Introduction to Non-Linear Optics and its Applications

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Lecture – 09
Basic Linear Optics (Contd.)

So, welcome student, to the next class of Non-Linear Optics. So, this is lecture number 9; so, let us see what we have in this lecture.

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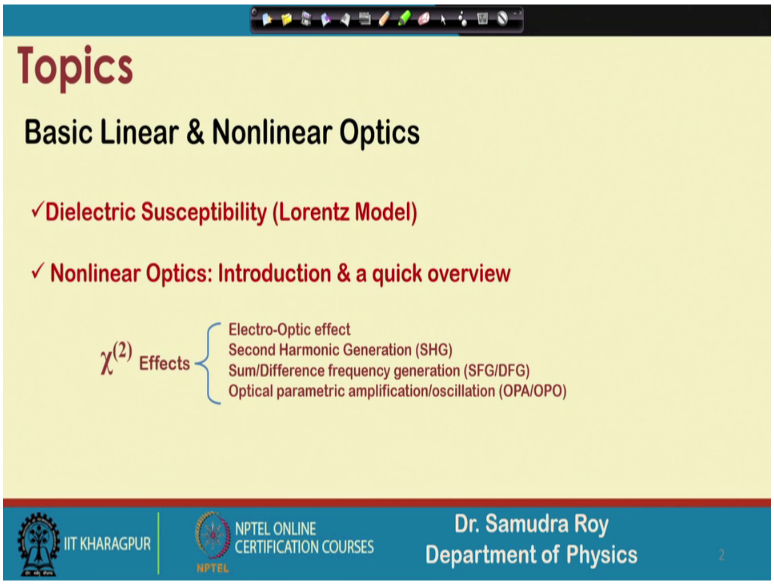

So, the basic topic of this lecture is the dielectric susceptibility we going to learn that. In the last class also we have started this part, but today we will going to learn this part in detail and the knowledge of Lorentz model is very important because we will going to use that in our future calculations which is very important in non-linear optics. So, Lorentz model we need to learn carefully.

So, that is why I emphasis on this dielectric susceptibility in this class. Also, we will start the non-linear optics, introduction and quick overview. So, what is non-linear optics which of the topics we will going to cover we will introduce in this class. So, let us start with the dielectric susceptibility Lorentz model. The same thing that we have done the previous class, again we will going to do the similar thing, but we will going to find out few extra elements and the calculation procedure that will be useful in our next class.

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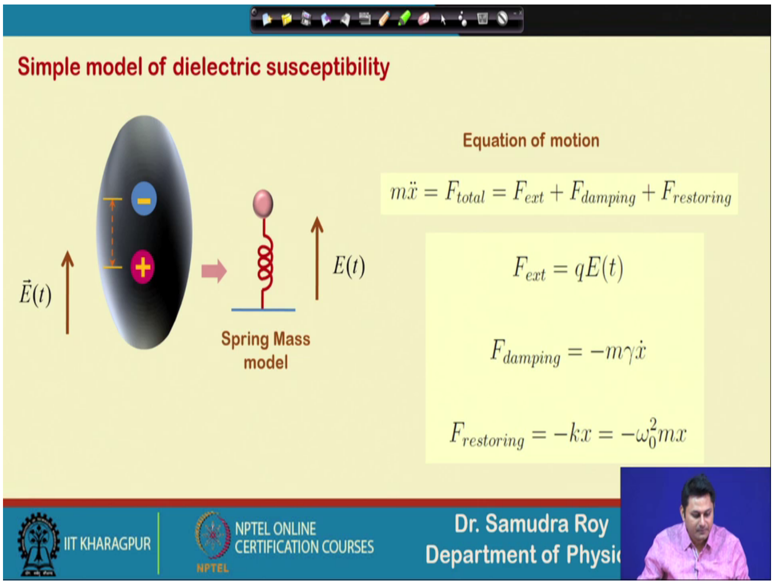

So, let us do that, ok; so, for so, some sort of recap. So, I would like to introduce again the simple model of dielectric susceptibility, which is nothing but, the electron cloud shifting of the electron cloud by application of some electric field. So, if I apply some electric field so, what happened? Is shown here in this figure. So, here we have electric field that is being applied. So, this is the electric field I am applying; this is the term.

Now, what happened because of the application of the electric field? The electron will going to shift and this electron cloud shifting leads to some kind of dielectric; dielectric means the separation of charge and the separation of charge basically now, will be oscillating if the launch electric field which is the function of time has some frequency say omega. So, if this is a constant electric field then we should have some kind of separation, but if it is a function of omega then what happened? That these things will going to vibrate and we can consider this as a spring mass model.

So, this is basically some sort of spring mass model, where I am launching the electric field here and as a result what happened, the things will vibrate. This things will going to vibrate with some frequency and this vibration can be modeled by the equation of motion; the straight forward equation of motion and this equation of motion contain few forcing term we have already explained these some external force which is by this external electric field and then the damping force that is the material property or the spring property here and then the restoring force. The spring should have some kind of restoring force and if I now these are the three forces in terms of electric field, damping coefficient and the characteristics frequency omega 0 of the spring.

So, after having the force then it is possible to form the equation of motion. So, after having this amount of force then we can have the equation of motion here.

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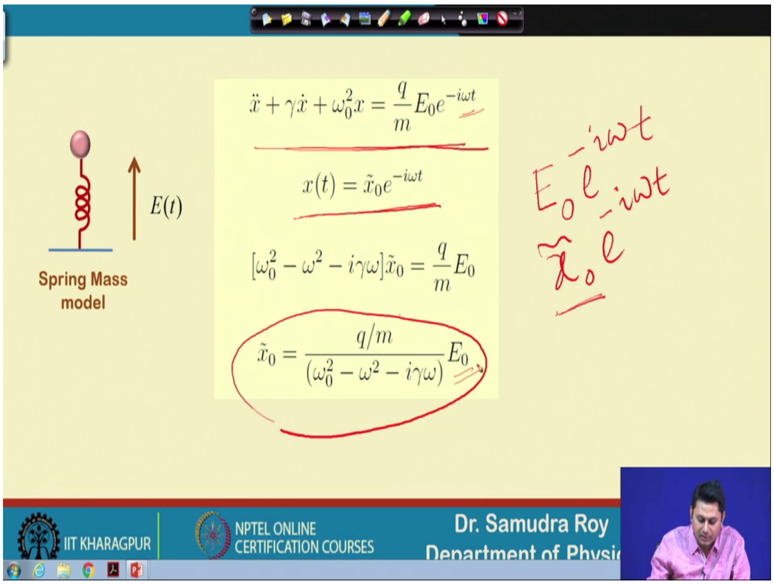

And, this equation of motion can be represented in terms of the displacement x and if I try to find out what is the displacement under some application of electric field then it should be like this. So, this is the equation of motion and this equation of motion contain a frequency term here as I mentioned earlier and if I do the calculation then we will going to find that my this is my displacement.

This is my displacement by application of the electric field E. So, I am not going to learn you in detail the calculation because the calculation is very straight forward. Once you have the equation of motion this will be your solution because your applied field with the form E 0 your applied electric field of the form E 0, e to the power of minus i omega t this is your applied electric field.

So, I want to find out the displacement in the same form and the same form is in this case x 0 tilde e to the power of minus i omega t. This is the form of the solution I want to find figure out; x 0 where is the separation peak separation or the peak value of the displacement because displacement is changing, it is the function of time; so, it is vibrating every time. So, if I do this calculation so, this is nothing, but the amplitude of thus this separation and this amplitude can be represented in terms of the amplitude of their launched electric field here E and once we have the launch electric field E with the other term I have this kind of expression. This is a very important expression and one thing we should consider here this is approximation that omega 0 is basically the resonance frequency of a single system.

So, now this is not necessarily be the true case because omega 0 can be different for the different molecules the different atomic systems. So, there should be some summation sign over that, but to make the system more simple we actually consider that the entire system had a single reasonance and this resonance is at omega 0.

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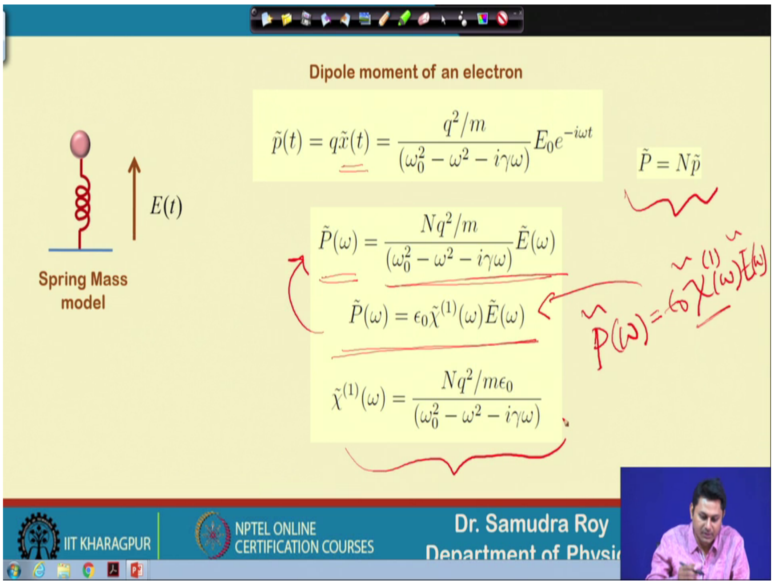

Well, from this x 0, now it is possible to find out what is the value of the susceptibility. How we figure out? Again, this is a very straight forward thing that once we have these value P 0, p is my dielectric p is my polarization which is the just the or in other term in other way I can say that p is my p is my dipole. So, dipole is nothing, but q which is the charge multiplied by the distance. So, charge multiplied by the distance is my dipole.

So, now the distance we have. This distance is nothing, but the x. So, this value of x I just calculate in the previous life. So, once we have the value of x then what we will do you will put this value here and if I put this value if you remember the value of the x, it was nothing, but q div q divided by m divided by this quantity with E 0 e to the power minus i omega t. So, mind it, here I am representing the value of x t which is a function of time it is not the value of x 0.

Well, after having that I have the total polarization which is dipole moment per unit volume. So, in here is the number of the number of dipole per unit volume once we have the number of dipole per unit volume then the next thing is to write this things in terms of N. So, my small p now become big P which is the function of omega and I just multiplied both the side with N and I will get this expression.

Again, P and E is related by this expression we know that the polarization is related to electric field. In frequency domain we have calculate this in earlier classes that P omega is equal to epsilon 0 multiplied by susceptibility term which is again a function of omega and multiplied by the E which is function of omega. Exactly the same equation is use I used here, so that I have a relationship between P and E.

Once we have the relationship between P and E, I can compare these two to find out what is the expression of thus chi first order susceptibility and my first order susceptibility term turns out to be this; N multiplied by q square divided by N epsilon 0 and then the whole divided by omega 0 square minus omega square minus i gamma omega. Here one thing you should remember that this is not a real quantity this is a complex quantity and here the term i is sitting. So, this i has a profound significance in terms of loss or absorption because this is related to the damping part of the system.

So, you can see that this gamma and I are related omega 0 is something which is the resonance frequency. So, if omega if I neglect this term then the rest term is where we can neglect the damping. So, if I neglect the damping then we can say that we are neglecting this i term when I neglect this i term then we have something quite real term then this real term basically gives you the amount of refractive index and this refractive index is essentially the thing we always consider this as a real quantity, but we will find that this is not the case refractive index at least the susceptibility terms is complex. So, refractive index has to be complex. So, let us see what happened when you consider the refractive index to be complex.

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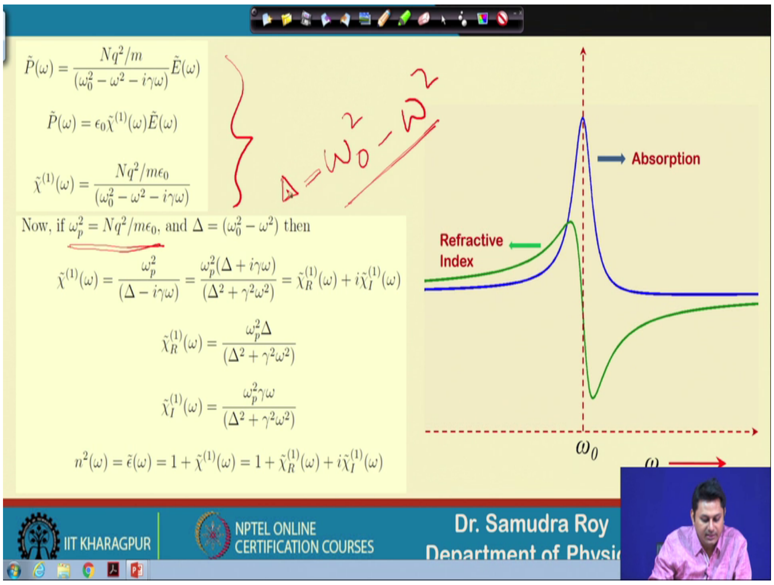

So, this is a very important thing. So, these are the calculations that we have already done. I have just written here as is note, in the last page we in the last slide we have calculate. So, now, here we can see that N q square divided by m epsilon 0 if this term is replaced by another this is a constant term another term say omega p square omega p is nothing, but it is called the plasma frequency and omega 0 square here omega 0 square divided by minus omega square this is the term we have in the denominator. So, this term I replaced by another coeff another term called delta. So if I replace this thing this to thing to make the expression more compact.

Then, we will have an expression like this. Now, this is a complex quantity. So, we know that when the complex quantity is there we always try to find this complex quantity in terms of x plus i y. So, this is the standard form to write a complex quantity. Here the chi 1 omega which is the first order susceptibility or the linear susceptibility is now become a complex quantity and this complex quantity I write try to write this complex quantity in terms of real plus i into imaginary in this from.

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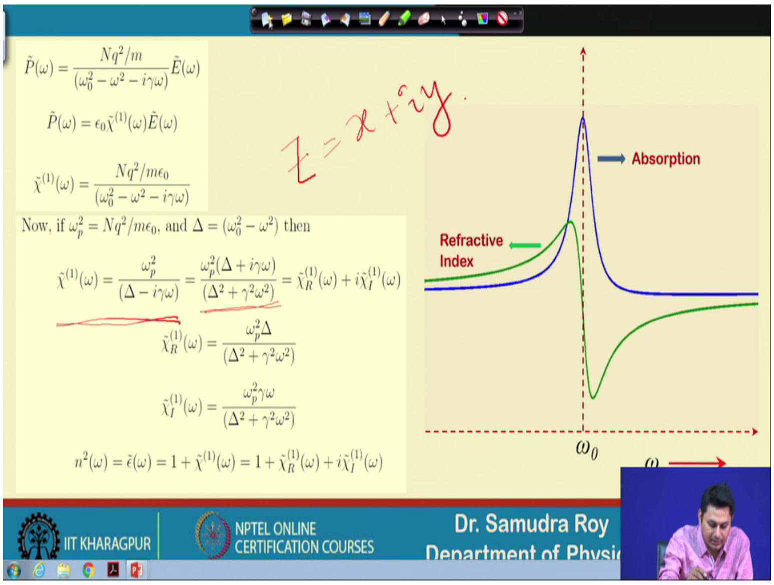

So you know that if Z is a complex quantity we always try to write it x plus i y. So, exactly the same problem we are doing here. So, we have something omega p square divided by delta big delta minus i gamma then omega. Then what we will do? We just multiply the complex conjugate of this quantity both the sides. So, that means, we multiply the numerator with gamma multi plus i big delta plus i gamma omega and in the denominator we will do the same thing. So, it will be a plus i b into a minus i b form. So, that eventually gives you a square minus b square. So, minus term will be absorbed because of this i term. So, eventually we will have delta square plus gamma square omega square the entire thing is now real in the denominator.

So, now we are in a position to write the thing in x plus i y form. Here x is the real part of the susceptibility term and x chi I is imaginary part of susceptibility. So, now, if I write the real and imaginary part we have two expression in our hand and this two expression is very important. The real part basically gives you some kind of some kind of thing which is important here you can see that the real part contain delta in the numerator and imaginary part do not have any kind of term like delta here.

So, if I try to plot this real part of the susceptibility which is defined as chi R. If I now try to plot that this real thing so, what happened, that I will have one zero value at delta equal to zero point.

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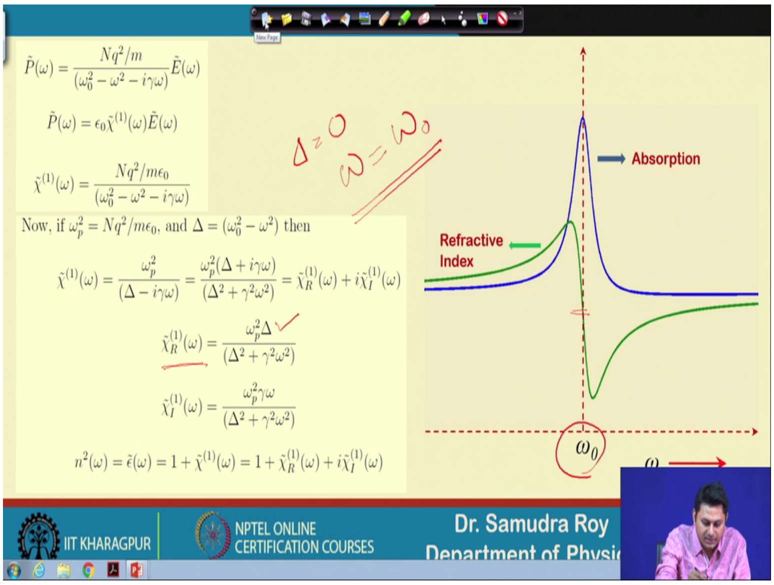

So, delta equal to 0 point is nothing, but when omega will equal to omega 0, that means, exactly at the resonance point what happened I will have a 0 value of R. So, if I plot this two function then this two function gives you something like that. So, this is the frequency and if I plot this frequency and in x axis and in the y axis if I plot the real and imaginary po part of the susceptibility term then we will have this kind of structure. This basically gives you absorption.

This blue curve and this blue curve is coming from the imaginary part of the susceptibility term. So, the imaginary part of the susceptibility and the real part of susceptibility we plot together and in the real part of susceptibility we find that there is this quantity is vanished at omega equal to omega 0. Here, if I see this is the omega zero; that means, at that point this quantity is 0, but in the right hand side of the omega 0 and left hand side of the omega 0, we have some value which is positive and negative respectively.

On the other hand positive and negative means if I go in this and this side in making omega 0 as my reference frame that means, the frequency of resonance then in the right hand side and left hand side we have two kind of distribution and this distribution basically gives you the coming from the real part of susceptibility which contain the big the real part is basic related to the real part of the refractive index which is in fact the refractive index. So the variation of the refractive index can be observed in this curve and here we can see something quite interesting that if I look carefully that here with omega increasing we have the increase increment of the refractive index from here to this region. From this region to this region we have the similar tendency that if omega is increasing then refractive index is increasing. Here also, omega is increasing that means, refractive index is increasing.

So, that is normally the case we know that when the omega is increasing, that means, when the frequency is high the refractive index is also high for the material, but in between this peak point to this peak point something different is happening. So, here if I draw a line here and a line here which is passing through the maxima and minima of these things in this region, this is the region I am making. So, this is the region where we can see by increasing omega the refractive index is decreasing. This is a this is very close to the region of absorption. We can see that absorption is the point where most of the energy can be absorbed, that means, we are very close to the resonance.

The frequency is very close to the resonance of the material. The resonance frequency of the material then what happened, most of the energy will be absorbed and if that is the case then what happened in this region the refractive index will behave in opposite manner. That means, the refractive index is now going to decrease with increasing omega. This particular region is called the anomalous dispersion.

This case where the refractive index is gradually increasing with omega is called the normal dispersion region and this is called the anamalous dispersion region where the opposite phenomena is happening that is why it is called anamalous anyway this is the part of refractive index we have discussed. And, the resonance condition at the resonance condition if I now plot these I part or the imaginary part this imaginary part basically give rise to some kind of absorption of the loss.

So, if I plot that we will find this is some sort of expression which we called the Lorentzian expression because we know that if I increase the denom the now if I increase the omega then there is a value of omega, where the delta function will going to vanish.

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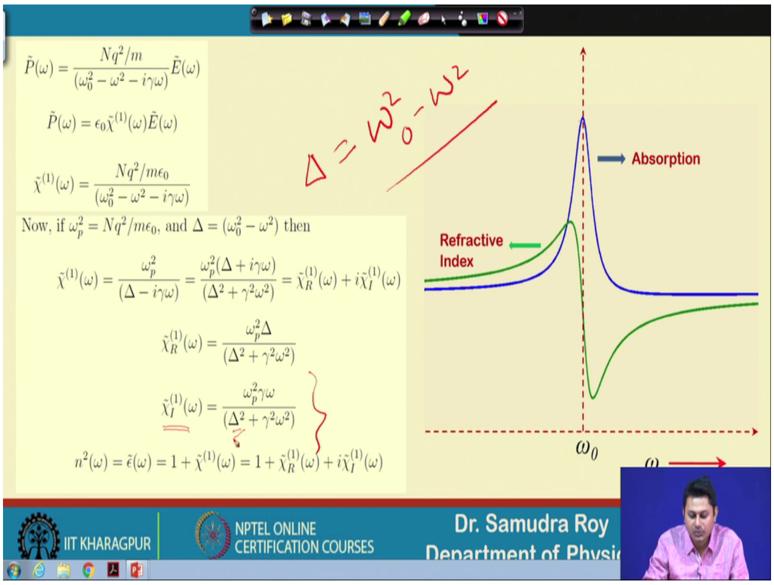

And, this delta which is related to omega like omega 0 square minus omega square. So, if I put this thing here then what happened that I will have omega at omega 0 point and I will have a maximum because this term will going to vanish. So, since this term is 0 here at omega equal to omega zero point we will have the peak value and this peak value basically suggest that at this point entire energy will going to absorb because we are launching the frequency electric field with the frequency that is exactly same of the frequency of the resonance frequency of the system.

So, this resonance frequency is matching with the external frequency the things will vibrate the molecules or the atom or the electrons will vibrate across the electric field and if it is this vibration frequency is very close to resonance frequency most of the energy will be absorbed by the system and that is why we will have a huge absorption line here, ok. This concept is important in linear optics as well as a non-linear optics because this calculation this Lorentz model and the mode of the calculation is important as I mention in the next class we will discuss in detail in next few classes.

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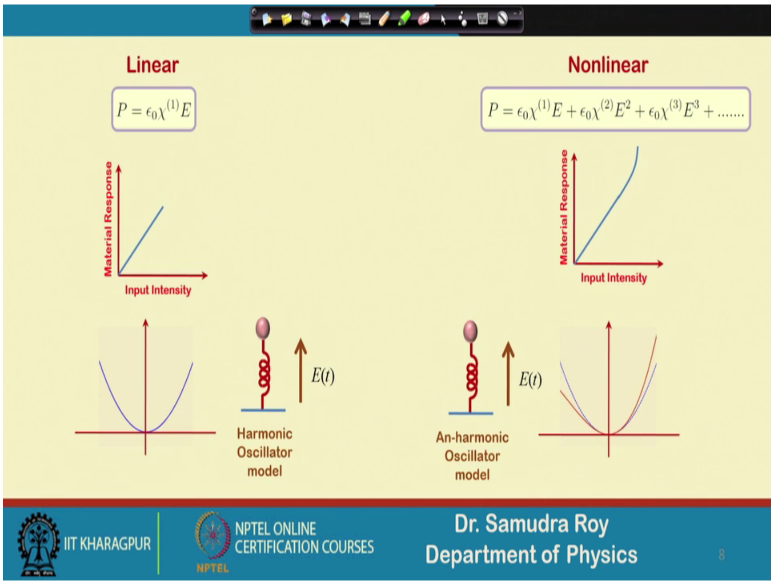

So, now we will go to our non-linear optics part. So, from here, now all the discussion will be non-linear. So, what is the meaning of nonlinearity in terms of optics we will going to study.

So, let us try to do in a simple way. So, far we are dealing with the linear optics and the linear optics essentially deal essentially have the expression P is equal to epsilon 0 chi and E. P is polarization and E is electric field if the polarization is proportional to electric field then what happened? In this particular plot if I increase the input intensity what happened, the material response is also responding and this response is linear in nature. So, there is no nonlinearity and everything will have like a linear curve here, everything has a linear that material response.

So, this thing one can also considered in terms of the potential term. So, when we launch the electric field, if my launch electric field is not that high then the potential that is generated due to the restoring term of the screen will be nothing, but simple harmonic in nature.

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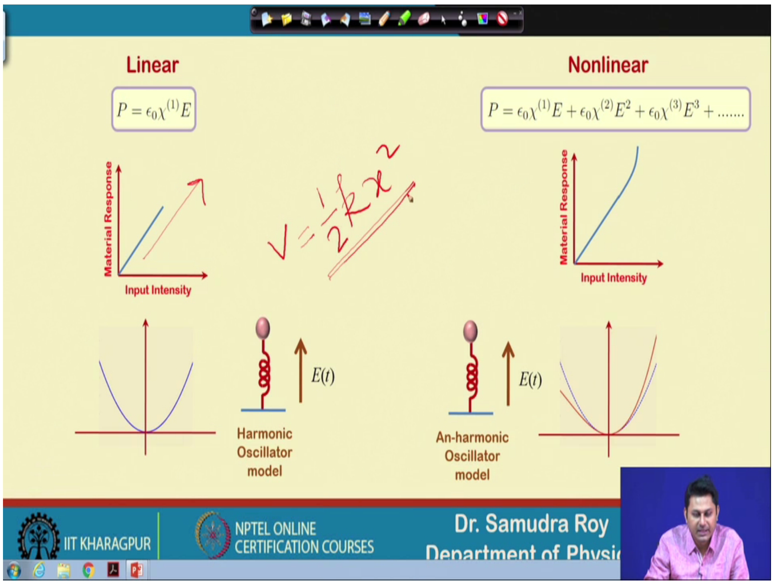

So, that means, here my potential V is something like half k of x squared, where k is the spring constant. So, the electron potential electron is vibrating under the potential which is in this form. Now, what happened in non-linear optics, try to understand compared to this that if I start increasing my electric field then the relationship between the polarization and the electric field will not remain linear anymore. That means, I need to introduce the higher order term.

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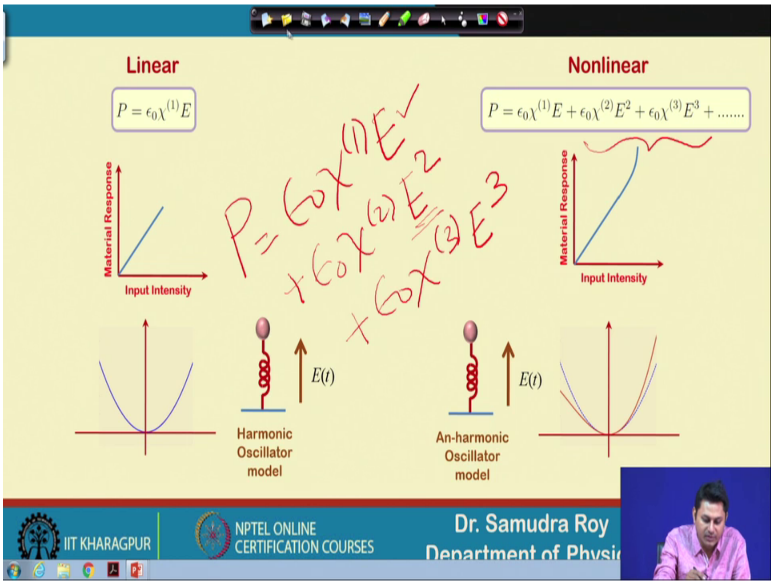

For example, here we have already introduced the higher order term. So, P equal to epsilon 0 chi 1 E this is the linear term up to this. Now, on top of that we need to write the higher order terms also. Now, E if I write the first higher order term then it should be epsilon 0 then chi 2 this is second order susceptibility and then E square.

So, what it means that my polarization is now not depends on only E, but also depends on the square term of that. If that is the case it is not restricted to with the square term we can also include end the term which is cube and so on. So, this is the next higher order effect and so on. So, this is essentially the equation which suggest that my polarization is now not depends on the electric field E the polarization is also related to E square and E cube.

Now, if the electric field is very high then this vibration this vibration or the electron mass model will not remain harmonic in nature some kind of an-harmonic effect will come because I am launching the electric field and this electric field is very high. So, that is why if I draw a potential here, if we look this curve here this potential defined by this rate line is some sort of an-harmonic potential.

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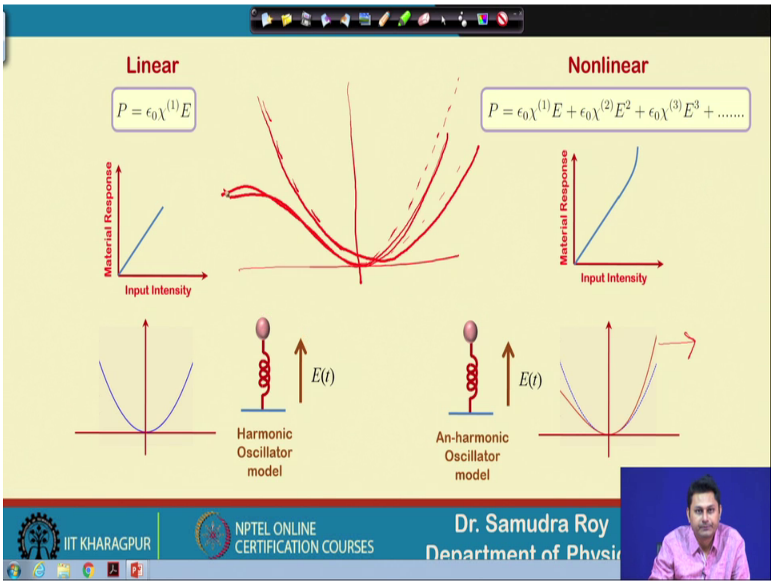

So, this an-harmonic potential is not something which is symmetric some kind of asymmetry is there. If I draw it should be something like this on top of that if I draw the potential related to harmonic one should be very much symmetry and it has some kind of value like this or the dot dotted line supposed to be; come like this, but it should be very much symmetric is something like this.

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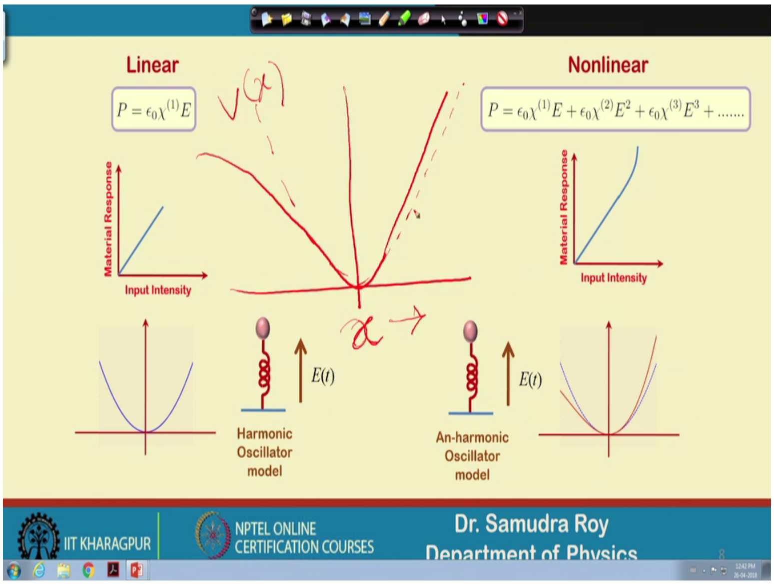

So, there should be some kind of asymmetry of this line. Let me know that once again to make it more clear. This is the potential related to harmonic oscillator for an-harmonic oscillator this solid line will be something like this something like this. There will be asymmetry over if I write this in terms of x then the potential V of x will be something like this. So, these an-harmonic nature the vibration of the electron under some an-harmonic potential will be given you will be now consider and if we consider that then will find that some kind of nonlinearity will appear because of this nature we will do that.

So, this is basically a very rough estimation of how this nonlinearity is appearing into the system and very simply we can say that since my polarization P is not depends on electric field E in a linear fashion some non-linear terms in terms of E square and E cube is there in the equation my system become non-linear. So, material response will be non-linear as shown here in this figure. So, input intensity if I increase then after some point the Hooke's laws Hooke’s law breaks. So, we will have some kind of an-harmonosity in the system and as a result what happened the things will vibrate in different way. The electron will vibrate in a different way and this basically give raise to some kind of nonlinearity in the system.

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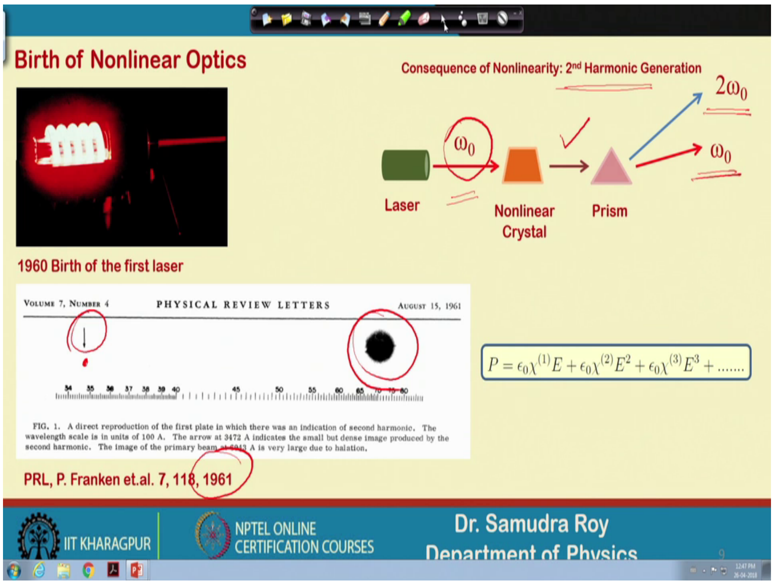

So, now we will also give you the idea the non-linearity and this is basically the birth of non-linear optics. So, non-linear optics is a subject which was known to people in the early stage because they know that the polarization is not related to E, but E square and E cube, but the effect of E square and E cube is so small that is very hard to find out some kind of affect out of that. So, what happened in 1960 the first laser was born. We know that laser is a light source which has a very high intensity. The directionality properties there, they are coherent in nature. So, if I consider all this things together then laser be a ideal material to introduce some kind of electric field which is very high in nature.

So, I can launch the laser into the material with this high intensity what happened that it will fall to fall into the system and the system will behave in a non-linear fashion and as output we will get some kind of effect. The first affect which is very important is a generation of second harmonic. We will discuss in detail second harmonic, but here in this slide I try to make you understand in a very simple way what is the meaning of second harmonic, that if I launch the electric field very high electric field the frequency omega here and it is falling on the non-linear crystal. This is a laser light laser source as I mention then what happened the non-linear when the light will come out from the non-linear crystal then instead of having omega 0, we will have other frequency component also.

Since the name is second harmonic generation we should have a frequency which is of the order of 2 of whatever the frequency I launch. So, that means, if I launch omega 0 then the frequency that will going to generate is double of that. So, that is way it is called the second harmonic of that. So, we will get a frequency 2 omega 0. It is the very fascinating phenomena that I am launching one particular frequency and I am getting something which is double of that.

So, this is a famous paper this is a famous paper by Franklin which was produced or which was published in 1961 just 1 year after the born of the laser the birth of the laser the laser the first introduced in 1960 and the paper was 1961, where they have shown the generation of second harmonic. This is the figure of the paper where you can see very dark spots here and this dark spot is the frequency omega 0 or the launching frequency which should be very high.

And, in that this particular figure they supposed to have a tiny dot here which basically the signature of the second harmonic, that means, the electric field that is generating at the frequency double of that, but unfortunately in this paper what happened that the editor by mistake the miss this dot and this dot was not there, but you can still see this line this line where they mentioned that here we should have some kind of dot which is basically gives you the idea that the second harmonic was generated in this experiment. They have experimentally verified the generation of second hand second harmonic. This is a very historic experiments that is why I thought I should show you this experiment, ok.

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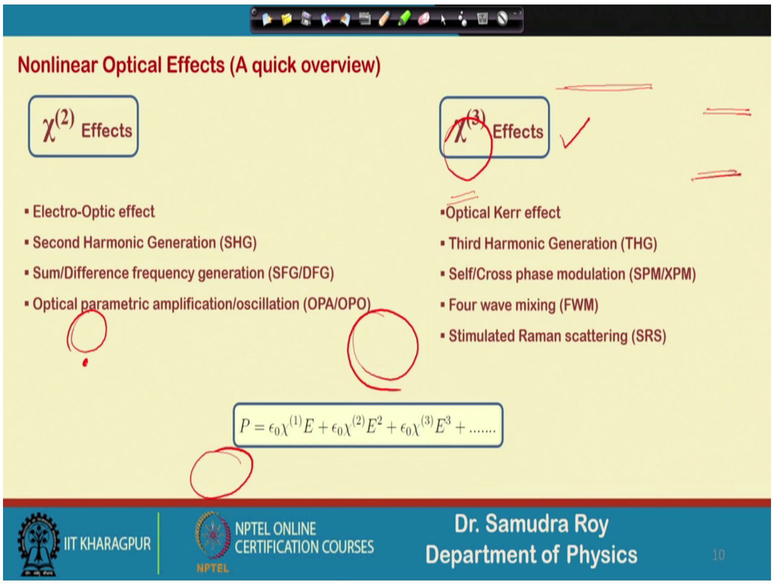

So, if I go back to the effects, these effects are something like this. So, the non-linear effects is basically subdivided into two parts; one is chi 2 effect and one is chi 3 effect. In chi 2 effect these are the effect we like to discuss and in this chi 3 effect in chi 2 affect this is basically the second, if I see all this considered to this term; that means, the term related to E square and in chi 3 effect we have something where we have everything related to the term related to E cube.

This is the first two terms of the polarization and we will going to roughly make a outlook of what is the meaning of all these terms like electro-optic effect, Second Harmonic Generation, Second Harmonic Generation I just mention and then sum and difference frequency regeneration and optical parametric application and so on.

In the chi 3 effect, again we will discuss few effects like optical Kerr effect, Third Harmonic Generation everything will be now discussed in a very simple way, so that you we will have a quick overview of that, but in detail we will going to study in the future classes. So, with this note let me conclude my class her. In this class we learn about how to calculate the linear susceptibility by classical Lorentz model, we have already started this part in our earlier class, but today again we have learn the technique and then understand what is happening in terms of if I plot this the real and imaginary part. The real part basically gives you the idea of refractive index whereas; the imaginary part gives you the absorption.

So, this is the linear part then in non-linear part we just started today, and we find that in non-linear part the polarization and now not function of electric field in a form like P equal to epsilon 0 chi E rather E square E cube term can also be added with that, if the electric field is very high. So, laser is something through which if I excite the medium then medium will start behaving in the non-linear fashion and chi 2 and chi 3 effect will be evolve in the next class we will start the chi 2 effect and then we will go to chi 3 effect and then we will start in detail what is the meaning of those effect that is today whatever we have discussed.

So, with this not let me conclude here. So, see you in the next class. We will start with the chi 2 discussion of the chi 2 effect and if it is completed in the next class we will do the chi 3 effect in after that class and then we will start detail calculation of non-linear optics. With this note so, let me conclude and see you in the next class.

Thank you for your attention.