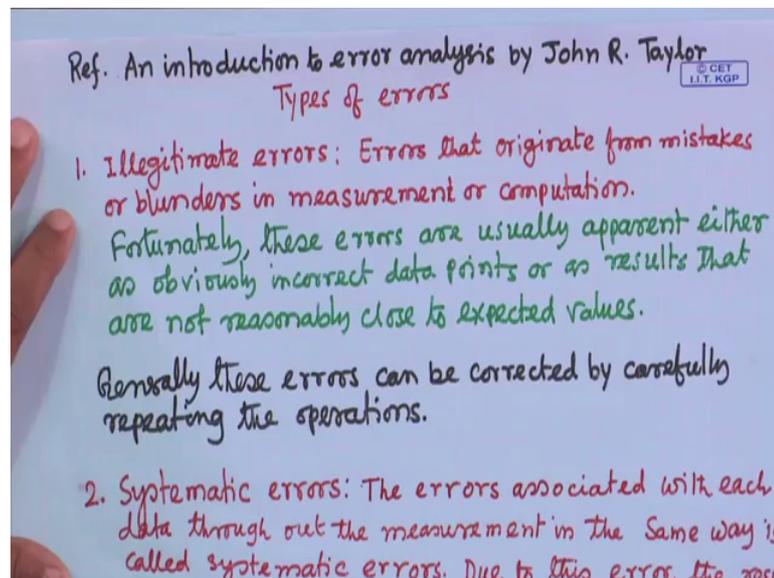


Experimental Physics I
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Lecture - 17
Basic analysis (Contd.)

So, we will continue our discussion on our Analysis. So, yesterday I have discussed some of the aspects. So, whatever this error analysis I am discussing and today also I will continue discussion. So, mainly I will refer one book which is very good book and you can consult with this book. So, this that book is basically here I have given this.

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So, basically an introduction to error analysis by John R Taylor, ok. So, this from this book mainly I am teaching about their analysis. So, we can we can get this book basically this book will be available I think this from Google search you will get this book digital books you will get.

So, today I will discuss about the types of error. So, when we do experiment, so, what are the types of error we face? So, it is a basically first type of errors is illegitimate errors. What is that? Errors that originate from mistakes or blunders in measurement or competition. So, these are this very careless error ok. So, that may happen and that very easily you can find out this type of error from your result because this error is will be obvious from the incorrect data points or results.

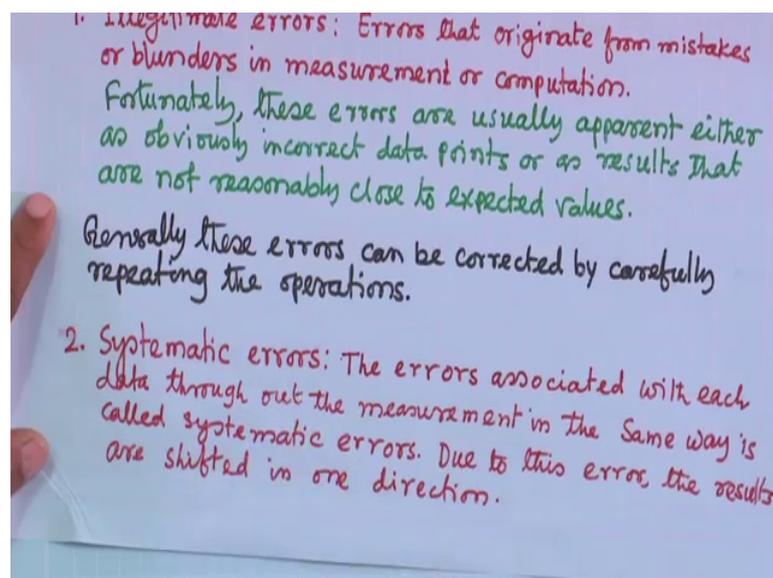
So, that then you can find out the reason for this error and repeat the experiment to avoid avoiding those errors. So, these are very careless errors. So, that is we tell this illegitimate errors. So, it is not under the analysis of the error because this error no way this result is correct. So, you cannot accept the result ok.

So, you have to just throw this results and you have to do the experiment again and but before doing experiment you have to find out what is the reason for that ok. So, now this one reason I can tell you this you have written you are suppose to write say in centimeter unit, but we have written in millimeter unit. So, these are very broad gross error this obvious error that only you will find out from your incorrect data points.

So, incorrect results and then if you pursue the reason for that. So, they be find out. So, some this obvious mistake we have done, yes for example, as I told in place of centimeter we have written millimeter. So, during calculation so, you have we have consider this unit millimeter. So, till one order of magnitude this that is error it will obviously affect your data points and result so that you can verify easily repeating the experiment ok.

Then next type of errors is systematic errors is systematic errors. So, what is that?.

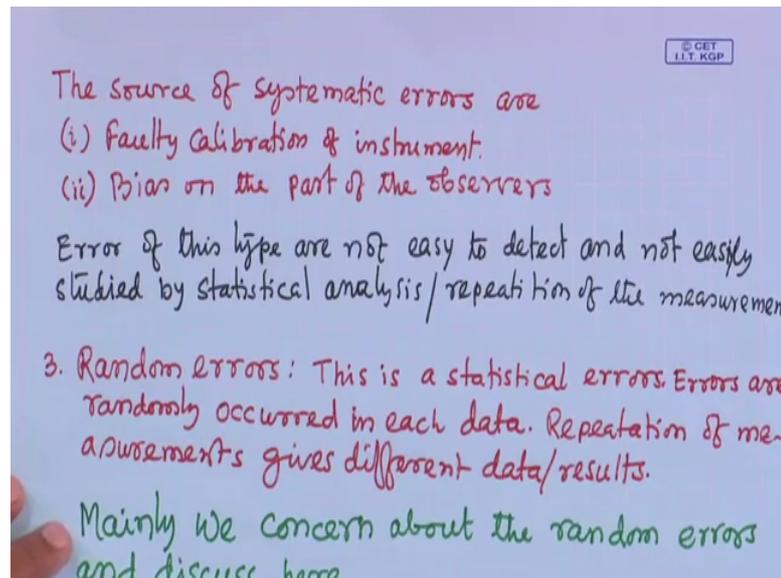
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So, from the name you can from the name you can tell that the errors associated with each data throughout the measurements in the same way is called the systematic error ok.

So, this type of error is associated with each data and in the same way ok. So, then this is the systematic error and due to this error the results are shifted in one directions. So, I will give you example that or what are the sources of this systematic errors. So, that; obviously, we can find out.

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So, this source of systematic errors generally it can be due to faulty calibration of the instrument ok. What does it mean faulty calibration of instrument? Say you are using ammeter or voltmeter. So, their scale is given 1 volt, 2 volt or between 1 volt 2 volt there are smaller divisions right. Now, so, someone calibrated this volt meter ammeter and if that calibration is wrong that means, whatever here 1 volt it is showing if you use another volt meter or meter ammeter. So, then it is showing say it is 1.2.

So, this 1.2 should be the accurate 1, but your meter it is showing 1. So, this faulty calibration basically will affect your all data's. So, this another type of systematic error could be this bias on the part of the observer. What does it mean bias on the part of the observer is? So, when you are taking reading, so, you are suppose this you are suppose to take reading just you see your eye you have to just in perpendicularly you have to put your eye with the scale, but you are taking reading from this side or the other side ok.

So, then whatever reading you will see, so, if you take this reading systematically. So, you do not know that you have to take perpendicularly, you have to set your eye perpendicular to the scale, but you are all the time that is why you are probably habit or

whatever you are or this you are think that you are in perpendicular position, but somehow you are tilted.

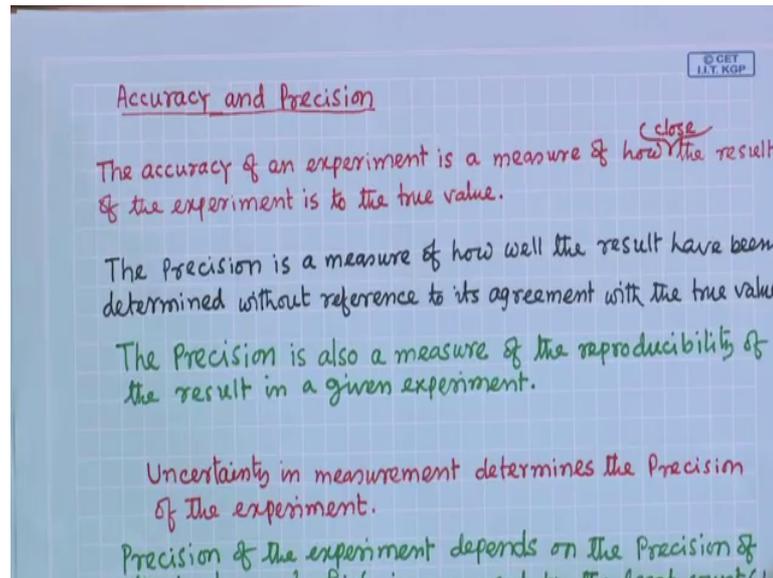
So, that way your all data will be affected because of this your position because of the position of your eye. So, this again this so, this will be propagate in all data points ok. So, there is a it is how basically. So, your if you plot this data points and if you know the true value of the curve or true curve you know and if you plot your experimental data then you will see your data may be very good, but you will see this overall there is a shift of this whole data ok.

So, from there obviously, you will find out this is because of some error. So, that basically you have to find out. Now here it is very difficult to find out. Repeating the experiment will not help you to find out this error or to minimize this error because, if you use the same faulty calibrated instrument, if you take the data in same way from sidewise it is ok. So, you will get the same result again and again. So, that is why we tell this type of error this type of error are not easy to detect and not easily studied by statistical analysis or repetition of the experiment, repetition of the experiment.

So, this so, but you can identify this is the systematic type of error and how to how to how to nullify that one, so, that you have to find out. So, it is not easy to find out, but, but we have to try to find out this type of error and nullify it or what we do? We know this is a systematic error reason I could not find out. So, just we shift this data just we shift this data. So, we removed this offset; we removed this offset. So, that way also we correct this data to nullify this systematic error.

So, then last this and this is very important error this called random error. So, name again this name is telling this basically statistical error, it is randomly occurred in each data. So, to improve the error, so, only way the repetition of the to minimize this error this is the repetition of the measurement is the only way to get better result. So, this random error is very important error and mainly we will concern about this random error in our laboratory because this is in our hand to minimize it ok. So, mainly lifting the experiment we can minimize this and yes. So, this is the error we will discuss in details.

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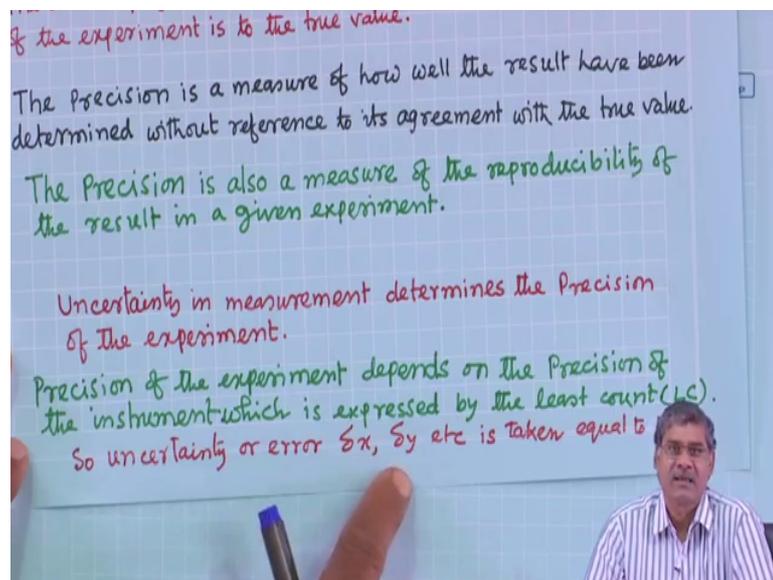
So, when we do experiment or study experimental physics then one obvious question is this or we use this two terms; one is accuracy and another is precision; one is accuracy and another is precision. So, what is accuracy and what is precision how to distinguish this two terms; what is accuracy? Accuracy of an experiment is a measure of how close the result of the experiment is to the true value. So, how close our experimental result to the true value ok.

So, if difference between this true value and this measured value experimental value are very close then we tell this accuracy of our experiment is very good. So, that is why we defined the accuracy. Then what is precision? We generally tell we have to we measure precisely our instrument is very precise and we did the experiment very precisely. What does it mean? So, the precision is a measure of how well the results have been determined without reference to the to its agreement with the true value. So, you did experiment very carefully that is called very precisely without bothering the agreement with the true value ok

So, this is the precision ok. So, this precision is also a measure of the reproducibility of the result in a given experiment. So, if you repeat the experiment; if you repeat the experiment then all the time you will get more or less same result, same kind of data if you get then this called the reproducibility ok.

You can reproduce your data you can reproduce your result by measurement in many times. So, if you get the same result same data then we tells the reproducibility of your experiment is very good and that is that we expressing terms of the precision. So, the precision of the experiment is very high. So, that way we express this terms precision. So, this a basically uncertainty in measurement uncertainty in measurement determines the precision of the experiment. So, how this the experiment will be precise so that we can we can determine basically through the uncertainty in the measurement ok. So that means, uncertainty and precision are related ok.

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So, precision of the experiment depends on the precision of the instrument which is expressed by the least count, precision of the instrument we expressed by the least count. So, and this precision depends on this on the precision of this least count. That means, so, this precision of the instrument that we expressed in terms of least count and that is what this least count we take as a uncertainty. So, thus the precision is related with the uncertainty. If your uncertainty is the smaller and smaller means least count is smaller and smaller then automatically precision of the experiment will be higher and higher.

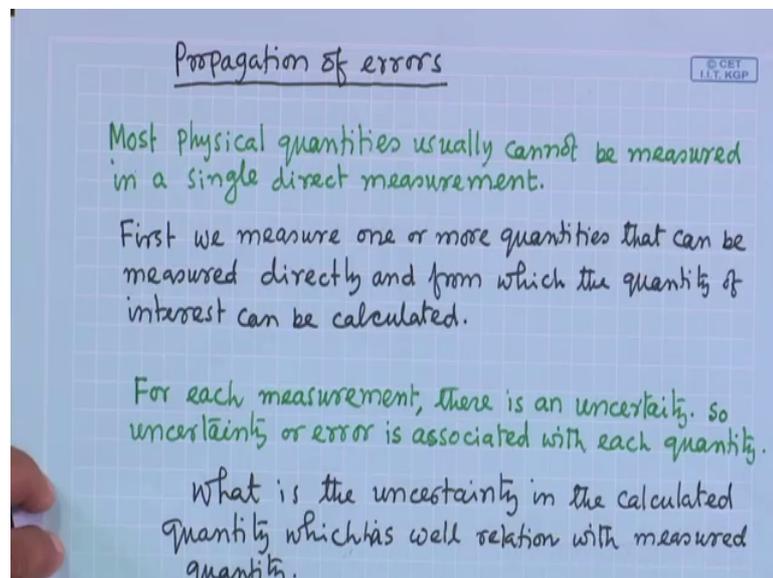
So, this uncertainty or error in the measurement of xyz if it is Δx Δy that is why this Δx Δy that we take which is equal to the least count LC, Least Count. So, as I told this I was discussing this just using when we are writing the result ok.

So, some x best value plus minus Δx ; what is that Δx ? So, that I told I will tell later on. So, this Δx is basically is related with the precision of the instrument and this Δx Δy etcetera this are equal to the least count of the instrument which is used to measure this parameter x and y .

So, now, when we are doing experiment in the laboratory basically our aim is to find out some quantity physical quantity. So that physical quantity most of the time we cannot measure directly. If you can measure directly, so then least count of the instrument which is used to measure that quantity so that will be the error or uncertainty in the measurement. But unfortunately that does not happen all the time we have to measure one or more quantities and then the physical quantity which is related with this all this measured quantity with some relations ok. And then basically from this measured quantity and from this relation we calculate the physical quantity of our interest.

So, final result whatever we are getting of a physical quantity, so, this is not from a direct measurement it is basically calculated from the relation of many parameters many quantities which are measured and for each quantities there is a error uncertainty.

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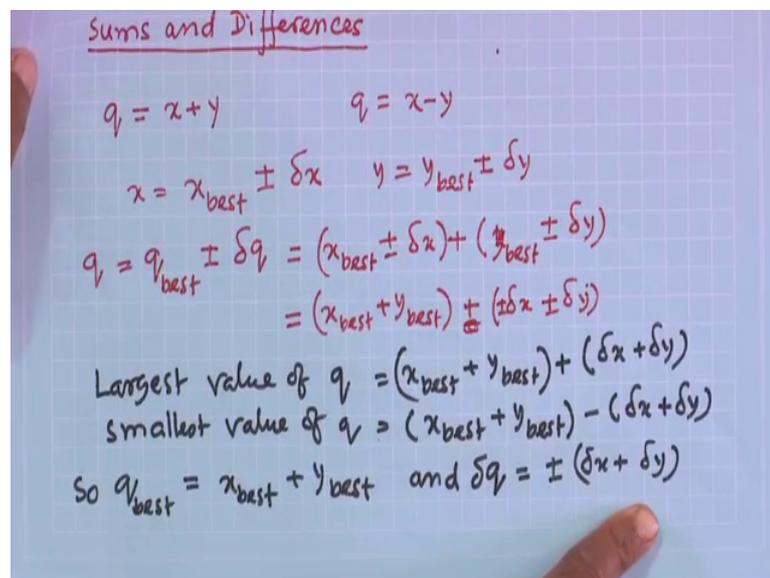
So, now question is now question is what is the uncertainty, what is the uncertainty in the calculated quantity what is the uncertainties in the calculated quantity which has well relation with measured quantity as I described. So, this is the question we have to answer ok; that means, I know the uncertainty or error for each quantity which I measured and

now I have calculated the some physical quantity which is relative with those measured quantity. Now, what will be the error of this physical quantity that we have to find out, ok.

So, these we tell the basically propagation of errors; propagation of errors and how to find out the error of the final quantity. So, that is what we will discuss. So, this physical quantity it has some relation with the measured quantity right. So, relation either it is this some of the measured quantity or difference of the measured quantity or multiplication of the measured quantities or division of the measured quantity. There will be relation with the physical quantity of the measured quantities.

So, as I told either some difference or multiplication or division ok. So, in this form it is this there will be relation between these two physical between the physical quantities and measured quantities. So, here in step by step we will find out the rules how the error propagate from measured quantity to the physical quantity which will which is which will be calculated.

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Sums and Differences

$$q = x + y \quad q = x - y$$

$$x = x_{best} \pm \delta x \quad y = y_{best} \pm \delta y$$

$$q = q_{best} \pm \delta q = (x_{best} \pm \delta x) + (y_{best} \pm \delta y)$$

$$= (x_{best} + y_{best}) \pm (\pm \delta x \pm \delta y)$$

$$\text{Largest value of } q = (x_{best} + y_{best}) + (\delta x + \delta y)$$

$$\text{Smallest value of } q = (x_{best} + y_{best}) - (\delta x + \delta y)$$

$$\text{So } q_{best} = x_{best} + y_{best} \quad \text{and } \delta q = \pm (\delta x + \delta y)$$

So, here when there is a sum or differences in the relation, so, this two parameter here x and y and physical quantity is q; physical quantity is q. So, this is the relation summation or this difference. So, this physical quantity q is related with the measured quantity x and y or this x and y to measured parameter and this physical quantity q this is the relation either in summation form or difference form.

So, if so, then what will be the error on q ? So, for error on x I know error on y I know ok. So, x I have measured. So, x we expressed as x best plus minus Δx y is y best plus minus Δy right. So, then q equal to basically we have to express q best plus minus Δq . So, now, you want to find out this Δq error on this q .

So, then we can write this is basically if you take this summation form. So, then x best q equal to x plus y x is basically x best plus minus Δx plus y best plus minus Δy ok. So, this I can write x best plus y best plus minus Δx plus minus Δy . So, why I have written this way because this x best plus y best then plus either this can be plus it can be plus Δx plus Δy plus Δx minus Δy minus Δx plus Δy minus Δx minus Δy ok. So, there are four possibilities ok.

So, among them among this four what will be the larger value of q ; obviously, this x best plus y best plus Δx plus Δy and smallest value of q will be this minus this x best plus y best minus Δx plus Δy ok. So, this is the largest value of q and this is the largest value smallest value of q and that depends on this on this on this part on this part. So, q best is equal to x best plus y best and this then Δq this is basically error this is Δq . So, we can write as a plus minus Δx plus Δy Δx plus Δy what does it mean. So, basically this error we define this lower limit and upper limit. So, this data will be this value will be between these two limit, ok

So, this so, from here we got that this when this plus then is a larger value when is a minus then is the lowest value. So, this limit is taken will be taken as a of this error Δq . So, that is plus minus Δx plus Δy . So, what does it mean what does it mean? So, here basically in case of some here we here we can tell that this error on this quantity q it will be just it will be just this summation in summation of the error of x and error of y right error of x is Δx and this Δy .

So, summation of this Δx and Δy will be the error on q and; obviously, that is plus minus. So, from this summation we got this one let us see this difference if this x and y this it is related with the physical quantity q it is a like this q is equal to x minus y .

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The image shows a handwritten derivation on a grid background. It starts with the equation $q = x - y$. Below it, the error propagation is shown as $q = q_{best} \pm \delta q = (x_{best} \pm \delta x) - (y_{best} \pm \delta y)$. This is then simplified to $= (x_{best} - y_{best}) + (\pm \delta x \mp \delta y)$. The next line shows the largest value of q as $(x_{best} - y_{best}) + (\delta x + \delta y)$. The following line shows the smallest value of q as $(x_{best} - y_{best}) - (\delta x + \delta y)$. The final line concludes that $q_{best} = (x_{best} - y_{best})$ and $\delta q = \pm (\delta x + \delta y)$.

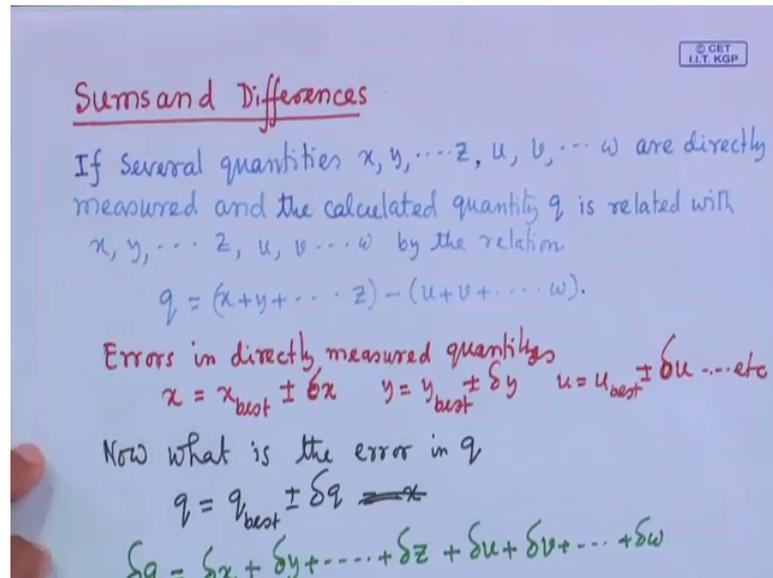
So, same way if you proceed same way if you proceed. So, q is q_{best} plus minus δq equal to x minus y ; so, x_{best} plus minus δx minus y_{best} plus minus δy , right. So, this we can write x_{best} minus y_{best} this is the q_{best} this is the q_{best} plus from here I have written here plus minus δx and here I have written minus plus δy right.

So, what will be the largest value of q ? So, obviously, this value x_{best} minus y_{best} plus δx plus δy . Here also four possibility here also four possibility. So, this I think this same four possibilities ok.

So, this among this four this two so, for this two one is plus and another is minus of the summation of the individual error. So, of that will give me the largest value of q and most smallest value of q so, two limits of the value of the q . So, then in this case also this error is basically coming as a summation of the individual error. So, for summation and difference in both cases the error of the physical on the physical quantity will be the summation of the individual error of the parameter x and y ok.

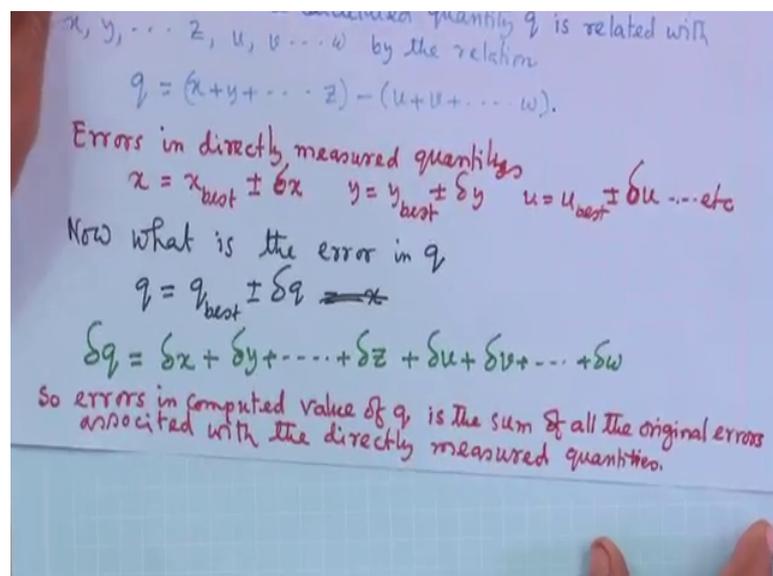
So, I think then I will discuss this I think this. So, in generally we can generalize this result because for summation and difference we saw the error will be on the physical quantity which is calculated. And if it is related in summation or difference form of the measured quantity or measured parameters then we found that the in both cases error will be the just summation of the individual errors of the parameters ok.

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So, we can generalize these findings. So, then here just for rules this now rules is for summation and difference. If several quantities x y etcetera z u v etcetera w are directly measured and the calculated quantity q is related with this x y z u v w like this ok; q equals so, here the summation is there also this difference also there. So, then error in directly measured quantities is that we know from the measurement and least count of the instrument x best plus minus delta x etcetera. So, then now what is the error in q ? So, error in q is q best plus minus delta q and that delta q will be just summation of the individual error on x y z u v w right.

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So, error in computed value of q is the summation of all original errors associated with the directly measured quantities ok. So, this one rule we could find out for the propagation of errors. So, then we will discuss I think next one I can discuss just I can take one examples to show you.

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Examples

Two flasks + Liquid =

$M_1 = \text{mass of 1st flask+liquid} = 540 \pm 10 \text{ gm}$
 $m_1 = \text{mass of 1st flask empty} = 70 \pm 1 \text{ gm}$
 $M_2 = 960 \pm 20 \text{ gm}$
 $m_2 = \text{2nd flask} = 130 \pm 1 \text{ gm}$

Total mass of liquid

$$M = M_1 - m_1 + M_2 - m_2$$

$$= 540 - 70 + 960 - 130 = 1300 \text{ gm}$$

$$\Delta M = \Delta M_1 + \Delta m_1 + \Delta M_2 + \Delta m_2 = 10 + 1 + 20 + 1$$

$$= 32 \text{ gm}$$

Final result: Total mass of liquid

So, here just I have 2 liquids in flask. So, this liquid it has weight say M_1 ; M_1 is the weight of flask and liquid. So, weight of this of this one liquid and this flask is M_1 capital M_1 and then I will just throw this water and then take the weight of the empty flask. So, this has smaller m_1 , mass of first flask empty then I think I should show you.

So, similarly M_2 and for this for this one M_2 and capital M_2 capital small m_2 . So, this value say here 540 plus minus 10 gram smaller small m_1 is 70 plus minus 1 gram, M_2 is 960 plus minus 20 gram m_2 is 130 plus minus 1 gram. So, now, what is the total mass of liquid that is what if I if you want to find out.

So, that obviously, this mass of the total mass of liquid M is M_1 if we add them together; if we add them together this two liquid. So, what will be the mass of this liquid? So, this is M_1 minus small m_1 that is the liquid mass plus M_2 minus small m_2 ok. So, this is the mass of the second liquid ok.

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Two flasks

$$M_2 = 960 \pm 20 \text{ gm}$$

2nd flask

$$m_2 = 130 \pm 1 \text{ gm}$$

Total mass of liquid

$$M = M_1 - m_1 + M_2 - m_2$$
$$= 540 - 70 + 960 - 130 = 1300 \text{ gm}$$
$$\Delta M = \Delta M_1 + \Delta m_1 + \Delta M_2 + \Delta m_2 = 10 + 1 + 20 + 1$$
$$= 32 \text{ gm}$$

Final result: Total mass of liquid

$$= 1300 \pm 32 \text{ gm}$$
$$= 1300 \pm 30 \text{ gm}$$

So this 1300 gram that is the mass of the liquid. So, now, find this is of my result. Now what is the error on it what is the error on it so, that I have to find out delta M. Now this is the relation it is the summation and the summation and the and the difference ok. So, rule I know. So, this error on this capital M will be the just summation of the individual errors. So, delta M 1 plus delta small m 1 delta capital M 2 plus delta small m 2 and there error is given plus minus here is 10, 1, then 20, then 1; so, 10 plus 1 plus 20 plus 1 equal to 32 gram.

So, final result then we can write the total mass of the liquid is 1300 gram plus 32 gram. So, here basically we can just round off. So, this 1300 gram plus minus 30 gram 30; so, instead of 32 grams generally we can write this 30 gram. So, because this two have because error is 32 itself so, this two 32 and 30 this is within here. So, we round it of and better to write 30 gram if you write 32 also it is not it is not harmful, but it is this are the well representation.

So, this way just one can find out the error of the parameter of quantity of interest. So, I think I will stop here I will discuss this product and quotients in next class.

Thank you for your attention.