

**Upstream LNG Technology**  
**Prof. Pavitra Sandilya**  
**Department of Cryogenic Engineering Centre**  
**Indian Institute of Technology Kharagpur**

**Lecture - 72**  
**Tutorial on refrigeration – II**

Welcome; in this lecture we shall be learning something more about the basics of the refrigeration and liquefaction.

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**What we shall learn**

- ✓ J-T expansion coefficient
- ✓ Expansion of an ideal gas
- ✓ J-T coefficient for a gas obeying van der Waals equation of state
- ✓ Isentropic expansion coefficient

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So, in this lecture we shall be doing some derivations and see that how to read the  $T_s$  diagram etcetera further. So, what we shall learn we shall learn about the J-T expansion coefficient, the expansion of an ideal gas the J-T coefficient for a gas obeying Van Der Waals equation of state and isentropic expansion coefficient.

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**Solution**

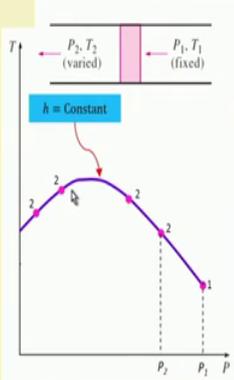
Applying the 1<sup>st</sup> law of thermodynamics for a steady flow through an expansion valve, with

- Zero heat transfer
- Zero work transfer
- Negligible kinetic and potential energy changes

$$h_1 = h_2$$

Although the flow within the valve is irreversible and not isenthalpic, the inlet and outlet states do lie on the isenthalpic curve.

$\mu_{JT}$ : Slope of the isenthalpic curve

$$\mu_{JT} = \left( \frac{\partial T}{\partial p} \right)_h$$


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So, let us take the first problem here we have that we derive an expression for the J-T expansion coefficient in terms of temperature and the specific volume. This particular expression we have given in our theoretical class in here (Refer Time: 01:04) we shall be deriving that particular expression. So, in this what we do that we take the first law of thermodynamics and we assume the steady flow through an expansion valve and we make some assumptions, one is that there is no heat transfer there is no work transfer and there is negligible changes in the kinetic and potential energies. So, with this let us first see that if we draw a temperature versus pressure curve, how it will be?

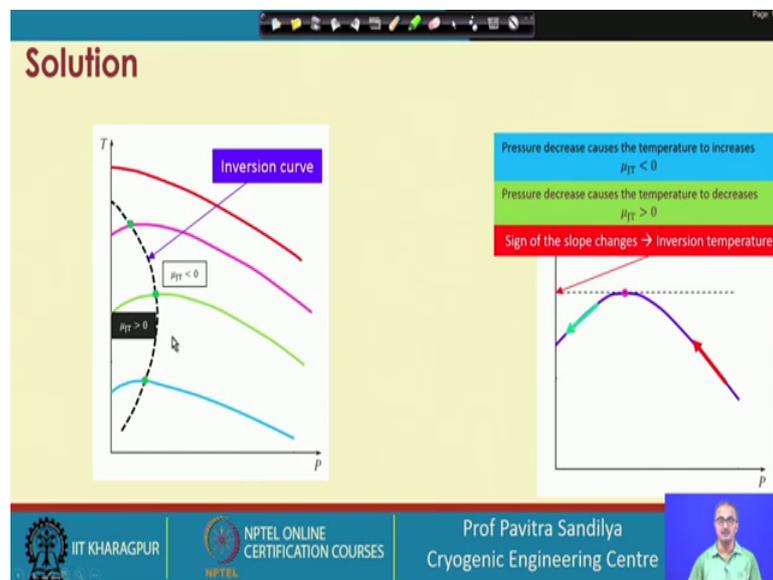
So, here we see that there is some fluid that is going this particular pink volume shows that it is the barrier or the expansion valve, we can say and here we have some fluid a gas maybe that it is coming at some pressure  $P_1$  and  $T_1$  which is fixed. And now it is going through this valve and it is expanded and here we have some other pressure and temperature and in this what we are saying that we shall be select the enthalpy is remaining constant. So, with this we shall see that this kind of this line we shall be getting and if you make it is isenthalpic line, then you see that this is the  $h$  is constant here and when  $h$  is constant and suppose here we have  $P_1$   $T_1$  and suppose here we have  $P_2$ .

Now, we will you see that here in this case, what happens that this  $h_1$  is equal to  $h_2$  because of this assumptions. So, this is from the first law of thermodynamics and here

we see that these 2 maybe at different places on this isenthalpic curve. Now depending on where it is in the gas will either undergo a temperature decrease or a temperature increase and the slope at any point whatever slope you take that slope will be this  $\frac{dT}{dP}$  at constant enthalpy, because we are dealing with a isenthalpic line.

So, if we find the slope of at any point we shall be able to get the value of the, this expansion coefficient at constant enthalpy. And now what we see here is this that on this zone in this zone, when we are decreasing , the pressure the temperature is increasing. On the other hand, whenever we are in this zone though when we are decreasing the pressure the temperature is also decreasing.

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So, when the temperature is increasing with decrease in the pressure, now  $\mu_{JT}$  as from the definition is coming out to be negative; that means, using this  $\mu_{JT}$  is coming to 0. It means that the gas will get heated up on expansion on the other hand, if we go to the other zone where with the decrease in the pressure the temperature also decreases, this zone we have  $\mu_{JT}$  more than 0 that is positive. So, for cooling we want the  $\mu_{JT}$  to be always positive. Now here whenever there is a change in the slope it is called the point of inflection. So, here this particular point this is what we call the point of inversion, so whatever temperature we read here this will be the inversion temperature of the particular gas. Now here, we see that in this particular figure there are several isenthalpic lines. Now you will see that if we draw this kind of things for each of the

isenthalpic lines and then we join the inversion temperatures. So, we get this particular dashed curve. This is the locus of all the inversion temperatures and this particular locus is called the inversion curve. So that is how we generate the inversion curve.

So, we see that on the left hand side of the inversion curve we have  $\mu_{JT}$  always more than 0 and on the other hand, on the right hand side of this inversion curve we have  $\mu_{JT}$  always less than 0. So, for cooling we would like to be on the left hand side of this particular curve and this particular thing wherever it is maximum wherever this particular inversion curve is going to a maximum that is the maximum inversion temperature.

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**Solution**

$h = h(p, T)$   
From cyclic relationship for state functions

$$\left(\frac{\partial T}{\partial p}\right)_h \left(\frac{\partial p}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_p = -1$$

$$\mu_{JT} = \left(\frac{\partial T}{\partial p}\right)_h = - \left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial T}{\partial h}\right)_p$$

$$= - \left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial h}{\partial T}\right)_p$$

But  $\left(\frac{\partial h}{\partial T}\right)_p = C_p$

For determining  $\left(\frac{\partial h}{\partial p}\right)_T$ , consider  $Tds = dh - vdp$

$$\Rightarrow \left(\frac{\partial h}{\partial p}\right)_T = \left[ v + T \left(\frac{\partial s}{\partial p}\right)_T \right]$$

From Maxwell's relations,

$$\left(\frac{\partial s}{\partial p}\right)_T = - \left(\frac{\partial v}{\partial T}\right)_p$$

$$\Rightarrow \left(\frac{\partial h}{\partial p}\right)_T = \left[ v - T \left(\frac{\partial v}{\partial T}\right)_p \right]$$

$$\mu_{JT} = \frac{1}{C_p} \left[ T \left(\frac{\partial v}{\partial T}\right)_p - v \right]$$

Now, let us see that how to derive the expression for the  $\mu_{JT}$ . So for this is what we do that we start with the not expression but the functionality of enthalpy with respect to pressure and temperature. So, here we find this we are writing this h is the enthalpy and p is the pressure, T is the temperature. So, we are writing h is equal to some function of pressure and temperature and then we are using the cyclic relationship for state functions. To drive all these expressions, you have to refer to some books on the thermodynamics. So there you will find all the basic relationships which we are going to use in this derivation.

So, here we have the cyclic relationship we find that it is  $\frac{dT}{dp}$  by  $\frac{dh}{dp}$  at constant h, then  $\frac{dp}{dh}$  by  $\frac{dh}{dT}$  at constant T, then  $\frac{dh}{dT}$  by  $\frac{dh}{dp}$  at constant p is equal to minus 1.

It is very easy to remember that you start with the one pair keeping the other one constant. So, we are doing  $\left(\frac{\partial T}{\partial p}\right)_h$  and then you can take this  $p$  in the numerator and this  $h$  in the denominator and keep this as constant and then you take this in the numerator and this in the denominator and take the other one constant. So, this is how we can remember this cyclic relationship and this is applicable only for the state function not the path functions.

Now we know that  $\mu_{JT}$  is equal to  $\left(\frac{\partial T}{\partial p}\right)_h$ . Now using the cyclic relationship what we find that we can replace  $\left(\frac{\partial T}{\partial p}\right)_h$  by  $\left(\frac{\partial h}{\partial p}\right)_T$  and  $\left(\frac{\partial T}{\partial h}\right)_p$ . So, from this relationship we are replacing with this, now it is simply that we have taken the reciprocal of these 2 terms. After this we are writing that  $\left(\frac{\partial h}{\partial p}\right)_T$  everything we are doing that with respect to  $h$  because,  $h$  is a function of temperature pressure. So, we are putting everything in terms of  $\left(\frac{\partial h}{\partial T}\right)_p$  or  $\left(\frac{\partial h}{\partial p}\right)_T$ . So, for this is remaining same as it is and this is  $\left(\frac{\partial T}{\partial h}\right)_p$  which we are writing as reciprocal of  $\left(\frac{\partial h}{\partial T}\right)_p$  and now we know that  $\left(\frac{\partial h}{\partial T}\right)_p$  is the specific heat at constant pressure.

So, we can replace this particular term with the  $C_p$ , now to find this  $\left(\frac{\partial h}{\partial p}\right)_T$ , we are again using as the thermodynamic relationship that is  $T ds = dh - v dp$  and these derivations are given in the thermodynamic books, so I am not going in to those derivations. So, now what we are doing using this particular expression we are doing that  $\left(\frac{\partial h}{\partial p}\right)_T$  is coming out to be  $v + T \left(\frac{\partial s}{\partial p}\right)_T$ . Now there is also there are some Maxwell's relationships which are also given in the thermodynamics book.

So, from there we find this  $\left(\frac{\partial s}{\partial p}\right)_T$  is equal to  $-\left(\frac{\partial v}{\partial T}\right)_p$ . So, what we are doing that we are replacing these term using the Maxwell relation here. So, we are getting  $v - T \left(\frac{\partial v}{\partial T}\right)_p$ , now what we are basically doing here that we have reduced all the terms in terms of the measurable variables ok. So that is what we have done by this particular thing. Now in here once we have obtained this we can put this value over here in this expression for  $\mu_{JT}$  and what we get ultimately, we get this particular equation which we showed you earlier in the lecture. So, that is how we can derive the expression for the expansion coefficient at constant enthalpy from the basic thermodynamic relationships.

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**Problem Statement 2**

✓ Explain why an ideal gas would not experience a temperature change upon expansion through an expansion valve.

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Now, we go to another problem in this we have to explain that why an ideal gas would not experience a temperature change upon expansion through an expansion valve, to answer this question we have to see that what is the value of the  $\mu_{JT}$  for the ideal gas. Because, when  $\mu_{JT}$  is equal to 0 it means that all expansion  $du_T$  by  $du_p$  at constant  $h$  is 0; that means, all expansion or on compression there is no change in the temperature if it is done at constant enthalpy.

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**Solution**

$$\mu_{JT} = -C_p \left[ v - T \left( \frac{\partial v}{\partial T} \right)_p \right]$$

For an ideal gas,

$$v = \frac{RT}{p}$$
$$\left( \frac{\partial v}{\partial T} \right)_p = \frac{R}{p} = \frac{v}{T}$$
$$\mu_{JT} = -C_p \left[ v - T \times \frac{v}{T} \right] = 0$$

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So, assuming that constant enthalpy condition we start with the expression which we have just derived about  $\mu_{JT}$  and we see that if we put this for ideal gas relationship, we put this a  $\mu_{RT}$  by  $p$  and we find that this is coming like this. So here we put the value we find that this is coming to 0.

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**Problem Statement 3**

✓ Determine the JT coefficient for a gas obeying van der Waals equation of state

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So, this is this proves that  $\mu_{JT}$  is 0 means it is that there is no change in the temperature on changing the pressure. Now we come to the another problem in this we have to find the JT coefficient for a gas obeying Van Der Waals equation of state.

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**Solution**

van der Waals equation of state

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$pv - pb + \frac{a}{v} - \frac{ba}{v^2} = RT$$

$$p \left(\frac{\partial v}{\partial T}\right)_p - \frac{a}{v^2} \left(\frac{\partial v}{\partial T}\right)_p + 2 \frac{ab}{v^3} \left(\frac{\partial v}{\partial T}\right)_p = R$$

$$T \left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p - \frac{a}{v^2} + \frac{2ab}{v^3}}$$

$$p - \frac{a}{v^2} + \frac{2ab}{v^3} = \left(p + \frac{a}{v^2}\right) - \frac{2a}{v^2} + \frac{2ab}{v^3}$$

$$= \frac{RT}{v - b} - \frac{2a}{v^2} \left(1 - \frac{b}{v}\right)$$

$$T \left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{\frac{RT}{v - b} - \frac{2a}{v^2} \left(1 - \frac{b}{v}\right)}$$

$$T \left(\frac{\partial v}{\partial T}\right)_p - v = \frac{RT}{\frac{RT}{v - b} - \frac{2a}{v^2} \left(1 - \frac{b}{v}\right)} - v$$

$$= \frac{1}{\frac{1}{v - b} - \frac{2a}{RTv^2} \left(1 - \frac{b}{v}\right)} - v$$

$$= \frac{1}{\frac{1}{v} \cdot \frac{1}{1 - \frac{b}{v}} \left[1 - \frac{2a}{RTv} \left(1 - \frac{b}{v}\right)\right]} - v$$

$$= \frac{1 - \frac{b}{v}}{1 - \frac{b}{v}} \left[1 - \frac{2a}{RTv} \left(1 - \frac{b}{v}\right)\right]^{-1} - v$$

$$= \frac{1}{\frac{1}{v} \cdot \frac{1}{1 - \frac{b}{v}} \left[1 - \frac{2a}{RTv} \left(1 - \frac{b}{v}\right)\right]} - v$$

$$= \frac{1}{\frac{1}{v} \left(1 - \frac{b}{v}\right) \left[1 - \frac{2a}{RTv} \left(1 - \frac{b}{v}\right)\right]} - v$$

$$= \frac{1}{\frac{1}{v} \left(1 - \frac{b}{v}\right) \left[1 - \frac{2a}{RTv} \left(1 - \frac{b}{v}\right)\right]} - v$$

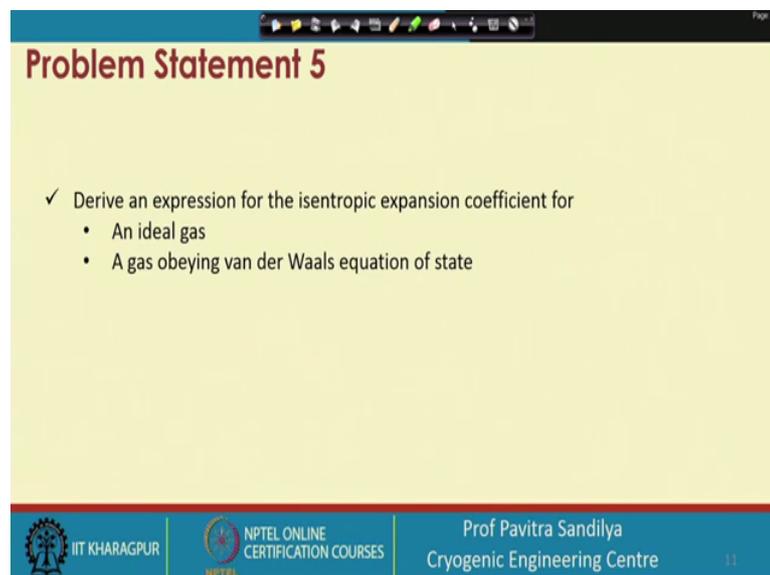
$$\mu_{JT} = \frac{1}{C_p} \left[ T \left(\frac{\partial v}{\partial T}\right)_p - v \right]$$

$$\mu_{JT} = \frac{1}{C_p} \left[ \frac{-b + \frac{2a}{RT} \left(1 - \frac{b}{v}\right)^2}{1 - \frac{2a}{RTv} \left(1 - \frac{b}{v}\right)} \right]$$

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Now, let us consider this Van Der Waals equation of state, so this is the famous equation of state in this pressure is getting modified this particular term and volume is getting modified by this particular term. And perhaps you know that these are accounting for the intermolecular forces and also the finite volume occupied by the molecules or the atoms in the real gas. So, those modifications have been taken into account and then we are just expanding this expression and from here, what we are doing? We are taking the derivative at constant pressure with respect to the temperature. So, we have derived derivative with respect to temperature at constant pressure, so we get this particular expression and then we have derived this thing.

Now, after this is what we are doing is that we are going with this we are just rearranging the equations this denominator has been taken here and which we are rearranging here. And after this rearrangement, what we get that if you do this rearrangements; you will slowly and slowly get all these expressions and ultimately you will get this particular expression. Now after taking this expression what we are doing that we have got that this  $T \frac{dv}{dT}$  at constant  $p$  minus  $v$  is coming out to be this and now we going to the basic difference  $\mu_{JT}$ . And from this we find that we are for this particular expression in the bracket has been replaced by this whole expression, here we can this term and this term get cancelled off. So, we are left with these expressions which we are putting here and this is how we are able to derive the expression for the  $\mu_{JT}$  considering the Van Der Waals equation of state. Now please note that this in a similar manner if you are given some other equation of state for real gases, you can get some other source expressions for this  $\mu_{JT}$ . (Refer Slide Time: 13:05)



**Problem Statement 5**

- ✓ Derive an expression for the isentropic expansion coefficient for
  - An ideal gas
  - A gas obeying van der Waals equation of state

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Now next problem is this we are having some an ideal gas and some real gas which obeys the Van Der Waals equation of state and for that we have been asked to derive the expression for the isentropic expansion coefficient. So, far we have looked into the isenthalpic expansion coefficient, here we are looking at the isentropic expansion coefficient which is there in the expansion engines.

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**Solution**

For an ideal gas,

$$\mu_s = \left(\frac{\partial T}{\partial p}\right)_s$$

$$s = s(T, p)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

$$\left(\frac{\partial T}{\partial p}\right)_s \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial p}{\partial s}\right)_T = -1$$

$$\left(\frac{\partial T}{\partial p}\right)_s = -\left(\frac{\partial s}{\partial p}\right)_T$$

$$\left(\frac{\partial s}{\partial T}\right)_p = \frac{C_p}{T} \quad \left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

$$\mu_s = \frac{T}{C_p} \left(\frac{\partial v}{\partial T}\right)_p$$

For an ideal gas,

$$pv = mRT$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} = \frac{v}{T}$$

$$\mu_s = \frac{T}{C_p} \frac{v}{T} = \frac{v}{C_p}$$

van der Waals equation of state

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

$$pv - pb + \frac{a}{v} - \frac{ba}{v^2} = RT$$

$$p \left(\frac{\partial v}{\partial T}\right)_p - \frac{a}{v^2} \left(\frac{\partial v}{\partial T}\right)_p + 2 \frac{ab}{v^3} \left(\frac{\partial v}{\partial T}\right)_p = R$$

$$T \left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p - \frac{a}{v^2} + \frac{2ab}{v^3}}$$

$$p - \frac{a}{v^2} + \frac{2ab}{v^3} = \left(p + \frac{a}{v^2}\right) - \frac{2a}{v^2} + \frac{2ab}{v^3}$$

$$= \frac{RT}{v - b} - \frac{2a}{v^2} \left(1 - \frac{b}{v}\right)$$

$$\frac{T}{C_p} \left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{C_p \left[ \frac{RT}{v - b} - \frac{2a}{v^2} \left(1 - \frac{b}{v}\right) \right]}$$

$$= \frac{C_p}{v} \frac{1}{\left(1 - \frac{b}{v}\right)} \left[ 1 - \frac{2a}{RT} \left(1 - \frac{b}{v}\right) \right]$$

$$\mu_s = \frac{v(1 - b/v)}{C_p \left[ 1 - \frac{2a}{RT} \left(1 - \frac{b}{v}\right) \right]}$$

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Now, for this again we first write the expression for the isentropic expansion coefficient that is  $\left(\frac{\partial T}{\partial p}\right)_s$  by  $\left(\frac{\partial s}{\partial p}\right)_T$  at constant entropy. Now here we start with the entropic relationship that entropy is a function of temperature pressure. And as we did earlier under the isenthalpic expansion the similar manner we are also taking this expression and we are using the cyclic relationship and from this we find that the  $\left(\frac{\partial p}{\partial T}\right)_s \left(\frac{\partial T}{\partial p}\right)_p \left(\frac{\partial p}{\partial s}\right)_T = -1$  at constant  $s$  is coming out to be this way and now what we find that from the thermodynamic relationship again.

We find that  $\left(\frac{\partial s}{\partial T}\right)_p$  by  $\left(\frac{\partial s}{\partial T}\right)_p$  at constant  $p$  is nothing but  $C_p$  by temperature and this  $\left(\frac{\partial s}{\partial p}\right)_T$  by  $\left(\frac{\partial s}{\partial p}\right)_T$  at constant  $T$  is this particular expression from the Maxwell relation. Now this particular than  $\mu_s$  we are putting replacing in terms of this expression, so we find that  $\mu_s$  is coming to  $T$  by  $C_p \left(\frac{\partial v}{\partial T}\right)_p$  at constant pressure. Now for an ideal gas we know that  $pv$  equal to  $mRT$  and we are getting the value of  $\left(\frac{\partial v}{\partial T}\right)_p$  by  $\left(\frac{\partial v}{\partial T}\right)_p$  at constant pressure and that is how we are getting this replacing it with  $T$  by  $C_p v$  by  $T$ . So, we are getting a nonzero value of the isentropic expansion coefficient for an ideal gas which is

unlike the case, when we got the isenthalpic expansion coefficient to be 0 for the ideal gas.

Now here what it means as I said in the lecture that in case of isentropic expansion even an ideal gas will undergo a temperature change and because  $\mu_s$  is always positive, so on expansion the ideal gas will always lead to cooling unlike the case of the isenthalpic expansion when the gas can either get cooled or heated up the isentropic expansion will always result in cooling.

Now what we do that we consider the Van Der Waals equation of state, in this we write the expressions like this and here what we do again we take the derivative with respect to temperature at constant pressure. And then we get this expression and we take this denominator and now we are rearranging the denominator and we find that on rearrangement. We are getting this particular expression for the right hand side of the isentropic expansion coefficient and that is how this particular thing leads to this expression for the isentropic expansion coefficient using the Van Der Waals equation of state. So, in a similar manner if you are given some other equation of state, then you can make use of the cyclic relationship and the Maxwell relationship and you can get the expression for the  $\mu_s$ .

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**Problem Statement 5**

✓ Nitrogen gas having an enthalpy of 120 kJ/kg is expanded through an ideal JT valve under the following initial and final conditions

a)  $p_{\text{initial}} = 200 \text{ atm}$  ;  $p_{\text{final}} = 100 \text{ atm}$   
b)  $p_{\text{initial}} = 20 \text{ atm}$  ;  $p_{\text{final}} = 10 \text{ atm}$

Determine the temperature change in each case

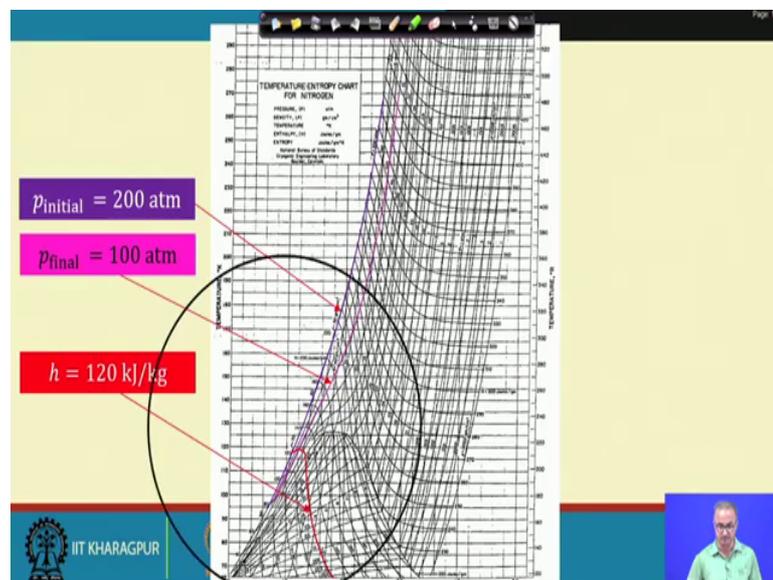
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Now, we go to the next problem, in this problem what we are doing that we will see these particular things in a  $T_s$  diagram whatever we are learnt. So, far we shall look into

those things again in the Ts diagram, so here we have nitrogen gas having an enthalpy of this particular thing is expanded through an ideal JT valve under the following initial and final conditions. So, here we are given the initial pressure to be this and final pressure to be this and initial pressure is this and this. Now please understand by changing these pressures what we are doing that we shall be having different isenthalpic curves ok.

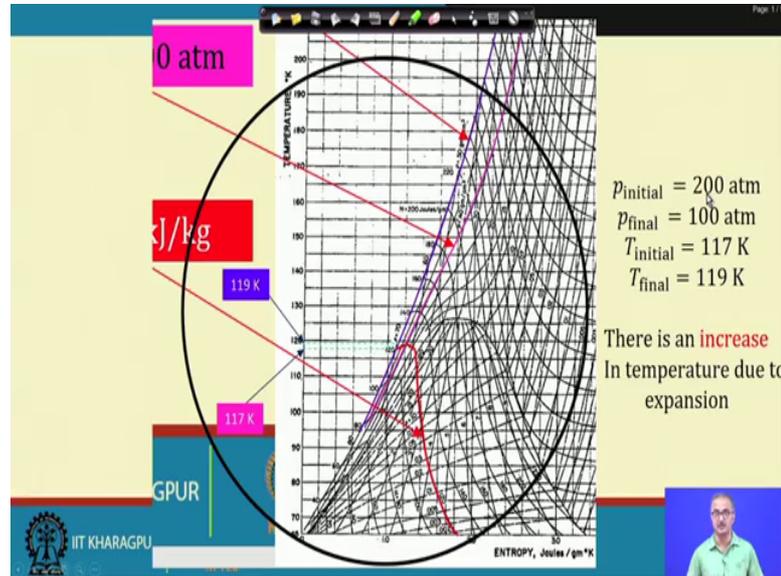
So, that is how these 2 problems would differ and because we have been specified the nitrogen the fluid is specified nitrogen. So, we have to make use of the Ts diagram or any other thermodynamic diagram for this and we shall be using here a Ts diagram and we have to determine the temperature in each case.

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particular figure that the both this curves are lying on the left hand side of the maximum of the isenthalpic line.

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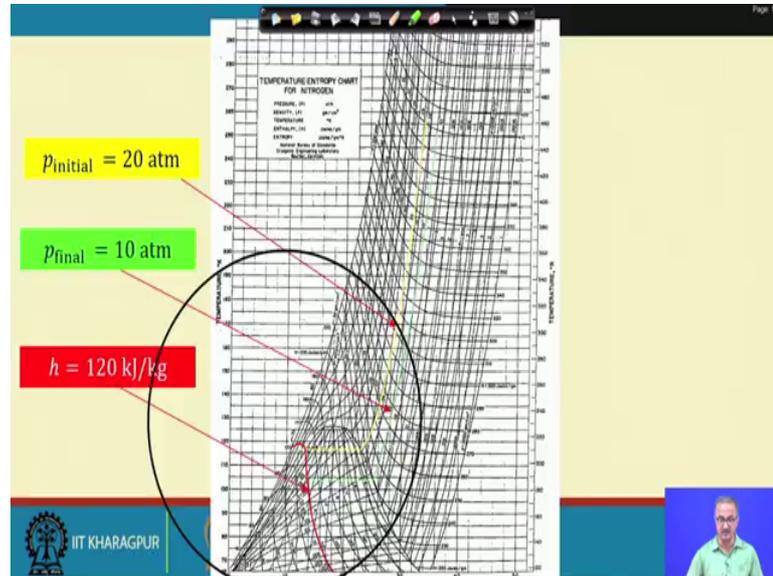
Now, if we simply zoom that particular part on zooming, we can see it much more clearly that how this isenthalpic line and the 2 isobars are placed on the Ts diagram. Now we find here that if we take this that at 200 atmosphere it is intersecting the isenthalpic line at about 117 k, that means the initial temperature of the gas is 117 k. In similar fashion we can also find out the gas temperature at the lower pressure that is 100 atmosphere and that is coming out to be 119 k.

Now, you see that in this case what we were finding that there is an increase in the temperature due to expansion, so that means that in on this region. So, earlier we found it on the p T curve here where you get Ts curve, so on the Ts curve what we find on the left hand side of this maximum we are getting the heating, on the right hand side of expansion we are getting the cooling. So, this difference you must be noting with depending on the type of the thermodynamic diagram you are using, you may find that the zone of the  $\mu_{JT} < 0$  and  $\mu_{JT} > 0$  may be changing.

So, here with respect to the Ts diagram we find that the zone that the  $\mu_{JT}$  is more than 0 is always on the right hand side of the maximum, where as the zone where  $\mu_{JT}$  is less than 0 that is a zone of increase in temperature with decrease in pressure will be always the left of the maximum on the isenthalpic curve. So, we find that on the

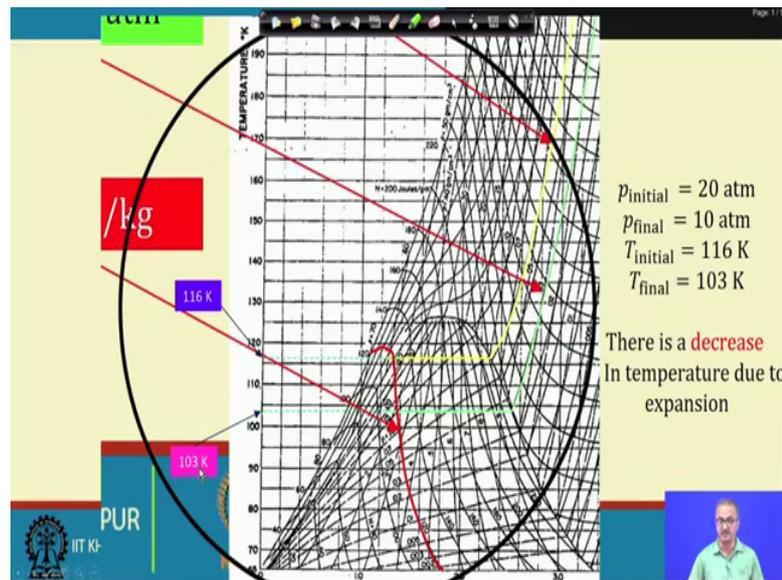
expansion this nitrogen gas is getting heated up at from this 200 atmosphere 200 atmosphere for this particular enthalpy.

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Now, again we look into this Ts diagram of nitrogen then for second problem, in the second problem we have been given that the gas is expanded from 20 atmosphere to a 10 atmosphere with the same isenthalpic zone, that is we have to consider this particular isenthalpic line and we locate this 2 curves isobaric curves one is this 10 20 atmosphere and it is the 10 atmosphere. Now here we can clearly see that on decreasing the pressure the temperature is decreasing. What it means that if we are changing the inlet pressure, then it has a direct effect on the cooling or the heating of the gas and for the same value of the enthalpy. So, value of enthalpy remains the same but we have simply changed the inlet pressure.

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So, what we are getting that we all zooming we can see that these 2 temperatures are coming out to be that is initially at 20 atmosphere it is 116 k whereas, at 10 atmosphere the temperature is coming down 103k. So, this particular thing as I told you earlier that is leading to the temperature decrease.

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### References

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Cryogenic Engineering Centre

So, you can find more of this fundamental from these books, so first book is on thermodynamics and the rest 2 books are on the cryogenic and there you can also find the particular Ts diagram which I have referred to.

Thank you.