Thermal Physics
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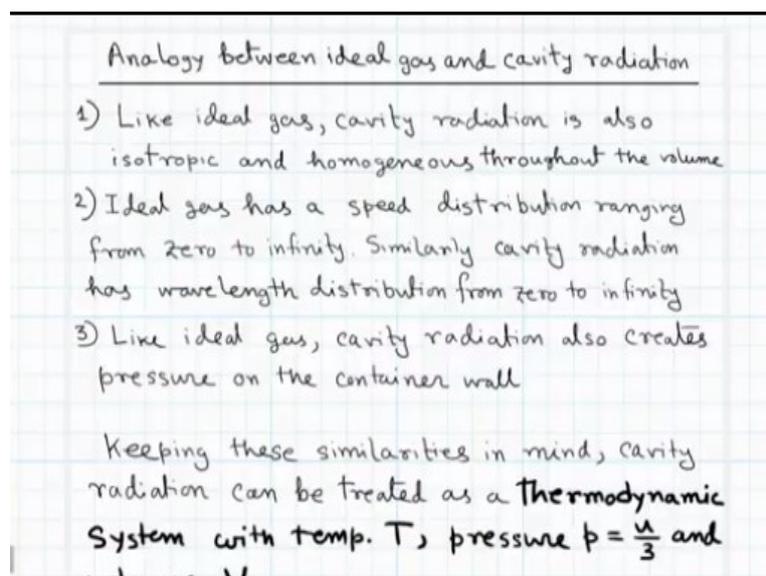**Indian Institute of Technology-Kharagpur**

Lecture - 58
Cavity Radiation as a Thermodynamic System: Stefan-Boltzmann Law

Hello and welcome back to another lecture of this NPTEL lecture series on thermal physics. Now in yesterday's lecture, we talked about radiation pressure. We talked about radiation pressure of directional radiation and diffuse radiation. And also we have discussed about different properties of radiation for example emissivity or rather different properties of an object under diffuse radiation.

For example, the amount of radiation that is the incident on it, we know how to calculate that. We know the amount of radiation that is been emitted by a body in a enclosure, how to calculate that, we have learned all that. Also we have come up to the conclusion that p is equal to u by 3, that is for cavity radiation.

Now in today's lecture, we will be trying to we will start by drawing an analogy between cavity radiation and an ideal gas assembly, okay.

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Analogy between ideal gas and cavity radiation

1) Like ideal gas, cavity radiation is also isotropic and homogeneous throughout the volume

2) Ideal gas has a speed distribution ranging from zero to infinity. Similarly cavity radiation has wavelength distribution from zero to infinity

3) Like ideal gas, cavity radiation also creates pressure on the container wall

Keeping these similarities in mind, cavity radiation can be treated as a **Thermodynamic** system with temp. T, pressure $p = \frac{u}{3}$ and

So what are the analogy? First of all like ideal gas, cavity radiation also isotropic and homogeneous throughout the volume. I mean like ideal gas, they are very random in movement, they are like moving everywhere.

But if you take one small volume at one part of the container and another small volume at another part of the container, the number density, the pressure that means the pressure and the distribution of molecular particles will be same inside one elementary volume taken in one part and another elementary volume taken let us say simultaneously in another part.

Similarly, in a cavity radiation if you look at any point of this inside this container, you will have you know the radiation will be completely diffused. So in one part you might find that radiation I mean okay I should not try to draw this analogy I mean I cannot make it a very hard analogy, but let me tell you that the like ideal gas although they are random in nature, but they are you know the molecular distribution is homogeneous and isotropic.

Similarly, the cavity radiation although they do not, the cavity radiation has no specific direction at any part of the container they are isotropic and homogeneous. Similarly for ideal gas the speed distribution that ranges from 0 to infinity. In a, we can draw the analogy. I mean for of course for cavity radiation for any EM wave, the speed is fixed. Speed is c the speed of light. But for this the wavelength varies from 0 to infinity.

So that is where we can draw an analogy. Similarly, like ideal gas, cavity radiation also creates a pressure on the container wall and or any object that is placed inside the container. Same as ideal gas. If you put an object inside an ideal gas container, it will also that there will be pressure on that item itself. Similarly, pressure on the cavity wall.

So keeping these similarities in mind, cavity radiation can be treated as a thermodynamic system with the following parameter. We have temperature T pressure p is equal to u by 3 and volume V. Now very soon, we will see that the temperature and pressure they are actually connected to each other.

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Emission from black body: Stefan-Boltzmann law

If u is the energy density of radiation and V is the volume, the internal energy

$$U = Vu$$

Applying energy equation

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

and writing $p = \frac{u}{3}$, and assuming $u = u(T)$

$$\left(\frac{\partial U}{\partial V}\right)_T = u \quad ; \quad \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3}\left(\frac{\partial u}{\partial T}\right)_V = \frac{1}{3}\frac{du}{dT}$$

and

$$u = \frac{T}{3}\left(\frac{du}{dT}\right) - \frac{u}{3}$$

Now next is we look into the emission from a black body that is given by Stefan-Boltzmann law. The law was initially, empirical law was given by Stefan and later on Boltzmann came up with a theoretical footing for that law. And also he has proved that the Stefan's law which was the Stefan's law is strictly valid, or the law we will learn very soon is strictly valid for a black body.

Now for other objects, there has to be certain modification to this law, okay. So the statement of the law I think we are all familiar with it. That is the emissive power per unit area per unit time is sigma d to the power 4. We will come to that. Also there is an order relative statement for non-black body. So let us look at it in a step by step manner. Let u be the energy density of radiation inside a cavity of volume V.

So the internal energy of the total radiation which is the total energy that is contained will be U times capital U is equal to small v sorry capital V times small u. That means, U is equal to V times u. Now we remember that and now we are treating this as a thermodynamic system. Now in a thermodynamic system we have a general certain general relations like TdS equations and energy equations.

So what do we do? We apply energy equation in order to correlate your our what you call the internal energy and so basically internal energy and pressure and volume and temperature this has to be correlated, right. So we apply the energy equation del U del V T is equal to T del p del T V minus p. Now we write p is equal to u by 3 and assuming u is equal to u of temperature only.

And that is kind of obvious because see wavelength dependence we already have integrated over it in order to derive the expression for energy density or pressure, we already have integrated over the entire wavelength range. Now what is the other parameter that we have in hand, this temperature.

Now of course, and from empirical observation also it has been found that the energy radiated from an object is dependent on its temperature. So it is only logical to assume that energy density u is a function of temperature only. So in this case del U del V T will be simply equal to u. Del p del T V will be 1 by 3 del u del T V is equal to 1 by 3 du dT. Because it is u is a function of u only.

So it will be simply du dT. And u is equal to T by 3 du dT minus. So if I yeah, if I substitute in the energy equation, so what do we get? We get u is equal to T by 3 du dT minus u by 3 that is for p, right? So we have this equation.

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We now simplify this. And finally, simplify this to this particular form that du dT is equal to 4u by T. And after integration we get u is equal to a T to the power 4, where a is the integration constant. Now this is the relation of energy density mind you, energy density as a function of temperature. We see that depends on the fourth power.

Now we have already discussed for any surface the total emission energy per unit surface area in the upper hemisphere is e a is equal to so please remember in the last

derivation, we had a 2 pi here, because we integrated over the entire 4 pi solid angle. But if we look only at the upper hemisphere, it will be integrated to half of the space available. So we have pi 0 to infinity e lambda d lambda integrating over all wavelength, which is pi e.

Now for a black body we have also proved that e emissive power of a black body is equal to K the surface luminescence of the container wall, right. So we and please remember u is equal to 4 pi K by c, sorry K is the surface brightness not the surface luminescence, basically the same thing, surface brightness.

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So if we correlate all of this we can write E a that is the total emissive power per unit area per unit time from the surface of a black body is sigma T to the power 4 where sigma is equal to a times c divided by 4 is another constant, right.

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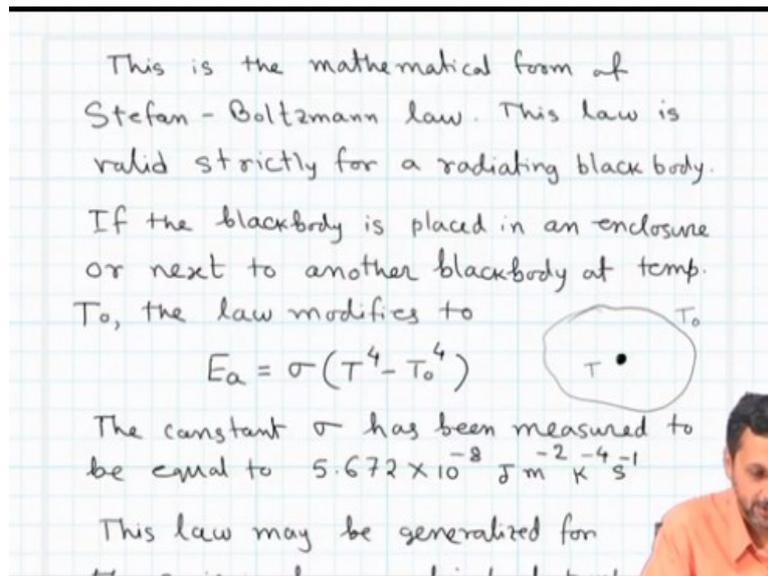
This is the mathematical expression which is, this is the mathematical form of Stefan-Boltzmann law. And please remember once again this law is strictly valid for black body, because of this assumption here that, not assumption I mean because of this consideration here that e b is equal to K. So that is why it is strictly valid for a black body in this particular case.

Now if this is this black body in question is placed next to another black body which has a different temperature, or let us say it is placed inside a cavity, yeah, so where the cavity has a different temperature, the cavity wall has a different temperature, then what happens is it will emit certain amount of radiation, but it will also absorb certain amount of I mean good amount of radiation from the other black body in the vicinity.

So if we have the temperature of the other black body as T 0, then the total emissive power okay, so total sorry not total emissive power, the total energy that is emitted is actually, the total energy that is emitted in absolute scale minus the energy that will be absorbed by this black body in absolute scale.

So it will be E a equal to sigma T to the power 4 minus T 0 to the power 4 assuming given that the entire energy that has been released by the other black body is totally absorbed by this one. So the better way of looking at it is let us say we have an enclosure, okay. Inside this we have a black body here, right. So this one is at temperature T and this enclosure it has a temperature T 0.

So that is where we can write for this particular black body the emission per unit area per unit time is sigma T to the power 4 minus T 0 to the power 4, right. Now this constant sigma is called the Stefans constant or sometimes also called the Stefan-Boltzmann constant. It is measured experimentally and the value is 5.672 into 10 to the power -8 joules per meter square per Kelvin to the power 4 per second, right.

So you have to be careful about this unit here, right. Now this law once again this is strictly speaking this is valid for a black body, right. But we can again generalize this law for any object at any finite temperature T, okay.

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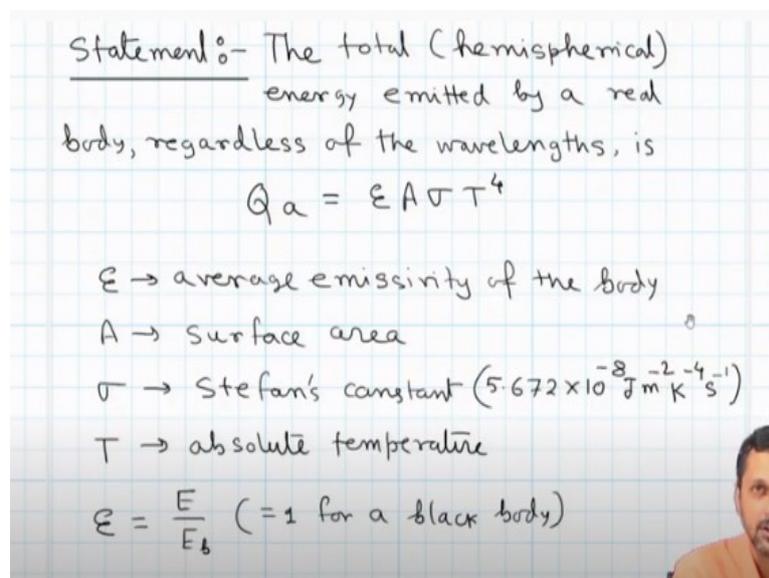
So the generalization says the total hemispherical energy emitted by a real body regardless of the wavelength is Q a is equal to epsilon A sigma T to the power 4 where epsilon is the average emissivity of the body, that is averaged over all possible wavelength. Strictly speaking emissivity like absorptivity a lambda is also a function of lambda, right? But this is the average emissivity of a body which can be measured experimentally.

A is the total surface area of the body. Sigma is the Stefans constant once again. T is the absolute temperature. Now epsilon is E by E b that is equal to 1 for the black body. So that is the emissive, total emissive power divided by total emissive power of a black body which is 1 for an ideal black body. Now we see for ideal black body this law will be equivalent to the Stefan-Boltzmann law.

Please remember this E, I mean this E here is the emissive energy per unit area per unit time. Whereas, Q here is the emissive energy per unit time only. So energy is already taken care of, right. So after this we can look into some applications.

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NPTEL On-line Certification Courses
Thermal physics
Classroom problems: Week 12

1. Sunlight falls on the surface of the Earth with a power per unit area equal to I=1370 Wm^{-2} Calculate the radiation pressure and compare it to atmospheric pressure.

2. The temperature of the Earth's surface is maintained by radiation from the Sun. By making the approximation that the Sun and the Earth behave as black bodies, show that the ratio of the Earth's temperature to that of the Sun is given by

$$\frac{T_E}{T_S} = \sqrt{\frac{R_s}{2D}}$$

where R_S is the radius of the Sun and the Earth-Sun separation is D.

3. A blackened cubical metal container of sides 5 cm and negligible heat capacity is with water at 310K. It is placed in an evacuated enclosure whose walls a 300K. Calculate the time in which water will reach the temperature of th Take $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}sec^{-1}$.

So let us, we have learned enough theory for last two and half lectures. So for the remaining time we will be focusing on applications. First of all, it is a very simple problem. Sunlight falls on the surface of earth with a power per unit area equal to I is equal to, I equal to 1370 Watt per meter square. Calculate the radiation pressure and compare it with, compare it to atmospheric pressure, right.

Now sun being very distant from earth, although it is a spherical object we know and it radiates in a spherical manner, we can for all practical purposes consider this as a directional ray.

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Classroom problem : Week 12

1) Sunlight on Earth's surface may be considered as direction beam, and we can write $p = u = \dfrac{I}{c}$

$I = 1370 \ w/m^2$, $c = 3 \times 10^8 \ m/sec$

$\therefore p = 4.56 \times 10^{-6} \ Pa$

Atmospheric pressure $\sim 10^5 \ Pa$. (10^{11} difference)

2) The energy emitted by sun per sec. is

$Q^s = 4 \pi R_s^2 \cdot \sigma \, T_s^4$

At a distance D from the Sun, this

So for assuming that sunlight, and I think we have discussed it already in the previous lecture, that if we are sufficiently away from the source of, point source of a radiation, then we can consider at a point which is far away, we can consider the radiation as directional in nature. Similarly, as we are quite far away from sun, we can consider this as a this one directional wave.

Now, so that means we can use p is equal to u is equal to I by c, where I is equal to 1370 watts per meter square, c is equal to 3 into 10 to the power 8 meters per second. And p if I just put this number p will be equal to this divided by this will be 4.56 into 10 to the power -6 Pascal. Now the atmospheric pressure is if you remember 10 to the power 5 Pascal. So what is that order of magnitude between this two?

The order of magnitude is 10 to the power minus or 10 to the power 11 is the difference. So we understand, this is a huge number. So that is why we do not feel the pressure due to solar radiation on us. Otherwise, I mean if it would have been somewhat comparable we would have a feel for it probably. But here we do not have a feel.

Of course, we can feel the warmth. If we are standing in the sunlight directly we can feel the heat, feel the warmth, but we do not feel the pressure, the mechanical pressure due to it. But rest assured that there is a mechanical pressure that is present for any radiation, right.

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The next question is a rather interesting question. Please pay attention, we have to understand this very carefully. The temperature of the earth surface is maintained by radiation from the sun, which is almost you know 100% true statement. Of course, there is certain amount of energy that is contained in earth's core, but primarily at the surface the energy comes from the solar system, right.

By making the approximation that the sun and the earth behave as black bodies show that the ratio of earth's temperature to that of sun is given by T E that is temperature of earth divided by T s is equal to root over R s that is the radius of sun and 2D, D being the distance between the two. So where R s is the radius of the sun and earth-sun separation is D, yeah. So the situation is like this, yeah.
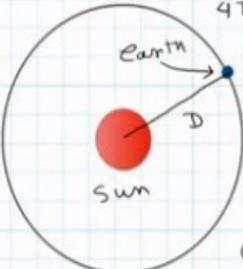
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So I will, once again I first have to refer to this picture then we can go to the problem. So we have sun here. Earth is a tiny dot as compared to sun placed at a distance D. And the, what we can assume or what we gather from this problem is see solar radiation goes in all possible direction. It will go, sorry it will go along this direction, this, this, this, all possible direction, right?

Now only a tiny fraction of this will be heading towards earth and will be finally captured by earth, right? So it will be, I mean I am just drawing it in 2D but it is, in reality it is in all around all 4 pi solid angle. So that means it will be uniformly distributed. If I draw a sphere of radius D, the solar radiation will be uniformly distributed over this 4 pi D square area.

Now out of that, what is the area that is that is offered by earth, that is covered by earth, if the radius of the earth, the average radius is e, R e, then it is only pi R e square, the cross section, okay. So not even the upper hemisphere area we have to consider. We have to just consider the cross section, roughly. I mean, it is once again it is a crude calculation.

But I think you understand that you have a tiny point somewhere far on the periphery of a large sphere. So the ratio of radiation that it will receive is actually the ratio of 4 sorry 4 pi D square divided or to the ratio of pi R e square, right. Now this is exactly what we have, we are doing here. So this is the ratio. So the total power incident on earth will be, so what is the total power that is radiated by sun?

The total power that is radiated by sun is 4 pi R square, R s square, right? 4 pi R s square being the surface area of the sun, times sigma T to the power T s to the power 4. So this is my total power that is radiated by sun per unit time or rather yeah, so that is the radiation per unit sorry, total radiation per unit time by sun. Out of that only a fraction pi R e square divided by 4 pi D square will be incident on earth.

Now earth will capture this radiation. And so we have this incident radiation. Now total power that is radiated by earth once again will be given by 4 pi R e square sigma T E to the power 4. Once again this radiation will be heading in all possible direction right, all possible direction.

But we do not care because at equilibrium, in thermodynamic equilibrium I mean it is, once again strictly speaking, it is not a two body system, two body problem. Because, although solar radiation is the primary source of radiation, there are other sources also but irrespective. But their contribution will be very small on earth's surface.

Also earth's radiation, some part will go back to sun but this is also a very tiny fraction. So we can ignore that because earth is I mean not even comparable, the radiating power of earth is not even comparable to that of sun, yeah? So but please remember that sun sorry earth will irradiate in all direction similar to the sun. So but at thermodynamic equilibrium, it is a many body system. So there are surrounding.

So these two are only part of the system. There is a huge surrounding which are also absorbing the energy and some other cosmic energies are also coming in. But at equilibrium this Q I and Q R has to be equal. So it is not a strictly speaking, it is not a two body problem. In a two body problem Q I and Q R will be the mutual heat exchange.

I mean, it will be equivalent to mutual heat exchange between two bodies. In this case it is not. But still we get a thermal equilibrium.

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$$R_S^2 \, T_S^4 \cdot \frac{R_E^2}{4D^2} = R_E^2 \, T_E^4$$

$$\text{or} \quad \left(\frac{T_E}{T_S}\right)^4 = \left(\frac{R_S}{2D}\right)^2$$

$$\text{or} \quad \frac{T_E}{T_S} = \sqrt{\frac{R_S}{2D}}$$

putting $R_S = 7 \times 10^8$ m, $D = 1.5 \times 10^{11}$ m

$T_S = 5800$ K, we get

$T_E = 280.16$ K.

which is not a bad result!

So we once I equate Q I to Q R and simplify we simply get this relation here. T E by T S equal to R S by 2D. Now in order to verify whether this crude approximation

what we have made here is it what determined. So what we can do is we can put RS is equal to 7 into 10 to the power 8 meters, which is average radius measured, experimentally measured for sun. D is the earth to sun distance, average distance.

And also the temperature of the sun which is once again spectroscopically measured is 5800 Kelvin. That is the temperature of the earth surface, okay. So if we put these values we get an average radius or sorry average temperature T E is equal to 280.16 Kelvin, which is once again is by I mean it is not a bad result by any means.

280 Kelvin means 7 degree centigrade assuming that we have a wide temperature distribution on earth's surface the poles are typically subzero. Whereas, in the equatorial region it is somewhere around 40 degree centigrade all throughout the year. So on an average if we get something like 7 degree centigrade, we should be not very upset about it. We should not be very upset about it, okay.

So we see that this crude approximation works and we get a reasonably good number for the temperature, average temperature of earth from this, okay.

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$$\frac{\overline{T_E}}{T_S} = \sqrt{\frac{R_S}{2D}}$$

where R_S is the radius of the Sun and the Earth-Sun separation is D.

3. A blackened cubical metal container of sides 5 cm and negligible heat capacity is filled with water at 310K. It is placed in an evacuated enclosure whose walls are kept at 300K. Calculate the time in which water will reach the temperature of the enclosure. Take $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}sec^{-1}$.

4. A container of 1000 litre contains equilibrium radiation at 100^0C. If the temperature is increased to 1000^0C, compute the heat requirement

5. Blackbody radiation in a cavity at 2000K and volume 100 cm^3 is subject to reversible adiabatic expansion through 10^3 cm^3. Compute the final temperature of the radiation.

6. A cavity with volume 10 litres is maintained at 3000K. For the system, compute:
   a) Specific heat at constant volume
   b) Absolute entropy

7. For isotropic cavity radiation, what is the value of Gibbs free energy G and potential μ? Can you explain the result physically?

8. A Carnot engine operates with a phonon gas between two reservoirs ke

So we have two more problems to finish in today's lecture, right? No sorry actually one more problem, because this other problems we can do it in the next class itself, right. So this is the last problem for today's lecture, okay. So blackened cubical metal container of side 5 cm and negligible heat capacity is filled with water at 310 Kelvin. It is placed in an evacuated enclosure whose walls are kept at 300 Kelvin.

So there is a temperature difference between the object and that of the surrounding, right. Calculate the time in which water will reach the temperature of the enclosure, sorry calculate the time in which water will reach the temperature of the enclosure. Take sigma is equal to this, so value of sigma is given. So we have to first understand how this problem looks like, okay.

So I have not added a drawing, but probably I should, okay. So let me just try to add a drawing here and see if this helps.

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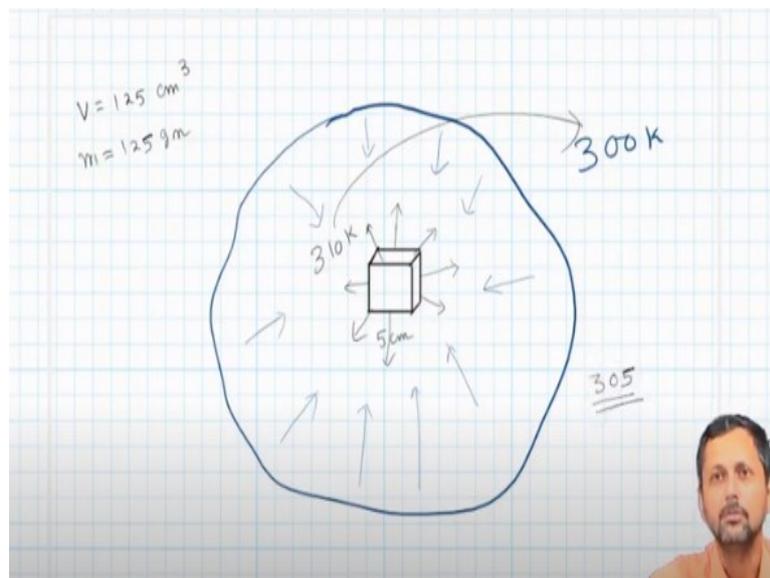
So we have an arbitrary cavity of arbitrary shape, which is at 300 K. So inside what do we put? We put a cube. Let me see if I can find a cube. No not from here, never mind. So we have a tiny cube and this cube is blackened. So that means we have let us say we have put some this one charcoal I mean carbon black or something on it, so that it behaves like I mean somewhat like a black body, right.

So we can consider this system as a black body. Now the sides of this cube is 5 cm. So the volume of this cube V will be 125 cm cube. Now this is filled with water and for water we can safely take the temperature sorry the density as 1 gram per centimeter cube. So yeah 1 gram per cc. So there is a mass m of 125 grams, right. So already it is mentioned that a blackened cubical container and negligible heat capacity.

So we do not have to worry about the heat capacity or the mass of the cube itself, okay. So it is placed in an evacuated enclosure where the walls are kept at 300 Kelvin. Now once it is kept in an enclosure, it will start radiating heat. It will start radiating heat from all its surface in all possible direction, right?

Now and at the same time, heat will be, there will be certain amount of heat will be coming from all directions in a diffused manner. It will fall on the cube. So there will be a, so basically it is a classical example of and it is an evacuated chamber. So we can assume that there is no conduction or convection of heat, especially no convection of heat because there is no air molecule to conduct heat, right.

So it is a classical example of reaching thermal equilibrium by means of radiation, right. By the way our universe is can be also considered as a huge giant container at least yeah. So different parts, they are still reaching thermal equilibrium by exchanging radiation. So the temperature of the universe is slowly decreasing. Please keep that in mind. Of course, I am not an expert to comment on this.

If you have courses on astrophysics, probably there you will have a detailed discussion on this topic. Anyway, so this is one example where the heat, the thermal equilibrium is being established by radiation. Now let us go back to the problem, yeah.

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So now finally, the container will reach 300 Kelvin. That is for sure, because thermal equilibrium has to be achieved. Now in that case, total heat that is lost during the process will be m s delta T. Now m is 125 grams and s is the heat capacity and that is typically 4.2 joules per gram, okay. So 1 calories per gram, so it is 4.2 joules per gram.

So altogether the heat requirement will be 125 into 4.2 into 10 joules, which is 5250 joules. Now during the cooling process, the mean temperature can be taken as, please remember that in a Stefan-Boltzmann law, we do not know what happens if the temperature changes. I mean, of course, during this process, every moment the temperature will reduce by certain amount.

Because, it is at 310 Kelvin and eventually it will reach 300 Kelvin. So very slowly the temperature, I mean that during the during this time before this equilibrium is reached, every time we have a new temperature and that means the Stefan-Boltzmann law in the present form cannot be applicable.

But what we can do is we can solve this by just or we can bypass this by just by introducing a average temperature that is 305. That is the arithmetic average of these two numbers, right.

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law, we may write

$$R = A \sigma (T_{av}^4 - T_0^4)$$

$T_0 = 300\,K, \quad T = 305\,K, \quad A = 6 \times 25 \; cm^2$
$$= 0.015 \; m^2$$

$$\therefore \frac{5250}{t} = 0.015 \times 5.67 \times 10^{-8} \times (305^4 - 300^4)$$

Simplyfing $\quad t \simeq 3 \; hr \; 6 \; min$

This 'average' temperature concept is an approximation. But this gives reasonably good answer.

So if we do that, we can write that the temperature that is yeah you know the rate at which the heat will be lost will be A times sigma times T to the power 4 minus T 0 to

the power 4. Actually I should write the average here. T average to the power 4, minus T 0 to the power 4. So this T average is 305 Kelvin. A is the surface area. We have in a cube we have 6 surfaces and this the total surface area is 0.015 meter square.

So we have in one side this quantity. On the other side, if this is the total heat that will be radiate, that will be yeah, this is the total heat that has to be lost by the cube. So cube will be I mean cube will radiate this many joules of heat over a time let us say T seconds. So the rate of radiation will be 5250 divided by t joules per second.

Now this 5250 by t has to be equal to this A sigma T average to the power 4 minus T 0 to the power 4. Once we equate that and we simplify, we get t is equal to 3 hours 6 minutes approximately. There will be few seconds, which I have ignored, okay. So this average temperature concept is once again is an approximation, but that saves us lots of trouble.

We do not have to integrate the Stefan-Boltzmann law every time assuming that the temperature is changing every time. So that would have been a tedious job. But instead what we have got is a average number I mean an approximation, but this works pretty well for us, okay. So that is where we will stop today.

In tomorrow's lecture, we will be trying to understand different properties of, other properties of black body assuming that it is it behaves like a thermodynamic system. Like for example, we will be talking about isothermal and adiabatic processes, heat capacity, free energy functions and other properties. Till then goodbye.