Introduction to Non-Linear Optics and Its Applications
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Lecture – 02
Basic Linear Optics (Contd.)

Welcome back to the next class of Non-linear Optics and Its Application. In the previous class we have learned something about the basics of Maxwell's question and its solution. Today, we will going to learn few more things related to basic optics and the outline of this topic are these.

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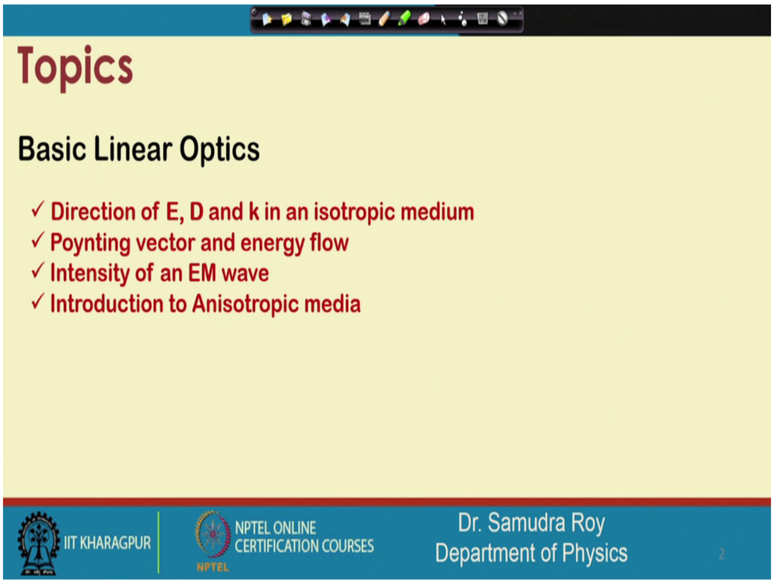

So, direction of the vector E, D, k in isotropic medium; So, we know what is electric field, what is magnetic field and what is the D vector and k is the preparation constant. So, how these things are related to each other, what is the director relative direction to this quantity we need to learn in an isotropic system.

Then pointing vector and energy flow that is very important, that when we launch an electric field into the system how the energy will going to flow and what should be the direction of this energy that we will going to learn, then intensity of an electromagnetic wave. So, when we launch a electromagnetic wave, the intensity is the very important parameter in non-linear optics. So, we need to know that how the electric filed amplitude and intensity are related to each other. So, we will find an expression for that.

And finally, we will introduce what is an isotropic medium. So, this the introduction of anisotropic, it is not going to say much about anisotropic in this particular lecture, but like to say something about than isotropic some introduction. So, let us start ok.

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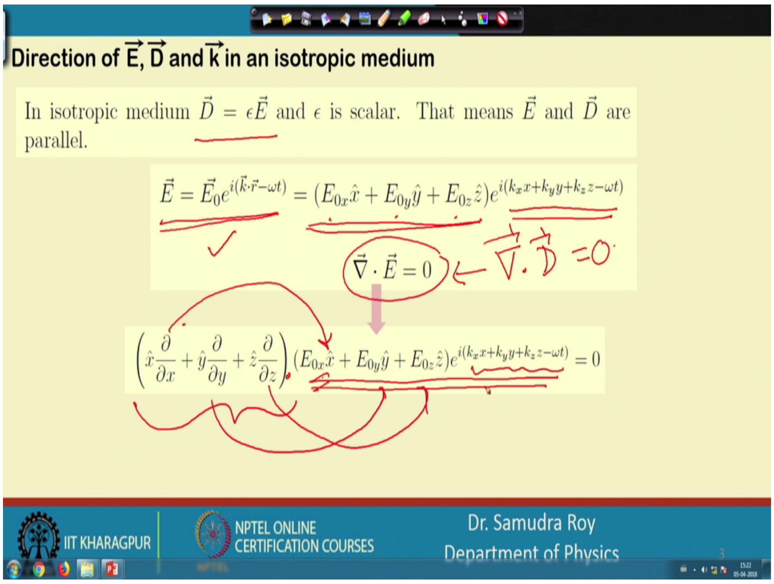

So, the first topic is direction of electric field vector D and k. So, here we can see that in isotropic medium D and E are parallel to each other. So, D is equal to epsilon, E where epsilon is this scalar if D and E is parallel, then epsilon has to be scalar.

Now, if D and E are parallel then I will have a plane with solution for E. So, E is easy to E to the power i k dot r minus omega t as usual. So, what we will do that, we will expand these because we are dealing with the direction. So, we will like to expand this E. So, E is expanded the vector E 0 is the amplitude of the system or the electric field. So, this electric field amplitude is a vector quantity. So, I expand this vector quantity into x y z, and if I expand this into xyz then it will be E x 0 x this is the x component, it is a y component and this is z component of this vector, with the unit vector x y z also k dot r x expanded here.

So, this is nothing, but the expansion of whatever is written here in component wise. Now, we know that grad dot E is equal to 0. So, that is if there is no free charge even in the material if there is no free charge, then we can write that grad dot E is equal to 0. See in isotropic case if you remember in the previous class that was our expression grad dot D is equal to 0, now D and E are parallel. So, I can write that grad dot E is also 0. Now, if this is the case grad dot E is 0 if this is the case, then I can expand this grad operator in this way we know that this is the unit vector x this is a vector operator. So, the operator is in this particular form, it will going to operate over this entire E, which is written this which is equal to 0.

So; that means, this d dx, there is there is a dot sign should be here somewhere. So, this d dx when having a x component will be when we make a dot products only this things will be there with exponential term, this is a pure scalar term. So, you should not bother about that. So, these will be related to this, these will be related to this and these will be related to this with this exponential term.

So, when we make this operation we will have a solution like this.

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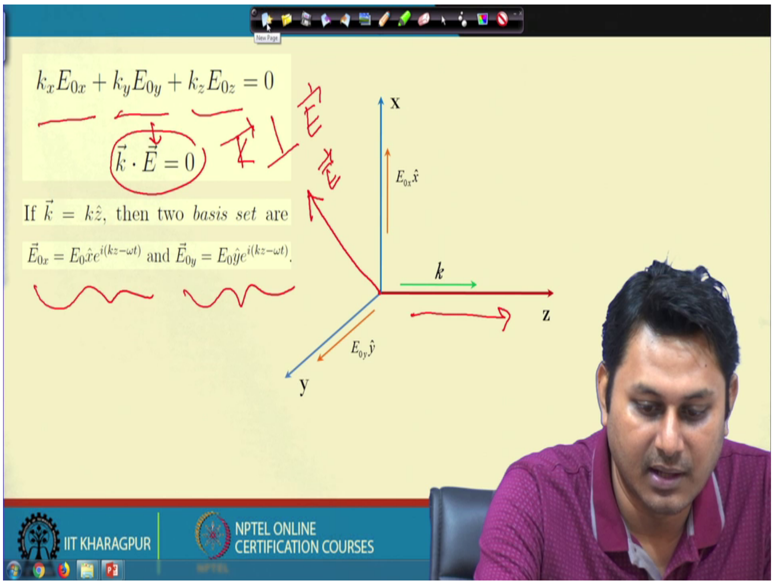

So, let me go back so; that means, when we make a operation over these, then what happened that the derivative will gives me ikx. So, if I look this, the derivative of del del x of this term will give me ikx, derivative with this term will give me i k y, and derivative of del del z will give me k z. So, this term will come out and when this term will come out, I will have a expression which suggest that k x E x plus k y E y plus k z e E 0 z is equal to 0.

So; that means, this quantity is 0, I can write this as in dot product. Because kx ky kz are the xyz component of k vector, E 0 x, E 0 y, E 0 z are the xyz component of the E vector.

So, this equation is nothing, but k dot E equal to 0. Now, dot E equal to 0 means k and E are perpendicular to each other; k dot may E equal to 0 means, vector k is perpendicular to E this the first out first thing we find that vector k is perpendicular to E.

Now, if k is one direction say a prefer direction say z here is shown, that k is moving along z direction, then E is perpendicular to k. So, two preferred direction can one can assume x and y, this x and y is the two basic set of the E. So, first we find that the propagation constant an E their perpendicular to each other. After that we say that if k is along Z direction, then what happened other two perpendicular directions are there X and Y.

So, in any whatever the k we have in if it is in Z direction, I will have a perpendicular direction which is in X and Y plane in the direction of E, which is in y and x plane we have a general electric field direction, which is perpendicular to k as well as Z direction. If that is the case then what happened? This E can be divided into two part one is along this direction X and one is along Y direction. Any E can be divided into two basis sets. So, these are the two basis sets which are perpendicular to k.

These are essentially two polarization, we called this two polarization state. So, once k is given to me, then what happened? I can understand that E is perpendicular to the vector k. So, there are infinite amount of possibility that k is perpendicular to E is perpendicular to the k, because E can be anywhere in this Y Z plane, but if I divide these vector into two preferred direction X and Y, I should divide in X and Y direction.

So, these two we can called the basis set of the electric field. This basis sets are nothing, but the polarizations of this. So, E has a polarization in this direction, but I can divide this into two basis set and this basis sets are the polarization along X and Y direction these two perpendicular direction ok.

So, the important information here is k and E are perpendicular to each other ok.

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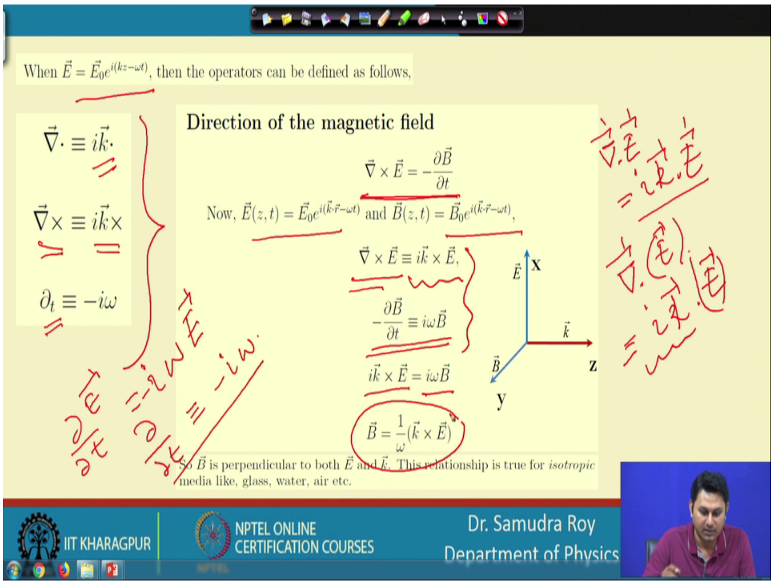

So, one very important thing now we will like to know I mean that is, what are the meaning of this operators. In the right left hand side of the side you will find some operators are given. These are the operators grad dot (Refer Time: 10:02) and del del ts are the operators this is a very useful identity. So, if I operate this over the electric field which is has a plane wave form, then what happened that this will give me this quantity.

So, the meaning is if I operate see grad dot E, where E is this form then this eventually give us ik dot E. If it is a one dimension then it should be ik E, but if it is a vector form this is the form.

So, from here I can see that if this is the operator, which is operating over k if k has a plane wave solution, this is equivalent to i k operate over E. So, from here we can say that i k is equal to equivalent to this operator, we will going to use this in several places. So, that is why it is important. In the similar way if I make a (Refer Time: 11:16) operation over that, I will have these things and if I make a derivative with respect to t of this thing then what happened.

So, let me make a derivative, I am try to make a derivative of this quantity E, then what happened? If E has this particular form if E has this particular form, we have i minus i omega and then E this minus sign because E have k z minus omega t. So that means, del del t operation is equivalent to multiplication of minus of i omega that is as simple as that.

So, after having the knowledge of this operation in terms of k and omega, we will going to use that. So, now, direction of the magnetic field we try to find out. So, direction of the magnetic field one can find out by simply the relationship with E and B, which is the third equation of Maxwell's equation called causes minus del B del t. E has this form, B has this form as I mentioned this is plane wave form.

Now, (Refer Time: 12:30) cross E is equivalent to i k cross E; when it is equivalent to i crate cross e. So, del del tB is also equivalent to i omega B, because this operator del del t is equivalent to minus omega, i omega. So, negative sign. So, that is why this minus term is not here.

If that is the case, then we combining this two we have another equation i k cross E is equal to i omega B. If I remove this item and rearrange, then B is related to k and electric field E in this equation. This is the final expression of B, where we have the value of v as well as the direction of B. So, direction of B is very important.

So, from this equation so, let me erase all this things and then only concentrate on this expression that B E and k are related with this. So that means, if k is one direction, E is the perpendicular direction that is in this direction, this is the direction of E this is the direction of k, then k cross E in the right hand rule screw rule gives me the B in this direction.

That means B is perpendicular to both electric field E and the propagation direction k. This is not a very very unique result because it is a very general thing we know that electric field and magnetic field both are perpendicular to the propagation of the light. So, light is a transverse wave. So, transverse wave the means that the light is propagating along one direction, and when the light is propagating along one direction the electric and magnetic field is perpendicular to that particular direction.

In this case light is propagating on k direction. So, electric and magnetic field are perpendicular to that direction that is one issue. Second thing electric field and magnetic field itself perpendicular to each other, both are perpendicular to k and E and B again perpendicular to each other.

So, that is the interesting thing say electric field and magnetic field are perpendicular to each other and also they are mutually perpendicular to k. So that means, they are forming a system of xyz; that means, a coordinate system can be formed with this let us go to the next slide.

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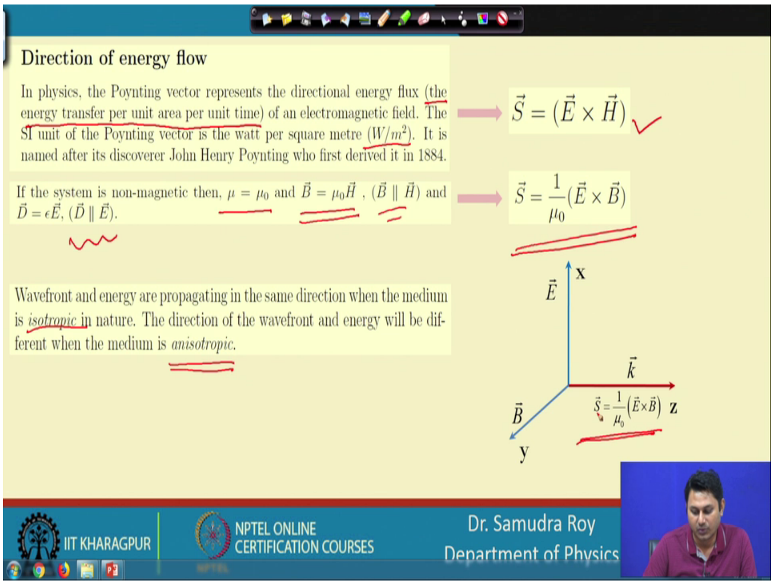

So, the direction of the energy flow; So, the direction of the energy flow is important, but before that we need to know what is the meaning of pointing vector. So, pointing vector is a term as we can here that, represent the directional energy flux the energy transfer per unit energy flux is nothing, but the energy transfer per unit area per unit time.

Amount of energy that is passing through per unit area, per unit time is called the energy flux. So, obviously, this quantity has some unit and the unit is straight forward omega watt divided by meter square. So; that means, the energy propagate per unit time is power and per unit area is the unit of area. So, that is why it is meter square. So, it is power divided by meter square.

So, now, for non magnetic system mu is mu 0 and B can be represented as mu 0 H and B and H are parallel. Also in isotropic system E and D vectors are parallel to each other and here the pointing vector is represented as this is the definition of the pointing vector S is a pointing vector it is represented as E cross H. Also S can be represent in terms of B by just replacing H. So, this is another form in many places we have this form also. So, now, if I look carefully the direction of S is E cross B.

So, previously we find that if k is direction, E and B are perpendicular to that when E and B are perpendicular to that when E and B are perpendicular to that, then E cross B is a vector which should be in the same direction of the k. That means, the pointing vector and the k vector are in same direction. If I take E cross B and using this right hand screw rule, we will find it is in the same direction of the k. But here I should mention very clearly that this is on the true for isotropic system. As shown here wave front and energy are propagating in the same direction when the medium is isotropic.

So, the wavefront and the energy is propagating in this particular direction, when the system is isotropic in nature, but in anisotropic case we will find that k and s may not be in same direction. That means, wavefront will propagate in a different direction and energy will propagate in another direction. So, there will be a mismatch we called as a walk off we will discuss in detail, for which we will have a different direction for k and s vector.

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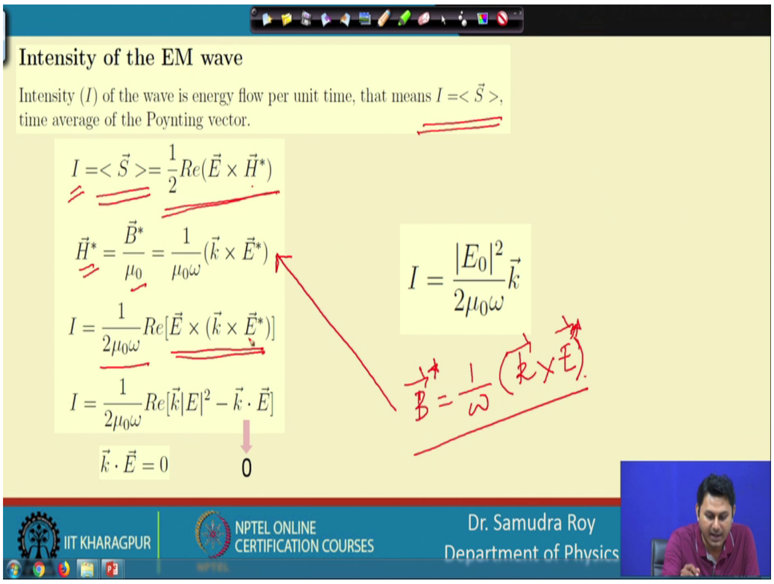

Let us go back to go to the next slide which is very important, that is the intensity of the electromagnetic wave. As I mention the non-linear optics is entirely based on the intensity, that if the intensity is very high then only we can excite the non-linear property. So, we always like to know what is the relationship between the intensity and the applied electric field amplitude; So, here is the treatment. So, intensity I of the wave is energy flow per unit time; that means, I can write this in this way, this bracket says the time average the time average of that. So, time average of my pointing vector is represented by bracket of S. It is nothing, but if I write in a proper way it should be (Refer Time: 19:45) part of E cross H and because of the time average we have a half term here.

So, real part because E and H maybe a complex quantity with the complex amplitude, but when I when I make H star then we can make it entire thing in a real. So, this is a definition of the intensity, but note that intensity is a quantity, which is which do not have any kind of vector. But here I am using a vector in the right hand side. In principle there should be more sign here, but for the time being I am not using the mod sign and that the reason is that, we need to find out in which direction the intensity will flow.

Obviously, it will flow on the direction of k, but that we will going to figure out. Now H star is B star divided by mu and then if I write B in terms of E and k which we have done in our previous calculation, then it should be. So, B was if you remember B was 1 divided by omega into k cross E that was the direction of vector B.

So, now if I make a B star star of that, so, we will put a star here and we have to put a star on E here. So, this is used here, this is the thing that we have used here and then we use this value of H here. So, I become 1 divided by 2 and then mu 0 omega 0 mu 0 omega term is sitting here, E is this E cross, H star is replaced by this quantity k cross E star.

Now, we have A cross B cross C this is the form we are now having if this is the case then I can expand A cross B cross C we know if we expand A cross B cross C, then we will have. So, what is A cross B cross C it is something like B C A minus C A B.

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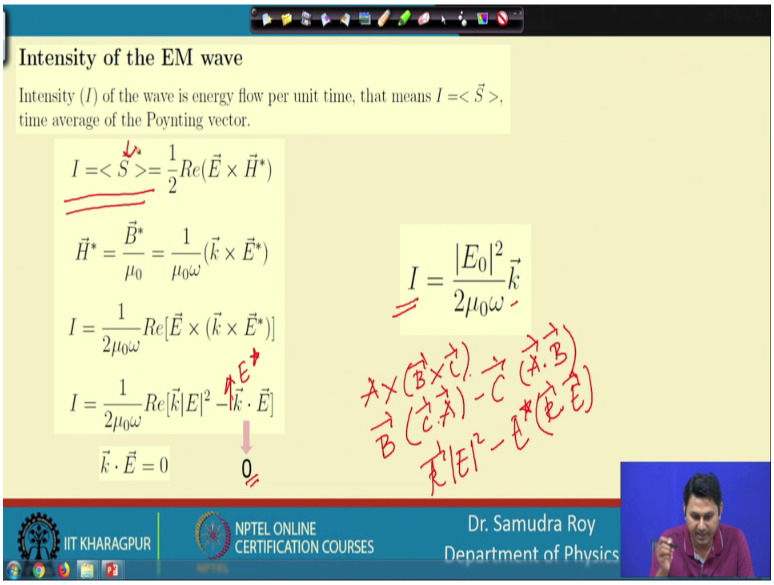

Now, E cross t cross E star if I do, then B here is E. So, my first term is B here is k. So, my first term is k, and c is E star and E. So, if I dot E star and E it will be mod of E square minus C here is my C here is my E star and then we will have A dot B; that means, k dot E.

So, I have a mistake here, here one E star term should be there be careful there some E star term should be should be there, but k dot E is 0 we find that in the previous calculation that k dot E is 0. If k dot E term is 0, then we should not bother about the next term.

So, then my expression simply comes out to be I is equal to epsilon E 0 mod of E 0 square that is amplitude square 2 mu 0 omega and k. So, now, I is a vector, I is an I is a intensity is not a vector, but I write this is in a vector form to ensure that the energy is propagating along k direction.

So, technically this equation is not right, but I just here I just put this S sign here, the vector sign here so, that I can know that in which direction the energy flowing. The energy is flowing along the k direction, but in the next slide we will remove that and eventually find out what is the expression, what is the final expression ok.

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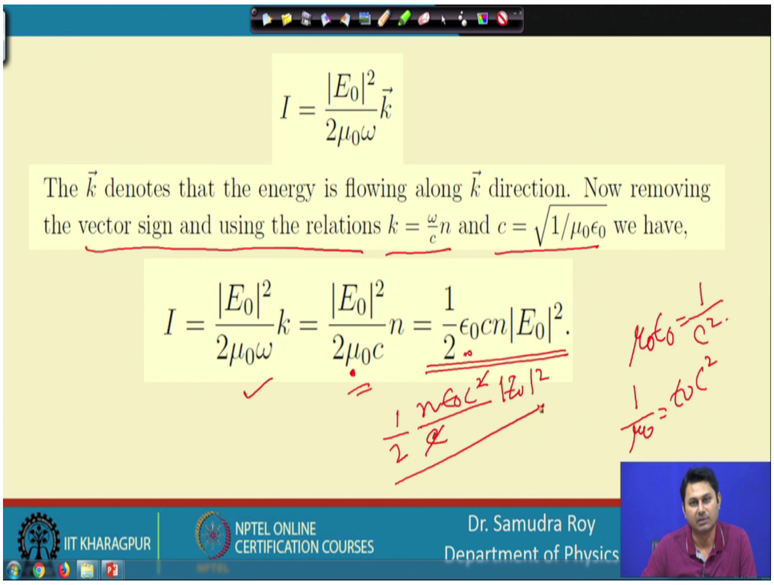

So, as I mention k denotes here energy. The vector k denotes that, the energy is flowing along the k direction. Now, removing the vector sign and using the relation k equal to omega c n and c equal to the we have this form. So, I replace k as omega c divided by m. So, when I replace k, then I have omega here and c here. So, this omega is canceled out and we have c and n sitting here.

So, now this is the form in many places people use, but I preferred this next form, I just do not want this mu 0 sitting here, because mu 0 is associated with magnetic system, but I can remove that and I can use a dielectric system and epsilon 0. So, easily one can do that. So, if I write these half of n and then mu 0. So, mu 0 epsilon 0 is nothing, but one by C square. So, 1 by mu 0 is replaced by epsilon 0 C square. So, when I replace 1 by mu 0 epsilon 0 C square, one C will cancel out.

So, n and then I write epsilon 0 C square in the denominator I have 2 C and this C this C will cancel out and mod of v square is there as usual I have half n epsilon 0 C square. So, that is the expression we have this is a very very important expression we will going to use this expression in many cases we should remember this expression, and this expression is tells us that how the intensity is related to the electric field ok.

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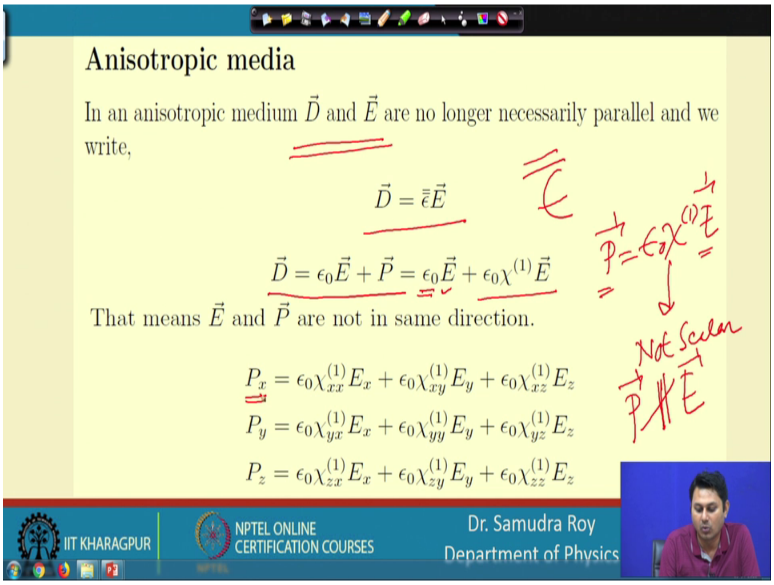

Finally, the Anisotropic Medium; We will just introduce today what is anisotropic medium quickly and in the later class, we will discuss in detail that what is anisotropic and what is the consequence of that. So, anisotropic medium where D and E the first thing is that, in anisotropic medium the physical property is not it depends on the direction. So, in all the direction the physical properties are not same.

As a result what happened that D and E are not necessarily parallel to each other. Since, it is not parallel to each other, we write D is equal to epsilon E, but now my notation is slightly change if you note, I write epsilon bar double bar. So, this is a notation of a tensor quantity. So, we will going to learn what is the meaning of this tensor quantity and what is that. So, the thing is that D and E are now not related to any scalar relation. So, the relation is more complex. So, you can see that why it is complex.

So, now D is equal to epsilon 0 E plus P. Where P is a polarization, epsilon 0 is a is a is scalar then E is there and P is now related to E. So, P we know that this is epsilon 0 chi 1 and E. So, P and E are related to this expression, but if I say D and E are not parallel; that means, these quantities are not scalar this is scalar, but this is not scalar.

So, this quantity is not scalar means, this is not scalar if this quantity is not scalar. That means, P and E are not in same direction. So, they are not parallel to each other which is very important. P is not parallel to E, P and E are not in the same direction. If that is the case then; that means, P x component will not be equal to E x component multiplied by some constant. Rather P x component is related to E x component E y component and E z component. In the similar way P y component is related to E y component E x component E y component and E z component and so, as for z. How it is happening? So, let me let me go back to the next slide and then probably things will be understood.

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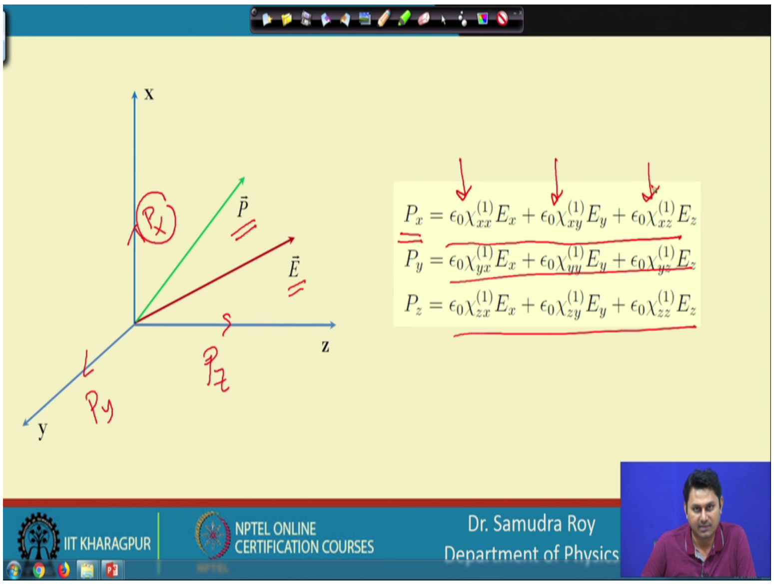

Here we have a figure and this figure tells everything; that if I launch the electric filed in one direction and we are saying that P and E are not parallel to each other; that means, P will generate in a different direction.

So; that means, the dipole will arrange not in the direction of E, but in some other direction. If that is the case then what happened? If I divide these things into component wise, then the x component of this P is generated because of the xyz component of E. So, x component of the P is generated due to the application of the eclectic field, but the electric filed x y and z all components are involved. So, that is why the equation is written in this particular form; these, these and this, this is true for z.

So, if I divide these x y and z. So, readily this component is not related to the x component of E, rather x y z component is there to form the x component of polarization vector P. So, if I write this entire thing in a general way, then we can write it and we will going to see that one can ride in a matrix form. And now since it is written in a matrix form, this coefficient the susceptibility coefficient is now not a scalar quantity rather this is a vector quantity or this is tensor quantity, it is not a vector quantity at all its a tensor quantity. So, how this tensor quantity will be related and how these things will going to imply, I mean what is implication of these tensor quantity we will going to explain this in our next class.

So, today we will just introduce the isotropic system and basic of the anisotropic system. Anisotropic system is very very important in our case and in the next class, we will learn more about anisotropic system; with this note let me conclude today's class so.

Thank you for your attention and see you in the next class.