Modern Optics
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Lecture – 45
Acousto-optic Effect (Contd.)

We were discussing the propagation of acoustic waves in isotropic medium. In the last occasion, we were discussing the propagation of longitudinal acoustic waves in isotropic medium and now we will continue this discussion for the acoustic waves which is of transverse type in a isotropic medium itself. After that, we will continue the discussion for the propagation of these waves in anisotropic medium and we will see how this longitudinal acoustic wave and these transverse acoustic waves the effects are different.

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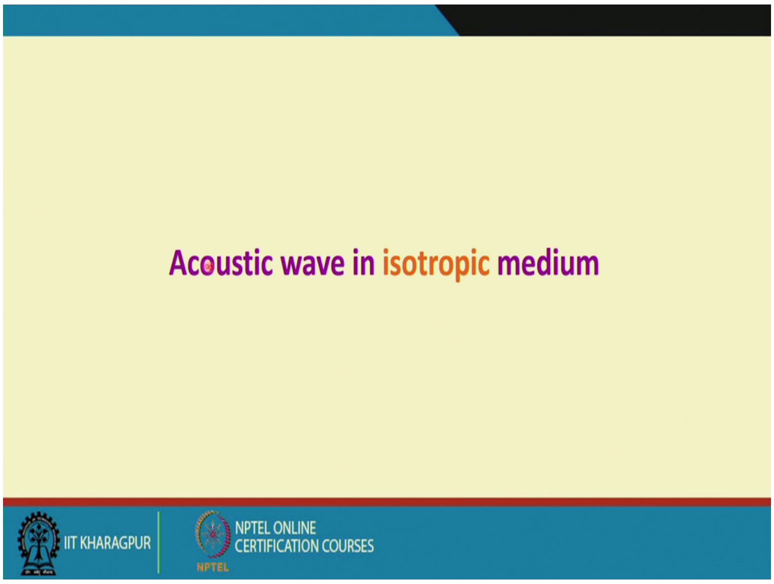

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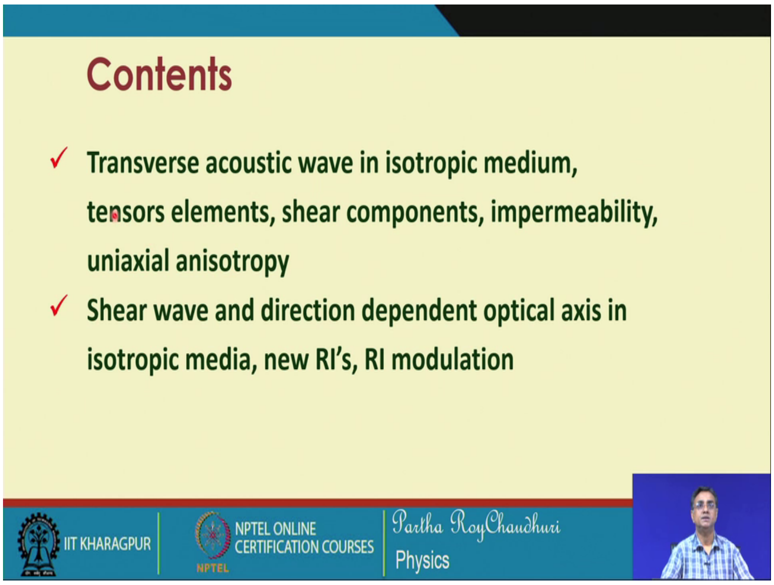

So, the discussion is under this following subtopics, that is transverse acoustic wave in the isotropic medium and we will see that this how this tensor elements shear components are coming into play in this particular case and how they are connected to this impermeability tenser and, then the general modified uniaxial anisotropy even though the medium is originally isotropic. Shear wave and the direction dependent optical axis in case of isotropic medium, we will see that everything remaining same. It is only the direction of the propagation of the acoustic wave. We will define the optic axis in the case of isotropic medium.

So, we will try to understand taking 2 different polarizations which are completely arbitrary whereas, the direction of propagation remaining the same how the refractive index changes modification in the permittivity takes place. We will study that.

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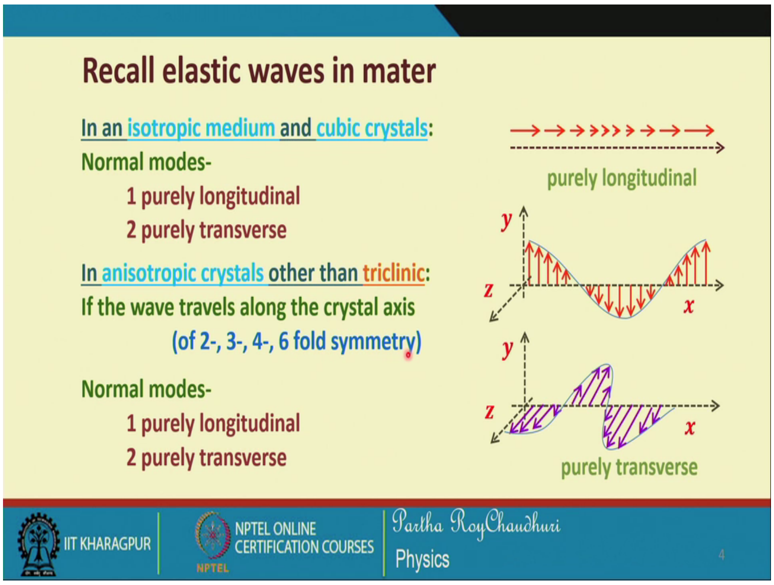

So, let us recall the elastic waves in matter which we were discussing in the previous section this isotropic medium and cubic crystal. There are normal modes, these are the normal modes which are purely longitudinal and there are 2 purely transverse modes. Let us suppose, the wave is traveling along this x direction, then there could be a y polarization as well as there could be x polarization. This is how this these are the shear modes, that is shear acoustic wave that is propagating through this medium.

So, in case of anisotropic crystal other than triclinic systems, the wave travels along the crystal axis. So, that is 2 3 4 and 6-fold symmetry; in that case the normal modes one purely longitudinal and 2 purely transverse. So now, having known this, we can analyze the propagation of acoustic wave in anisotropic medium when the wave is of transverse nature that is either it is polarized along y or it is polarized along z and it is travelling in the along the x axis.

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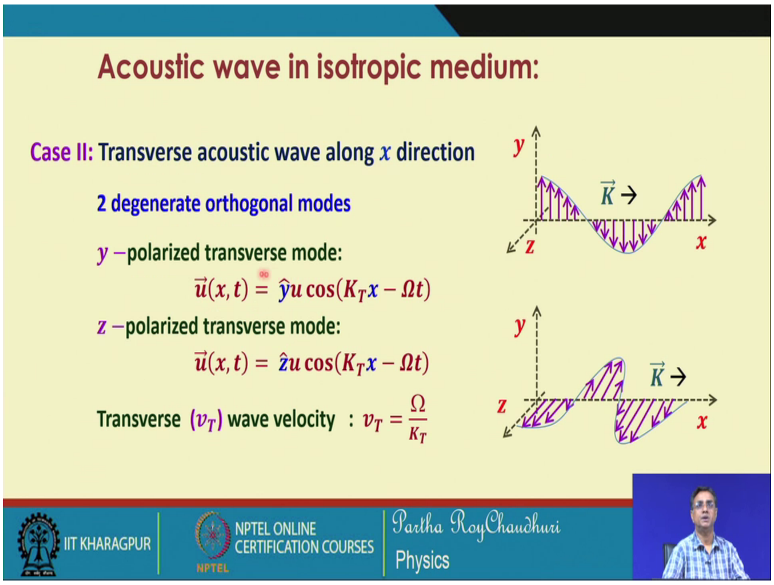

But, the acoustic wave can be in general x in anywhere between x and y or it could be a mixture of x and un polarized light, that is both x y and z polarizations are coexisting. That is also possible and we can always analyze that case as well.

But, we will first discuss that if the wave is y polarized and it is propagating along x and if the wave is z polarized and propagating along x that is keeping the propagation direction the same. We will just flip the polarization direction that we will consider both the polarization and we will see the consequence that how toggling this polarization that is the polarization of the shear waves the refractive index properties are changed. What is the differential? What is the change in the properties that we would like to identify.

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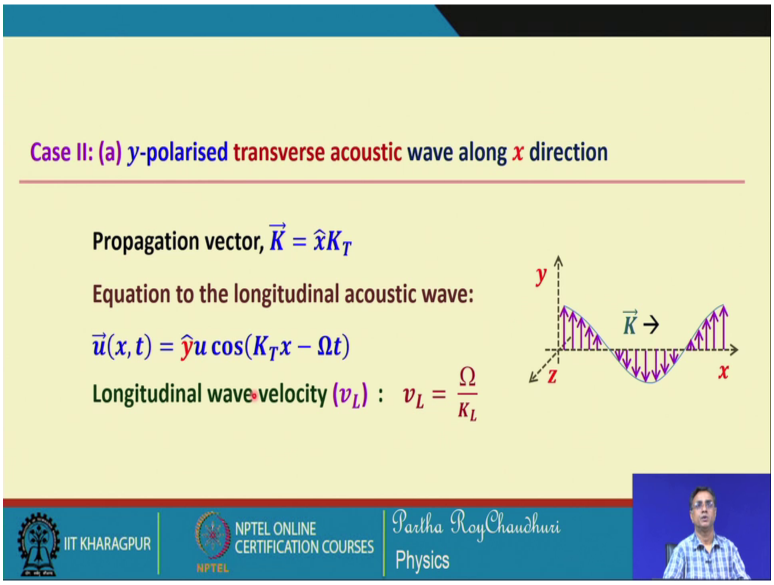

So, for this y polarized case we have this propagation vector which is directed along x equation to the longitudinal acoustic wave. We have seen we can write in this form and the longitudinal velocity is connected to the acoustic wave frequency and the propagation vector in this form. So, this is also a very well-known L is to represent that longitudinal wave velocity, but this is the case which. So, it should be K T and this should be v T, this should be v T. It is not the longitudinal velocity; it should be the transverse velocity sorry.

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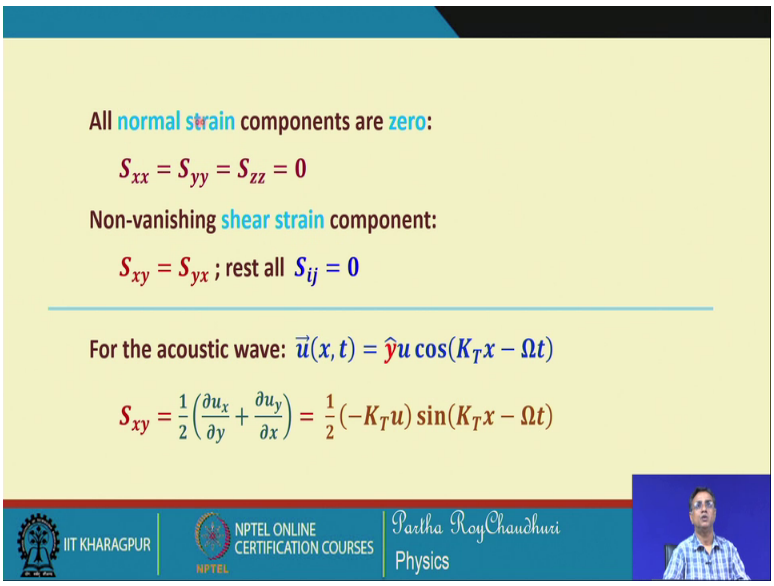

So, all normal strain components are 0, then in that case you see because there is no normal, there is no compression and rarefaction along the along the x axis. So, this normal strain components are 0 that is S 1 S 2 S 3 which are represented by S x x S y y and S z z are equal to 0. Only non-vanishing shear strain component that is existing is S x y and y z see the direction of propagation is x and the polarization is along y.

So, as a result, only y because this u as a function of x, we are to look at the changes u as a function of. So, you can see that non vanishing shear strain component, that is S x y equal to S y x symmetric nature.

So, they of diagonal elements are all equal. Then, rest all other component strain components are 0 only there will be one strain shear element and for the acoustic wave of this, this is u x. If we as u y that is the amplitude along the y direction u y. So, this quantity will be u y with respect to S will be existing, but there is no u x because, there is no change in the x direction. So, as a result, this S x y will be given by this equation. It comes from here, and we can call this as we have seen we can call this is the amplitude of the strain wave.

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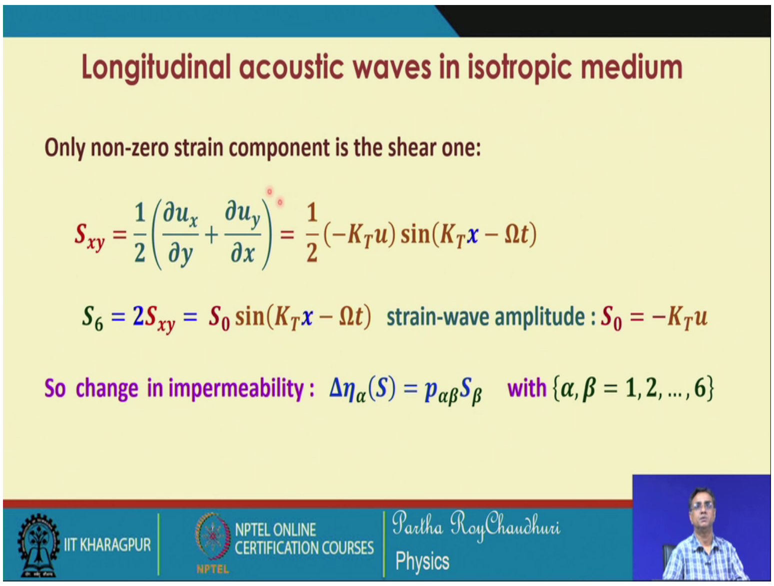

So, we can write this equation in this form. So, S 6 is actually this in the compressed notation, S x y is equal to 2 S x y which is equal to S 0 sin K T x minus omega t where this amplitude is given by this quantity. So, the change in impermeability this minus sign does not matter because it is only the phase from where we have to start the wave. So, change in impermeability is given by this delta n alpha function of x, this is the strain optic tensor and this is the strain tensor.

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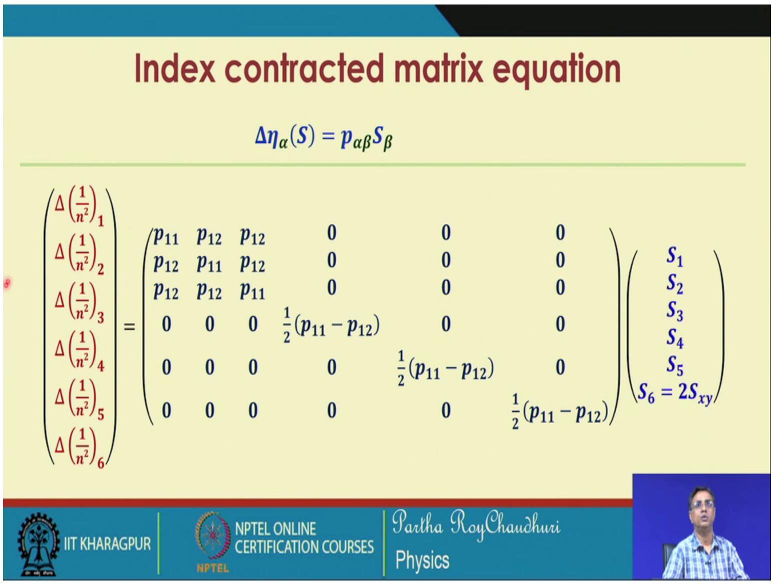

So, we can now evaluate this for anisotropic material. We have seen this is the form of the strain optic tensor. You have 2 independent strain optic elements p 1 1 and p n 1 2 and rest all of them are composed of only these 2 independent elements. So, these are the changes in the impermeability and these are the strain elements. In the present case, it is only the S 6, that is S x y that is effective rest all of them are 0. Therefore, through this equation we obtain that delta n 1 delta n 2 delta n 3 you see only this S 6 element will give you this multiplication with this column.

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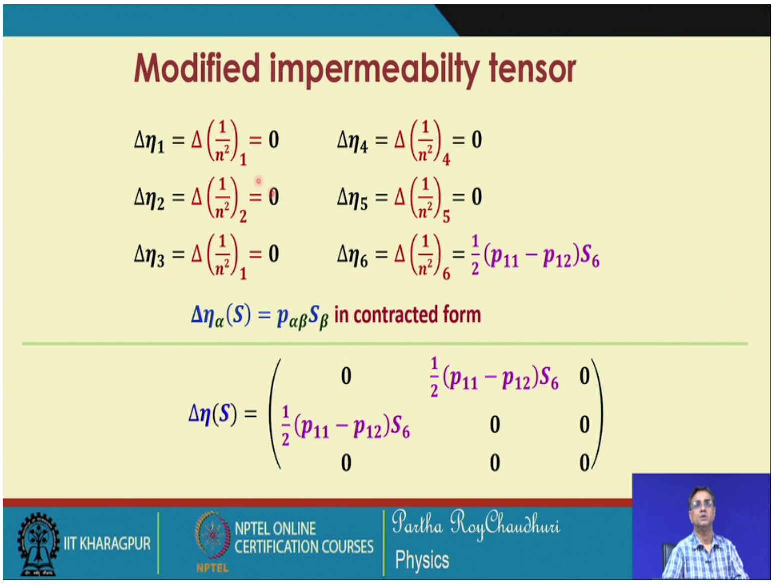

So, therefore, all this delta n 1 delta n 2 delta n 3 all of them are 0 except this. delta n 6 you can see from here; this is the quantity multiplied by this will give you this delta 1 by delta eta 6. Therefore, we have only the change in the impermeability that is attached to this delta eta 6. And therefore, in the contracted form we can write this change in the impermeability tensor in this form. These are the this is the x y element this is the x y element and therefore, it occupies these positions and rest all of them are 0.

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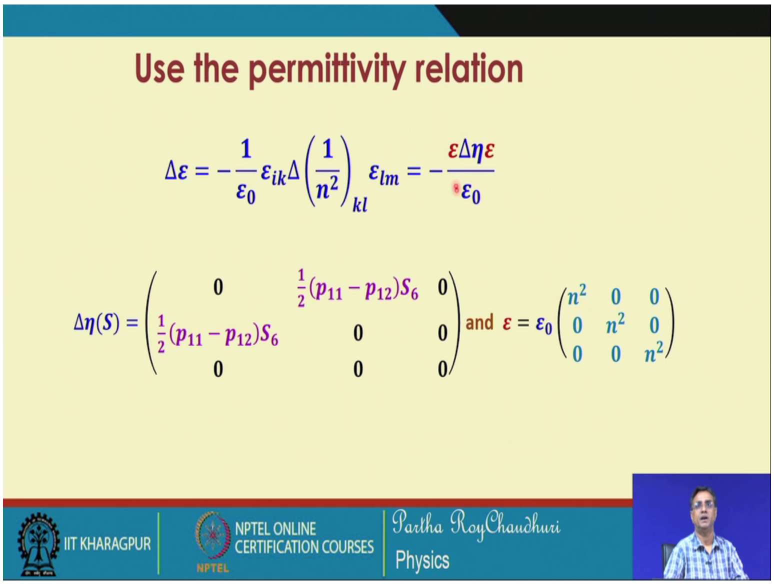

Now, we will use this we have seen that how we can represent the change in the permittivity when we know the change in the impermeability through this matrix equation. So, this we know that this you know the permittivity tensor which is given by this for an isotropic material.

These are all the diagonal elements are the same because this is a sphere the index ellipsoid is a sphere and all the refractive indices seen in the 3 principal wave directions are the same. So, and this is the impermeability change in the impermeability tensor. So, if I now multiply these 2 post and pre multiplication and then we divide by epsilon 0.

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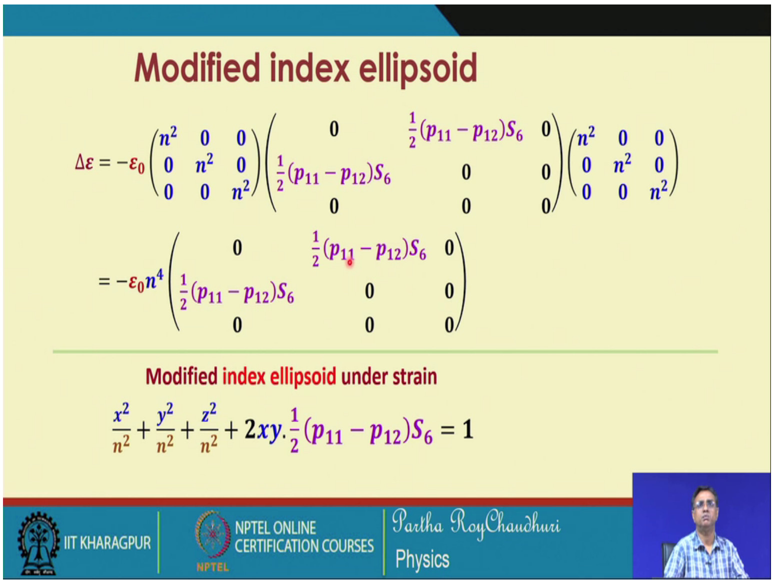

We will get this equation. You can see from here this is your pre multiplication, this is your post multiplication and after that we will get this because it is only the 3 diagonal elements that will be multiplied. So, this will be there this will be 0 this will be there and rest all of them are 0. So, only these 2 again when you pre multiply with this; that is after this multiplication this quantity is again multiplied with this, then we get this all the diagonal elements are 0 only these quantities. So, which is of the similar form as that of the impermeability tensor.

But now, that you have this S 6 multiplied by this n to the power of 4 which is the refractive index of the isotropic media. So now, this modified index ellipsoid in this case can be represented in this form because the diagonal elements are all 0. Only the x y only the x y elements are effective are non zero.

Therefore, this x y component they have come this has come twice 2 into x y into this change in the impermeability has come into the index ellipsoid, which is very simple and very clear that when there is an acoustic wave which is traveling along x and it is polarized along y direction, then x traveling wave direction is x. And it is polarized along y, then you have a cross term which is the x y term appearing in the index ellipsoid.

We will see that if it is otherwise a different polarization, but the direction of propagation is still x, then that polarization will be appearing. Particularly, the z polarization will be appearing here.

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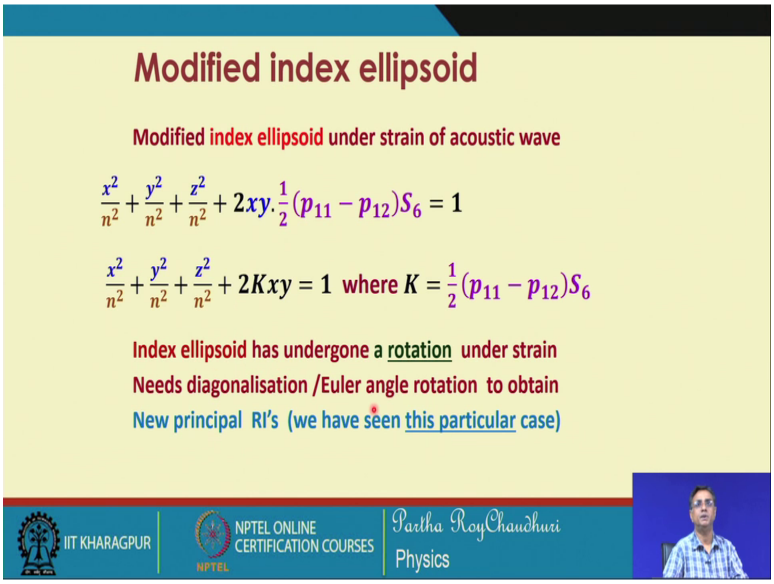

So, that will be the difference. So, the modified index ellipsoid under strain will be written as in this form and so, this quantity if I write as equal to twice K x y. This is a very familiar equation we have worked several times and we know how to bring it to the this ellipsoid to the principal axis system by giving a rotation of the coordinate axis or it can be done by a matrix diagonalization. So, in any case, we know that this K equal to this. So, we know that this K plus will be attached to this and K minus will be attached to this because it is a rotation about z axis and the cross term is cross term is with x y.

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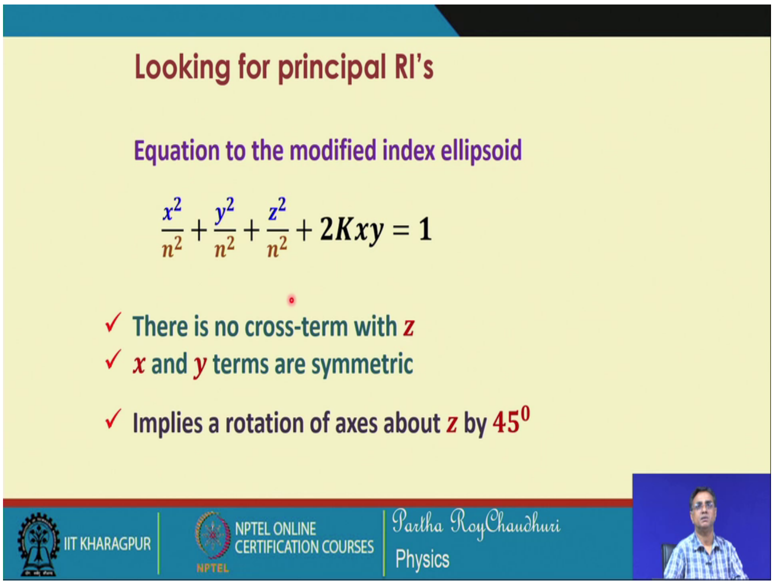

So, there is no cross term with z and x and y terms are symmetric. So, it requires it implies that there is a need of rotation about z by an angle of 45 degree

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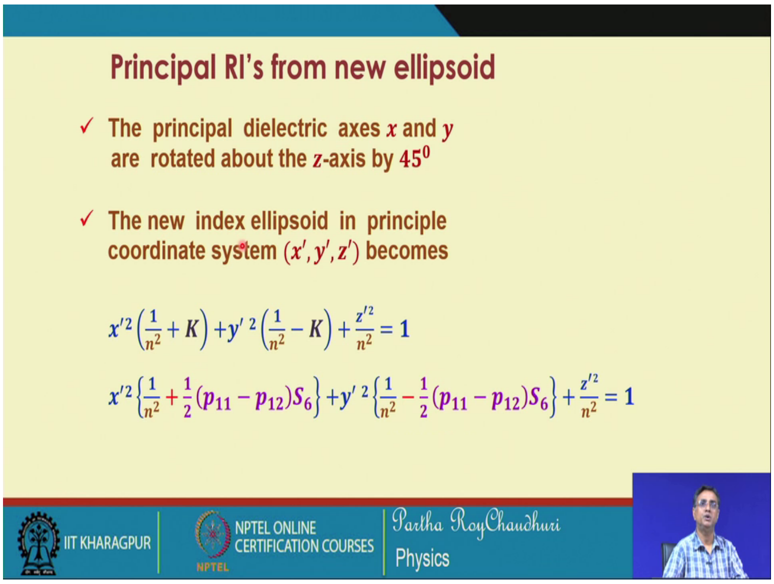

So, by doing that we will transform this index ellipsoid to the principal axis system and, we know that this after transforming, this coordinate axis we get the new index ellipsoid, that is the modified index ellipsoid which will look like this. I have seen that the plus K is attached to this and minus K is attached to this and this we have seen how it comes into this ellipsoid equation the cross term coefficient.

Therefore, because now, this K we have called we have seen that this K is actually representing this term that is half of your p 1 1 minus p 1 2 into S 6. Therefore, substituting that value of K, we get the a 1 by n x square this new refractive index along x new x axis new y axis and anyway there is no change in the z axis. So, it remains the same since the wave is polarized along y direction and it is traveling along x direction. So, this both the refractive indices are now affected by the wave.

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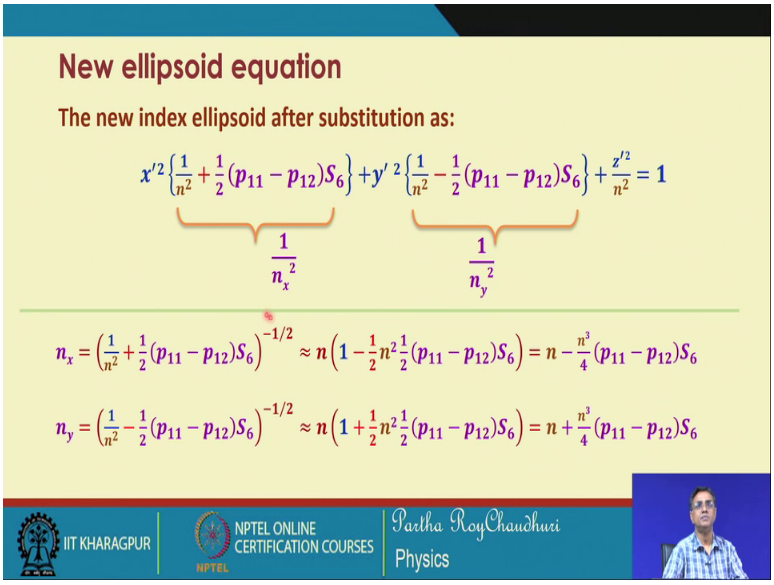

Therefore, your effective 1 by n x square is this effective 1 by n y square is given by this n z square. There is no change it remains as it is and we know that we can approximately write this value as equal to this and n y is approximately. We can write as equal to this. So, this is the if I have a wave which is polarized any electromagnetic wave or optical wave which is polarized along x or along y or halfway through this, then they can see birefringence due to the changes in the refractive indices in an isotropic medium as a consequence of the propagation of acoustic wave mechanical wave in the medium.

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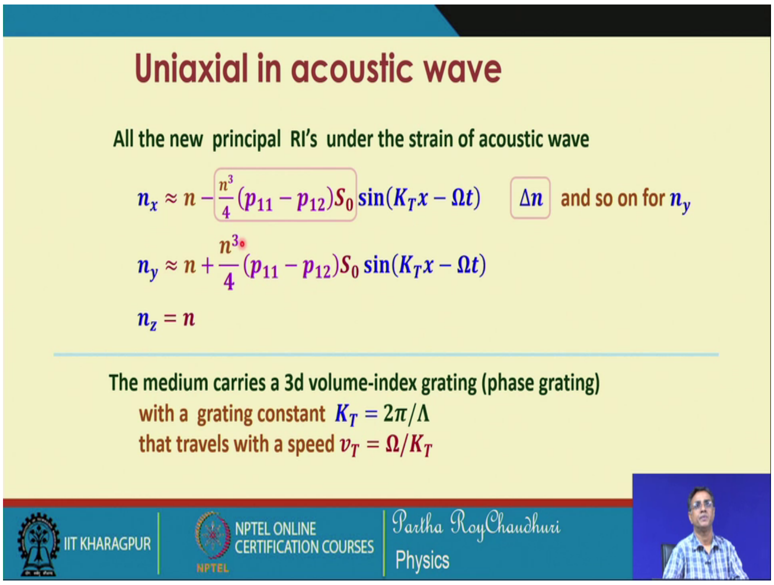

So, we can see that all new refractive principle refractive indices are this. So, one thing that we want to note here because, this result we will carry forward when we will study the acousto optic effects in details that this term because S 6, we can replace S 6 by the original quantity that is the S 0. The wave form of the strain wave equation that is S 0 sine K T minus x minus omega t and this is the peak change in the in the dielectric a peak change in the refractive index which will we can call is equal to delta n.

So, n minus delta n sine, then this phase quantity K T x minus omega t and similarly for y also this quantity can be represented as delta n and we get a similar expression. In this case, it is plus in this case it is minus. So, there will be a change in the phase only for when we look at the phase that is associated with this.

So, we can observe that because of the because of the presence of the acoustic wave, the medium refractive index along x direction is now modulated with a frequency and periodicity of the acoustic wave and the amplitude the peak value of the modulation is delta n, which is the property of the refractive index of the isotropic medium and the strain optic tensor element associated with this component of the strain.

And similarly, the change in the refractive index will be of the periodic nature which is which is a travelling change which is, it is a traveling waveform because it changes the position and time with position and time. So, this delta n is the peak change in the refractive index on top of the base value the original value of the natural refractive index in the y direction. So, these 2 are similar, but there has been change in the sine n z is not affected.

So, by looking at this result, we can see that this medium carries a 3 dimensional volume index grating a phase grating is a 3 dimensional volume index grating where the grating is the periodic grating is formed along y direction as well as along x direction.

And this grating is the grating has a constant that is K T equal to twice pi by lambda which will come from here and the periodicity of the grating will be given by the wavelength of the acoustic wave the speed of the grating. The way it moves within the medium, it actually moves with the speed of the acoustic wave. So, it is the acoustic velocity with which the grating moves in the medium. So, by applying an acoustic wave, we can see in a that when it is a transverse that a shear wave that is traveling in the medium will generate a periodic refractive index grating.

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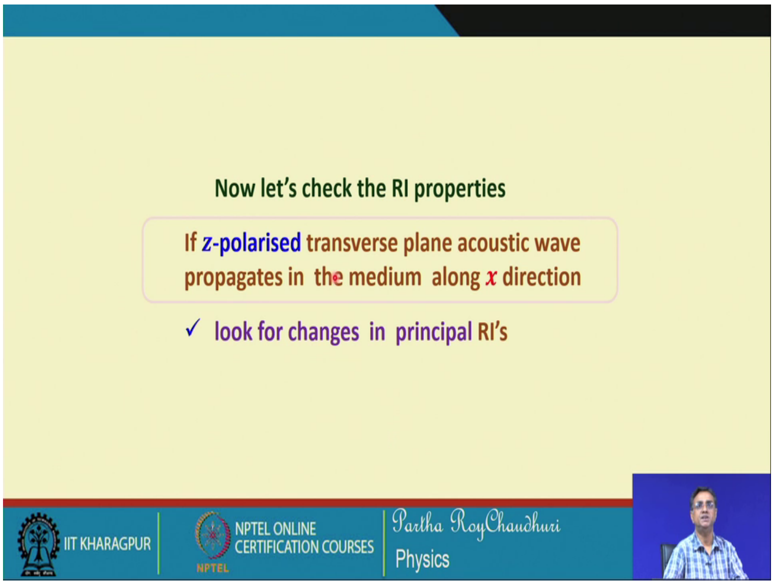

Now, we will see if we change the polarization from x, this is just to see where is the difference for this y polarization to z polarization and how you can see that here I should mention that this the medium has become uniaxial the medium has become uniaxial and the medium is anisotropic rather the medium is anisotropic where your this z axis about the z axis. There are there is a change in the refractive indices along x and y direction.

But, if we if we change this polarization direction to from y to z, then the change in the refractive index will take place along y which is very similar. But, that will redefine the 3 dimensional volume grating in terms of the directionality. So, let us now just quickly check that, how the z polarized shear wave acoustic wave in the propagating in the x direction makes the changes in the principle refractive indices.

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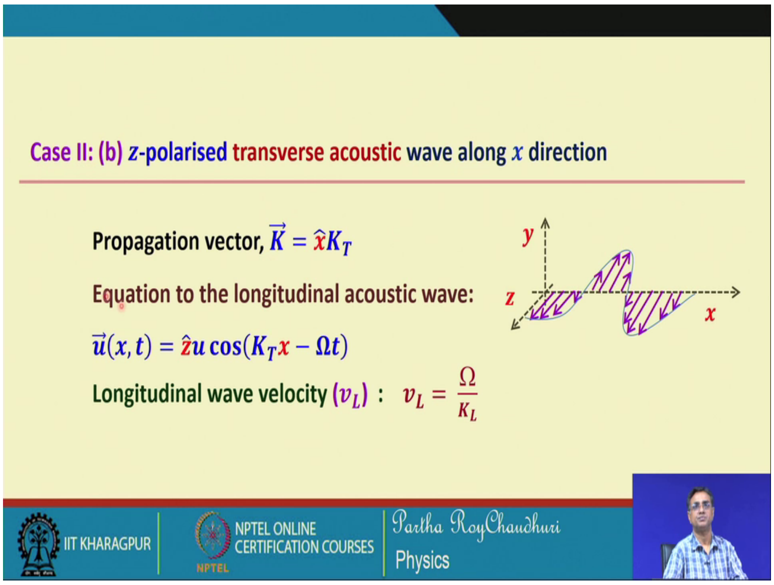

To do that, we again rewrite this equation x propagating wave. Now, polarized in the z direction. So, this is again it should be transverse wave velocity. This is the wrong.

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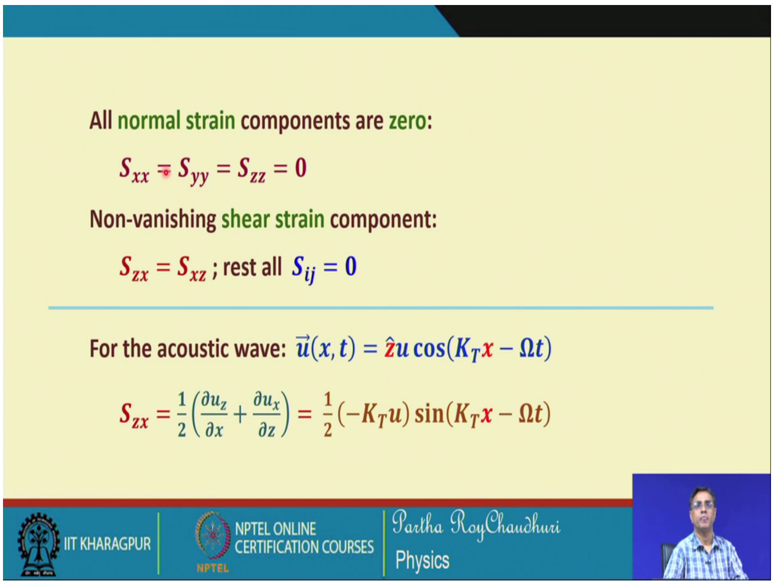

And then, in this case also the normal strains are all 0. Only non-vanishing strain component this time will be z x or x z which is equal to this; it is all the similar way.

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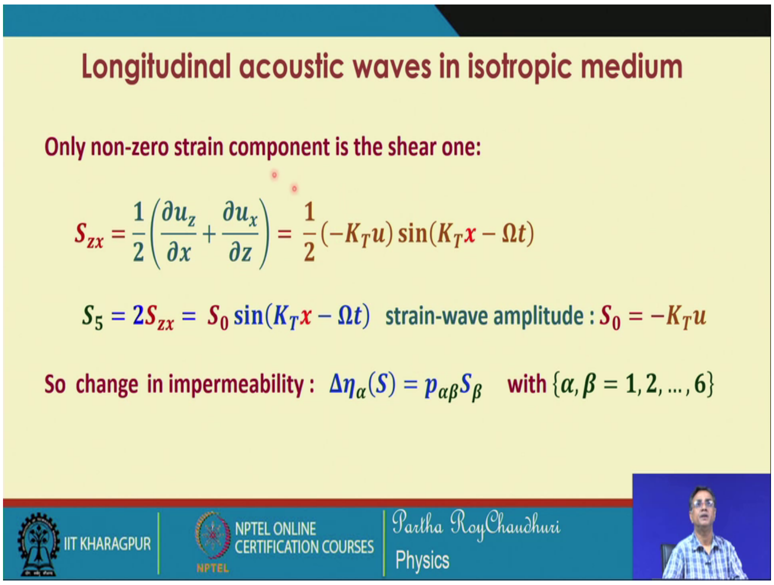

And then, longitudinal acoustic waves will generate this S 5 matrix S 5 element of the strain matrix the shear component and we can write this S 0 as the amplitude of the of the shear strain wave. So, the change in the impermeability that will be same, which is a just the repetition of the earlier configuration of the shear wave only that this time S 5 this matrix is now non zero.

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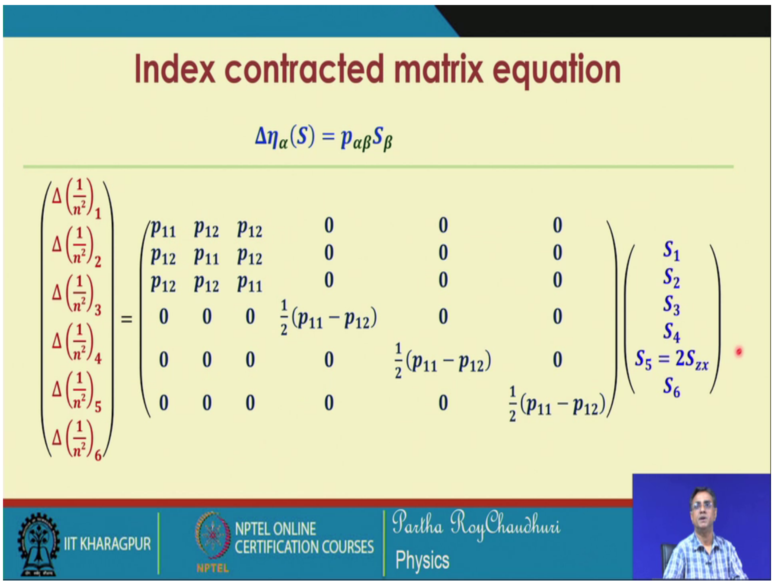

So, this will be multiplied with this elements as a result only this delta eta 5 will be non 0. Rest all of them are 0. So, this delta eta a change in the permeable impermeability tensor we will have only 2 off diagonal components only 2 off diagonal components you can see that.

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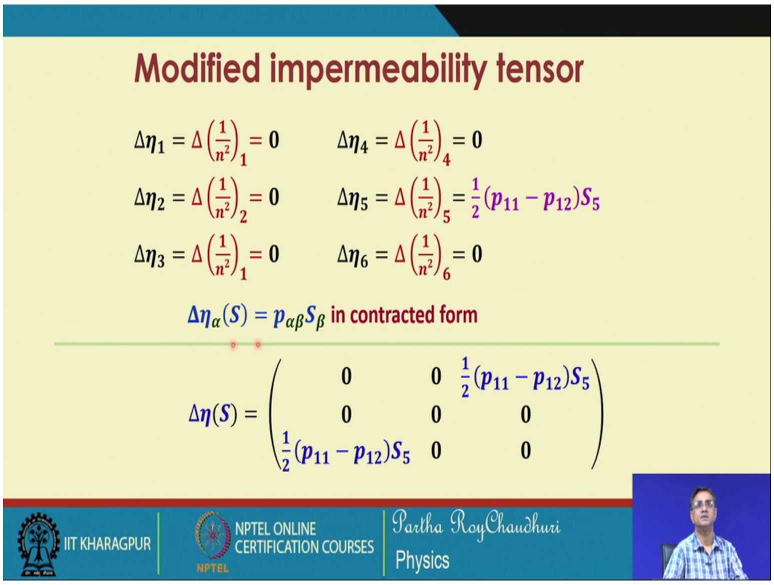

Delta eta 1 delta eta 2 all of them 3 all of them are 0, except this delta eta 5 which is associated with the strain S 5. And as a result, these are the 2 off diagonal elements in the impermeability tensor change in the impermeability tensor.

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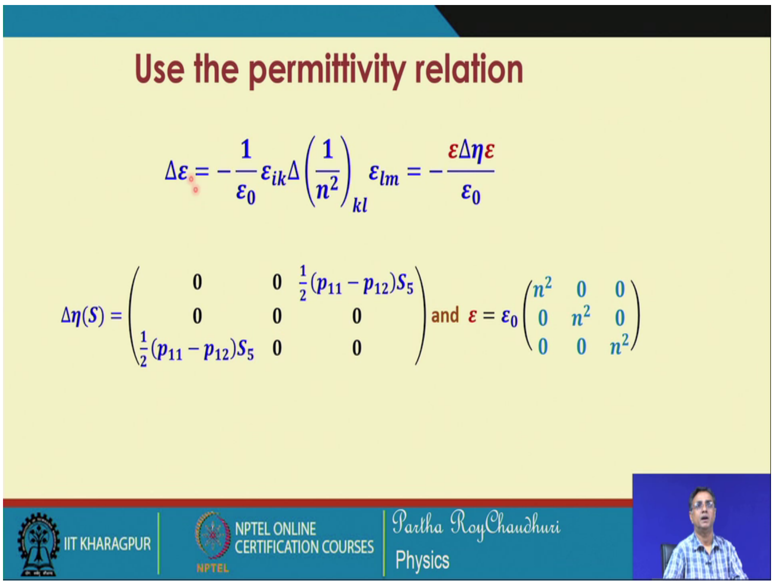

And now, again we will use that permittivity relationship to find out the change in the permittivity through this, and having known this natural having known this um, permittivity of the isotropic medium which is all identical n square and then, we multiply with this equation.

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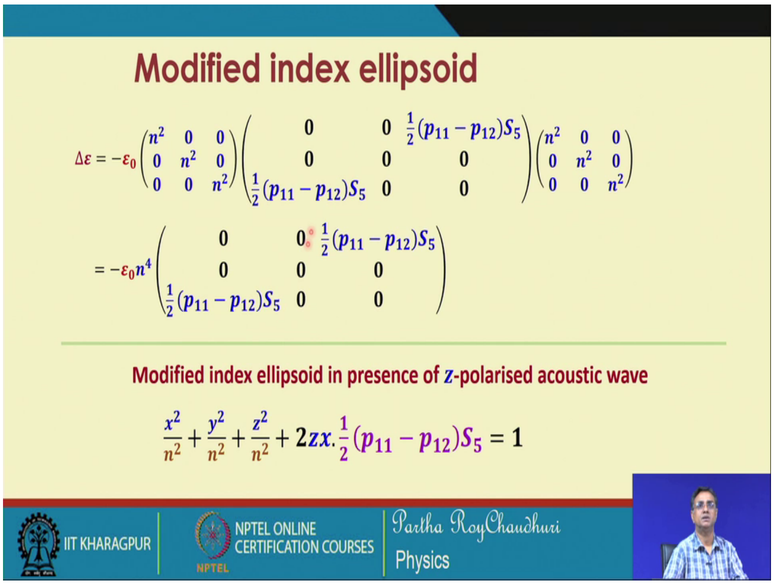

Once again, we will get these 2 these 2 elements which are non 0 and rest all of them are the same. So now, this time as we mentioned previously, this modified index ellipsoid in presence of the z polarized acoustic wave will have a cross term which is associated with this z x and this is the coefficient of the cross term which can be represented as k. So, earlier it was y x or x y. Now, it is z x; x is the direction of propagation and z is the direction of polarization.

So, we conclude that everything remains same. So, while studying all the individual cases, we find that everything remains same only that the relevant strain shear strain component is effective and non zero and that defines the new birefringence of the system.

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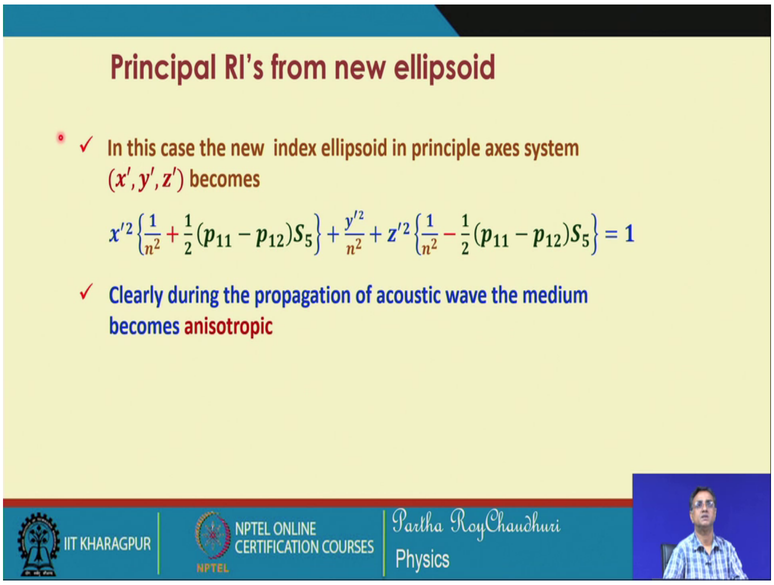

So, in this case, the new index ellipsoid in the in the principal axis system it becomes this because now it is attached to x prime square and z prime square. So, this is the new principal refractive index here the x prime component of this principal reflect and clearly during the propagation of the acoustic wave, the medium you can see that originally it was isotropic. But, just because of the acoustic wave, the medium has become anisotropic. It has a different refractive indices along x prime and y prime direction x dash and y dash direction.

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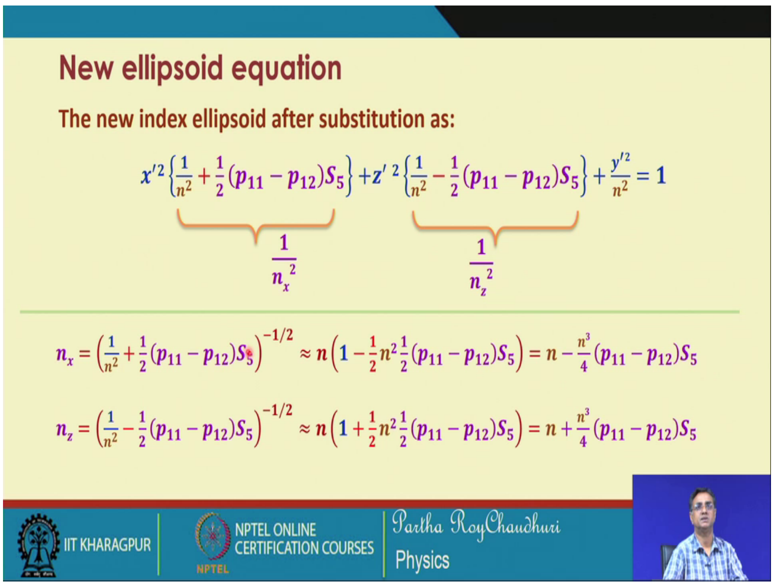

The new index ellipsoid after substituting this value, then we can write this is approximately equal to equal to this which is which can be written as this and for n z it is this. So, we see that n y the refractive index along y direction does not undergo any change. But, there has been a change in the n z there has been change in n z and n x which defines the birefringence along the 2 polarizations which are in the x and y direction x and z directions. So, this is the refractive indices seen by the 2 polarizations.

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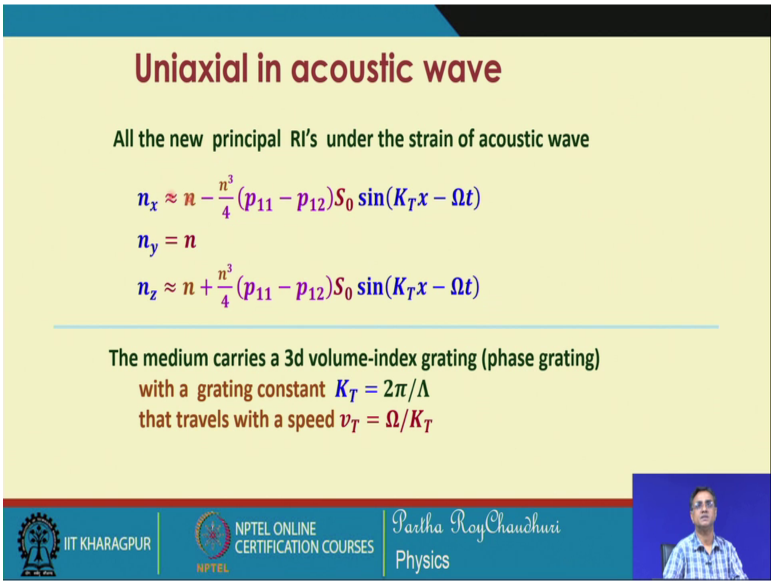

So, n x can be approximately written as this and this quantity again is going to define the peak value in the change in the refractive indices. So, there is an acoustic wave. Because of that, there is a periodic deformation and because of the periodic deformation, there will be a periodic change in the refractive index which is now straightforward to compute knowing the strain optic coefficients that is p 1 and p 1 2 in the case of isotropic material and having known the refractive index of the material.

So, there will be a periodic change and the change is also traveling in the medium with the acoustic wave. So, once again, we find that the medium carries a 3 dimensional volume grating phase grating. And the grating constant is again defined by the same quantity which is just nothing but the connected to the periodicity the wavelength of the acoustic wave capital lambda and it travels with the same speed as the acoustic wave.

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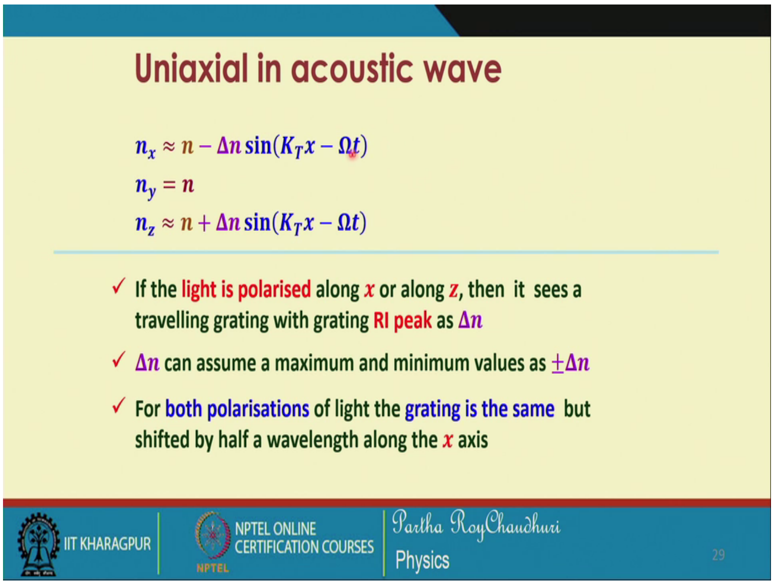

Therefore, the we can write this change as I mentioned this is equal to delta n. So, there is a plus n and there is a minus n delta n. These are the peak values of the change in the refractive indices. If the light is. So, this is these are the conclusions from after studying these 2 cases there are 2 polarizations while keeping the direction of propagation the same, there are 2 degenerate modes as we have seen and the consequence consequences are that if the light is polarized along x or along z is polarized along x or along z, then it sees a traveling grating with the grating peak which is equal to delta n i.

So, if the light is polarized along x, it can see a traveling grating. If the light is polarized along z, then also it can see a the same grating same grating because these 2 gratings are identical, but they are shifted by a wavelength only, they are shifted by a wavelength.

So, delta n can assume a maximum value of plus minus delta n. So, because of the sine that can assume plus minus 1 so, these 2 are periodically the same these 2 gratings traveling gratings are the same, but they are shifted by one wavelength half a wavelength shifted by a half a wavelength and for both polarizations of light the grating is the same, but shifted by half a wavelength along the x axis.

So, these are very interesting and very useful finding that in an isotropic medium. When there is a shear wave that is traveling whether it is the a refractive index variation, the perturbation is the same which is independent of whether it is y polarized or it is z polarized. So, these are the 2 possible polarizations the refractive index formation that is that grating formation is identical in both the cases.

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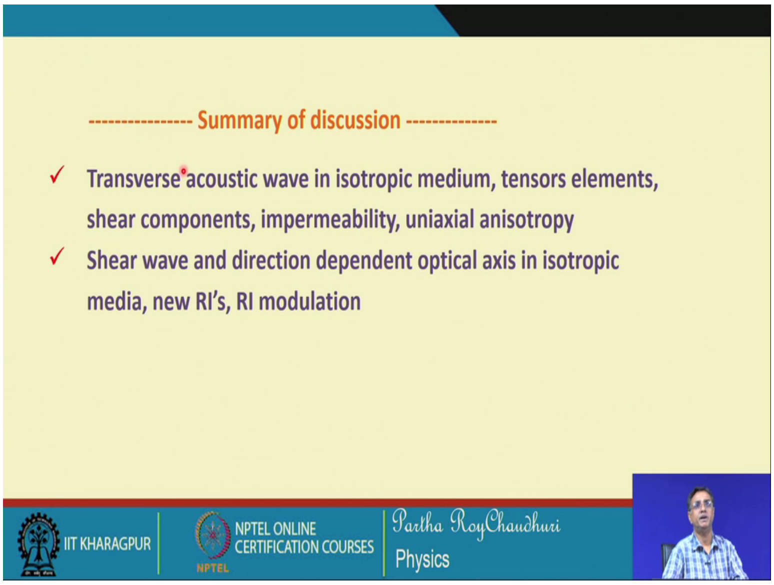

So, the summary that we now consider the shear acoustic wave in anisotropic medium and in that case, we utilize the strain optic tensor, for acoustic medium. Then, we saw that it is only the shear components which are non 0 for the for the shear waves. And then, we looked at how this impermeability change takes place and anisotropic material medium becomes an anisotropic and the shear wave the look at the direction dependent optical axis in the case of isotropic medium. Because, there will be one axis which is the direction of propagation about which the 2 polarizations are different the n y and n z.

So, that is also we discussed and now we will consider in the next section the acoustic wave that is traveling through an anisotropic medium. We will consider that how this the longitudinal acoustic wave will be very straightforward, but the shear acoustic wave in an isotropic medium can give rise to several possibilities. We would like to study that in the next discussion.

Thank you very much.