic process in which we just increase the temperature.

I mean you know compress the cavity so that volume changes, temperature also changes, yeah? So or let us say independently we just keep the volume same and increase the temperature. Then we actually have the value of N changing, right. Also we have a equation of the form p is equal to p of temperature only. Because p is equal to what 1 by 3 a T to the power 4.

So that means p is the, I mean p is a function of temperature only. This is a characteristics. When we have in a P-T diagram when we have a line where which means that p is a function of temperature only, this is the characteristics of a first order phase equilibrium line. Please remember, please recall your lectures on, recall the lectures on phase equilibrium.

So p as a function of T means that this is a first order phase equilibrium line which follows the Clausius sorry Clapeyron equation. Now what are, so that means there has to be two phases, right? Now what are those two phases? Those two phases happen to be the phonon which can be taken for example, I mean you can take it as a gas phase, phonon gas and the wall of the cavity, which are the so called solid phase.

Now this cavity wall, this works as the reservoir of the photons, not the phonon sorry, if I said phonon I am wrong, it is photon. So the cavity wall works as the reservoir for photons, when the temperature increases. Let us say we keep the volume constant, we increase the temperature of the system. So that means the cavity will inject, the walls will inject more photon into the cavity.

So the radiation, the energy density and pressure everything will go up, right. Similarly, if the temperature goes down keeping the same I mean same volume. So that will lead to a process which will withdraw photon from the system, right.

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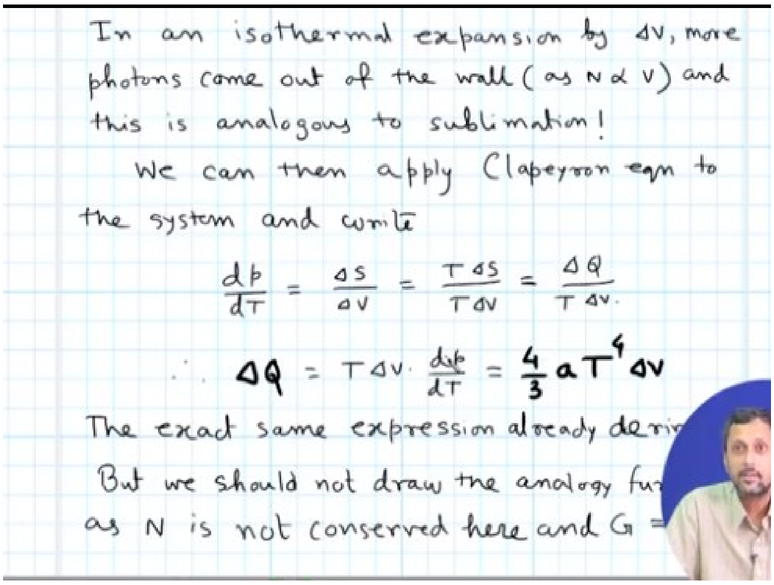

In an isothermal expansion by ΔV, more photons come out of the wall (as $N \propto V$) and this is analogous to sublimation!

We can then apply Clapeyron eqn to the system and write

$$\frac{dp}{dT} = \frac{\Delta S}{\Delta V} = \frac{T\Delta S}{T\Delta V} = \frac{\Delta Q}{T\Delta V}.$$

$$\therefore \Delta Q = T\Delta V \cdot \frac{dp}{dT} = \frac{4}{3} a T^4 \Delta V$$

The exact same expression already deri

But we should not draw the analogy fu

as N is not conserved here and G =

So this is equivalent to a solid gas phase equilibrium which is equivalent to sublimation. Now given that p is a function of T only, we can try to apply Clapeyron equation in here. Now what was Clapeyron equation? Clapeyron equation was dp dT is equal to dp dT along the phase equilibrium line is equal to delta s by delta V. Now what was delta s?

And now what we do was we multiply the numerator and denominator both by T. So we write TdS divided by TdV is equal to dp dT. Now TdS is equal to delta Q. TdV remains TdV. After rearrangement we get delta Q is equal to T delta V dp dT right? Now what is dp dT? dp dT is equal to 1 by 3 a T to the power 4. And we have a temperature coming from here.

So it will be sorry 1/3 a sorry what was p once again? Just a minute, yeah. So p is yeah so dp dT will be 4/3 a T cube and when I multiply this with T it will be 4/3 a T to the power 4 delta V. So and this is exactly the same expression we have found for heat, we have found for this isothermal expansion of the system, right.

So that is why we basically get back the familiar expression for the heat required for a given process in this system. Now the exact, so but we should be careful here. We should not draw an analogy any further because N is not a conserved quantity here and G is equal to 0. So that is where the analogy stops.

But what I wanted to tell you that in this system I mean we can take it as an equivalent system as if we have a solid and a gas phase in equilibrium with each other. And we can even compute the temperature required for the photons from going from solid to the gas phase or vice versa, right. Now let us try to solve some problems before we conclude this lecture, okay.

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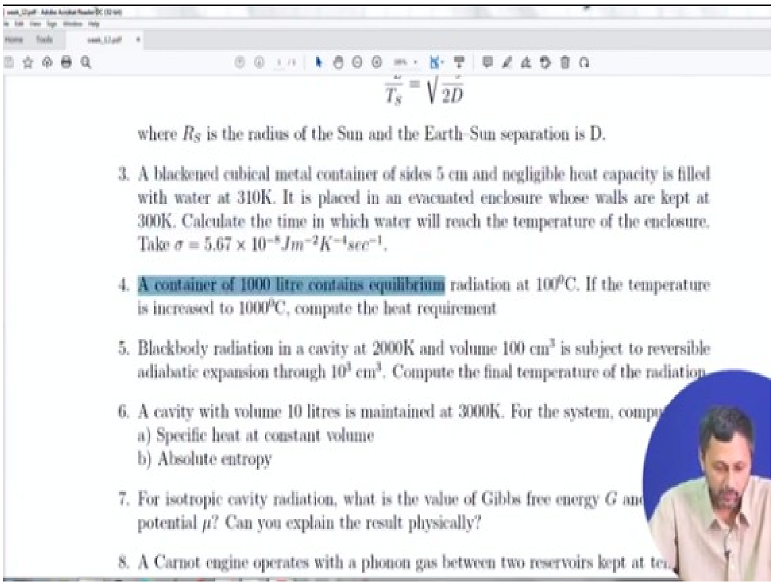

$$\frac{}{T_S} = \sqrt{\frac{}{2D}}$$

where R_S is the radius of the Sun and the Earth–Sun separation is D.

3. A blackened cubical metal container of sides 5 cm and negligible heat capacity is filled with water at 310K. It is placed in an evacuated enclosure whose walls are kept at 300K. Calculate the time in which water will reach the temperature of the enclosure. Take $\sigma = 5.67 \times 10^{-8} Jm^{-2}K^{-4}sec^{-1}$.

4. A container of 1000 litre contains equilibrium radiation at 100^0C. If the temperature is increased to 1000^0C, compute the heat requirement

5. Blackbody radiation in a cavity at 2000K and volume 100 cm^3 is subject to reversible adiabatic expansion through 10^3 cm^3. Compute the final temperature of the radiatio

6. A cavity with volume 10 litres is maintained at 3000K. For the system, compu
   a) Specific heat at constant volume
   b) Absolute entropy

7. For isotropic cavity radiation, what is the value of Gibbs free energy G an potential μ? Can you explain the result physically?

8. A Carnot engine operates with a phonon gas between two reservoirs kept at te

So the first problem is, sorry the fourth one. We have done up to problem number 3 in the previous lecture. A container of 1000 litre contains equilibrium radiation at 100 degrees centigrade. If the temperature is increased to 1000 degree centigrade, compute the heat requirement. So for this problem, what we need to do is we need to first use the relation that we have. Just a minute, right.

So in this case the temperature changes. This is not an isothermal change, right. So what we have to do here is, okay. By the way, all these things that I have covered in today's lecture is I mean it will be there in some of the standard textbooks.

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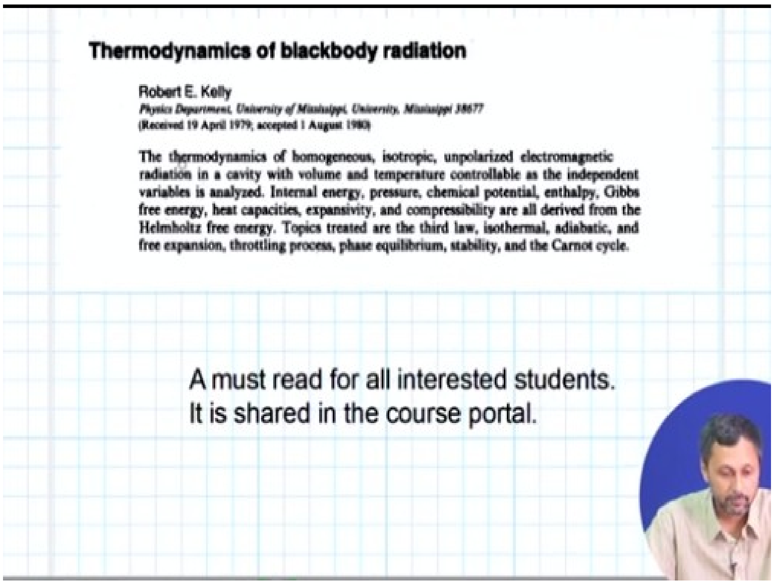

Thermodynamics of blackbody radiation

Robert E. Kelly
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(Received 19 April 1979; accepted 1 August 1980)

The thermodynamics of homogeneous, isotropic, unpolarized electromagnetic radiation in a cavity with volume and temperature controllable as the independent variables is analyzed. Internal energy, pressure, chemical potential, enthalpy, Gibbs free energy, heat capacities, expansivity, and compressibility are all derived from the Helmholtz free energy. Topics treated are the third law, isothermal, adiabatic, and free expansion, throttling process, phase equilibrium, stability, and the Carnot cycle.

A must read for all interested students.
It is shared in the course portal.

But also there is a beautiful paper. It is not exactly a research paper, it is a paper that is written for undergraduate students, keeping undergraduate students in mind. This is a paper that was published back in 1980. It is a thermodynamics of black body radiation, right? So this is already uploaded in the course portal. So interested students, and I would request everyone to at least go through this at least once.

So it is a very interesting paper where all this concept that I have discussed is given in some more details, okay. And it is a nice read, okay. So let us go to the problem, right.

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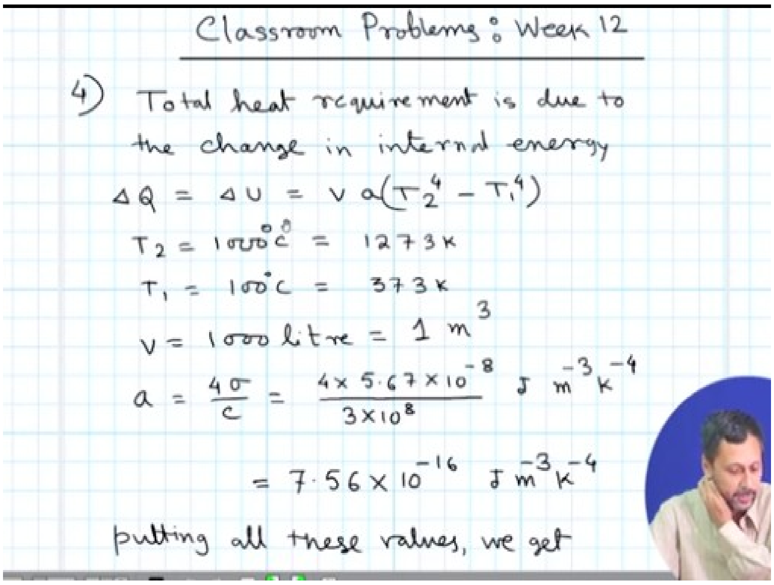

Classroom Problems : Week 12

4) Total heat requirement is due to the change in internal energy

$\Delta Q = \Delta U = V\, a(T_2^4 - T_1^4)$

$T_2 = 1000^\circ C = 1273\,K$

$T_1 = 100^\circ C = 373\,K$

$V = 1000$ litre $= 1\, m^3$

$a = \frac{4\sigma}{c} = \frac{4 \times 5.67 \times 10^{-8}}{3 \times 10^{8}}\ J\, m^{-3} K^{-4}$

$= 7.56 \times 10^{-16}\ J\, m^{-3} K^{-4}$

putting all these values, we get

So what happens is when the, it is not an isothermal process exactly, right. So when the heat, I mean heat goes into the system, it will change the internal energy of the system, right. So the heat requirement, heat required for the process will be equivalent to the change in internal energy so that delta Q is equal to delta U. Now what is U? U is equal to small u times capital V.

So it will be, the volume does not change, right? Because, if we go back to the problem we see, the container volume is 1000 litres, which the temperature was 100 degrees and now it is moved to 1000 degree centigrade. So keeping the volume same. So it will be V times a T 2 to the power 4 minus T 1 to the power 4. Now what is T 2? T 2 is 1273 Kelvin.

What is T 1? This is 373 Kelvin. And V is equal to 1000 litre which is 1 meter cube. So that means, a is and what is a? a is 4 sigma by c. Please remember that, during the discussion on Stefan’s constant, when we derived the expression for Stefan’s constant we have used 4 sorry ac by 4 is equal to sigma. Now sigma is experimentally measured. So that means, we have a way of determining a from sigma.

And it turns out that it is a really tiny number, it is 7.56 into 10 to the power -16 joules per meter cube per Kelvin to the power 4, right? So putting all these values, so we put all these values into this expression here.

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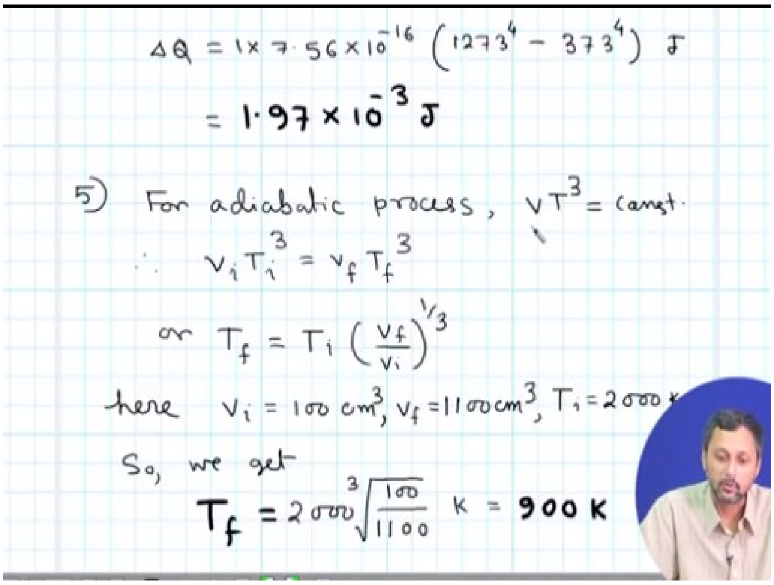

$$\Delta Q = 1 \times 7.56 \times 10^{-16} \left(1273^4 - 373^4\right) \text{ J}$$

$$= 1.97 \times 10^{-3} \text{ J}$$

5) For adiabatic process, $VT^3 = $ const.

$\therefore \quad V_i T_i^3 = V_f T_f^3$

or $T_f = T_i \left(\frac{V_f}{V_i}\right)^{1/3}$

here $V_i = 100 \text{ cm}^3, V_f = 1100 \text{ cm}^3, T_i = 2000$

So, we get

$$T_f = 2000 \sqrt[3]{\frac{100}{1100}} \text{ K} = 900 \text{ K}$$

And finally, we get delta Q is equal to 1.97 into 10 to the power -3 joules. So we see that energy requirement is typically very small here. And this is because of the, you know the smallness in this number a.

So that means for a cavity radiation if we want to increase the internal energy, the heat requirement is small as compared to you know, if we take water as a standard for example, so it will be an interesting you know exercise to compute the specific heat ratio of C V versus the specific heat ratio of water for and see what is the factor that is coming out. And let me tell you the factor will be really small, right.

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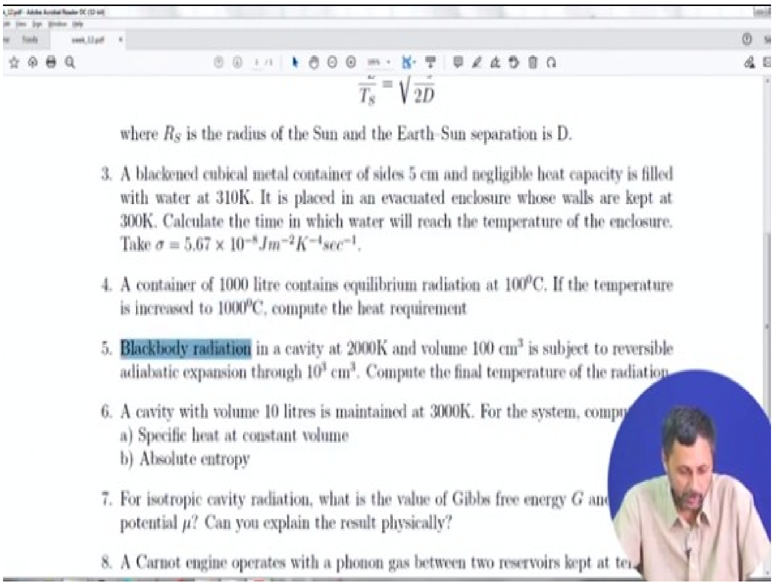

$$\frac{}{T_S} = \sqrt{\frac{}{2D}}$$

where R_S is the radius of the Sun and the Earth–Sun separation is D.

3. A blackened cubical metal container of sides 5 cm and negligible heat capacity is filled with water at 310K. It is placed in an evacuated enclosure whose walls are kept at 300K. Calculate the time in which water will reach the temperature of the enclosure. Take $\sigma = 5.67 \times 10^{-8} Jm^{-2}K^{-4}sec^{-1}$.

4. A container of 1000 litre contains equilibrium radiation at 100^0C. If the temperature is increased to 1000^0C, compute the heat requirement

5. Blackbody radiation in a cavity at 2000K and volume 100 cm^3 is subject to reversible adiabatic expansion through 10^3 cm^3. Compute the final temperature of the radiation

6. A cavity with volume 10 litres is maintained at 3000K. For the system, comp
   a) Specific heat at constant volume
   b) Absolute entropy

7. For isotropic cavity radiation, what is the value of Gibbs free energy G an potential μ? Can you explain the result physically?

8. A Carnot engine operates with a phonon gas between two reservoirs kept at te

Now question number 5. Black body radiation in a cavity at 2000 Kelvin and volume 100 degree 100 centimeter cube is subject to reversible adiabatic expansion through 10 to the power 3 centimeter cube. Compute the final temperature of the radiation. So this is an adiabatic process in which initial temperature is 2000 Kelvin, the final volume the ratio of the initial and final volume is 10 to the power 3, right?

So, sorry the reversible adiabatic expansion through 10 to the power 3 centimeter cube. So that means, if the initial volume is V 1, the final volume is V 1 plus 10 to the power 3 centimeter cube, okay.

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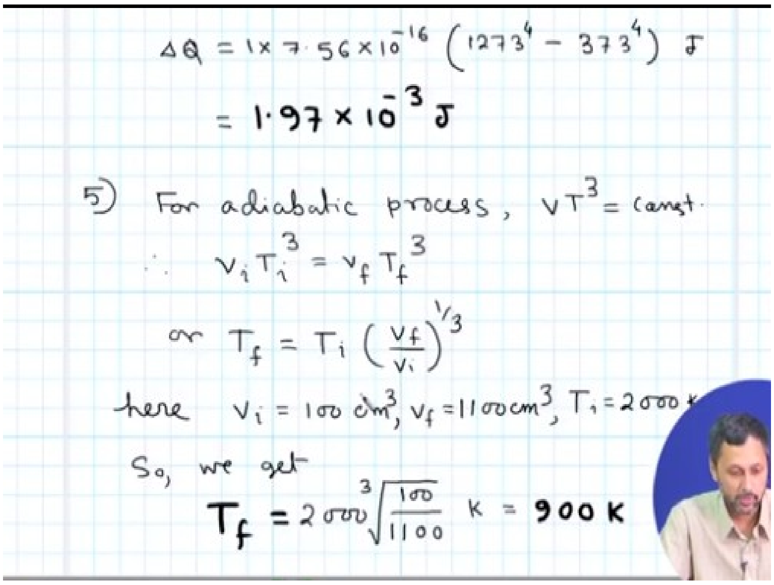

$$\Delta Q = 1 \times 7.56 \times 10^{-16} \left(1273^4 - 373^4\right) J$$

$$= 1.97 \times 10^{-3} J$$

5) For adiabatic process, $VT^3 = \text{const.}$

$$\therefore \quad V_i T_i^3 = V_f T_f^3$$

$$\text{or} \quad T_f = T_i \left(\frac{V_f}{V_i}\right)^{1/3}$$

here $V_i = 100\ cm^3$, $V_f = 1100\ cm^3$, $T_i = 2000$ K

So, we get

$$T_f = 2000 \sqrt[3]{\frac{100}{1100}}\ K = 900\ K$$

So now, let us use the relation that V i V T cube is equal to constant. So we have V i T i cube is equal to V f T f cube. So we have T f is equal to T i V f by V i whole to the power 1 by 3. Now V i is equal to 100 centimeter cube. V f is 1100 centimeter cube. So V i plus 10 to the power 3 and T i is equal to 2000 Kelvin. Now we do not need an unit conversion here primarily because the volumes are in ratio to each other.

So we can simply write T f is equal to 2000 cube root of 100 by 1100. After simplifying we get 900 Kelvin. So we start with 2000 Kelvin, we expand the volume by 10 to the power 3 centimeter cube and the final initial temp yeah, so initial temperature was 2000 and finally, we end up in 900 Kelvin. Also one thing is, no actually that is it does not matter, yeah. So it is a general property of any cavity radiation, okay.

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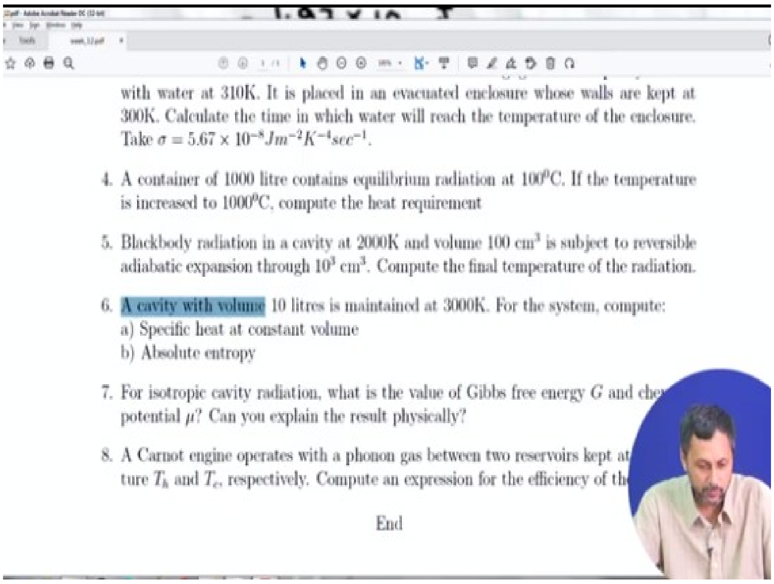

with water at 310K. It is placed in an evacuated enclosure whose walls are kept at 300K. Calculate the time in which water will reach the temperature of the enclosure. Take $\sigma = 5.67 \times 10^{-8} Jm^{-2}K^{-4}sec^{-1}$.

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a) Specific heat at constant volume
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7. For isotropic cavity radiation, what is the value of Gibbs free energy G and che
potential μ? Can you explain the result physically?

8. A Carnot engine operates with a phonon gas between two reservoirs kept at
ture T_h and T_c, respectively. Compute an expression for the efficiency of th

End

So problem number 6. A cavity with volume 10 litres is maintained at 3000 Kelvin. For the system, compute specific heat at constant volume and absolute entropy, okay. So once again here we need to we have an idea of specific heat and see what we get. So cavity with volume 10 litres.

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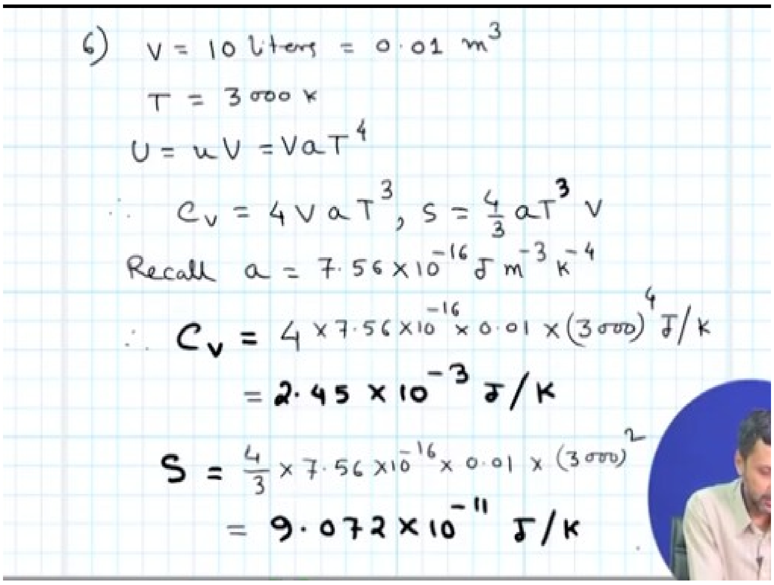

6) $V = 10 \text{ liters} = 0.01 \text{ m}^3$

$T = 3000 \text{ K}$

$U = uV = VaT^4$

$\therefore C_V = 4VaT^3, \; S = \frac{4}{3}aT^3 V$

Recall $a = 7.56 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$

$\therefore C_V = 4 \times 7.56 \times 10^{-16} \times 0.01 \times (3000)^4 \text{ J/K}$

$= 2.45 \times 10^{-3} \text{ J/K}$

$S = \frac{4}{3} \times 7.56 \times 10^{-16} \times 0.01 \times (3000)^2$

$= 9.072 \times 10^{-11} \text{ J/K}$

So what is given? V is equal to 10 litres, which is 0.01 meter cube. T is equal to 3000 Kelvin. So U, which is the internal energy which is small u times V cube is equal to V a T to the power 4. And C V is equal to 4 V a T cube. S is equal to 4 by 3 a T square V. So these are the expression that we have derived. I hope the expression is correct. Let me just verify. Yeah, so see it should be T cube actually, 4 by 3 a V T cube. 4 by 3 a V T cube.

So this is wrong. a V this should be T cube actually. Yeah, right. Okay, so we recall that a is equal to this, so C V will be simply, yeah. So, yeah. So a is equal to this. So C V is equal to 2.45 into 10 to the power minus 3 joules per Kelvin. Now if we if you want to compare it with for example, with water, then you have to also scale it either, you know you cannot do it per mole here, but you can do it per unit volume, right.

That is how the scaling is. So you can you know scale it further for the volume of this cavity which is 0.01 meter cube and then compare this with the value of water. Now S it will be 4/3 this a times V times it should be T cube once again. So the calculation is not right. So whatever it is, I will update this in the final version. So maybe one order of it should be two three orders of magnitude less compared to this number, okay.

So whatever it is, I will I think there is a yeah there is a calculation mistake. So I will just update this in the uploaded version. Kindly check, okay.

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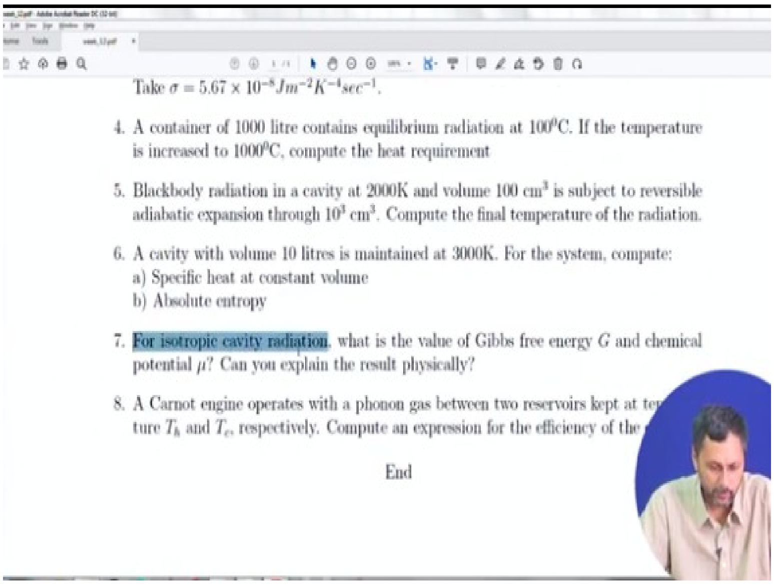

Take $\sigma = 5.67 \times 10^{-8} Jm^{-2}K^{-4}sec^{-1}$.

4. A container of 1000 litre contains equilibrium radiation at 100^0C. If the temperature is increased to 1000^0C, compute the heat requirement

5. Blackbody radiation in a cavity at 2000K and volume 100 cm^3 is subject to reversible adiabatic expansion through 10^3 cm^3. Compute the final temperature of the radiation.

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a) Specific heat at constant volume
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7. For isotropic cavity radiation, what is the value of Gibbs free energy G and chemical potential μ? Can you explain the result physically?

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Next, problem number 7. For isotropic cavity radiation what is the value of Gibbs free energy G and the chemical potential mu. Can you explain the result physically? Now for G we have already seen that G is equal to 0. Now what is chemical potential? Chemical potential mu is such that that that G is equal to N times mu, N being the number of particles into the system.

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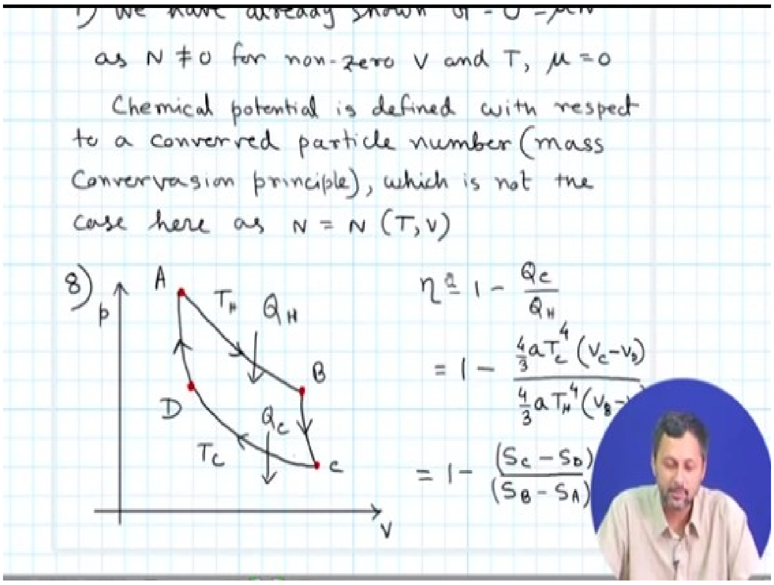

7) We have already shown $G = 0 = \mu N$

as $N \neq 0$ for non-zero V and T, $\mu = 0$

Chemical potential is defined with respect to a converred particle number (mass Convervasion principle), which is not the case here as $N = N(T, V)$

8)

$$\eta = 1 - \frac{Q_C}{Q_H}$$

$$= 1 - \frac{\frac{4}{3}aT_C^4(V_C - V_D)}{\frac{4}{3}aT_H^4(V_B -}$$

$$= 1 - \frac{(S_C - S_D)}{(S_B - S_A)}$$

Now we see that, we already see that G is equal to 0, which is equal to mu N and N cannot be 0 because then the system does not exist. I mean if N is equal to 0 that means there is no system. So that means, for any nonzero value of V and T, we must have mu is equal to 0. So that means not only the Gibbs energy but also chemical potential is equal to 0 for cavity radiation.

Now chemical potential and I mean that way chemical potential is Gibbs energy per particle. So chemical potential is defined with respect to conversion conserved particle number, which means mass conservation actually. So when, now for cavity radiation we see that N is a function of V and T. That means, the mass or the number of particle is not even conserved.

So for this type of a system that is why the chemical potential and Gibbs free energy they are not even defined. And I mean similar to C p, but for C p we cannot, but for C p we cannot just assign any arbitrary value to it, but it so happens that the calculation, simple calculation shows that the chemical potential and Gibbs free energy both are 0 in the system, okay.

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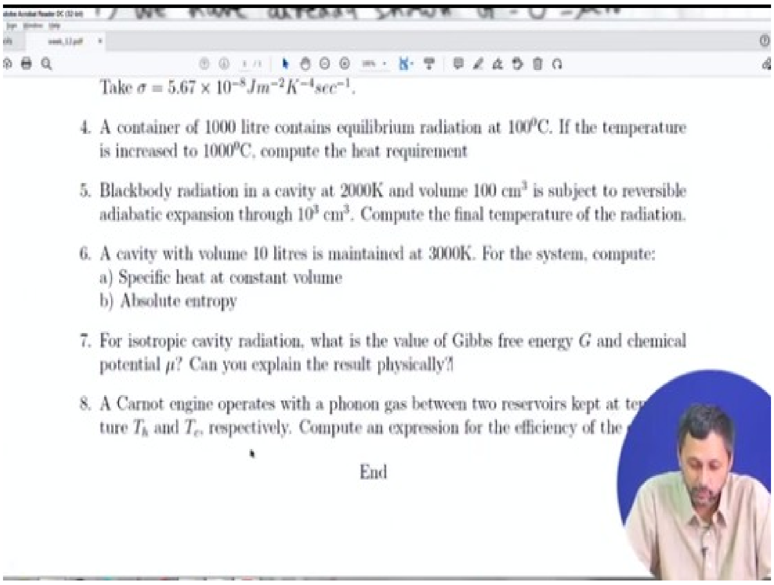

Take $\sigma = 5.67 \times 10^{-8} Jm^{-2}K^{-4}sec^{-1}$.

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7. For isotropic cavity radiation, what is the value of Gibbs free energy G and chemical potential μ? Can you explain the result physically?!
8. A Carnot engine operates with a phonon gas between two reservoirs kept at te
   ture T_h and T_c, respectively. Compute an expression for the efficiency of the

End

Now with this we come to our last problem, which is a Carnot engine operates with a phonon gas, sorry it should be a photon gas. Once again it is a mistake here, I will correct that. It should be a photon gas between two reservoirs kept at temperatures T h and T c respectively. Compute an expression for the efficiency of the engine, right.

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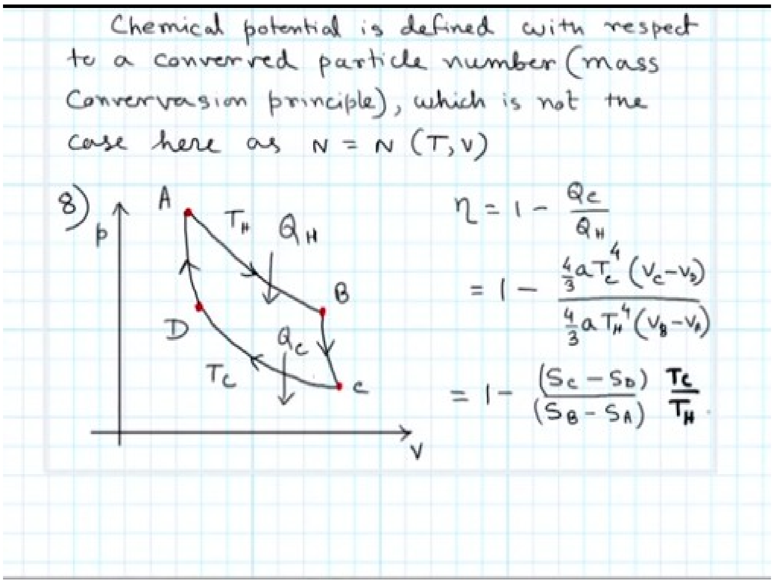

Chemical potential is defined with respect to a converred particle number (mass Convervasion principle), which is not the case here as $N = N(T, V)$

8)

$$\eta = 1 - \frac{Q_c}{Q_H}$$

$$= 1 - \frac{\frac{4}{3} a T_c^4 (V_c - V_D)}{\frac{4}{3} a T_H^4 (V_B - V_A)}$$

$$= 1 - \frac{(S_c - S_D)}{(S_B - S_A)} \frac{T_c}{T_H}$$

Now. So let us say this is the Carnot cycle we have. These two are the isothermal cycles and sorry A to B is isotherm. C to D is isotherm. D to A and B to C these are adiabatic process. Now for isotherm, the relation we have not computed. For adiabatic we know it is V T cube is equal to or p what was it, p V to the power, where is it? Yeah. So it is p V to the power 4 by 3 is equal to constant. But for isothermal we do not know what is the relation.

And we really do not need to know because we already know how to compute the heat the, sorry amount of heat exchange in an isothermal process. So if the heat that is been taken from the hot reservoir is Q H and that given to the cold reservoir is Q C, then eta is equal to 1 minus Q C by Q H. Now what is Q C? Q C is 4 by 3 a TC, T C being the temperature of the cold reservoir, T C to the power 4 V C minus V D, right?

So V C minus V D that is because we have already taken the sine of Q C as negative. So we are writing V C minus V T. Similarly, Q H is 4 by 3 a T H whole to the power 4 V B minus V A, right. So if we now cancel this out we see sorry, we do not have to cancel, just a minute. Now if we compare this with the change in entropy, right. So the change in entropy delta S is 4 by 3 a V T cube minus T 0 cube, right?

So our T f cube minus T i cube. So if I compare with this one, what do we get? We get we take 1 T C, sorry it should be T C by T H. Once again my mistake. It should be, so T C here by T H here, right. 1 minus T 1 by T C sorry. So we take this T C by

T H out. So this quantity will be delta S going from C to D. And this quantity is delta S going from A to B, right?

So we have S C minus S D divided by S B minus S A, right? Right. So this is what it is. Now please remember that these two processes B to C and D to A these are adiabatic process which can be also be called the isentropic process. Now in this case, so because these are isentropic process S B is equal to S C and S D is equal to S A.

The entropies of these two points and these two points are identical. So if we just substitute S B here so it will be S B minus S D. And if I substitute right for S A also if we substitute S D, it should be S B minus S D.

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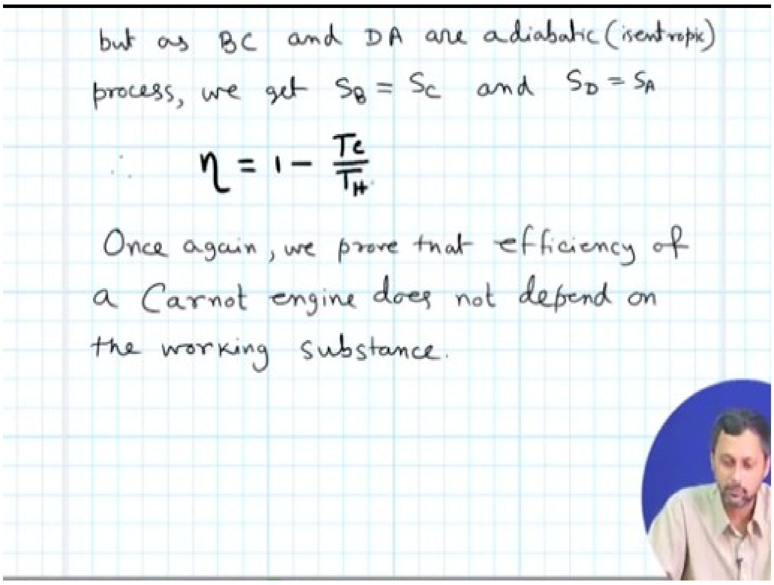

but as BC and DA are adiabatic (isentropic) process, we get $S_B = S_C$ and $S_D = S_A$

$$\therefore \quad \eta = 1 - \frac{T_C}{T_H}$$

Once again, we prove that efficiency of a Carnot engine does not depend on the working substance.

So whatever it is these differences in entropy they nicely cancel out leaving behind, it should be T C by T H, 1 minus T C by T H, right. So once again this is the familiar expression for efficiency of a Carnot engine working between two fixed temperature T H and TC, T H being the high temperature T C being the low temperature.

Now this demonstrates again that the Carnot engine, the efficiency of Carnot engine working between any two reservoirs does not depend on the working substance, okay. So with this we conclude this lecture on radiation. For the remaining single lecture that we have in hand, for that we will be focusing on little bit of conceptual discussion on low temperature physics and also the third law of thermodynamics. Till then, goodbye.