Thermal Physics
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Lecture - 35
General Discussion on Heat Conduction and Elastic Properties

Hello and welcome back to the last lecture of week 7 in this NPTEL course on thermal physics. Now in the last class we have discussed about and we ended the class with the discussion on the polytrophic process. What is the polytrophic process? It is a process where there could be a heat flow into the system there could be work done on the system or by the system and it is not a constant volume process is not a constant pressure process.

It could be a general I mean it is a general process and we have seen that the equation is given as pV to the power n is equal to constant where n is any number which is greater than one.

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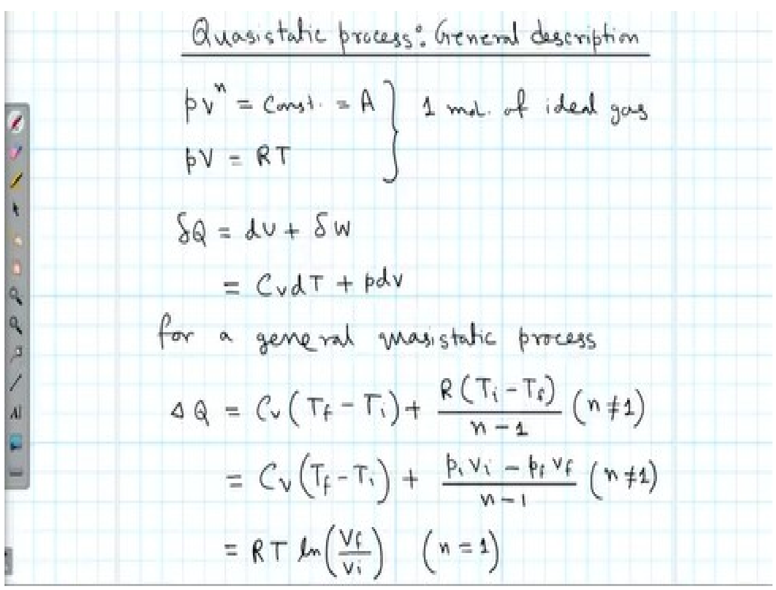

Now continuing on this on the same line we talk about different polytrophic processes or rather we talk about quasi static process a general description. And we have already discussed we have already computed the work done for different types of quasi static process and what we do will do here we will start with this general polytrophic process of this particular form and we assume ideal gas which is the equation of state for one mole of ideal gas is pV = RT.

Now we know that we can write dQ = C v dT + pdv for a gaseous system or a hydrostatic system. So, we can write dU as C v dT and d w we can write as pdv. So, the general quasi static process are given and this we have already discussed in the last class it is given as delta Q the total change in total heat flow in a general quasi static process is given by C v T f minus T i plus this is that work done in a general polytrophic process.

That we have derived in the last class which is given by R T i minus T f divided by n minus 1 when n is not equal to 1. And this is for an ideal gas and the general form which is not common to ideal gas as to say we can use it for any type of polytrophic processes let it be a real gas or let it be of any type of system the general form is p i v i minus p f v f divided by n minus 1 once again n is not equal to 1 because when n is equal to 1 this denominator becomes 0 and we have a basically this term blows.

At n = 1 which is a special case of polytropic process which is an isothermal process we have seen that delta Q is given by RT ln v f by v i. Now from these two expressions like this line and from this particular expression and from this particular expression, we can compute the general heat exchange for different types of quasistatic process which is realized in a ideal gas system.

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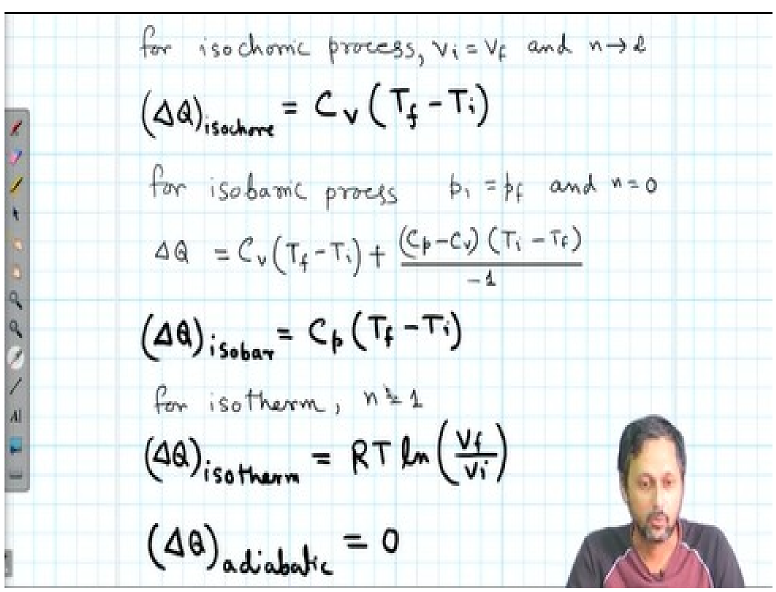

So, one is the isochoric process for example where the volume does not change and if we look at the general form then n tends to infinity general form of a polytrophic process, we can say that n tends to infinity and we have Q delta Q for an isochoric process is simply C v times T f minus T i. So, what happens to the second term? If we put n is equal to infinity here this term becomes 0 in so that is why the second term does not contribute anything in the expression.

For isobaric process p i is equal to p f and n = 0. So, if we put n = 0 here in this particular expression and please remember R = C p - C v for an ideal gas so we can write delta Q = C v T f - T i + C p - C v T i - T f divided by minus 1 n = 0. So, after simplification we simply get delta Q isobar is equal to C p T f minus T i. Similarly, for isothermal process which is n = 1 delta Q is simply RT ln v f by v i because for this very reason that C v dT, so the other term which is present here C v dT rather C v delta T for isothermal process delta T = 0.

So, there is no contribution from this term in an isothermal process. So, the work done is equal to heat exchange which is RT ln v f by v i and for adiabatic process of course by definition of adiabatic process there the heat flow = 0, so we always have delta Q adiabatic = 0. So, these are the four relations and of course we have already done we have computed four work done in these four different cases for hydrostatic systems that is ideal gas assembly or a fluid assembly.

And this combining that previous result which we have discussed with this one we actually can compute the change in internal energy for any types of quasistatic process for an ideal gas system. So, we will do that example towards the end of this lecture.

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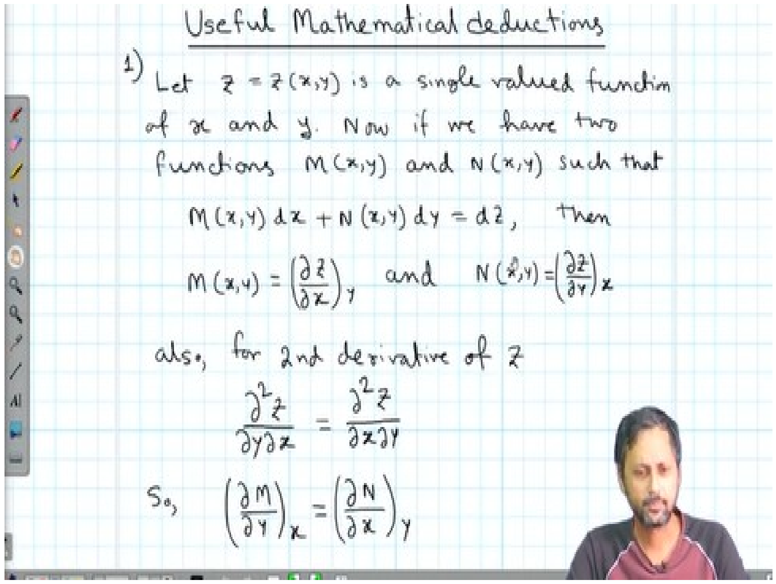

Now let us move on and before we go to the last theoretical discussion on this week's content, we would need to know little bit of or rather we need to revise little bit of mathematics. So, the mathematics we are going to do now is something that you all know only thing is we will just put it in into a slightly different form which is useful for the upcoming discussion. So, let us assume z is a function of x and y.

So, that we can write z is equal to z x, y and z only condition is z is a well-behaved function which is a single valued function of x and y. Now if we have these two other functions M and N which both are functions of x and y such that M x, y M dx + N dy = dz then we must have M = del z del x, y and N is equal to del z del y, x. This is because if we take right z is equal to z of x and y and compute the full derivative so dz will be del z del x at y constant dx plus del z del y at x constant dy.

So, we all know this is the chain rule of partial I mean computing differential and so for the second derivative of z please remember please recall that del 2 z del y del x = del 2 z del x del y so del x and del y or del del x del del y these operations are commutative in nature. So, that is for any well-behaved function this is a valid form so we can take it one step forward and we can write del M del y x will be equal to del N del x y which is very intuitive from here.

So, please keep this in mind that if we can write dz in this particular form then that means M = del z del x y and N = del z del y x. So, we will be using this relation over and over again so it is very simple I know but better to keep this in mind.

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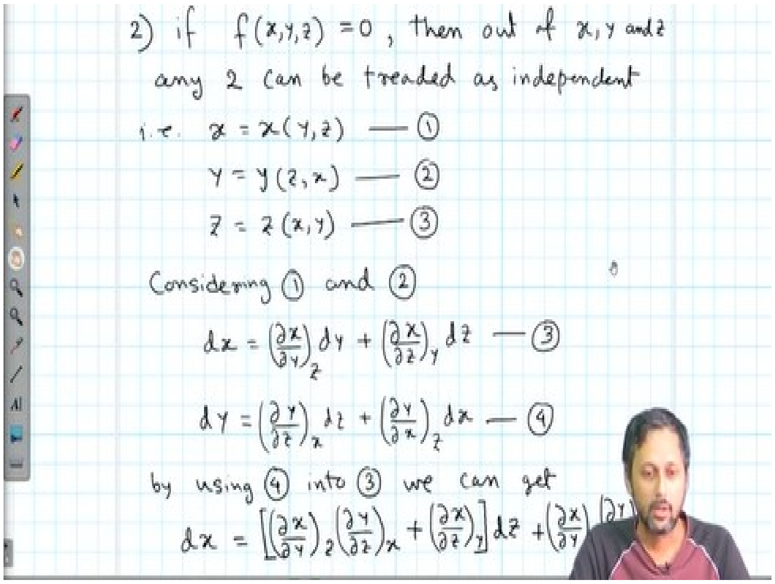

Next if we assume that there is a function f well behave function f x, y, z is equal to 0 exist so that means if x, y and z they are connected by some relation that is given by this function particular function small f. Then out of x and y and z we can choose any two as independent variables and once again this is nothing new because in one of the earlier lectures, I think in the last class itself we have used this relation.

That if we have a equation of state and if we have pressure volume and temperature connected by this equation of state we can treat any two of those as independent variables. And I have just generalized it here writing it x, y, z and instead of equation of state I have written f of x, y, z. So, considering the first two let us say now we will take it one step forward from here and we will derive two very useful relations from starting from this very simple consideration.

And this, please remember this is very general. So, it you can apply it anywhere in mathematics anywhere in physics or wherever applicable when this particular criteria is satisfied that f has to be a well behaved function and completely derivable function. So, and of course x, y, z also has to be continuous throughout the space and differentiable. So, this is the only criteria so if we just consider these first two relations, we can write dx is equal to del x del y z dy del x del z y at y constant dz.

And similarly, we can write dy is equal to del y del z at x constant dz and del y del x and z constant dx fantastic and up to this we call this relation three and four we just took relation one and two, I should actually name it four and so let us do this, so this is there is a 3 here already so I should name it 4 and 5. So, from putting now this will be putting using 5 into 4 we can get so basically what we do is we just take the value of dy from here and substitute it there.

So, if we do that and see the first term will be del x del y z multiplied by del y del z x times dz and there will be another term which will be basically del x del y z times del y del x z whole multiplied by dx. So, we just separate this out and there is already a desired term associated here. So, what do we do? We simply rearrange this and write dx is equal to del x del y z del y del z x plus del x del z y times dz and del x del y z del y del x z times dx.

And we have to name it relation 6 here so this is our relation 6. Now what we have to do now is once again very simple we have to just equate the coefficients of dx and dz from both sides. Now dx has a coefficient of 1 in this left-hand side. So, on the right-hand side this quantity this the multiplication del x del y z and del y del x z has to be equal to 1. Similarly, the coefficient of dz on the left-hand side is equal to 0 and this whole thing inside this bracket here has to be equal to 0. So, we first focus on this quantity.

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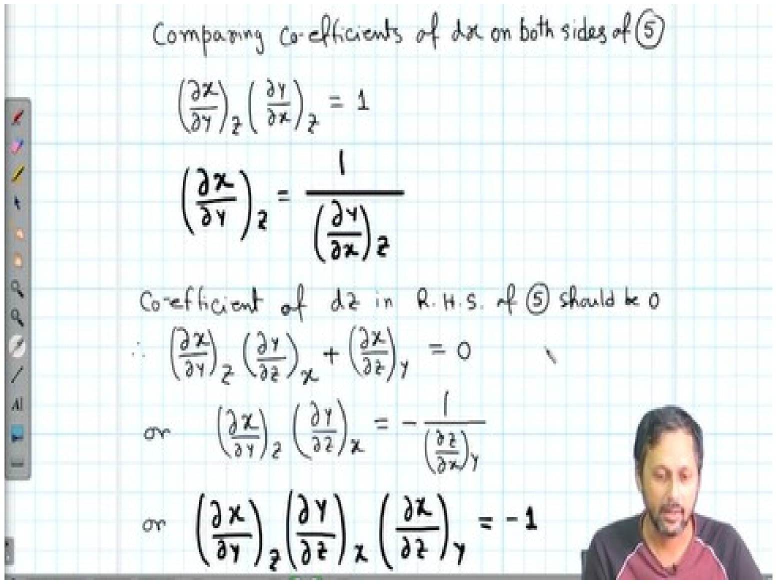

And we write del y del x del y z and del y del x z times del y del s z is equal to minus 1 which gives you del x del y z is equal to 1 over del y del x z. So, it is just saying that del I mean you can actually invert the differential or invert the whole differential which will be essentially the invert of the differential or rather you can invert the whole differential which will be the inverse differential. So, del x del y z is equal to 1 upon del y del x z.

Now if we concentrate on this term over here as I have said this will be equal to 0. Now what do we do? We write we keep these two terms here take this term on the right-hand side which is minus of del x del z y using this relation, what we can do is? We can simply invert it and we can write 1 upon del z del x y. Now all we have to do is we need to take this on the right hand on the left-hand side once again in the numerator just a simple multiplication.

And we get this very useful relation which is once again like a chain rule please follow my pen marker here del x del y z is equal times del y del z x times del x del z y is equal to minus 1. And this is a very general relation for any function and we started off with a function f x, y, z I mean we so there has to be a relation of the form f of x, y, z that exists. But other than that, there is no other need here mathematically that is the only need we have.

And this is for and this is true for any three functions or any three parameters x, y, z which are connected by a function f x, y, z by this type of a relation. So, now let us apply this in the case of thermodynamics and see how useful it can be.

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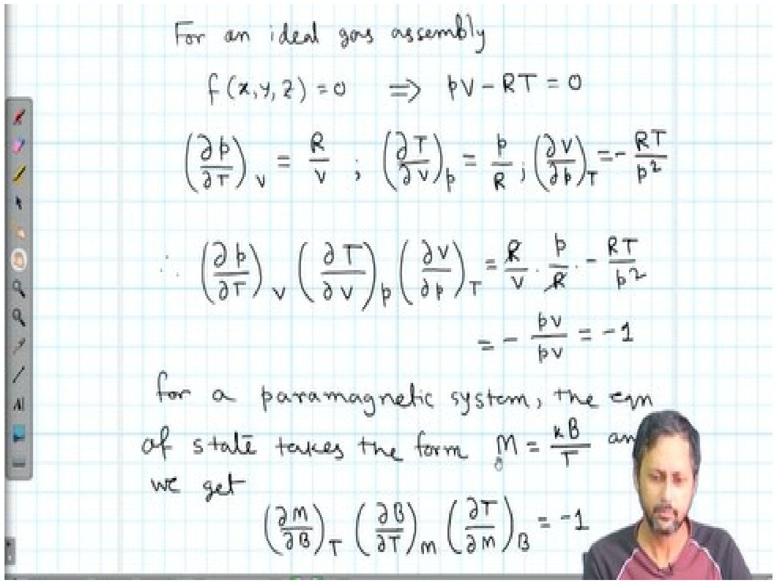

We will use it somewhere else but let us first verify whether this is true. So, we will start with the ideal gas assembly now f x, y, z is equal to 0 is equivalent to pV = RT which we can write pV - RT = 0. So, x, y, z will be pV and T or any other order we can define x as T, y as v, z as p or anything else but there has to be these three parameters. So, x, y, z is equal to pV and T. Now let us so our derivative for example this derivative del x.

So, all we have to do is we have to have this type of a relation. So, del x del y computed at z constant multiplied by del y del z computed and x constant and del x del z computed with y constant, the product has to be equal to minus 1. So, let us compute del p del T v for from this equation which will be R by v del T del v p which will be p by R and del v del p t which is minus RT by p square.

Multiply this 3 and you will see you will get RT divided by pV and write RT = pV because this is the master equation we have or equation of state we have and then we get miuns 1. So, for an ideal gas assembly del p del T v times del T del v p times del v del p T is equal to minus 1 and you can try it yourself you can do the we can prove the exact same thing for a Van der Waals assembly as well. So, the calculations are slightly complicated it is just one line there will be you might have to write two lines or three lines sometimes.

But finally, you will get the exact same result for that equation of state as well. And that has to be because at this point it does not matter whether it is Van der Waals equation whether it is ideal gas equation or Berthelot's equation or Saha and Bose's equation as long as there exists a valid equation of state that connects pV and T, we should have this relation in general. Similarly, for a paramagnetic system where there exists an equation of state of this form M is equal to k B by T which is the well-known Curie law from para magnetism.

We should have del M del B T where, B being the magnetic field and M being the moment T is the temperature we should have del M del v T times del v del T M times del T del M B is equal to minus 1.

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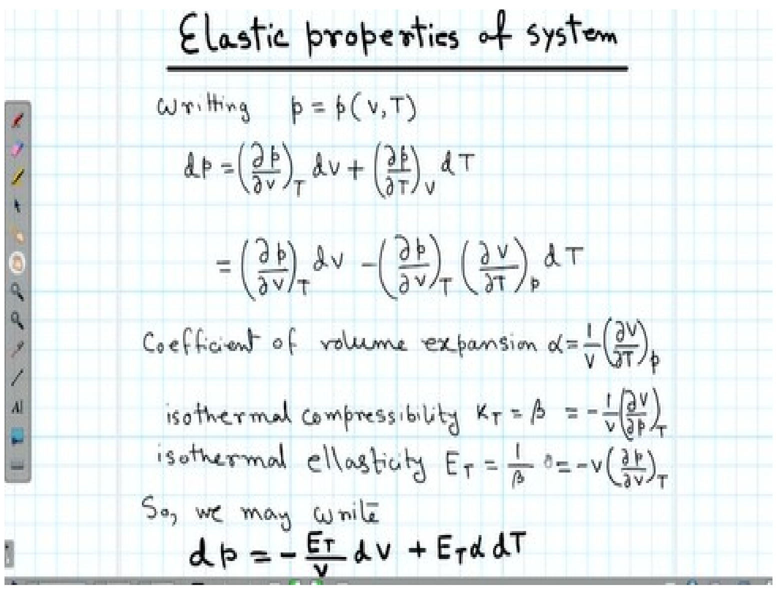

Now with this mathematical background let us go into the elastic properties of a system. So, what are the elastic properties of a system? We will come to that in very shortly. So, let us start with this simple equation we write p is equal to p of v, T so, we express pressure as a function of volume and temperature. So, we can write dp = del p del v T dv + del p del T v dT which once again the second term del p del T v will be del p del v T - of del p del v T times del v del T p because of this relation.

So, what we did here is del p del T v you see del p del T v will be equal to minus of inverse of del T del v p which is del v del T p times del p del v T. So, exactly that is what we have done here. Now we define the elastic coefficients of our system. First of all, it is coefficient of volume expansion which is given by alpha and in some books, it is mentioned with gamma as well but typically we use alpha.

So, this is the expansion per unit volume under a temperature change at constant pressure. So, this is mathematically written as 1 by v del v del T p please remember there is only not a negative I mean it does not come with a negative sign which means as the temperature increases as at constant pressure the volume increases. But when we keep the temperature constant and we apply pressure then the volume decreases.

So, isothermal compressibility which is defined which is written with the symbol beta is equal to minus 1 minus 1 over v del v del p T that means so this is the unit volume per or you know change in volume per unit volume under application of pressure P at a constant temperature T. So, as the volume changes negatively with increasing pressure we have a negative sign here and the opposite of isothermal compressibility is called the isothermal elasticity which is written as E T which is.

So, isothermal compressibility sometimes written with K T or K suffix T sometimes with beta and isothermal elasticity is the opposite of the isothermal compressibility written with E T which is basically 1 over beta is equal to minus v times del p del v T. So, keeping this in mind what we can do is we can go back to this expression for dp and we can write this as minus ET by v dv plus E T alpha dT we can do that.

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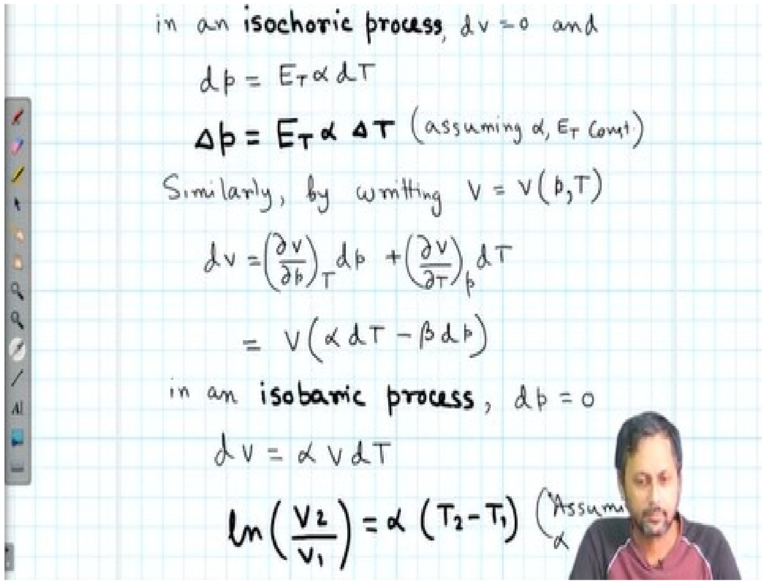

And if we are talking about an isochoric process in which the volume does not change then we have then this dv term goes to 0 and we have dp is equal to alpha E T alpha dT. So, the total pressure change delta P is equal to E T alpha times delta T where delta T is the temperature change. So, this is a linear correlation with pressure change and temperature change. So, first of all why you I would like to do that?

I mean so far, we did all the calculations without considering these elastic coefficients, why all of a sudden, we need this elastic coefficient? The answer is using elastic coefficients are easy use of elastic coefficient makes life more convenient. Because you know these are already measured for different types of standard gas assembly different specially for solid materials, we will come to that in a moment.

For solid materials we have these elastic coefficients well I mean measured with good accuracy and they are well documented. So, if we have these things in hand the life becomes lot easier. Similarly, we can write V is equal to volume is equal to function of pressure and temperature and dv = del v del p T dp + del v del T p dT which once again after some rearrangement you can look at it yourself, I am just going through the final result dv is equal to v times alpha dT minus beta dp.

So, how does this come? You have to just carefully look in I mean actually it is very simple you just have to divide 1 over v here 1 over v here multiply with v and you get this relation. Now for a isobaric process we have dp is equal to 0 and we have dv is equal to alpha v d T which on integration gives ln v 2 by v 1 is equal to alpha times T 2 minus T 1. Now during both these cases both these integrations we have assumed that alpha the elastic coefficients alpha T in this case and alpha.

In this case they remain constant during the process which once again is a first order approximation and in reality, it also changes slightly the elastic coefficients also changes slightly with the condition every time you change temperature or pressure or tension in a string for example the corresponding elastic coefficients keep changing. But in general, if the changes are changes can be negligible this integration becomes straight forward.

And we have one relation here and for isochoric process and one relation here for isobaric process which is given by this.

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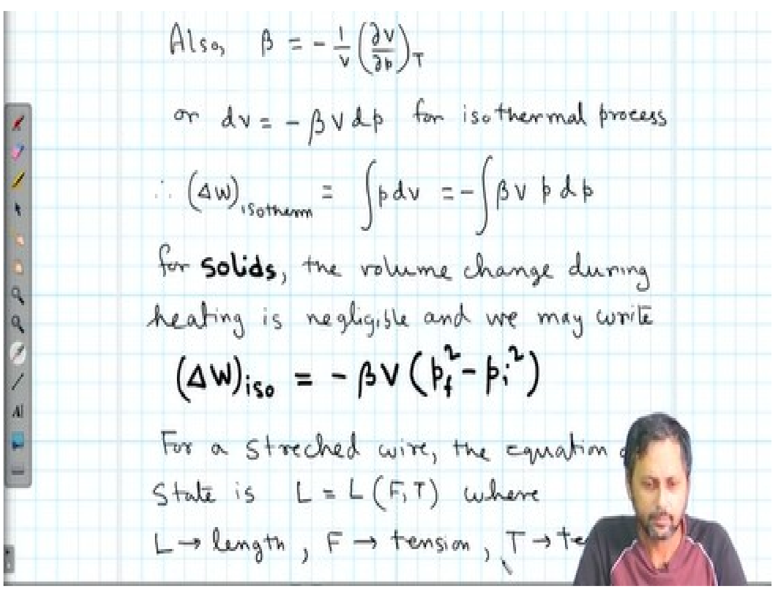

Now please remember beta is equal to minus 1 over v del v del p T. Now for a isothermal process we can write you know we can instead of writing this partial differential we can simply write it as a complete differential and we can write dv is equal to minus beta v dp that is for an isothermal process. So, for an isothermal process the work done which is given by p dv is equal to minus instead of dv we simply write minus beta v dp and there is a p already here.

Now for solids what happens I mean this relation I mean writing this for a you know hydrostatic system might not be that advantageous. But for solids typically the volume change is really small when even if you apply very high pressure the volume change you get as compared to a hydrostatic system, I mean a gas assembly or a fluid the volume change becomes really negligible.

So, we can neglect this volume change we can consider v is equal to constant beta is equal to constant during the process and we can compute the isothermal work done as minus beta V p f square minus p i square. So, this is for a solid material. For a stretched wire let us say we have a wire stretched between two points the length L can be expressed as a function of the tension F and the temperature T, I mean in some places temperature is written with theta and tension is written with T.

But I like to keep temperature as T so that is why I have written tension as F which is typically used for force not tension. But for our convenience I think we if we just use this notation that f is the tension and T is the temperature this will be good.

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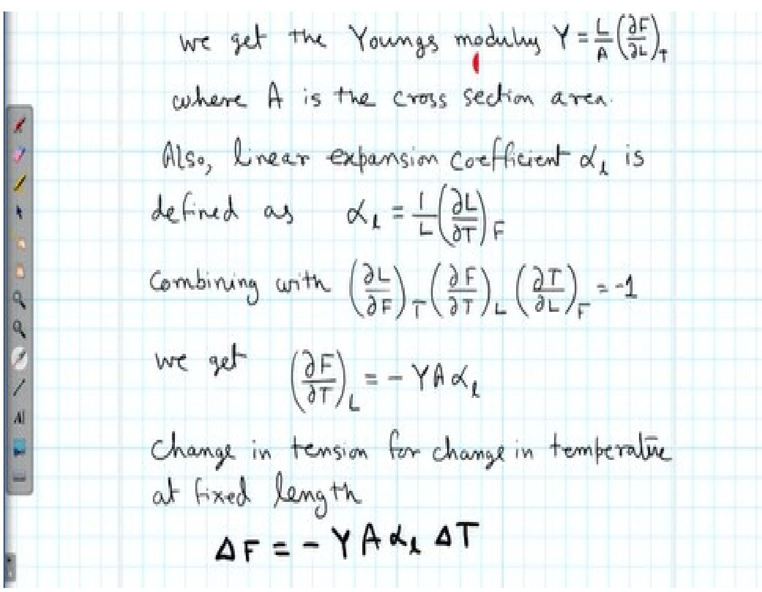

So, the young's modulus which is Y is equal to L by A del F del L T where A is the cross-sectional area, so we can if we know the equation of state, we can always compute the young's modulus. Similarly, there is a linear expansion coefficient which is given by alpha l just to separate it out from the bulk expansion coefficient, L stands for the linear is defined as alpha l is equal to 1 over L del L del T F.

Combining with this relation for a stretched where which will be del L del F T at times del F del T L times del T del L F is equal to minus 1 we get del F del T L is equal to minus YA alpha l where Y is the Young's modulus, A is the what you call the cross section, alpha is the linear expansion coefficient. So, the change in tension with temperature for change in when we have a fixed length when we change the temperature then the tension changes following this particular relation.

And we have delta F is equal to minus YA alpha l delta T. Please remember there is a minus sign here that means if we increase the temperature the tension decreases, if we decrease the temperature and the tension increases. And this is very obvious because when we are increasing the temperature due to this linear expansion term the wire stretch, I mean in the length increases of course the tension has to decrease.

As the temperature drops it becomes the wire shrinks become shortens in length and we have an increase in the tension so this is one relation that we get directly from here.

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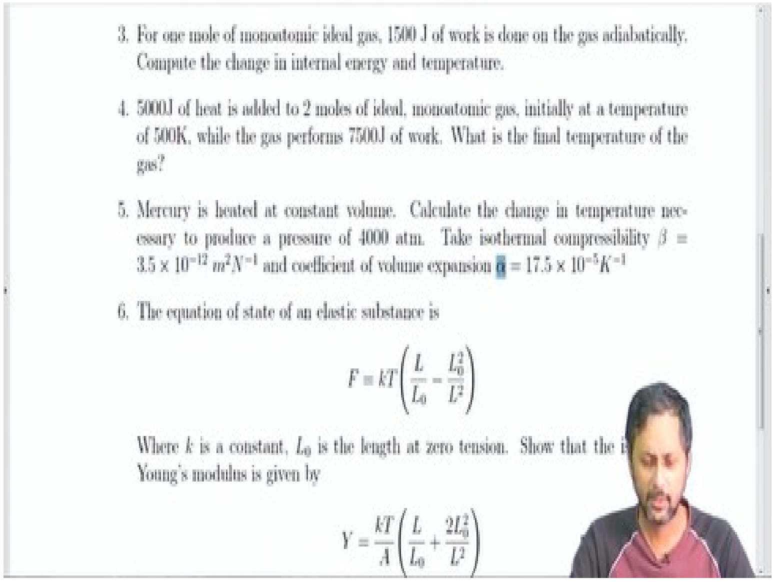

So, let us move to the last part of this week's lecture which is where we will be solving remaining problems these four problems. So, the first the third problem for the first two problems we have already solved. Third problem for one mole of monatomic ideal gas 1500 Joules of work is done on the gas adiabatically and compute the change in internal energy and temperature.

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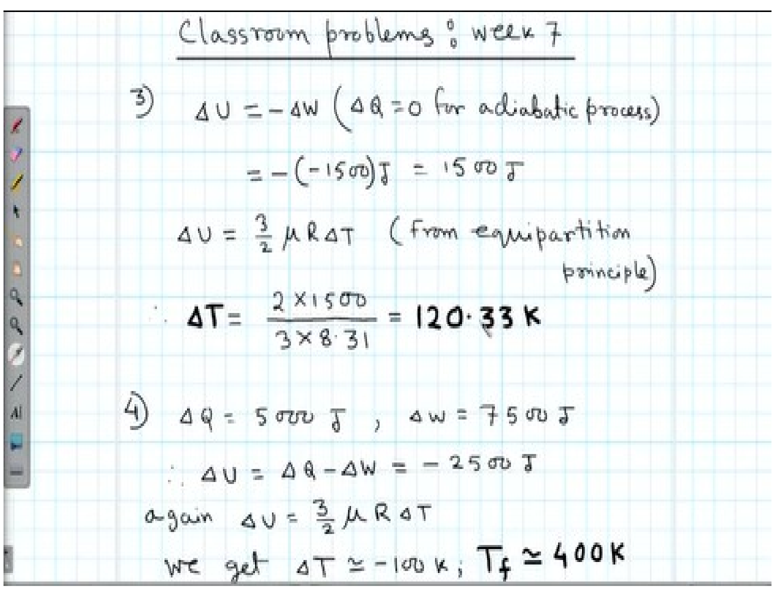

So, this is a very easy problem all we have to do is we have to keep in mind that delta U is equal to minus delta w in case of an adiabatic process because delta Q is equal to 0. So, delta U is equal to minus of minus 1500 Joules. Minus because the work is done on the system not by the system so, on the system work done is always negative so we have delta U = 1500 Joules. So, as a result of compression adiabatic compression delta U has increased by 1500 the internal energy has increased by 1500 joules.

Now for ideal monatomic gas delta U is equal to 3 half mu R delta T that is from equipartition principle, so from there we can directly compute so we can in this case mu = 1 so we have delta T is equal to 2 third into 1500 divided by 8.31 which is 120.33 Kelvin. The second problem I mean the fourth problem is also kind of similar where we have 5000 joules of heat is added to two moles of ideal monoatomic gas initially at a temperature of 500 kelvin while the gas performs 7500 Joules of work.

So, gas performs work so that means work done by the gas the question is what is the final temperature of the gas. So, once again this is a very straightforward problem here, we have to keep in mind that delta Q = 5000 Joules delta w = 7000 Joules so delta U = - 2500 Joules. So, as a result the internal energy decreases and as delta U = 2 third mu R delta T we get a delta T which is also negative because delta U is negative delta T is equal to minus 100 Kelvin approximately.

I mean its 100 minus 100.2 if I am not very wrong. Then initial temperature was 500 kelvin the final temperature is approximately 400 Kelvin for the gas assembly. The next problem the problem number 5, mercury is heated at constant volume calculate so it is a isochoric process calculate the change in temperature necessary to produce a pressure of 4000 atmosphere. Take isothermal compressibility beta which is 3.5 into 10 to the power minus 12 meter square per newton. And coefficient of volume expansion alpha is equal to 17.5 into 10 to the power minus 5 per Kelvin.

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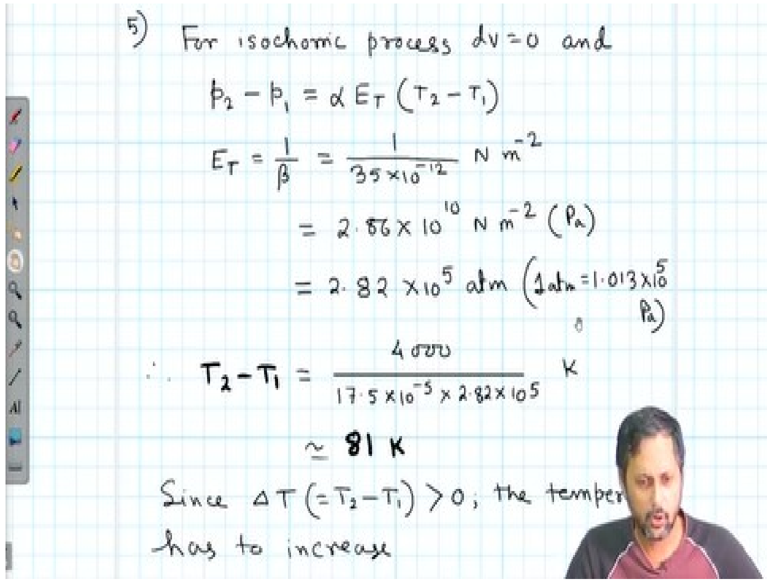

So, if you recall the isochoric for isochoric process, we have delta p is equal to alpha E T delta T all we have to do is we have to put the value of alpha and delta T and alpha and E T and delta T, delta T is given as alpha and E T and delta p is given as 4000 atmospheres. So, first thing is beta is given so we have to compute alpha T by taking 1 over beta which is 1 over 35 into 10 to the power minus 12 newton per meter square which is the units of pressure by the way.

So, 2.86 into 10 to the power 10 newton per meter square which is in pascals. What we can do is? We can simply convert this into atmosphere by dividing it by 1.013 into 10 to the power 5 pascals, so it will be 2.82 into 10 to the power 5 atmosphere. So, delta T = 4000 divided by 4000 is p 2 minus p 1 divided by alpha which is 17.5 into 10 to the power minus 5 and E T in the atmospheric unit which is 2.82 into 10 to the power 5 kelvin which is approximately 81 kelvin.

And since we see that delta T which is T 2 minus T 1 is greater than 0 that means the temperature has to increase which is obvious. If we want to increase the pressure on a substance, we need to increase the temperature. So, this is also coming out from this calculation. So, we come to the last problem of the day which is the equations of state of an elastic substance is given by F is equal to kT L by L 0, L 0 square by L square where k is a constant L 0 is the length at 0 tension.

And we need to show that the isothermal young’s modulus is given by kT by A L by L 0 plus 2 L 0 square by L square. Also, there is a part where A is the cross section also find the value of young’s modulus at the limit of 0 tension. So, let us first come start from this equation of state here and compute the young's modulus.

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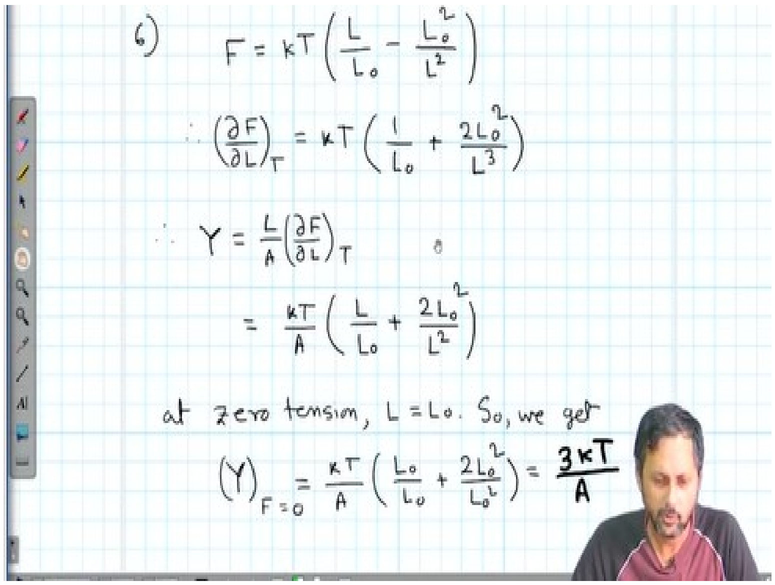

What is the young’s modulus? Y is equal to L by A del F del T del F del L at constant temperature so F is given if we compute del F del L at constant temperature which will be equal to k T times L by L 0 plus 2 L 0 square by L cubed. So, it is a simple differential and immediately we put this value here we get Y is equal to kT by A L by L 0 plus 2 L 0 square by L square which is very, very easy. Now for the value of Y at 0 tension we need to put please remember at 0 tension L is equal to L 0 which is already given.

So, all we have to do is in this expression we have to put L is equal to L 0 doing that we get Y at F is equal to 0 which is 3 kT by A. So, this concludes the week's lesson and this week we have been discussing first law of thermodynamics and all the necessary relative necessary discussion associated with that. Next week we will be starting with cyclic processes. We will be discussing about engines, heat engines and from there slowly we will go into the second law of thermodynamics. Till then good bye.