Modern Optics
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Lecture - 49
Acousto-optic Effect (Contd.)

We were discussing Acousto-Optic Effect and, last time we discussed this Raman Nath diffraction and, the condition for Raman Nath diffraction was, the width at the acoustic waves would be very small, preferably less than 1 centimetre.

We, could see through, small calculation and if it is more than, 2 or 3 centimetre width and for those numbers that we have used then, it falls into the category of Bragg diffraction.

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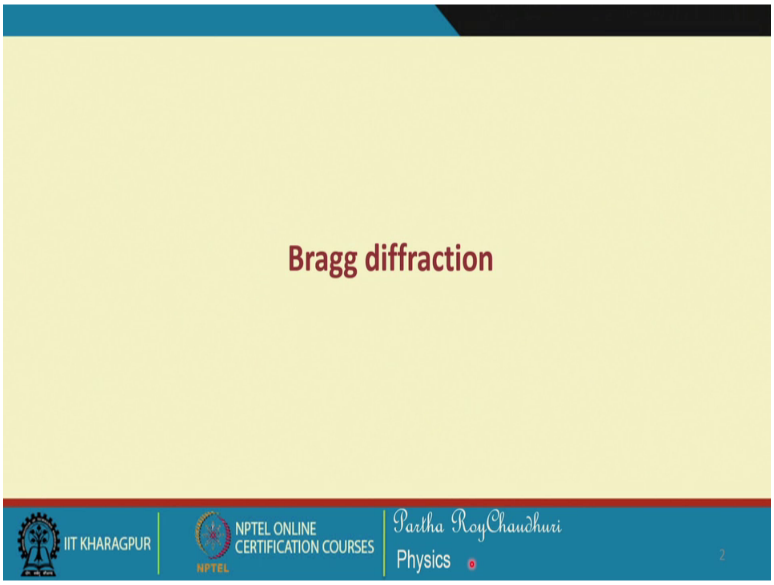

And in the case of, Raman Nath, diffraction, we saw that, the, the formulation was, based on this the interaction length of this acoustic interaction ength of this, electromagnetic wave, that is the light wave with the acoustic wave was small, but when this interaction length is longer then, that formulation is no more valid and, we will look at the couple more, theory couple more equation for this coupling and then the Bragg diffraction under, different configurations under different categories, that we are going to discuss.

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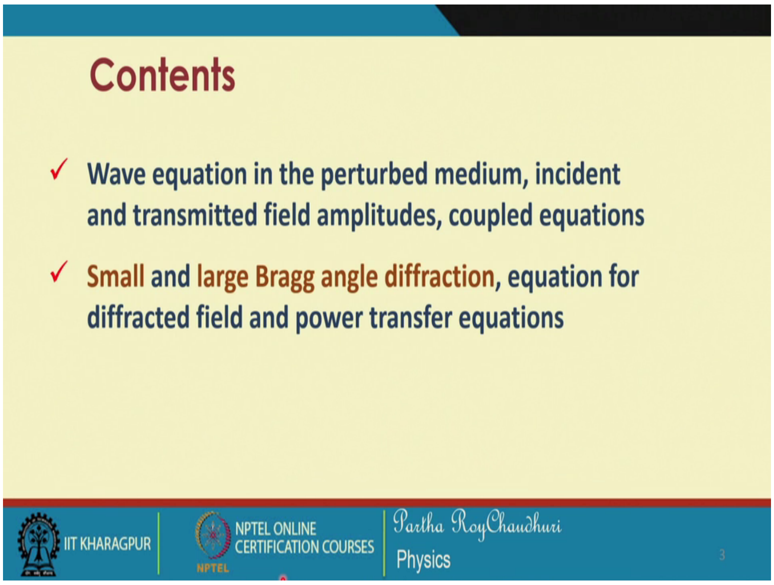

So, the content of the discussion today is we will be, starting with the wave equation for the perturbed medium.

Basically this perturbation is due to the, Refractive Index Periodic perturbation of the refractive in the medium through which this, light wave is, passing through and then we will look at the transmitted field amplitude, formulate this coupled wave equation. We will see, that this coupled wave equation is not very simple to solve so, we will make the approximation for small and large Bragg angle condition then, we will be, this discussion we will be focusing more on the small Bragg angle diffraction.

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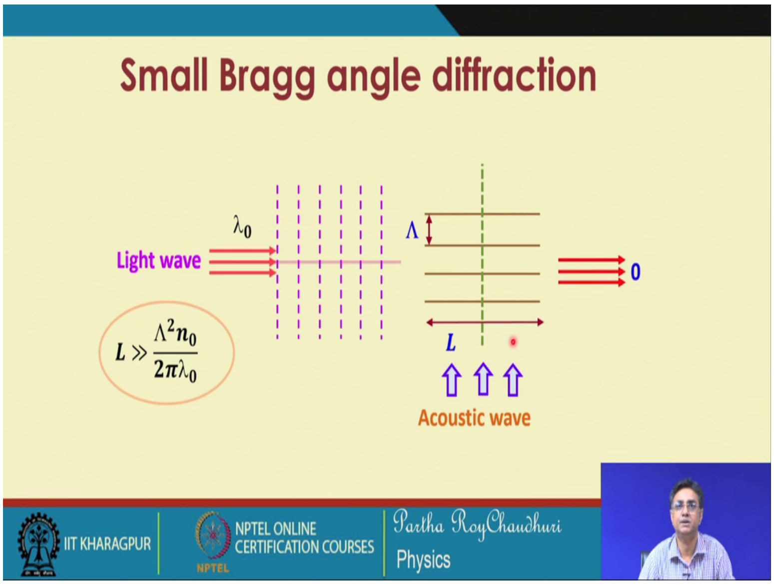

So, the as we e have learnt by now, that the length of interaction this L this is the acoustic wave, which is travelling through the medium and, because of the propagation of this acoustic wave the medium undergo was a change in the refractive indices.

Periodic change in the periodic refractive index change and that forms this travelling grating the peach of the grating is the, the periodicity of the grating is the, wavelength of the acoustic wave and this width of the acoustic wave, that is what is represented by L.

If it is large and quantitatively that is, given by this if the width of the interaction if the acoustic wave that is the effective width of interaction of the light wave with the acoustic wave that is, given by l which is more than this quantity where.

This lambda, capital lambda is the wavelength of the acoustic wave. This lambda is the wavelength free space wavelength of the light wave and n 0 is the unperturbed refractive index of the medium.

So, if we satisfy this condition that is this width of the acoustic wave is relatively large, in that case, we will end up with Bragg diffraction. The light wave, we will see a volume grating volume phase grating and in that case we will have only one order of diffraction plus order or minus order. In addition to the zeroth order, there will be a coupling between this zeroth order and the first order diffraction.

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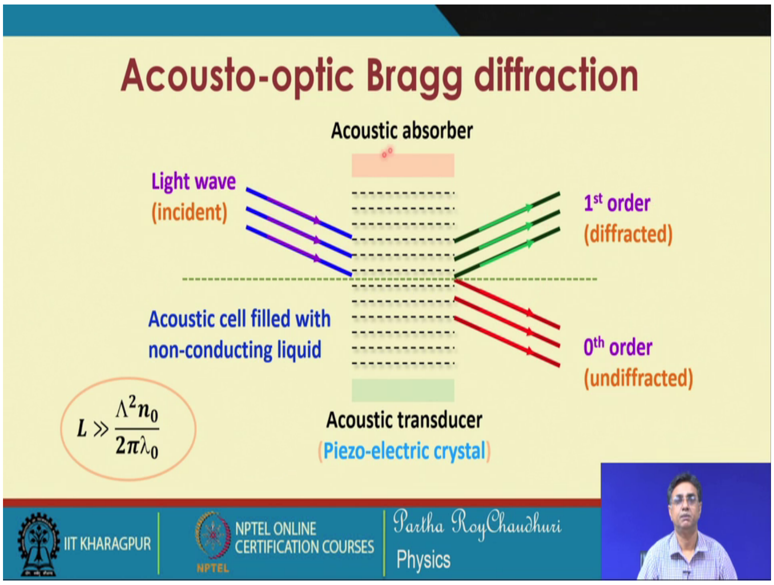

So, the, schematic of the arrangement of such diffraction Bragg diffraction is like this, you have this acoustic transducer which is usually a Piezo crystal mounted at one side, there is an acoustic, wave absorber.

So, that there is no reflected wave else this reflected wave will form a standing wave pattern and the grating will no longer be a travelling grating, but will be a fixed grating. And then this width of this acoustic wave is important, which will be fairly large, in this case and the light wave will be inclined. The case that, we will be discussing is that, the light wave will be inclined with an angle which will be small so, that we call it a small Bragg angle diffraction.

In that case, there will be a coupling of light from the zeroth order that is the undiffracted light. Let us suppose, there is no, no perturbation periodic perturbation no grating formation in the medium, in that case this light will go undeviated, there will be small and lateral shift, because of the width of this.

So, this light will be effectively undiffracted and once we switch on this acoustic wave then, there will be a coupling between this unperturbed, undiffracted wave, that is the zeroth order wave, zeroth order diffraction with the first order diffraction.

So, there will be a coupling between these two and we will see, not surprisingly that this coupling is just the same it is similar as the coupling between two optical waveguides and we will end up with similar set of equations, which will be very interesting and we will be seeing different conditions for the coupling, from this undiffracted zeroth order beam to the first order beam.

So, we for this case, because the length of interaction is fairly large so, that the thin -thin, phase grating formulation which was valid for Raman Nath diffraction is no longer valid and we will rewrite this wave equation.

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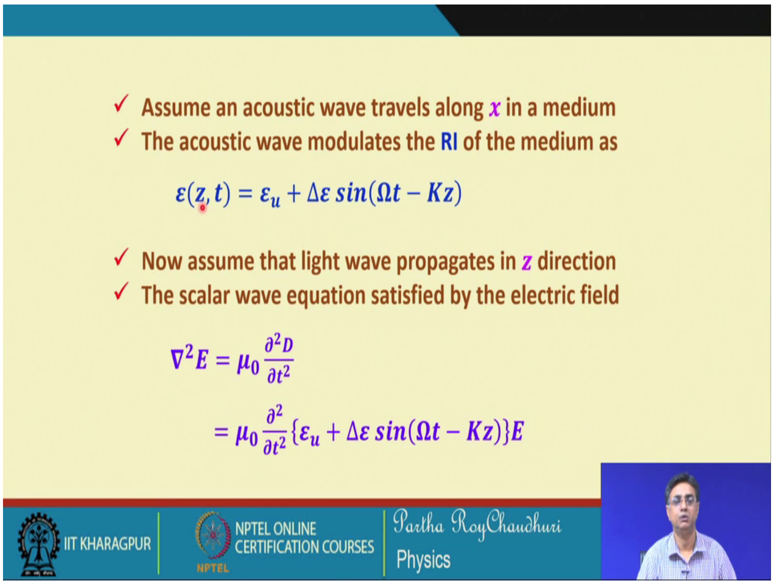

So, let us first consider that to look at the couple more theory, how we arrive that, let us first assume that an acoustic wave travels along x in the medium.

So, this is your x direction and the wave is travelling along the z direction. So, this is your z direction. So, this is x and this is z direction and for such a situation the refractive index perturbation, will be given by the acoustic wave modulus the refractive index of the of the medium, because it is travelling along the z direction. So, I think this, this should be z actually the acoustic wave travels along z.

So, the periodic perturbation in the medium will be given by this, the light wave is travelling along this. So, now assume that the light wave propagates in the z direction. So, these are actually, not correct this should be z and this should be x.

So, in that case, we can write this equation del square E equal to mu naught del square D by del t square and then we write this equation for this travelling acrostic wave, which will give you that this is the unperturbed permittivity and this is including the effect of the acoustic wave.

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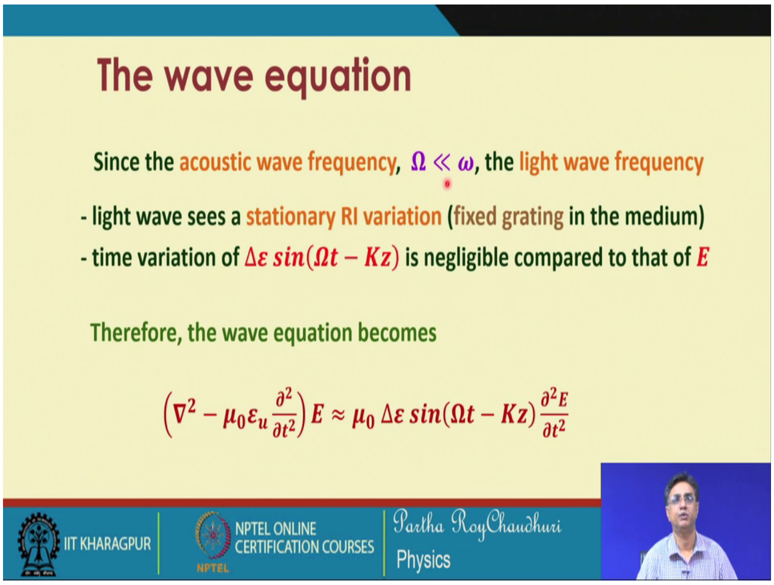

Now, since the acoustic wave frequency this omega is much much less than, the light wave frequency, this situation we have discussed earlier. Also, this is of the order of, some megahertz or some tens or hundreds of megahertz whereas, this is 10 to the power of 14, 15 hertz. So, this, much more then this omega is much more many times more than this frequency as a consequence.

This light wave will see a stationary refractive index variation a fixed grating in the medium and therefore, the time variation of this that is plus minus of delta E. This time variation will be negligible, compared to the time variation of the electric field of the light wave.

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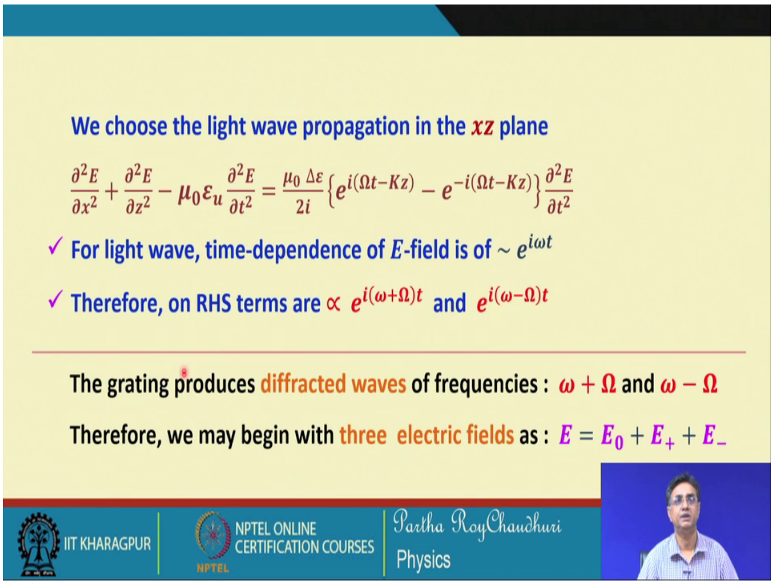

So, therefore, the wave equation can be written in this form, you have this wave equation and now, we will see that, because we are assume that the, the light wave propagates in the xz direction; x z direction we have to go back and see this, situation. So, this is your x direction and this is your z direction.

So, the acoustic wave is propagating along the z direction and the light will be propagating in the xz plane. So, that is the so, this thing this is not correct this should be z and the light wave propagates in the xz direction xz plane.

So, that we have to rectify then, we can write this equation in this form del square, minus mu naught epsilon u del del t of E and, because we can bring this quantity the fixed the stationary unperturbed part of the, permittivity to the left-hand side and we have this on the right-hand side.

So, this is a direct consequence of this we take this quantity to the left-hand side and leave this quantity on the right-hand side. Now, look at this if I use this for sine omega t minus Kz, if I use this identity e to the power of omega t minus Kz minus e to the power of minus omega t minus Kz divide by 2 i to be equal to sin of omega t minus Kz. Using this, I can rewrite this equation where this quantity is the x and z dependent variation of the electric field that is represented by this.

Therefore, because it is there is no y dependence of the electric field we have assumed that the electric field is confined within the xz plane. So, you have only xz variation of this. For light wave time independence of the electric field is of E to the power this is, the, the we know that e to the power of i omega t is the time dependence part of the light wave and therefore, this quantity on the left-hand side wherever it appears must be proportional to the e to the power of i omega t on the right-hand side.

So, RHS terms, they are proportional to this. So, with this is the factor which will tell us, who will be equal to whom, because, when we will analyze this equation, we will see that the terms will be proportional to e to the power of i omega t, e to the power omega t omega minus omega t and so on.

So, that a grating produces diffracted waves of frequencies omega plus omega, this has come from here, this will come from here and this will come from here. You can see because in the E field you have e to the power of i omega t. If you take into account, then it will be omega minus omega or omega plus omega. In this case it will be minus, because you have a minus sign outside.

So, on the right-hand side you will have terms which are proportional toe to the power of i omega plus capital omega t and e to the power omega minus capital omega t. So, these are the terms which will be on the right-hand side, right.

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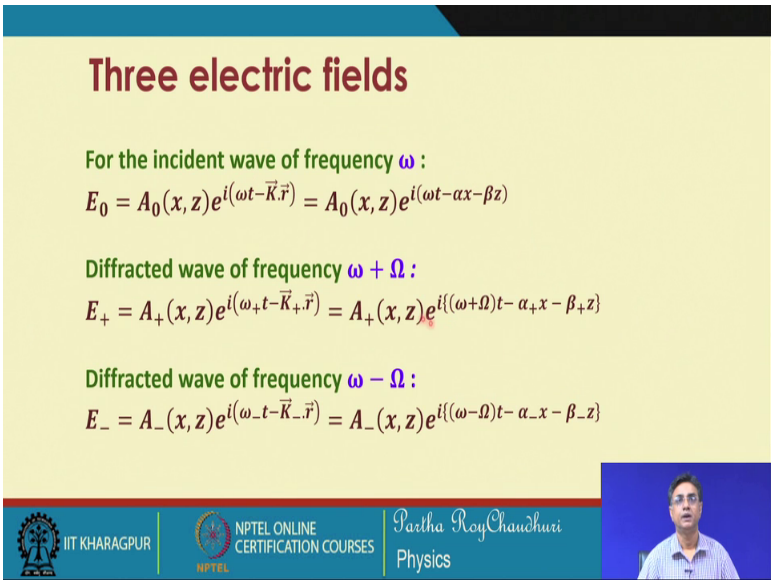

So, we can decompose this equations into three fields that is you have an incident field, which is given by this K dot r is the direction of the incident wave. You have seen because it is confined in the xz plane.

So, we can write it as alpha x and beta z where alpha is the x component of the propagation vector of the, light wave and beta is the z component of the propagation vector. And for the diffracted wave in the same way, we write this amplitude A plus which is a function of x z and omega plus is the frequency of the diffracted wave and this must be equal to this.

We can write this in this form, this K plus will be K plus dot r which is given by this, because the diffracted wave is also confined in the xz plane.

So, you have x component of this propagation vector z component of the which are different from here, because of the direct change in the directionality. And in the same way we have diffracted wave of frequency which is given by this and in this case we will have the propagation the components of x and z components of propagation vector which are given by alpha minus and beta minus.

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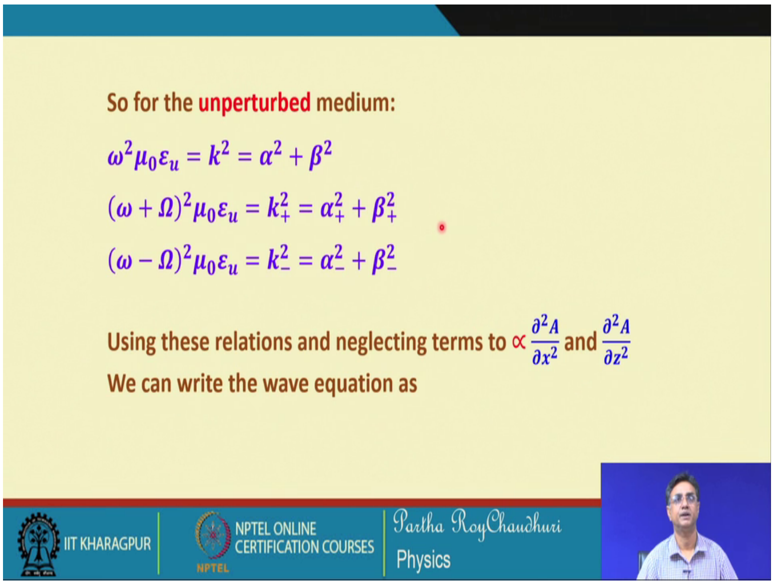

So, this is, these two are similar terms and this represents the incident wave. So, now we can equate these two for the unperturbed medium you have omega square mu naught unperturbed permittivity of this.

So, this put together we get k square which is equal to alpha square plus beta square that is for the incident beam. And then you have for the diffracted beam with plus order. We will see; we will call this will correspond to the plus order this will correspond to the minus order we will see that, but initially we have this as a consequence of this equation, because we can see, that there will be terms which will be minus omega t plus omega plus plus omega with plus small omega.

So, that is why we have, this quantity corresponds to the plus will be corresponding to the plus order and this will be to the minus order and now using these relations and neglecting the terms which have proportional to this, that is the, because the, amplitude variation along, the x and z directions are very small. It is only the, small change in the, so, that is why the the, second order terms we can neglect and then we can write. This wave equation, because we assume that, they are almost in the same direction.

So, we can write this wave equation this equation if I substitute in this equation this quantity is 0 E plus E minus. So, this we have to use this equation. So, this E, this electric field is the sum of these three the incident field the plus order field and the minus order field. So, all these three terms are appearing here.

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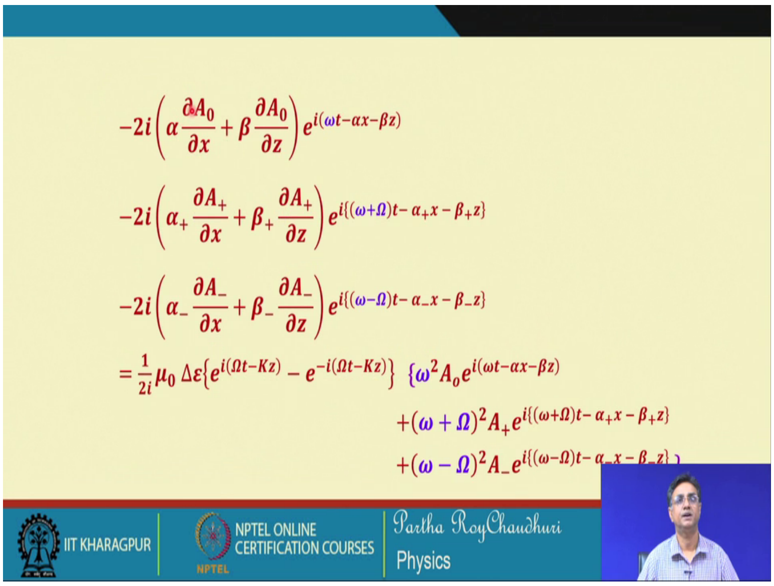

So, if we substitute this E 0 equal to this E plus equal to this E minus equal to this in the wave equation. Then, we can write down this equation of course, you have neglected the second order of, variation of this x and z component of the amplitude of the electric fields. Then we can write this equation in this form, which is very straightforward we have to work out this algebra and then you can compare the terms, because this is the right-hand side. And this part is the left-hand side. You can see that, you have terms omega small omega plus capital omega t and it is also there small omega plus capital omega t, you have omega minus capital omega you have this term also you have omega t omega t so, there will there will be a one to one correspondence of the terms.

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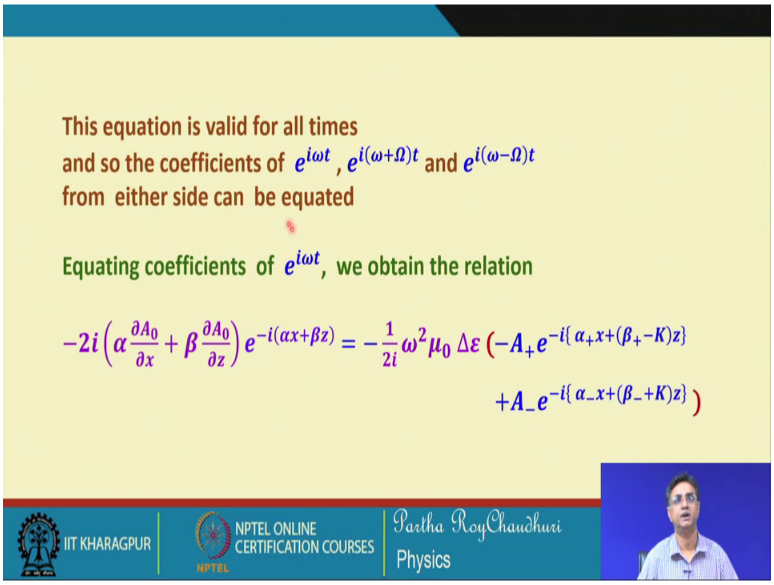

So, this equation is valid as long as these quantities individually each of them are equated, that is this quantity the coefficient of this the objects.

The factors attached to this quantity and the factors attached to this quantity, they must be same and similarly for this and similarly for this.

So, therefore, if we equate the coefficients of this and this separately from either side of this equations, then we can write down that this part is into e to the power of i omega t and this quantity into e to the power of i omega t so, therefore, we can write this equation

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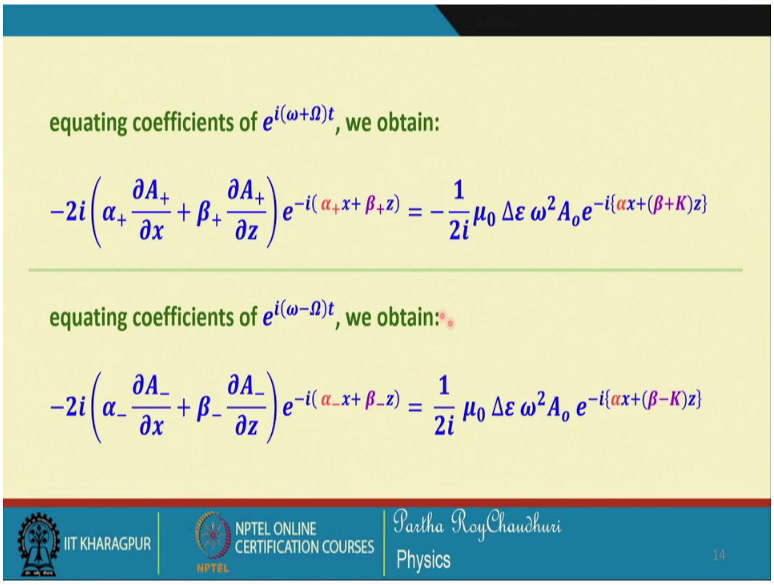

And similarly for this and for this you have a set up to similar equations just plus is replaced by minus and you have this beta plus K, here it has become beta minus K.

So, you have waves with this plus sign which are represented by this plus sign alpha plus beta plus amplitude plus and you have waves which are having the minus signs, for amplitude for the coefficient of for the x component of the propagation vector z component of the propagation.

So, we have three equations. So, set of three equations one is the coefficient due to this coefficient of this, the other two are due to the coefficients of this.

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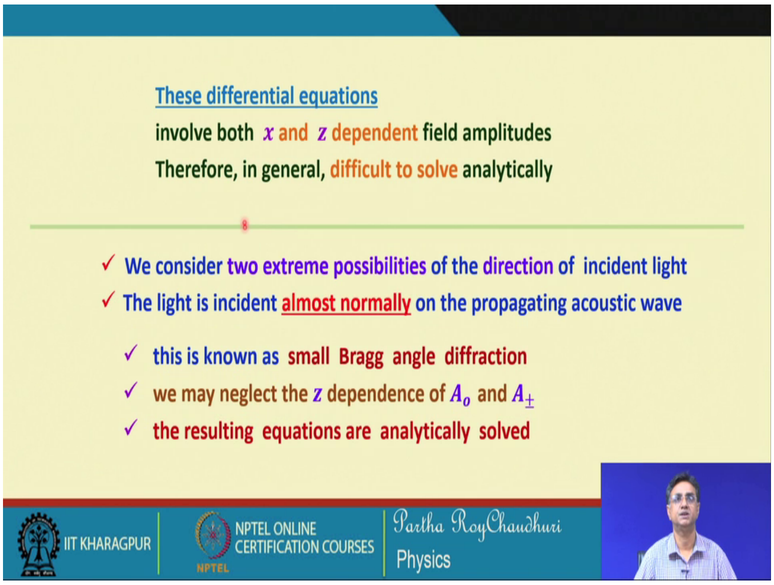

Therefore, you see these differential equations are actually not very simple to solve, because both the equations, they involve x and z dependent field amplitudes and in general, analytically it is really difficult to solve this it involves x and z dependent amplitude here x and.

So, therefore, we need to make some approximation so, that, we can have a feel of how this coupling is taking place.

So, we consider to extreme possibilities of the direction of incident light wave, that is, let us go back and see this situation that is, either the light wave can be incident, very close very close to this, x axis that is this angle is very small, this angle is very small.

So, in that case, we can approximate that the electric field is now, not depending on the z, but it is depending on x only so, that is the approximation. And the other situation is that if the electric field is very close to this direction that is, it is like this it is incident like this or it is incident like this that is this angle is very small this angle is very small so, that will be a large angle incidence.

So, in that case, we can make that approximation that the electric field is only a function of z, but not a function of x.

So, under that assumption, we will have two extreme cases, that is the we consider two extreme possible cases depending on the direction of the incident light, the light is incident almost normally this is one possibility that is the one which I, I described almost normally on the propagating acoustic wave and that is what is known as small Bragg angle diffraction and in that case, as I mentioned you can see this. This is the acoustic wave this is z direction.

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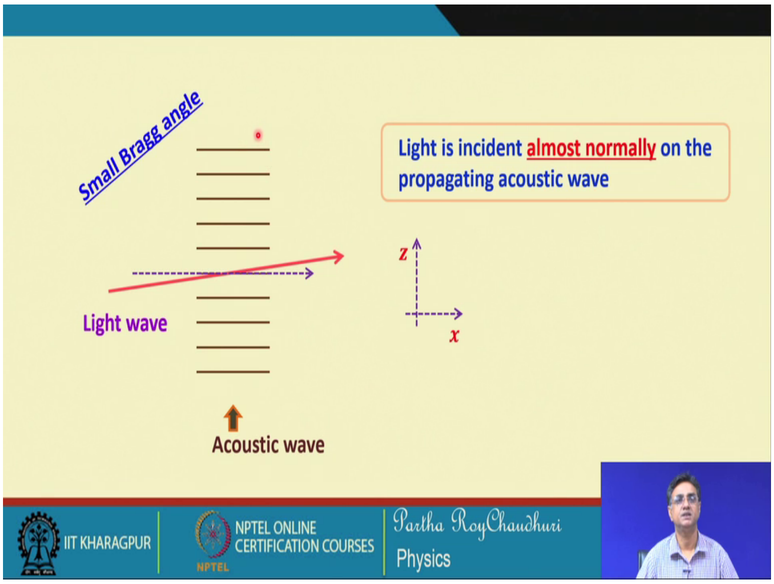

And now, this angle is very small it is almost normal almost close to this axis, this x axis and we call this is small Bragg angle diffraction.

In that case, it is like the light wave is trying to penetrate in the through the transverse direction and in that case this angle being very small, we can assume that the z dependence of the electric field is negligible, it only depends on the x, x it only depends on the x value. that is.

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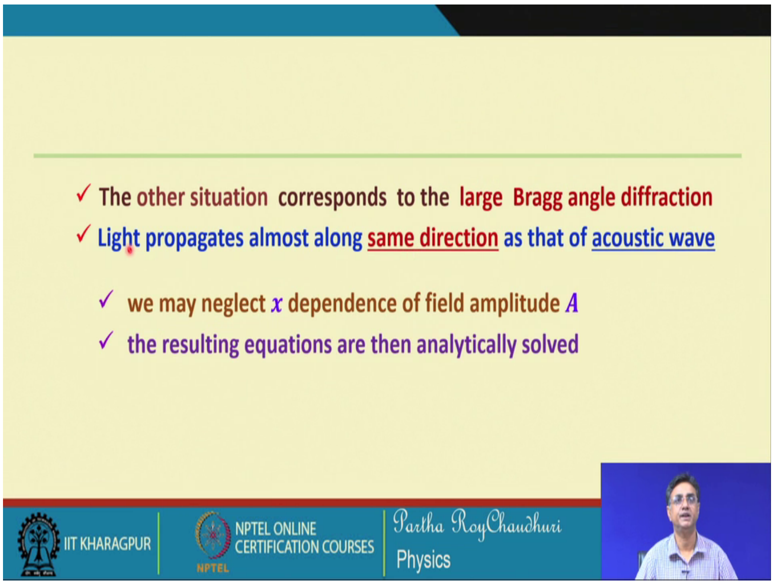

So, we have x dependent field amplitude which is nonzero and then the resulting equations can be analytically solved.

So, the other situation is the other situation that corresponds to large Bragg angle diffraction, light propagates almost along the same direction as that of the acoustic wave. That is this situation this is what, I just mentioned that.

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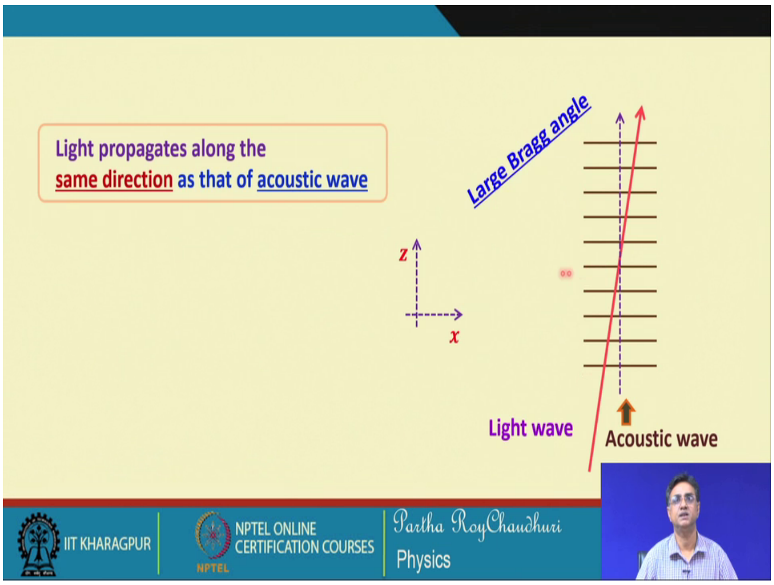

If you have an angle which is very small that, it is almost along the same direction as the direction of the propagation of acoustic wave. Light wave and the acoustic wave, they are travelling in the same direction almost. Then, you can neglect the as, x dependent, x dependence of the electric field you can consider that the electric field depends only on the z.

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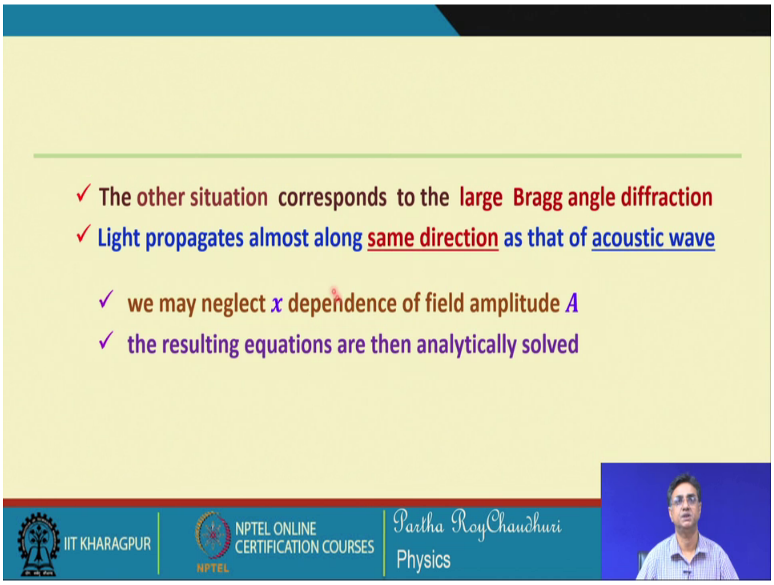

Therefore, these two situations, these two situations can now be analytically. So, over and above we made two approximation and that, that either the light wave is, normal to the acoustic wave.

So, that you can get rid of, the dependence of x and z simultaneously. We can have only x dependent, connection between these two or we can have only z dependent connection between these two.

So, then the equations can be; so, they become uncoupled. In that case, then the equations can be solved. So, these are the two situations, you have small Bragg angle where the angle of incidence is very small, angle of incidence is very large. That is why it is called large Bragg angle. This angle is the large Bragg angle.

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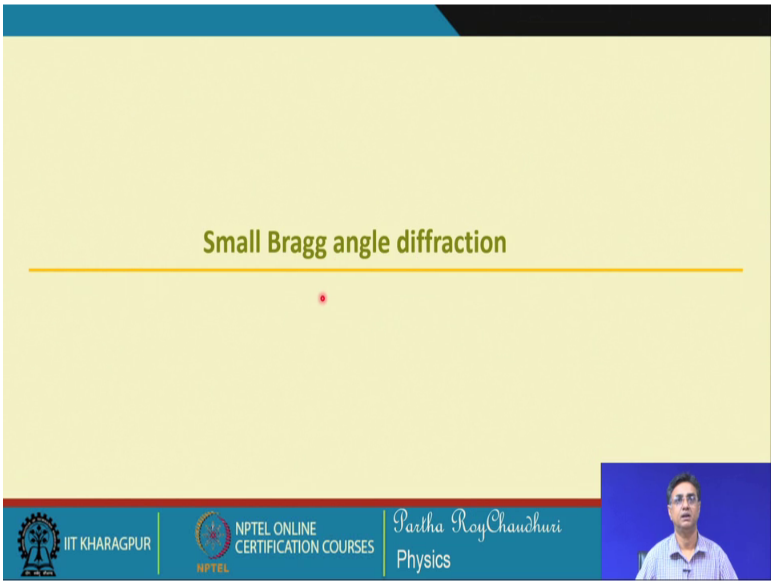

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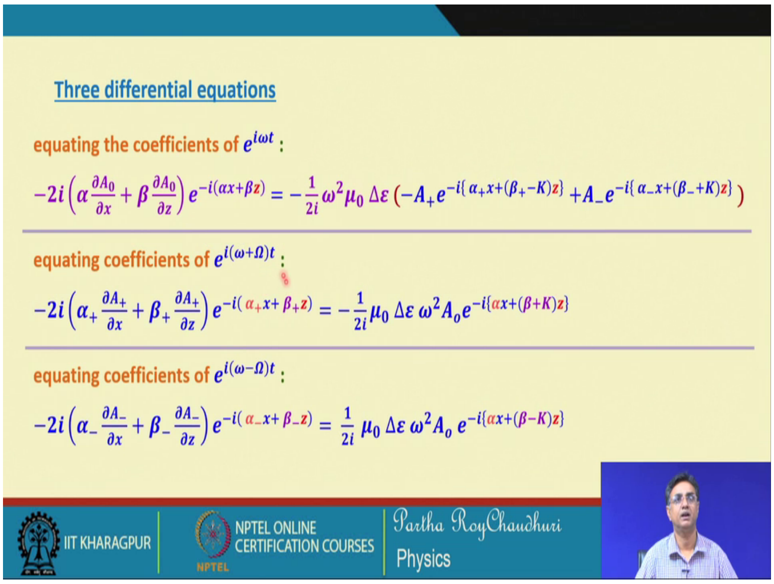

So, first we will consider the case of small Bragg angle, equating the coefficients of this thing. We have already seen, that we can now, because after equating these coefficients, we have, this quantities. Now, we will, because of the small Bragg angle there is no z dependence of the field.

So, field amplitude; so, we can now neglect this z dependents from here. So, for small Bragg angle light beam is approximately along x and if you neglect the z dependence, then dell A 0 del z we have removed from here dell A 0 del A plus del z del A minus del z these are the quantities, which are no longer appearing here.

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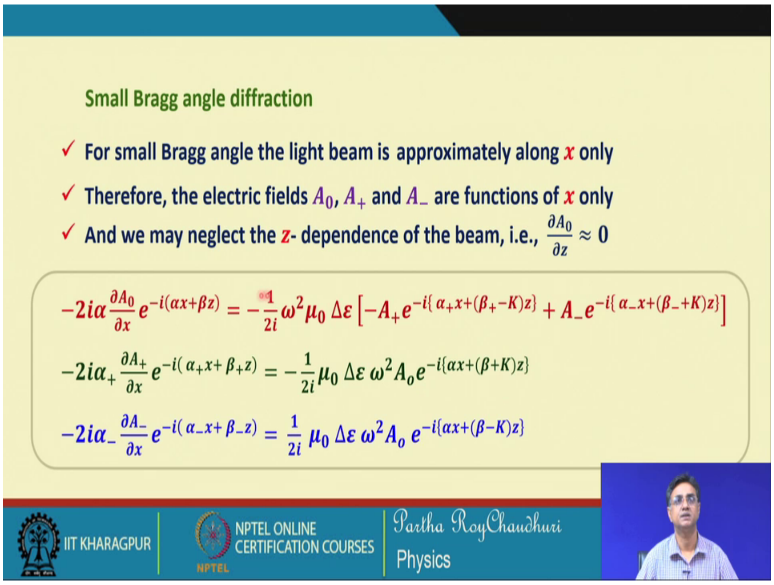

Then now, these two equations, can be solved analytically. In the above equation z dependent factors should cancel out and then we must have.

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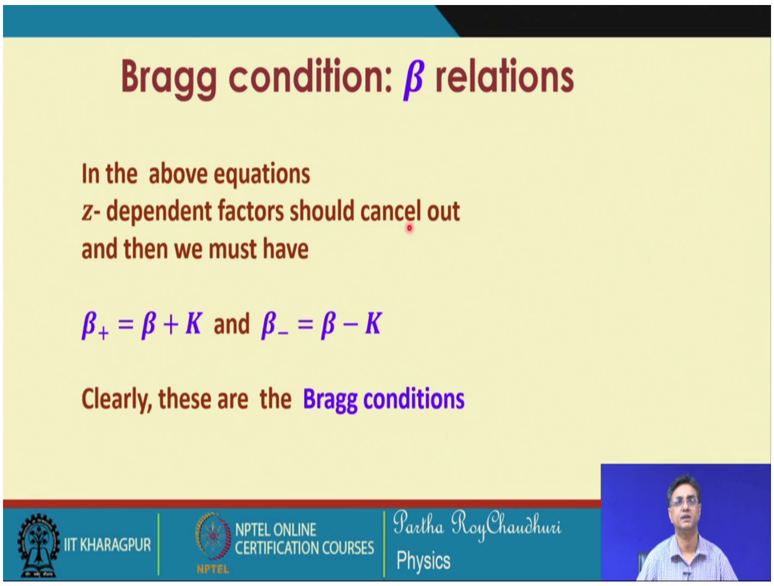

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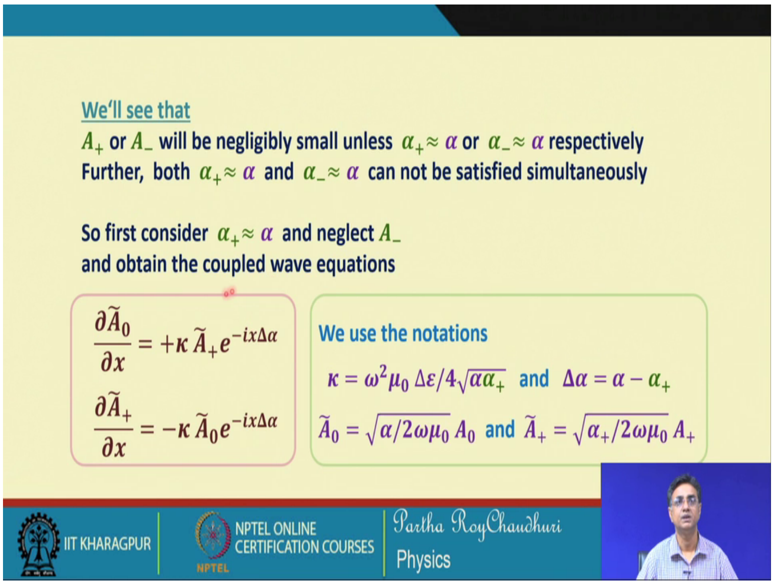

So, now, if you look at this equation, you have E to the power of i alpha x into this quantity and also that is multiplied by e to the power of i beta z on the right-hand side also you have the similar quantity.

So, this quantity which is attached to beta, that is e to the power of i beta z and e to the power of i beta plus minus z or e to the power of i beta minus plus K z they must cancelled each other. So, that gives you the condition that beta plus must be equal to beta plus K beta minus must be equal to beta minus K.

So, clearly these are the Bragg conditions for coupling of light. So, we will see that A plus and A minus will be negligibly small. If this alpha and alpha plus or alpha and alpha minus, they are not very close in terms of there values, if they are very close then only this terms will be surviving. So, both alpha plus will be close to alpha and alpha minus will be close to alpha these two things cannot be satisfied simultaneously.

So, first we consider alpha plus equal to a and neglect alpha minus. Then, we can obtain this set you can see, that this is much more simplified equation, where we have used this K kappa equal to this alpha alpha plus with a bit of algebra. We can arrive at this relation where kappa will call that coupling coefficient and A 0 that is the, incident amplitude will be written in terms of this and similarly for a plus.

Now, you can see that A 0 plus A 0 and A plus mod square of that will represent the intensity of the respective incident and we have considered that both the equations cannot be satisfied, that with beta plus and beta minus. So, at a time one will be present this is very clear from here.

So, if we consider the plus order only then we end up with the equation and you can see that, using these two equations, first you differentiate one, this one, then you substitute into the second one if I differentiate this and then substitute this one. So, del A or if I differentiate this and put it into this equation, we can write down this equation in this form.

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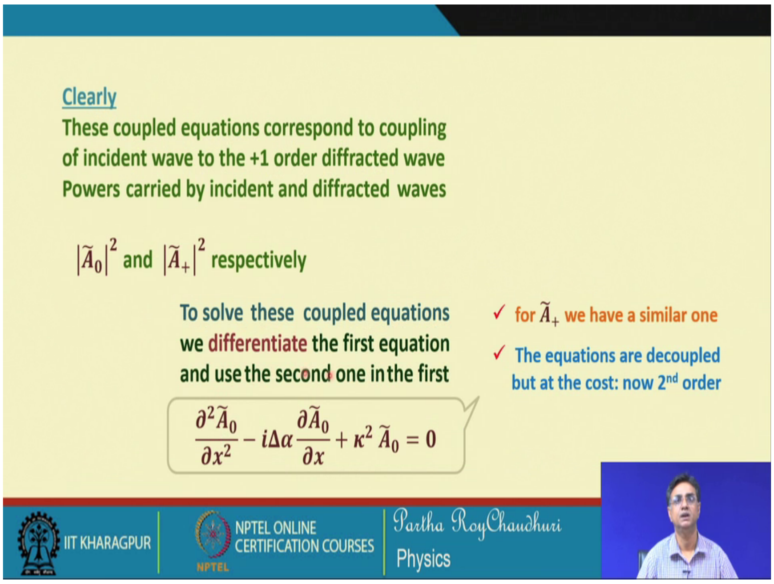

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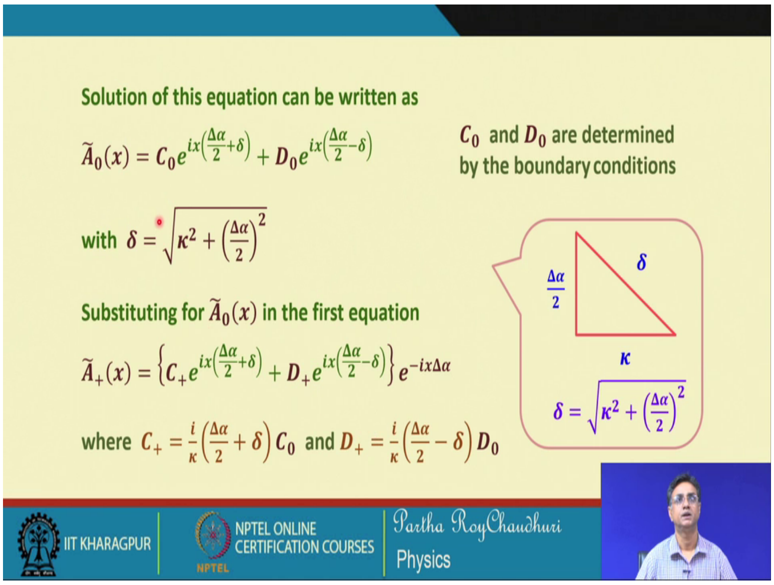

So, now, you can see, that these equations are now decoupled equations are now decoupled, but at the cost of the second order. Because earlier this equation was first order and it has become a second order equation differential equation. Solution of this equation you see this equation can be solved and the solution of this equation can be written in this form, where delta is equal to this.

You can see this delta the detuning factor with the K and this delta alpha by 2 are related by this and then substituting for A 0 plus x in the first equation, we get that A plus x equal to this.

So, you have two equations which will be simultaneously satisfied and from here combining these two equations we got one equation which is a second order differential equation and the solution of this equation can be written in this form having known all the quantities. Then, if you substitute for this equation, that is you differentiate this del A 0 del x and then substitute back into this equation del A 0 del x this quantity will give you A 0 plus.

If you put it back into this equation, then we will get this a plus another equation which will be the solution of that will be equal to this. And here C plus and D plus they are given by this it can calculate these values and, this c 0 and d 0 that is more important to know, because if you know, C 0 and D 0 then, we can calculate C plus and D plus, because they are in terms of C 0 and D 0. But C 0 and D 0 they come from the boundary conditions of this way, will see that considering some.

So, now, this equation corresponds to the amplitude of the electric field for the zeroth order that is the undiffracted order, but this quantity, we will correspond to the, electric field amplitude corresponding to the plus order plus 1 first order diffracted field.

So, we; after simplifying this equation, we actually end up at the amplitudes of the two diffracted waves zeroth order and the first order and we will see these two are now coupling and the they are coupling through this coupling constant kappa and we will see that how this coupling. Consider a special situation that at x equal to 0 A 0 you have the field amplitude in the incident beam is unity, but there is no field amplitude, because there is no at x equal to 0 there is no, interaction no diffraction.

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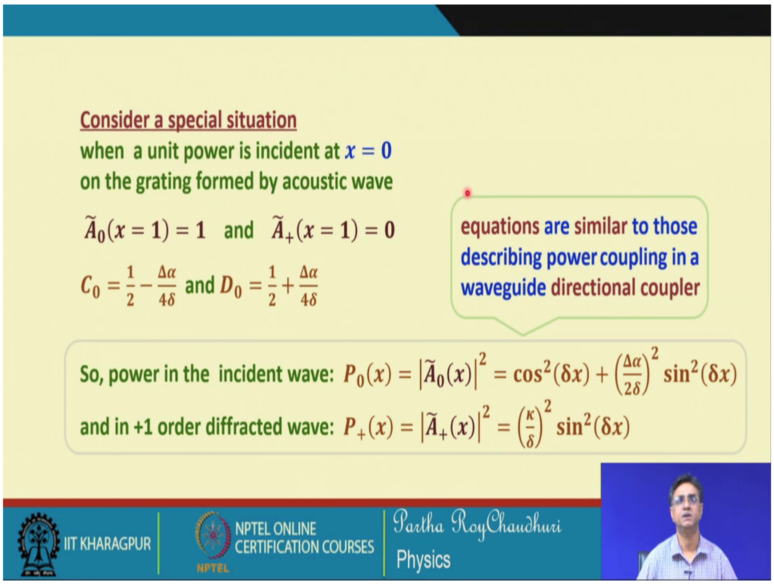

So, we can calculate C 0 and D 0 in terms of delta alpha delta and the power transfer, power in this incident wave can now be represented by this. Because this we can simplify ; once I use this condition A 0 equal to 1, you can write this power which is the mod square of amplitude.

You can see, this is a sin square cosine square variation of the phase that is this delta x. Delta E is also nothing, but a, kappa represented you have kappa gets into delta. So, this you can see that P 0 equal to this and the power in the diffracted wave is this. This is a the same similar set of equations, we obtained while analyzing the power variation power coupling in the case of directional coupler.

So, that reminds and it is of course, the same nature of coupling, that is why there coming out to be the same. For minus order also we can have a similar set of equations, but this one that we have discussed is the small Bragg angle diffraction, where we assume that the light is travelling almost parallel, almost perpendicular almost normal to the direction of the acoustic wave. And the consequence of that you have a periodic variation of the power.

Let us suppose this delta alpha equal to 0, then delta becomes equal to kappa and if so, in that case, this delta is equal to kappa. So, it becomes kappa x this becomes 0, because delta alpha equal to 0 and this becomes sin square delta alpha. So, there will be a sin square cosine square variation of the power., we will continue with this discussion in the next topic

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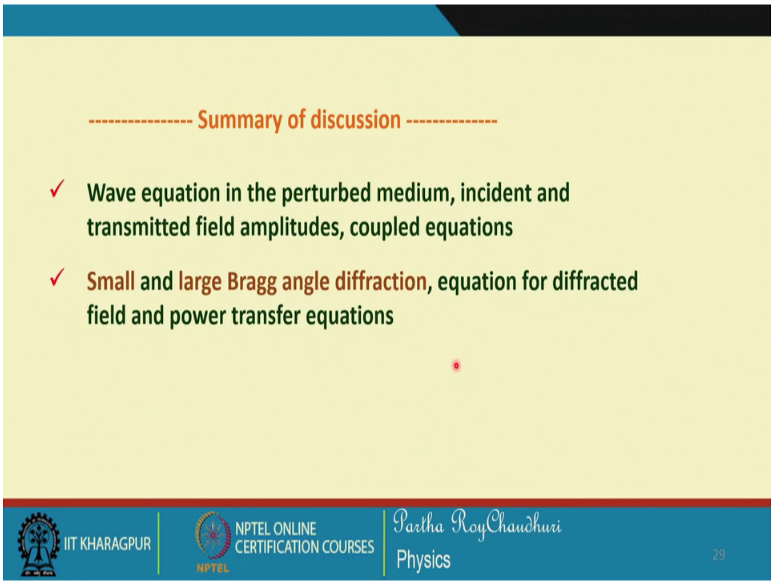

So, we discussed this wave equation in the perturbed medium incident and transmitted field amplitudes, then we organized this coupled wave equation, consider the approximation for small Bragg angle.

Of course, we discussed this large Bragg angle configuration, which we will be followed later and the equation for the diffracted field, we also considered the power transfer sinusoidal variation of the power transfer; periodic exchange of power that also, came out as a consequence of the small Bragg angle diffraction.

Thank you very much.