Experimental Physics I
Prof. Amal Kumar Das
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 25
Theory Regarding Moment of inertia of a flywheel

So, today I will discuss about the determination of Moment of inertia of a flywheel. So, moment of inertia you know this what is the significant of moment of inertia and how to calculate moment of inertia of regular shaped body?

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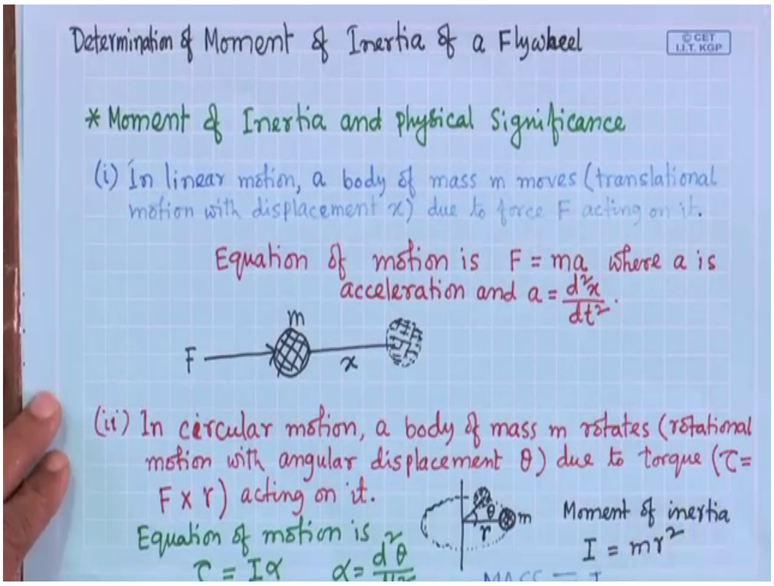

So, moment of inertia and its physical significance you know in linear motion a body of mass m moves that is basically translational motion with displacement x due to force F acting on it. So, equation of motion is F equal to m a, where a is the acceleration and a equal to the differential form d 2 x by dt square right. So, frequently we use we write equation of motion is basically m d 2 x by d t square equal to that force applied on the body.

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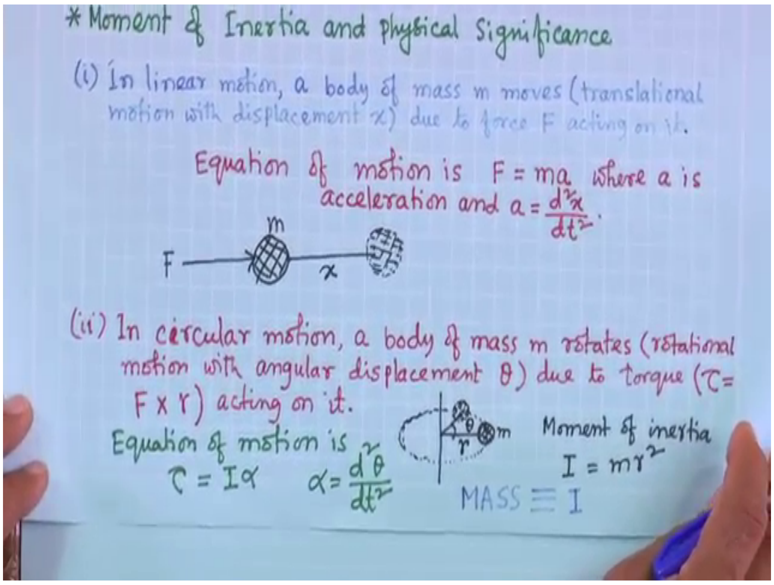

So, now in circular motion a body the same body of mass m rotates that is rotational motion with angular displacement theta due to torque. So, here linear motion and in this case circular motion of a body of mass m; of a body of mass m ok. So, the here displacement this linear displacement ok so, this we present with x and in this case angular displacement so, in case of angular motion or circular motion so, that is theta. So, due to torque in this case, it is force due to force and in this case due to torque and relation between force and torque is basically tau or tau equal to F cross r so, this torque acting on this body.

So, in this case equation of motion, this similar equation of motion F equal to ma here equation of motion is tau equal to I alpha; tau is torque, I is moment of inertia of the body and alpha is the acceleration angular acceleration. So, that is that basically change of rate of change of angular velocity. So, an angular velocity is again rate of change of angle with time.

So, if you compare these 2 equation F equal to ma and tau equal to I alpha. So, here a is acceleration here also alpha is acceleration here F is force here equivalent force is this tau. Now then you can see this m is equivalent to I, mass m is equivalent to I. So, whatever the function of mass in case of linear motion, in case of cancellation motion, this in case of circular motion, rotational motion this I plays role in same way as mass plays role in case of this motion. So, whatever m in translational motion the same function or equivalent function does by this moment of inertia. So, you know the definition of moment of inertia is basically mass into distance square.

So, rotational motion means something rotate with respect to a centre or with respect to an axis. So, here just we see this mass m is rotating in a circular path about an axis which is perpendicular to the plane of this circle and passing through the centre. If so, then its so moment of inertia I will be basically mass in to distance square. If it is single mass and its distance is all the time it is a remains constant r so, then it is just a mr square. If you have different mass a system of masses having different distance having different masses, then actually moment of inertia of the systems of this masses will be I equal to summation over m i r i square

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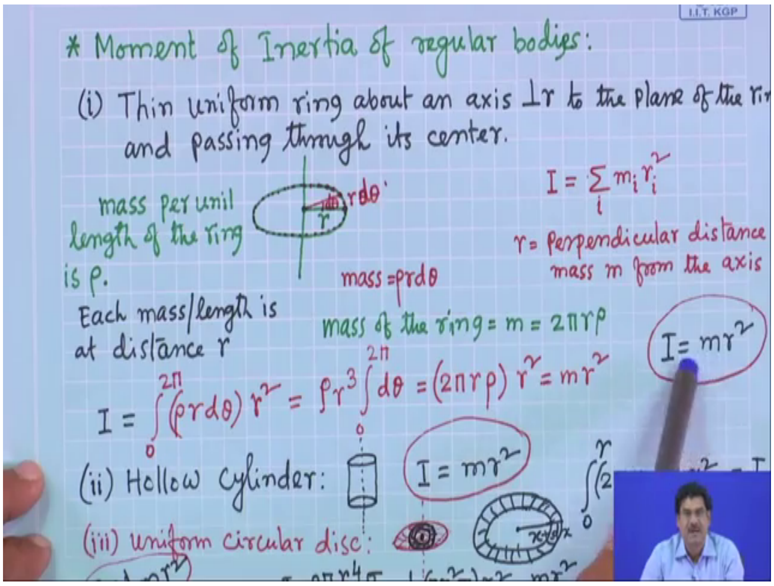

So, basically the definition of moment of inertia is I equal to summation over m i r i square. So, this summation over this i is for different masses. So, just simple example so in 11 class we have calculated moment of inertia of different regular bodies. So, generally we prefer to use the integration method. So, if something this mass is distributed continuously or distance is distributed continuously so, then the summation we replaced with the integration. So, in case of continuous distribution of mass or continuous distribution of distance, then we use integration instead of summation. But in case of discrete system of masses in that case so we use the summation right.

So, depending on the system, one has to decide whether summation is or one can use integration method. Integration method is very convenient way to find out the moment of inertia. So, but every time this its it we calculate using this basic definition of moment of inertia. So, that is moment of inertia equal to mass into square of the perpendicular distance of the mass from the axis of rotation ok.

So, let us see some calculation simple calculation. So, if you have a thin uniform ring about an axis which is perpendicular. So, this is the symbol of perpendicular this axis is perpendicular to the plane of the ring to the plane of the ring and passing through the centre of the ring ok.

So, basically if a ring of wire so that is thin uniform ring, thin uniform wire is used to made this ring. So, this ring has a mass and this ring this mass is distributed over the ring and all if we just represented by this point ok. So, as if this point masses are arranged on this ring so, then all masses in equal distance. So, when this ring will rotate around this axis; ring will rotate around this axis; ring will rotate around this axis so, this whole mass of the ring all the time it will its distance is r. So, directly one can tell if mass of the ring is m, then its moment of inertia will be with respect to this axis because with respect to this axis perpendicular distance of all masses is r. So, one can tell this moment of inertia is m r square; m is the mass of the mr square, m is the mass of the ring. But you can calculate following these from the basic definition of the moment of inertia.

So, say we have to consider that mass of mass per unit length of the ring is say, rho mass per unit length of the ring is rho. So, this what is the length of this thing? 2 pi r. So, mass of this ring will be 2 phi r rho; 2 phi r rho mass of the ring right.

Now, to calculate it just let us take a small length. So, if it is r, then if just if you take some angle this d theta small angle. So, from here just it is the small angle change is d theta. So, then this arc will be r d theta ok. So, now, this is the so, you know this per unit length is mass is rho. So, if I multiply this rho with this one so, that will be the mass of this small portion arc which makes angle d theta. So, this is the mass then r square so, distance is r so, r square. So, this will be moment of inertia with respect to this axis of this mass.

So, now over the d theta over the theta, you can so this whatever this angle we have taken d theta. Now if you just take integration over the 2 phi so, 0 to 2 phi, then it will cover the whole mass of the ring and this distance all the time it is r. So, r is constant basically so, this integration will give you moment of inertia.

So, integration easily if you can do so, this will be r and this r square. So, it is r cube, it is constant and only we have to integrate over the theta. So, this will give 2 pi r rho is there. So, r cube r I have put here and this r square r cube. So, this is the basically as I told this is the mass of the ring so, this I can write mr square ok. So, this is just simple calculation, but all body having regular shape one can calculate the moment of inertia just this is the method rho a one has to do.

So, similarly hollow cylinder about this axis ok. So, you can think this is the basically there are many thin uniform ring. So, you can take divided this hollow cylinder into many number of rings and for each ring you know, this each ring this moment of inertia mr square right.

So, in this case also so, for each ring so, ring 1 2 3 4 5 100 whatever so, for each ring so, mass if it is m 1, m 2, m 3, m 4, m 5. So, m 1 r square m 2 r square m 3 r square m 4 r square if you take summation. So, then basically m 1 plus m 2 plus m 3 etcetera into r square ok. So, that is the m 1 m 2 m 1 plus m 2 plus m 3; this is nothing, but it is the total mass of this hollow cylinder. So, one can find out the moment of inertia just using this knowledge or directly also you can find out calculating like this.

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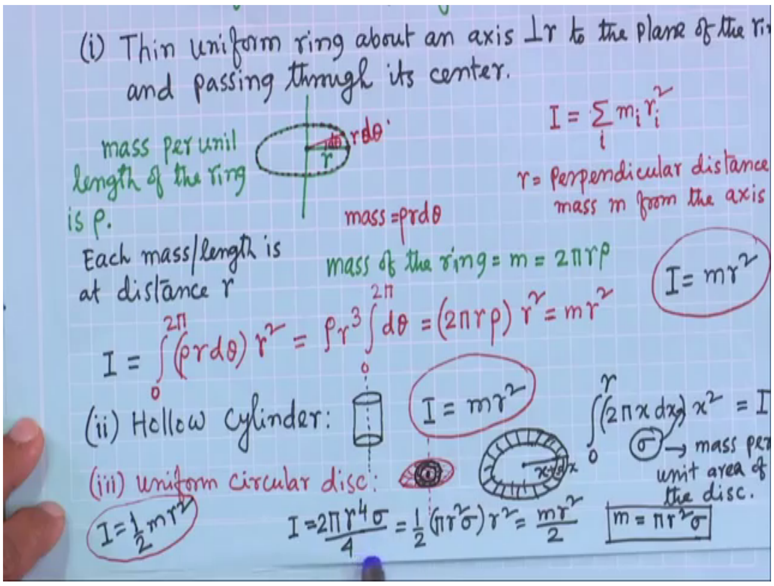

So, similarly for uniform circular disc is the solid circular disc. So, this is the example where r is changing. So, you have a circular disc; you have a circular disc of radius r. Now to calculate it so, it is the we want to calculate moment of inertia about this axis which is perpendicular to the plane of the circular disc and passing through the centre ok.

So, now you take concentric to circle at distance x and x plus dx. So, this I have drawn here in a large form. So, this x and x plus dx so that is the 2 concentric circle. So, this circle inner circle, outer circle this width is basically d x. Now here again if you consider that here you consider this now this circle if it is very thin its a like uniform ring type of things; uniform ring kind of things.

So, this distance is we can consider that this the distance is x. So, for any point on this ring ok, this distance is x. Now it is mass this ring mass will be. What is the area? Area of this one this is basically 2 pi x into dx that will be the area of this one into this sigma. Sigma is basically I have taken mass per unit area of the disc. So, this will be the mass of this ring and distance is x. So, a mass into x square distance square so, that is the definition of moment of inertia.

Now, I can integrate over 0 to r over the whole ring whole disc I can integrate from 0 to r so, x will vary from 0 to r. So, if you integrate so, that will be the moment of inertia of the disc. So, if we integrate so, basically integration over the x so, x square so x cube. So, it will be x to the power 4 by 4. So, now here we put limit r so up to your 4 by 4. So, this basically area total mass for the this disc will be total area is pi r square into the sigma mass per unit area. So, pi r square sigma this is the mass of the ring into so, r 2 there 4 r square here. So, r square here and then this 2 and here 4 so, basically half so, half mass r square; so, half m r square so, that is the moment of inertia of this disc about the axis passing through the centre of the ring and it is a perpendicular to the plane of this ring ok.

So, moment of inertia of this disc about different axis of rotation you can find out. If axis is in on the plane of the disc so, what will be the moment of inertia? If axis is not passing through these it is not on the disc, but it is on the plane of the disc, but it is it has some distance from the centre of the disc or it is passing through the disc and it is perpendicular, but it is not passing through the centre it is passing through the another point ok. So, there are theorem you know so, I will not discuss them.

So, the just from 11 class you know this how to calculate moment of inertia different angular body so, just I remind you. So, that I can now come to my experiment. What I want to do? I want to find out the moment of inertia of a flywheel, I want to find out the moment of inertia of a flywheel.

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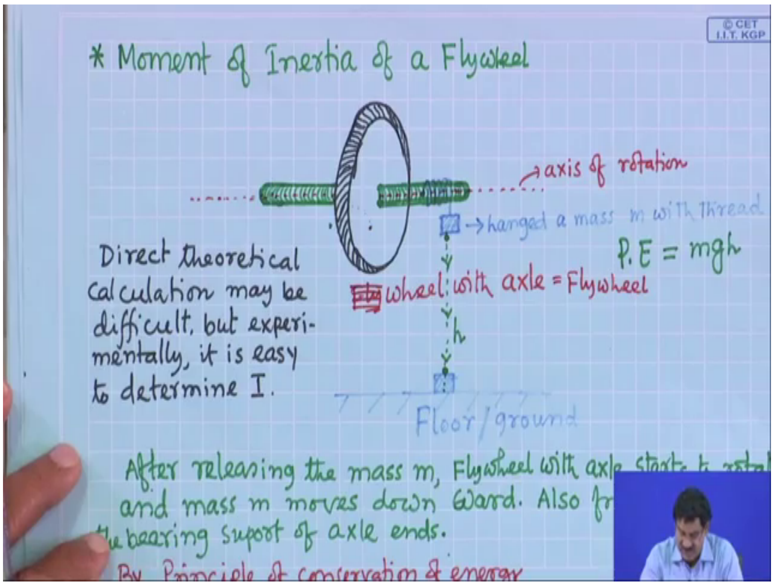

Now, this is the flywheel basically is a wheel there is wheel it is a wheel with axle. So, and it is this whole things rotate this whole things rotate ok. It is the you can see rickshaw or cycle have this ring with some axle ok. So, and this when you move so, this whole things it is rotate ok. So, this we tell this fly wheel it is rotate with axle. Now this axle is in bearing support basically this axle are in bearing support these 2 end there is a bearing support. So, to make this friction less so, we use ball bearing arrangement.

So, this is the flywheel wheel with axle is the flywheel. We want to determine the moment of inertia of this flywheel ok. So, theoretically direct theoretical calculation may be difficult to find out the moment of inertia, but experimentally it is very easy to determine moment of inertia. So, basically here I will discuss how to find out the moment of inertia of this kind of flywheel ok.

So, this wheel basically flywheel its axis of rotation is this dotted red line whatever and this is heavy. So, and its centre of mass of this wheel with axle is basically on the if it is on the axis so, then this wheel it is not on the floor it just we keep at a height from the floor; we keep at a height from the floor this. And this wheel its supported with ball bearing arrangement and this wheel not rotate. If its centre of mass is exactly coinciding with the axis of rotation, then you can keep wheel at any position; you can keep wheel at any position. If centre of mass is not coinciding with this is not passing centre of gravity is not passing through this axis of rotation so, then it in the cannot keep it at any position. So, automatically it will just try to rotate.

So, for experimental arrangement we have to arrange perfectly. So, that its axis of rotation and the centre of mass of this wheel flywheel wheel will coincide; will coincide in the sense, it is not will be in this direction. So, if it is center of gravity center of mass is if you are taking it just of from the floor so, centre of mass centre of gravity it should pass through the; it should pass through the axis of rotation. So, it should pass through the axis of rotation so, it will not rotate you just keep in one position ok, but if you just slightly rotate it so, it will try to it will continue rotation.

So, here we have made one arrangement. So, one mass we have taken mass m and this mass is tied up with the thread and this mass is hanged; this mass is hanged with thread hanged with thread from this axle ok. So, we have just we have taken the length of this thread in such a way so, this from floor this length of from the floor the length of this height of this axle is say, whatever the height. So, this thread length is more or less equal to this height ok.

And here just we have this thread is taken its a turned on the axle so, this I have shown here its. So, its the is basically this thread whatever the length that is we have taken its just turn on this axle and how many turn is taken that one can count. So, say n 1 number of turns of thread is taken on this axle and at that time this height of this mass is h ok.

So, now if you just rotate or if you just now you see, now this mass is attached with this axis with this flywheel and it is. So, now overall this centre of mass or centre of gravity will not pass through this axis of rotation. So, if you just leave this mass, then this wheel will start to rotate and mass will come down it will rotate because mass if it is in this side so, centre of mass will be shifted in this side of this axis of rotation.

So, due to the centre of gravity centre of mass, it is now of axis and because of that it will start to rotate in this direction and this it will be then unturned you know. So, and then mass will come down; mass will come down and this number of turns will decrease and then finally, before just touching this floor this here. This arrangement has made in such a way just we have when this all will be unturned so, this thread will be detached from this wheel ok. So that means, just from the so, this mass will move this height h and when it will just touch the floor so, then there is no connection with this mass.

So, now wheel is free from this mass and actually what will happen. So, wheel when you will leave this mass just and it is in a position because at static position when you will leave it so, wheel will start to rotate. So, when it is touching the floor just that times so, what is the angular velocity of this wheel? So, angular velocity of the wheel will be maximum say it is omega and this simultaneously there will be this was also at rest at height h it was rest. Now this will have also velocity v. So, it is these velocity is related with this angular velocity of this wheel. So, if this flywheel this axle radius is r, then v it is v will be omega r. And at this position, what is the potential energy of this mass the it is at height h? So, potential energy mgh.

So, at the starting point so, only this is the energy potential energy. Now when you release, then its potential energy decreased because height is decrease and then this mass as well as this wheel both are having the kinetic energy. So, basically this is a very good example of that conversion of energy from potential energy to kinetic energy.

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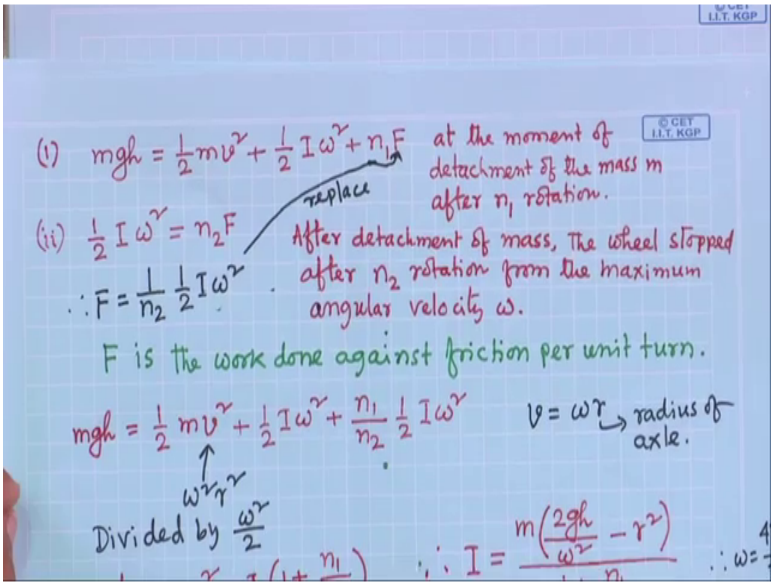

So, that is what happening. So, from conservation of energy one can write that this; one can write that this potential energy mgh equal to half kinetic energy of this mass m which is coming down. So, half mv square plus kinetic energy of the wheels. So, that is half I omega square ok, I is moment of inertia, omega is the angular velocity of the wheel flywheel and plus additional since this wheel is rotating and it is supported with the ball bearing arrangement ok. So, there will be you cannot make this frictionless, there will be slight friction and that friction if it is that what done against the friction; if it is F that is this F is for per unit turn, if this work done to for done against the friction when it rotate once so, per unit turn or for unit rotation if work done against the friction if that is F.

So, as I told this when this mgh will be 0, when this mass will touch the ground and that time so, it will how many turns this it took? The n 1 ok. So, that is how we are counting from the thread or you can just count before touching the floor how many turns it took this wheel. So, the if it is this n 1 so, this energy will be spend for kinetic energy of the mass, kinetic energy of the wheel and the friction energy n 1 F ok.

So, this is at the moment of detachment of the mass m after n 1 rotation and then after detachment, then wheel is having this omega square omega this angular velocity. So, its kinetic energy is half I omega square and this omega square omega is the maximum velocity at this moment. And so, now, this mass is detached. So, only this wheel is there and it is rotating so, its kinetic energy this and after some time it will stop the because of the friction.

So, after how many turns rotation it will stop? So, if it is n 2 and so, energy will be spent work will be done against the friction so, n 2 F. So, this kinetic energy will be spent for this for overcoming these friction. So, another equation you are getting half I omega square equal to n 2 F ok. So, from here you can find out F and this F you can replace here F you can replace here. So, basically then you can write mgh equal to half mv square plus half I omega square plus n 1 by n 2 half I omega square.

And now v equal to omega r as I told this r is radius of axle so, you write v equal to v square equal to omega square r square. Now if we divide by omega square by 2, then and then you take this part this other side. So, you will get 2 mgh by omega square minus mr square equal to I 1 plus n 1 by n 2. So, I equal to here just take we taken m out m into 2 gh by omega square minus r square divided by 1 plus n 1 by n 2.

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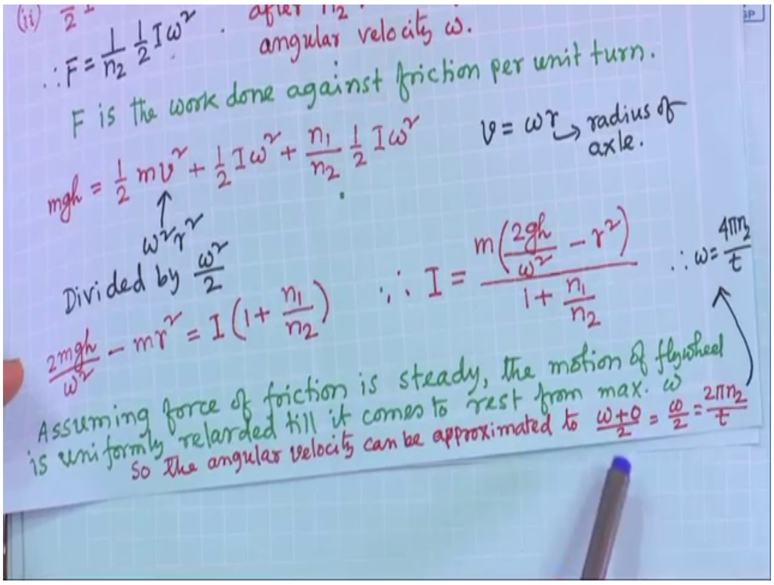

And now here h this is the basically height of the mass initially and omega is the velocity of the wheel maximum velocity of the wheel basically, this omega and r is the radius of axis axle; n 1 and n 2 this is the counting number initially this before touching the floor this mass before touching the floor. So, how many turns it took? So, that is n 1 and n 2 after detachment before stopping the rotation how many turns is took.

So, now how to find out omega? So, height I can measure, r I can also measure using screw gauge ok. This n 1 n 2 also using the just I can count just seeing the rotation and using the thread turns we can find out n 1 and n 2.

Now, omega how to get omega? So, here you see this omega now difficulties is that at this moment after just detachment of mass after just detachment of mass so, that time maximum omega and after some it is stored stopped so, omega then is 0. So, one can take average omega. So, that is the in that case we have to assume that these force of friction is steady, the motion of flywheel is uniformly retarded till it comes to rest from maximum omega ok.

So, basically angular velocity can be approximated to omega plus 0 by 2 that is basically omega by 2 and omega by 2 that you see for n o 2 number of turns. For each turn, what is the for each turn what is the angle 2 pi? For n 2 number of turns, what is the total angle 2 pi n 2? And if I take time if I count just used stopwatch and how much time it took to come to the rest if it is t so, from here basically so, omega by 2 average omega that is omega by 2. So, this 2 pi n 2 by t; so, from here omega equal to 4 pi n 2 by t ok.

So, just to come to the rest, how many turns is took we have to count and we have to take time for that duration using the stopwatch. So, then one can find omega ok. So, this is the working formula for our experiment so, I equal to this. So, we have to. So, here what we have to do? We have to measure height is so, we will use meter scale. So, height generally it is 50-60 centimeter and we have to measure r. So, we will use slide callipers, we have to measure r of this one this axle.

So, we will use slide callipers and then using stopwatch we can count n 1 and n 2 and this for n 2 we can measure the time t so, then we will get omega. So, this the working formula for the experiment where we want to determine the moment of inertia of a flywheel. So, based on this working formula so, I will next class I will demonstrate the experiment and this explained in our laboratory.

So, thank you for your attention.