# Thermal Physics

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## Lecture-12
## Topic-Viscosity: Transport of Momentum

Hello and welcome back to another lecture of this NPTEL course on thermal physics. Now in today's lecture, we will continue from where we have left it in the last class. So, that means we will be discussing transport properties within the lights of mean free path concept. Now we have briefly I mean we have already described what is the meaning of transport properties, we are going to do it in more details very soon.

But let me quickly remind you that we basically assumed that transport properties are present in a system in a fluid which by fluid here we mean kind of an ideal guess assembly, when there is a gradient. So, that gradient could be a velocity gradient, which could be a temperature gradient, this could be a density gradient. But the concept is whenever there is a gradient that means in some part of the gas assembly or fluid assembly as we might call it, there is an excess of some physical quantity.

Whereas, on the other part there is a reduced amount of that particular quantity. Now the system wants to drive itself towards equilibrium, so that is why the molecules or that whatever quantity is we are talking about the area where it is in excess from there it will flow towards the area where it is less in quantity reduced quantity. And this transport will be taking place by means of molecules of the fluid assembly or the gas assembly. So, this was the model that we have discussed.

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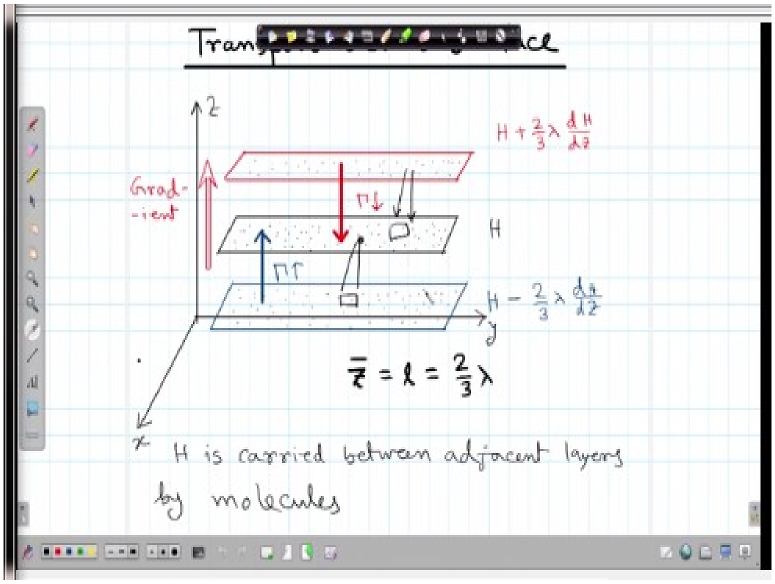


Now let us focus on our system. So, we have 1 layer which is our reference layer and the quantity of interest we do not know what it is, it could be any of the above mentioned quantities just before I have mentioned 3 different quantities. So, let us call it H, H is the physical quantity. Now above this reference layer we have a hot layer, what do we mean by hot layer? Hot layer means where the quantity H is has is present in greater amount as compared to the reference layer.

And then below here we have a cold layer which is where the quantity H is present in a lesser amount as compared to the reference layer. Now what is lesser and what is greater? And this is given by this relation that H is equal to on the upper layer, this is the quantity is present in an amount of H is equal to two third lambda d H dz, where d H dz is the gradient present along positive z direction.

So, in a similar manner in the lower layer it will be present by an amount H - two third Lambda dH dz. Now, this description is true for any 3 adjacent layers, we can take any one of them as the reference layer, one on topper as the hot layer one and or bottom as the cold layer, but why this quantity two third lambda? This is because this two third is the average distance which is given by either z bar or l along z direction by to which a molecule from one layer can be travelled uninterrupted to the next layer.

So, we have chosen 3 layers where the interlayer separation is two third lambda, lambda being the mean free path. And we have proved it formally in the last class; we do not want to do it again. We just mentioned that result here that z bar is equal to l is equal to two third y. So, H is the quantity is carried between adjacent layers by molecules. So, let us first finish this calculation whatever we are doing and then we will be talking more about what could be this H and how is it carried between layers by molecules.

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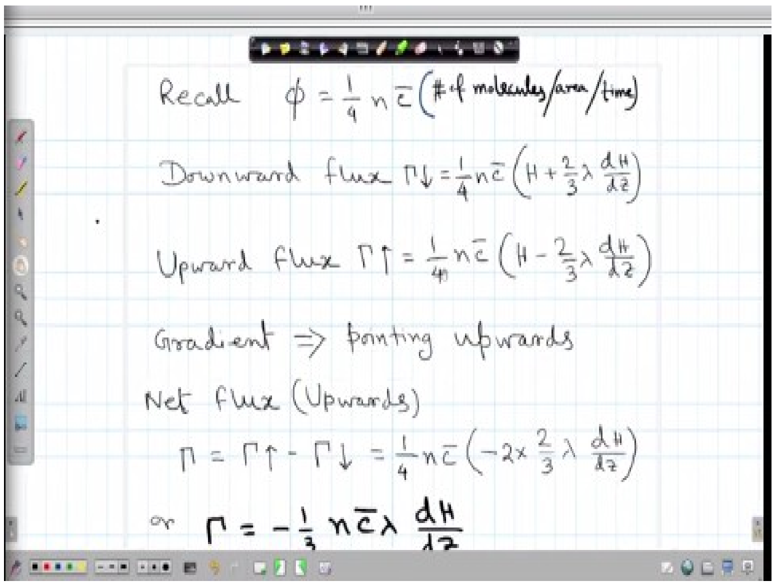


So, let us move on from here. So, now we recall the result that which we derived in one of the earlier classes, I think lecture 9. That the flux phi is 1 4 nc bar, what is the flux? This is the flux, that is this is the flux or rather when I say flux that already implies number of molecules per area per time. So, basically it is the number of molecule per unit area per unit time which is given by 1 4 nc bar.

This is we have derived this for number of particles hitting the wall of the container. So, 1 4 nc bar is the number of or flux of molecule that is hitting the wall of the container in an gas assembly. But I mean it could be and if you remember the choice of the area was pretty arbitrary, so we could might as well take the area in one of these layers. So, for example, if we are talking about the reference layer here, so I can take this volume from where the molecules start from on this hot layer and eventually they end up hitting this unit area on this reference layer and the flux will be given by 1 4 nc bar.

Similarly I can take a volume element on this one and the surface reference area on this cold layer and the number of molecule that will be going from here to here, the flux will once again given by 1 4 nc bar. So, keeping that in mind we can compute the net downward flux and net upward flux of this system. So, what is given by the downward flux and what is called by the upward flux?

So, if 1 4 nc bar is the number of molecule that is hitting unit area per unit time and each of them carrying this physical quantity H with them then that net downward flux which is going from the so called hot layer to the reference layer will be simply given by 1 4 nc multiplied by H plus two third Lambda dH dz which is the value of the parameter or physical quantity H in the hot layer.

Similarly upward flux towards the layer. So, basically we mentioned here this is given by gamma down which is the downward flux, this is here and this is given by gamma up which is the upward layer. So, this is simply upward flux is given by 1 4 nc bar H minus two third lambda dH dz. So, basically the flux multiplied by the physical quantity H that is the characteristics of the cold layer. Now we see that the gradient is pointing upwards, what do you mean by gradient pointing upwards?

That means the quantity H is increasing in the upward direction, this H is less here, H is slightly more here, H is slightly more here. So, that means H is increasing in the upward direction. So, what we do is we compute the net flux along the gradient which is gamma is equal to gamma upward minus gamma downward along the gradient means upward. So, we take upward minus downward and this is you see if you just compute this subtraction one third nc bar H cancels out nicely from both the terms leaving behind minus of 2 into two third lambda dH dz.

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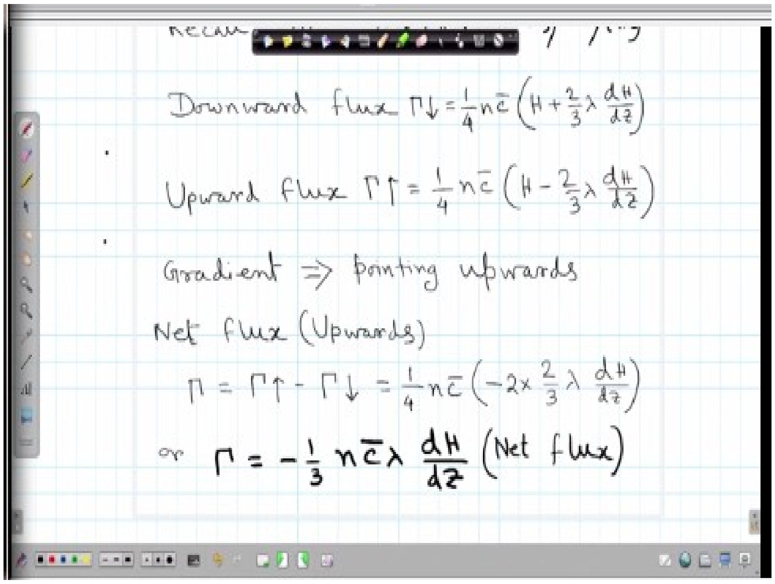


So, the net physical quantity or flux which is basically the net flux which is given by gamma is equal to minus one third nc bar lambda dH dz. So, this is a very generic expression for any quantity of interest, any quantity that is transported by the molecules between adjacent layers. Now up to this point we have been very, very, very generic, now it is time to be specific. What do you mean by specific? I think in the previous lecture or one of the previous lectures we have discussed that we are going to talk about 3 different physical quantity of interest or 3 different phenomena of interest.

One is viscosity which is the phenomena of shear friction between adjacent layers of a fluid. We are going to talk about thermal conductivity which is where if there is a temperature gradient present how the heat transformed from one layer to the other and 3rd is diffusion when there is what we call the density gradient present. So, up to this point H has been very, very generic, now it is time to specific.

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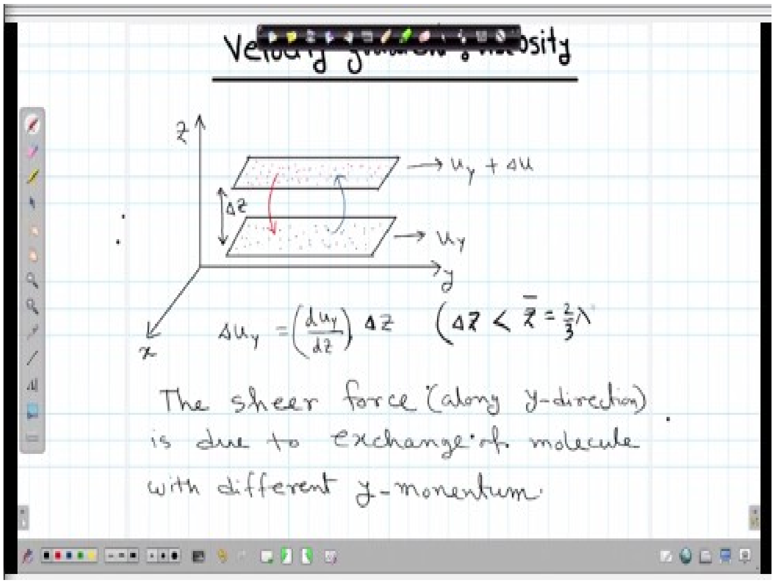


So, let us take up the case of velocity gradient or I should actually should not call it velocity gradient, I should call it the momentum gradient.

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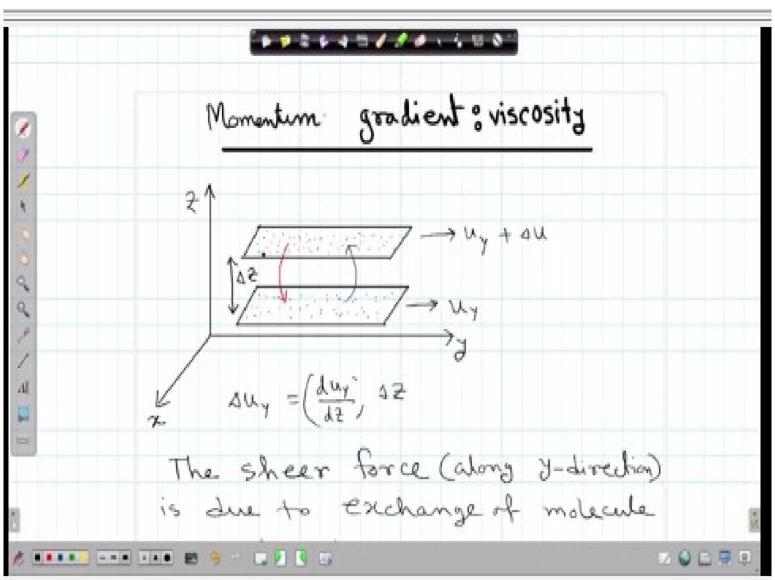


Because this will be a more accurate term for it. So, I mean velocity is actually okay, on second thirds velocity is actually okay, because here the masses are the same. So, velocity gradient essentially talks about a momentum gradient but that is okay, we will do it like this only. So, now in this case first of we have to understand what do we mean by viscosity, what is the origin of viscosity?

See, the viscous forces they are basically the forces between 2 adjacent layers. So, let us assume that they are 2 layers moving in the same direction, not the opposite direction, they are moving in the same direction but assume that the layer up here is moving slightly faster and the layer down here will be moving slightly slower. Now, what happens? The slow layer will eventually try to pull the fast layer let us assume that they are in contact with each other.

So, if the one layer is moving slower as compared to the other that layer will pull the fast layer backwards and from a similar perspective the fast layer will actually pull the slow layer forward. So, basically there is a frictional force that is acting in what you call the along the velocity, if the velocity is along this direction then the velocity gradient we are talking about is perpendicular direction.

Now let us consider this picture more carefully what we have drawn here. So, we have drawn once again 2 adjacent layers not 3, okay, so this layer is moving let us say x, y and z axis. So, both layers are moving along the y direction. Now when they are moving along the y direction one layer is moving with u y and the other layer is moving with u y + d y u y + delta u. Now delta u is given by d uy dz multiplied by delta z where delta z is the mutual separation between these 2 adjacent layers.

So, we are assuming that delta z we just make one more assumption, that this delta z is actually less than the average distance which is basically two third lambda we are talking about which is equal to two third lambda which we have discussed already. So, we have just taking any 2 adjacent layers between which the molecules can move freely. Now once again as the upper layer is moving faster we draw it as if it is the fast layer, so the molecules are marked in red and the lower layer the molecules are marked in black.

So, they are blue, so they are the slow layers. So, what happens because the molecules are free to move between these 2 layers because of this particular condition over here? So, hot molecules will jump into the cold layer and cold molecules will jump into the hot layer. As a result what happens is there will be a shear force which will act because shear force along y direction which will act because of the molecular exchange between these 2 layers, you understand this.

So, this will happen because there is a shear force acting between these 2 layers. So, basically the exchange of molecule with different y momentum make this shear force possible, so this is the origin of viscous forces. Now the question is what should be the quantity H that is being exchanged between these 2 layers? Obviously the quantity is nothing but the y momentum.

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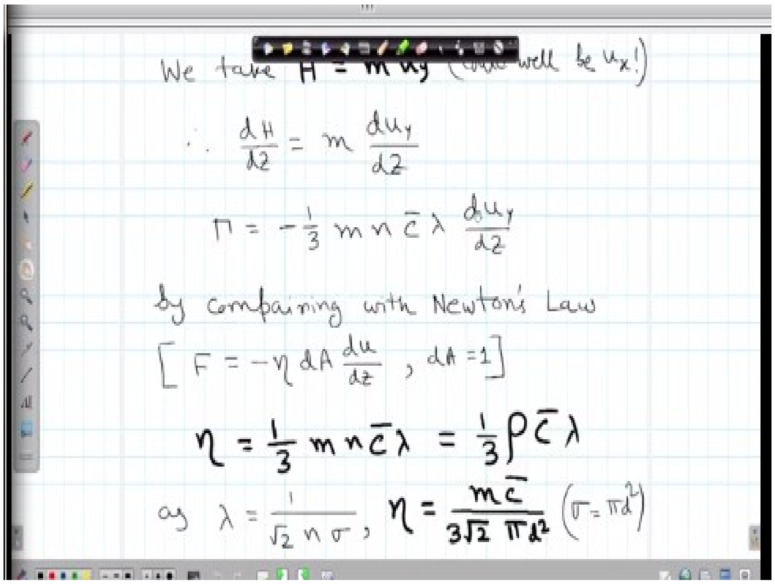


And y momentum is given by m u y, so H is equal to m u y. Now please understand that this u y could be might as well be u x I mean it does not matter whether it is an x or y momentum, only thing is the gradient it has to be perpendicular to the gradient, the gradient is present in along the z direction. So, the momentum could be anywhere between x and y, it does not really matter. So, dH dz is nothing but m du y dz, so the gamma which is given by minus one third nc bar lambda dH dz is this gamma is equal to one third m n lambda c bar du y dz, clear. So, this is the expression of gamma. Now we compare it with the Newton's law of viscosity, the shear's viscosity.

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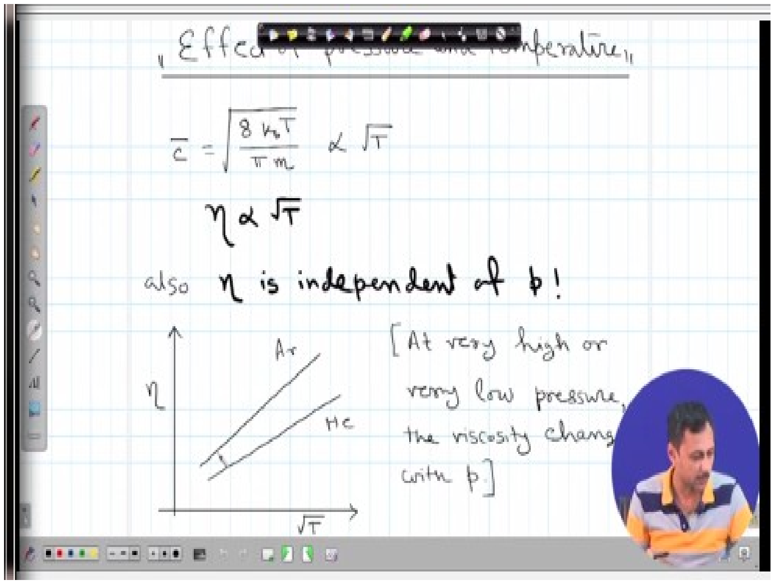


And we get F = -eta dA du dz but here we remember that we are talking about molecular flux already. So, we are discussing everything in terms of unit area, so we have to put dA is equal to 1. So, actually we can omit this term dA from this expression as well, but anyway we understand that dA = 1. Now if we compare this so the 1 to 1 correspondence actually tells you eta is equal to one third m n c bar lambda.

And once again remember that m n is nothing but the density rho. So, rho eta = one third rho c bar lambda. And also we have to put the expression lambda is equal to 1 by root 2 n sigma and we immediately see that eta is equal to m c bar divided by 3 times root 2 pi d square. So, basically what we did was we wrote sigma is equal to pi d squared here. And we see here eta in this particular expression over here and or also this particular expression over here eta is.

So, basically it is a macroscopic quantity which is macroscopically measurable through experiments, there are various experiments and I am sure in some of your lab courses you will have experiments where you have to measure the viscous coefficient eta. So, that can be directly correlated to the microscopic quantity like lambda in this upper equation here and d which is present in the bottom equation here.

So, these are the molecular quantity. So, d is a molecular diameter, lambda is molecular mean free paths which are directly connected to this macroscopic phenomena. So, once again the transport properties we are talking about they are correlating a macroscopic phenomena with the microscopic one. Now let us discuss the effect of pressure and temperature on the coefficient of viscosity.

So, let us go back to this expression once again we see in that expression the bottom expression down here we see m is a constant, the lower quantity everything is a constant that, given that we are talking about a given molecule, so m and d both are constant. So, n is proportional to c bar, now what is c bar here? c bar is we know is equal to root 8 K T by m which is once again is a function of temperature only and of course the molecular mass.

So, we can write that c bar is equal to proportional to root T, so we can write eta is also proportional to root T. Also we see all of this expression that eta is independent of pressure; there is no mention of pressure in any of this expression. So, experimentally it has been verified that eta is indeed proportional to root over of T. So, these are the some depiction of the experimental data of eta versus root T for argon and helium.

And for a wide range of temperature, the linearity of this plot is maintained. Of course there are for certain gases the linearity is not properly maintained or sometimes for very high and very high temperature we see that there is a deviation from this linearity. And that primarily happens, because you see if we go back to this expression over here we have sigma in the denominator. So, this entire expression is derived keeping in mind the hard sphere approximation, if the hard sphere approximation breaks then the T dependency of eta changes from root over of 2.

So, that happens and that has been verified experimentally as well. And this independent of pressure thing is also been verified experimentally by many scientists. And it has been found out once again that at very low or very high pressure, there is a notable deviation or notable change of eta with p but in the moderate temperature range starting from 10 to the power minus 3 millibar up to several atmosphere pressure, eta remains almost constant with varying pressure. Now what happens at very low or very high pressure?

At very low pressure, let us go back to this expression here. So, you see eta is actually proportional to lambda and also there is a number density. Look at the first expression one third m n c bar lambda. Now what happens is when the pressure is very low, the lambda becomes so large that it is almost equivalent to the length of the container, the dimension of the container. And if we reduce the pressure even further lambda cannot change because that is the upper limit of lambda.

Between 2 collisions, so when we are talking about when we say that lambda is compatible to the length of the container. So, basically we talk about a situation where the one collision is taking place on one side of the wall and it is going towards the it is changing direction and hitting the other wall in between there is no collision. So, that means even if we reduce the pressure further the lambda will not change, will not increase any further, so lambda remains constant.

Now what happens? When we reduce pressure further what changes? This quantity n changes, you see, let us assume that lambda is constant, now you reduce pressure further the number density changes, mass remains constant, c bar remains constant because c bar is just an average velocity for any Maxwellian distribution, it does not change. So, it changes only with temperature. So, keeping the temperature constant when we reduce from I mean when the pressure is already in a very low regime, we reduce the pressure further and eta starts decreasing with pressure because n decreases.

And next what happens at very high pressure? Very high pressure lambda becomes comparable with the intermolecular separation. In the last lecture or one of the earlier lectures where we solved problems I showed you that at STP the average intramolecular separation is of the order of nanometer, whereas lambda is at least 2 orders of magnitude higher than that. But when we start compressing the gas more and more, there will be a point when the lambda and the intermolecular separation or a the average intermolecular separation are comparable to each other.

Then the entire description breaks down whatever we have discussed so far that description breaks down and we have some variation of lambda or some. We start having some variation of eta with pressure which is totally outside the domain of this discussion here. But it is good to know that when lambda change or when eta changes with pressure and what are the reasons for it.

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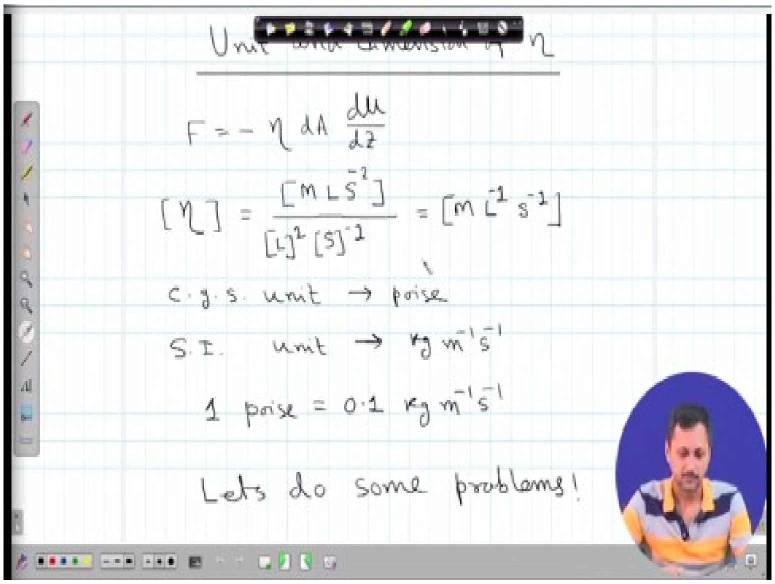


Next is we will take up the units and dimension of eta. So, let us first look at the dimensions, let us start from the very fundamental equation that F = -eta dA du dz. So, you see if we take eta we write eta is equal to omitting the negative sign F divided by dA divided by du dz then F has the dimension of M L S to the power minus 2, dA is basically that area which has a dimension of length square and du dz is the velocity gradient.

So, what is velocity gradient? Velocity gradient is you have velocity versus second divided by something is wrong here fine, whatever. So, it is du dz divided by, so basically the dimension will be length per second into 1 by length, so length, length will cancel out, it will leave you 1 by second. So, we have 1 by second in the denominator and finally the unit is M L minus S minus.

Now the c.g.s or the dimension is M L minus S minus. So, the c.g.s unit is poise and the SI unit is kg meter inverse second inverse and the conversion between the 2 is 1 poise is equal to 0.1 kg meter inverse second inverse. Also sometimes we express it as Newton's second meter to the power minus 2 which we will be discussing one, we will do some problems. So, just keep in mind that 1 poise is equal to 0.1 kg meter inverse second inverse.

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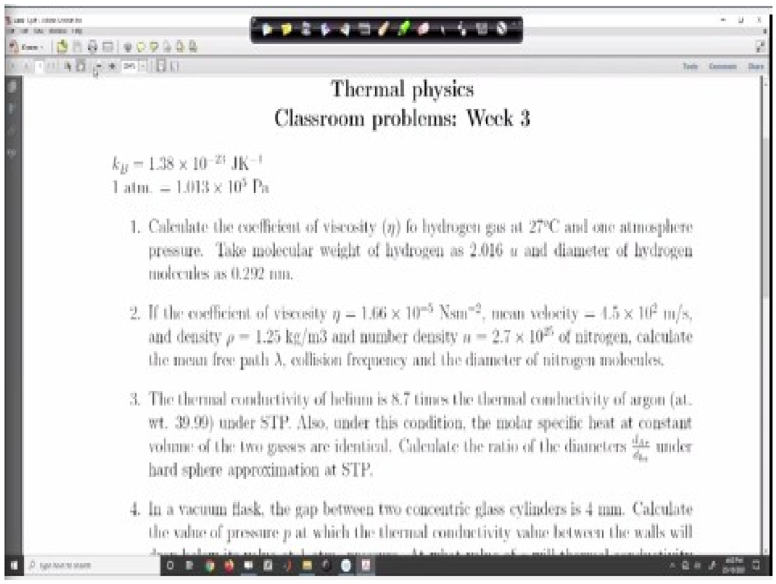


So, let us look at 2 quick problems from the classroom problems that of week 3 the first and the second one. The first problem is pretty straightforward where we have to calculate the coefficient of viscosity for hydrogen gas at 27 degrees centigrade and 1 atmosphere pressure. Take molecular weight of hydrogen as 2.016 u, u means the atomic unit and diameter of hydrogen molecule is 0.292 nanometer. Now this is a pretty straightforward problem.

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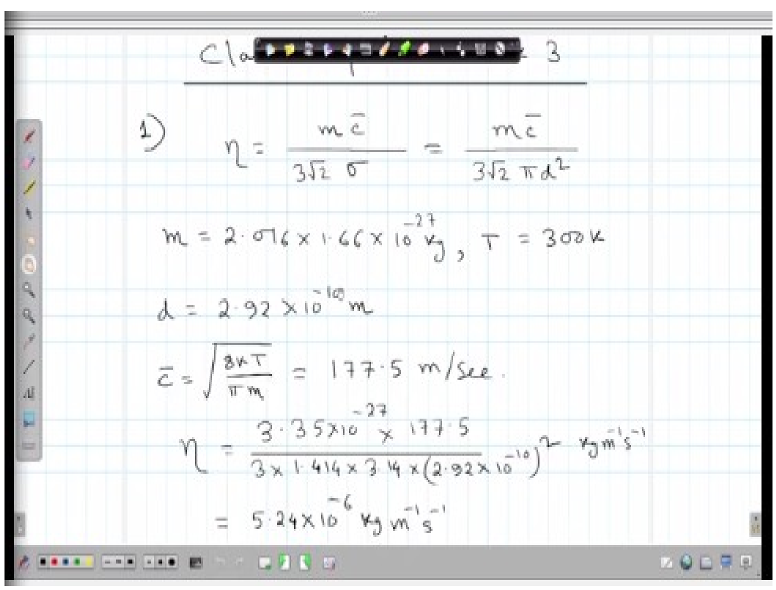


Here we simply have to start from this relation here which is eta is equal to mc bar divided by 2 root 3 sigma which can also be written as 2 root 3 pi d square, m is given as 2.016 atomic unit which is 2.016 into 1.66 into 10 to the power minus 27 kgs, T is equal to 300 Kelvin, d is equal to the molecular diameter is 2.92 into 10 to the power minus 10 meters, c is a simple calculation which is 8 kT by pi m, T is given we know that the values of kT and m, m is also given.

So, we simply have c = 177.5 meters per second. Next, we have to just put the numbers here; we have to put the value of, what we have to do? we have to put the value of m, we have to put the value of c **c** bar, we have to put the value of sigma, sigma is pi d square and finally we get 5.24 into 10 to the power minus 6 kg meter inverse second inverse. So, it is a very straightforward, very simple problem but better to practice such problems.

So, that we have a feel for the number. So, we see that for a typical gas, the viscosity is pretty low, the coefficient of viscosity I should not call it viscosity it is basically the coefficient of viscosity which is a measure of the viscous force is pretty low. Now for the second problem, if the coefficient of viscosity is see here the unit is 1.66 into 10 to the power minus 5 Newton second meter to the power minus 2 which is exactly the same as kg second inverse meter inverse, think why it is?

Just write this and express Newton in kg in terms of fundamental units you will get back the same unit. So, the mean velocity which is c bar is given by 4.5 into 10 to the power 2 meters per second, density is given and number density of nitrogen is also given. Calculate the mean free path, collision frequency and diameter of nitrogen molecule. So, we have to compute the mean free path from the viscosity data in this case which is once again very simple.

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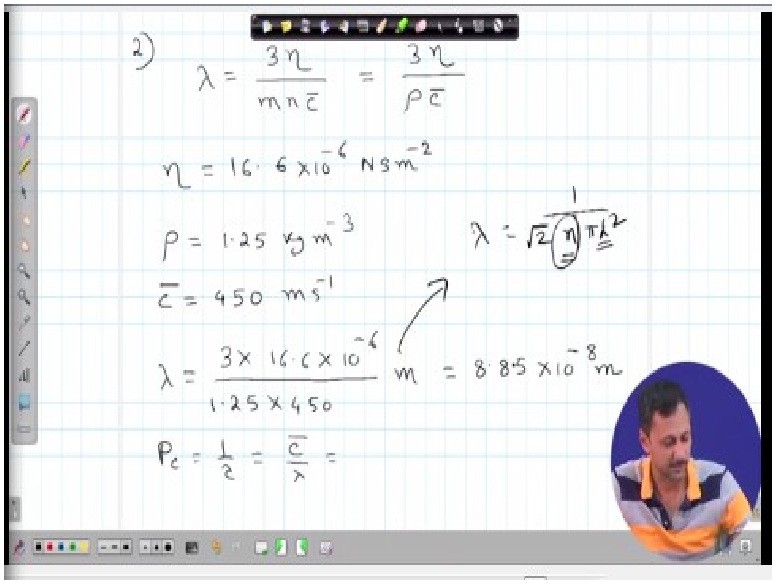


Because you see if you go back here the relation is eta = one third rho c bar lambda, rho is given c bar is already given, so lambda is equal to 3 eta by rho c. We apply that and we get 3 eta by rho, rho is m n c basically rho c, rho is already given. So, c calculating lambda is a very straightforward calculation and which gives you 8.85 into 10 to the power minus 8 meters. Now P c is the collision frequency which is 1 over tau which is given by c bar divided by lambda. I have not done the calculation here but you can always find out the number and as before I will given you in the final version of this notes, you will get the full calculation done.

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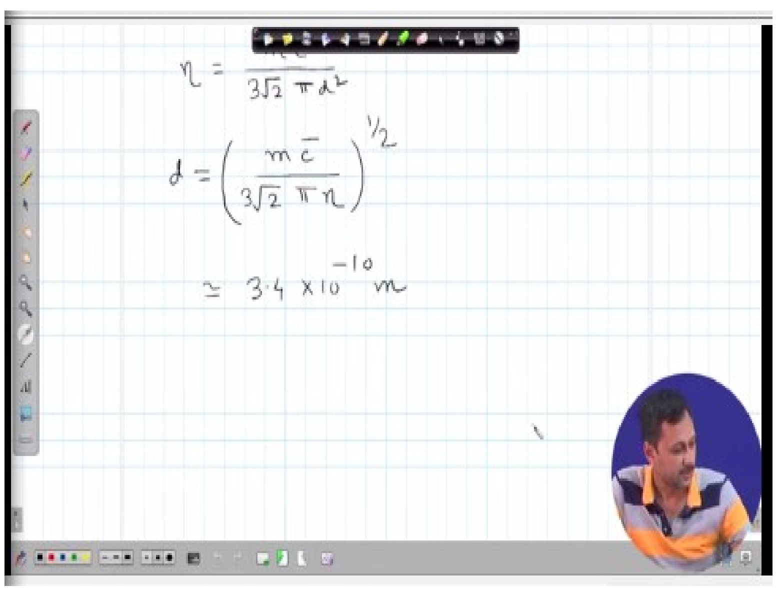


And for the second part you see number density is given, n is given. But I can show you that without n also we can compute this number. So, what you have to calculate the diameter of the nitrogen molecule. Now one way of doing it is once we have lambda we can simply from here we can write lambda is equal to 1 by root 2 n pi d square, use the value of n and compute d. This is straightforward, this we have done in the last week itself many times but I will show that there is another way even without knowing n that is the number density we can compute. So, I show you that this data is actually redundant, see. So, eta is is equal to m c bar divided by 3 root 2 pi d square, 3 root 2 sigma. So, that will give you d is equal to m c bar divided by **2** 3 root 2 pi times eta whole to the power half.

Now everything is given, we have c bar, we have actually my mistake m will be necessary, n will be necessary because without n we cannot compute m. Because what is given here is rho, so if we in order to my mistake. So, basically this and this technique is basically the same is just writing it another way, my mistake. So, we finally get 3.4 into 10 to the power minus 10 meters. So, what we are learned in today's class, we have started from the very fundamental concept of mean free path in this for last 2 lectures.

And we have learned how to compute the parameter of interest or the transport property of interest. In this case, for today's lecture it has been the shear viscosity but also we have derived the general expression in terms of the quantity H which enables us to compute the other 2 quantities of interest that is thermal conductivity and diffusion coefficient also. So, we will do it take it up in the next class onward, thank you.