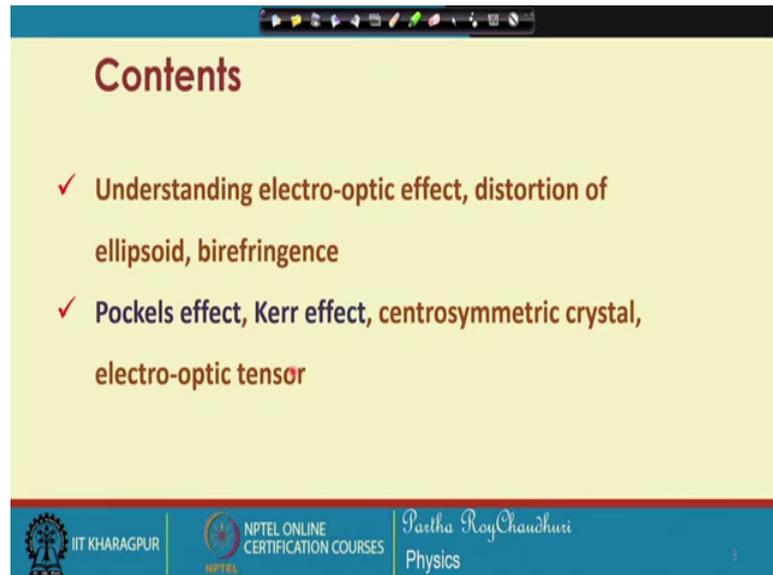


Modern Optics
Prof. Partha Roy Chaudhuri
Department of Physics
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Lecture - 29
Electro-optic Effect

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Electro-optic Effect; Earlier we have discussed the electromagnetic waves propagation an isotropic and anisotropic medium. Now, we will see when the medium is influenced by an external electric field, what happens to the birefringence, whether it is isotropic or anisotropic, there may be a change in the optical properties.

So, we have organized this discussion, first we will look at the understanding point of electro-optic effect, then the index ellipsoid its distortion under electric field corresponding induced birefringence, then we will categorize the two kinds of electro-optic effects the Pockel's effect and Kerr effect. We will look at a certain class of crystals centrosymmetric crystal which will remain unaffected as regards this Pockels effect, but will exhibit Kerr effect then we will look at the electro-optic tensor.

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Electro-optic Effect

We studied
light propagation in **isotropic** and **anisotropic** media
In **anisotropic** media the state of polarisation may change with propagation

In presence of an external electric field
RI properties of the medium may alter
may induce birefringence in otherwise **isotropic** medium
may alter **existing** birefringence property of the medium

This effect is known as **ELECTRO-OPTIC** effect

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So, as I have mentioned that earlier we studied the propagation of light in isotropic and anisotropic media, in anisotropic media there may be a state of polarization change with the propagation depending on the orientation of the principal axes with respect to the direction of propagation. But in presence of an external electric field the refractive index properties of the medium may undergo change. There may be some changes in the refractive index properties of the medium. And as a result it may induce birefringence in otherwise isotropic medium and it may alter the birefringence properties of the medium which is already a birefringence medium.

That is to say that if it is an isotropic medium it may become anisotropic, with certain changes in the refractive index properties and if it is already an anisotropic medium there may be changes in the refractive indices. So, that the new medium under the influence of external field is again anisotropic, but the properties are different. This effect that is the effect caused by an external electric field on a medium with respect to propagating electromagnetic waves in terms of the changes in the permittivity that is the refractive index properties of the medium is called electro-optic effect.

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Light modulator

Electro-optic effect
causes change in RI's of a medium with an applied electric field
using right configuration/orientation, birefringence of the medium
can be electrically controlled

Anisotropic medium can alter the state of polarisation

A half-wave plate between two crossed polarisers can yield an amplitude modulator

Retardation of waveplate can be controlled electrically to make optical modulator, light switch

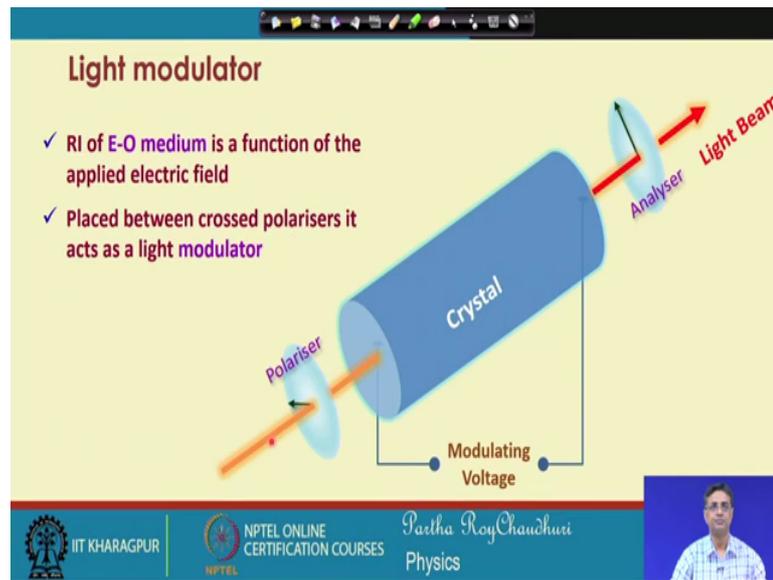
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Light modulator, electro-optic effect causes changes in the refractive index of a medium with regard to an externally applied electric field and if the medium is placed with right configuration or orientation with a suitable positioning of the medium with respect to the propagating electromagnetic waves. The birefringence that will be induced in the medium can be electrically controlled by the external by change in the magnitude of the external electric field.

In the case of anisotropic medium this electrically controlled anisotropic may altered the state of polarization of the electromagnetic wave that is propagating through the medium. A half wave plate placed between two crossed polarisers can yield an amplitude modulation, that is if we have an analyser and the polariser and in between there is a half wave plate then if the property of the half wave plate is modulated the amplitude of the light which is exiting from the analyser will be modulated.

A retardation of the wave plate can be controlled electrically, the wave plate that will be placed which in this case will be an electro-optic crystal, electro-optic medium that can be controlled electrically and that will result in optical modulation switching off light. We will see this things in details.

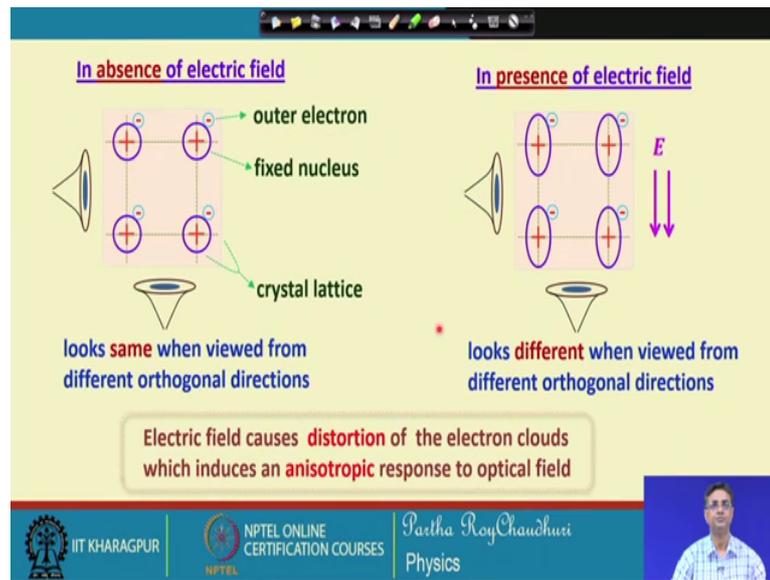
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Now, let us consider a medium a crystal which is electro-optic crystal that is if I apply a voltage that is an electric field across the crystal and the light is propagating in this direction with a polariser and an analyser who are mutually at cross position. So, they make an angle of 90 degree with each other then we can see that this system can be used as a as an amplitude modulator. Refractive index of the electro-optic medium is a function of the applied electric field.

And this effect when this strange refractive index profile will cause a change in the modulation of the light which are polarized linearly polarized at one end and you look at the light coming out of the analyser which is which used passed pass axis used at 90 degree with this. So, this configuration is very useful and very widely used in amplitude modulation.

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Now, let us try to understand simply how this electro-optic effect comes into the picture. Let us suppose you have a medium these are the fixed nuclear and you have outer electrons, this is the crystal lattice. And if you view this crystal from two orthogonal directions so lattice from this side or from this side it appears identical when there is no electric field. So, there is no effect of the electric field on to the crystal as a result when you look at the crystal from any direction it does not appear to be different, so it looks same when viewed different orthogonal directions.

On the contrary in presence of an electric field there will be distortion in the electron clouds that is simply looking at the electron orbits the there has been a modification there has been a change in the shape of this. So, if you view from the two orthogonal directions that is from this side and from this side they will appear different. If you are applied the electric field in this direction the modification the distortion of the electron cloud will appear like this. So, it looks different when viewed from the different orthogonal directions. So, this electric field cause distortion of the electron clouds which indices an anisotropic response to the optical field.

So, this is a very simple way of understanding the basic mechanism of electro-optic effect you have a field, you have no field, there is no change you have a field there is a change in the in the electron cloud and as a result you look at the same crystal from different directions will appear to be different.

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x, y, z lab frame

In absence of electric field

Isotropic $n_x = n_y = n_z = n_0$

$$x^2 + y^2 + z^2 = n_0^2$$

Sphere

In presence of electric field E

Becomes anisotropic $n_x \neq n_y \neq n_z$

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} + \frac{2yz}{n_{yz}^2} + \frac{2zx}{n_{zx}^2} + \frac{2xy}{n_{xy}^2} = 1$$

General ellipsoid

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Now, we have already learnt the index ellipsoid associated with the with the medium, with an optical medium and if it is an isotropic medium, and that to in absence of an electric field the index ellipsoid is a sphere - n_x , n_y and n_z all of them are equal. So, it represents a sphere that is in all directions the electromagnetic waves will see the same refractive index so which is represented by this equation. But in presence of an electric field E the same medium may become anisotropic there may be distortion in the index ellipsoid as shown here and consequently there may be a change in all the n_x , n_y and n_z values representing the refractive indices along x , y and z direction. As it is obvious from this picture in that case the index ellipsoid will be represented by this equation which is the general ellipsoid equation, ok.

This was the case when the medium was isotropic and it may become anisotropic under the influence of an external electric field, but if you already have an anisotropic medium in absence of the electric field it may be that all the it may be a biaxial system that is n_x not equal to n_y , not equal to n_z , then it will represent the ellipsoid equation by this, equation n_x , n_y n_z all are different but this is in the principal axes system.

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x, y, z lab frame

In absence of electric field

Anisotropic $n_x \neq n_y \neq n_z$

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

Ellipsoid principal axes system

In presence of electric field E

modified anisotropic $n'_x \neq n'_y \neq n'_z$

$$\frac{x^2}{n_x'^2} + \frac{y^2}{n_y'^2} + \frac{z^2}{n_z'^2} + \frac{2yz}{n_{yz}'^2} + \frac{2zx}{n_{zx}'^2} + \frac{2xy}{n_{xy}'^2} = 1$$

General ellipsoid

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Now, if I apply an external electric field along this direction there may be a distortion of the ellipsoid. And there may be a corresponding change in the n_x , n_y and n_z the new values are n_x prime, n_y prime and n_z prime all of them may be unequal and the general equation of the ellipsoid it will be represented by this in this case. From here we can again find out the principal refractive indices by making some transformation of this equation we will learn and we will see in details with examples.

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Distortion of ellipsoid

- ✓ When a steady electric field $E(x, y, z)$ is applied to medium
- ✓ a change in $\left(\frac{1}{n^2}\right)$ i.e., $\Delta\left(\frac{1}{n^2}\right)$ occurs in all six terms of ellipsoid

For example: an isotropic medium $\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_0^2} = 1$

responds to the electric field by modifying the ellipsoid

$$x^2 \left(\frac{1}{n_0^2} + \Delta\left(\frac{1}{n^2}\right)_x \right) + y^2 \left(\frac{1}{n_0^2} + \Delta\left(\frac{1}{n^2}\right)_y \right) + z^2 \left(\frac{1}{n_0^2} + \Delta\left(\frac{1}{n^2}\right)_z \right) + \Delta\left(\frac{1}{n^2}\right)_{yz} \cdot 2yz + \Delta\left(\frac{1}{n^2}\right)_{zx} \cdot 2zx + \Delta\left(\frac{1}{n^2}\right)_{xy} \cdot 2xy = 1$$

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The distortion of the ellipsoid when a steady electric field is applied to the medium there is a change in $1/n^2$ term, that is the change will be represented by $\Delta(1/n^2)$ that occurs in all 6 terms of the ellipsoid. Look at this we have all 6 terms, everywhere there may be a change in the property of $1/n^2$ in x, y, z square.

For example, if I have an isotropic medium which is represented by this because all the refractive indices principal refractive indices are n_x, n_y, n_z same they may respond to the electric field by modifying the ellipsoid. A very simple example to understand that there will be an additional incremental change in the refractive index value in this way in this form, in this form. So, it is associated with all the 6 terms in the index ellipsoid.

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Distortion of ellipsoid

For an **anisotropic** medium in **principal axes system**: $\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$

responds to electric field by modifying the ellipsoid:

$$x^2 \left(\frac{1}{n_x^2} + \Delta \left(\frac{1}{n^2} \right)_x \right) + y^2 \left(\frac{1}{n_y^2} + \Delta \left(\frac{1}{n^2} \right)_y \right) + z^2 \left(\frac{1}{n_z^2} + \Delta \left(\frac{1}{n^2} \right)_z \right) + \Delta \left(\frac{1}{n^2} \right)_{yz} 2yz + \Delta \left(\frac{1}{n^2} \right)_{zx} 2zx + \Delta \left(\frac{1}{n^2} \right)_{xy} 2xy = 1$$

Be it isotropic or anisotropic,
electric field **modifies** the ellipsoid by **altering RI's** associated with directions

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For an anisotropic medium in principal axes system this equation will be like this n_x, n_y, n_z maybe in general all of them are different; n_x, n_y if they are equal then will call uniaxial if all of you have seen that and if all of them are different that is a biaxial system. So, this anisotropic medium may respond to the electric field by modifying the ellipsoid in this form you have incremental change in the which is because of the presence of the electric field $\Delta(1/n^2)$ in x, y, z, yz, zx, xy . So, these are the terms which will be which are the new coming into the index ellipsoid. So, be it an isotropic or anisotropic the electric field modifies the ellipsoid by altering the refractive indices associated with a direction.

So, this is the summary that whether it is an isotropic system or an anisotropic system the presence of the electric field may alter the refractive index properties. So, isotropic may become an anisotropic, anisotropic may become again anisotropic, but the properties are now different from the one which was in absence of the electric field.

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New principal RI's: birefringence

$$x^2 \left(\frac{1}{n_x^2} + \Delta \left(\frac{1}{n^2} \right)_x \right) + y^2 \left(\frac{1}{n_y^2} + \Delta \left(\frac{1}{n^2} \right)_y \right) + z^2 \left(\frac{1}{n_z^2} + \Delta \left(\frac{1}{n^2} \right)_z \right) + \Delta \left(\frac{1}{n^2} \right)_{yz} 2yz + \Delta \left(\frac{1}{n^2} \right)_{zx} 2zx + \Delta \left(\frac{1}{n^2} \right)_{xy} 2xy = 1$$

We may now look for

- ✓ A new principal axes system by suitably rotating coordinate axes
- ✓ This can be performed by diagonalisation of the ellipsoid matrix
- ✓ By inspecting and providing suitable Euler angle rotation of axes
- ✓ Will result in new principal RI's of the medium and birefringence

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So, the new principal refractive indices and hence the birefringence that can be evaluated from this equation because under the influence of the electric field we have the ellipsoid equation, and from there we know how to find out the principal axes system and that the corresponding principal refractive induces.

So, now once having this equation in hand we can look for the new principal axes system by suitably rotating the coordinate axes. And this can be performed by diagonalisation of the ellipsoid matrix. So, this is again associated with a 3 by 3 matrix which we have seen and you have also seen an example how this matrix can be diagonalized as a tutorial in the anisotropic cases. Otherwise by inspecting and providing by inspecting this equation looking at this equation and providing suitable Euler angle rotation of the axes we can again find out the new refractive induces of the medium and hence the corresponding birefringence.

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The slide is titled "Two effects" and discusses two types of electro-optic effects. It states that when the change in refractive index is proportional to the external electric field ($\Delta n \propto E$), the effect is known as the Pockels effect, also called the linear electro-optic effect. When the change in refractive index is proportional to the square of the electric field ($\Delta n \propto E^2$), the effect is known as the Kerr effect, also called the quadratic electro-optic effect. The slide includes logos for IIT Kharagpur, NPTEL Online Certification Courses, and the presenter, Partha Roy Chaudhuri, a Physics professor.

So, by doing this we can find out the new refractive indices in the principal axes system. Now, the effect of the external electric field can be categorized in two ways we will experience two situations if the changes in the refractive indices are proportional to the external field simply linearly proportional that is Δn is proportional to E then such effect is known as Pockels effect after the name of the discoverer, Pockels effect. This is also known as linear electro-optic effect because the change is proportional to the electric field, but in some situations the refractive index change is proportional to the square of the electric field that is Δn is now, proportional to E square. The effect is known as Kerr effect after the name of the discoverer.

This is also known as the quadratic electro-optic effect. So, under this electro-optic effect we have two groups of effects to study that is Pockels effect and Kerr effect. So, we will first discuss the Pockels effect that is the linear electro-optic effect.

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Light modulation effect

Under certain geometrical configurations

an applied electrical field **acts differently** on two orthogonal linearly polarised waves passing through the medium

this produces **electric field dependent retardation/delay** between the two linearly polarised waves

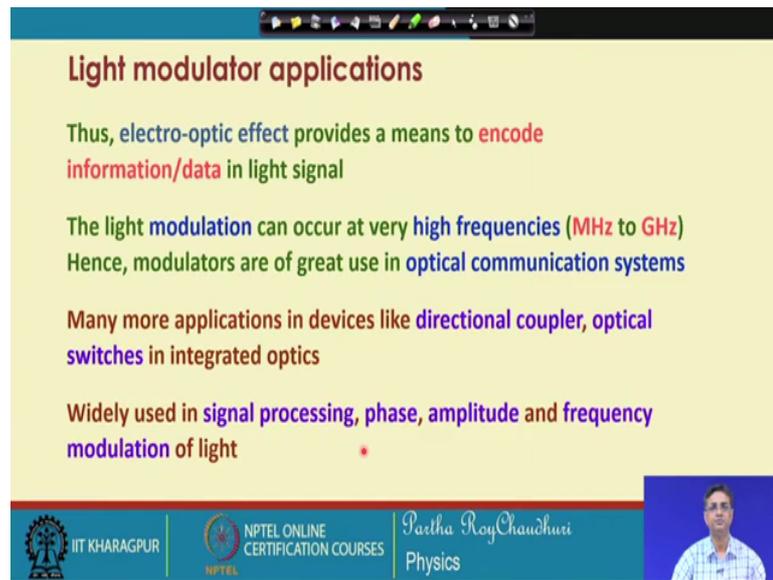
Hence, this effect can be used to **control the amplitude** of light in accordance with applied electric field

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So, under this Pockels effect or Kerr effect certain geometrical configurations of the medium are available. So, that if one applied an electric field then this electric field may act this in a different way on two orthogonal linearly polarized waves passing through the medium. So, let us suppose that there are two orthogonal linearly polarized waves which are passing through the medium and an electric field is acting on it, in that case the two polarized wave to linearly polarized waves will be affected in different waves and as a result there will be an electric field dependent retardation because there will be an induced birefringence because of this and which will cause a delay between the two linearly polarized waves.

If there is a delay then by controlling the electric field we can actually modulate the amplitude of the light according to the magnitude of the applied electric field.

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Light modulator applications

Thus, electro-optic effect provides a means to encode information/data in light signal

The light modulation can occur at very high frequencies (MHz to GHz)
Hence, modulators are of great use in optical communication systems

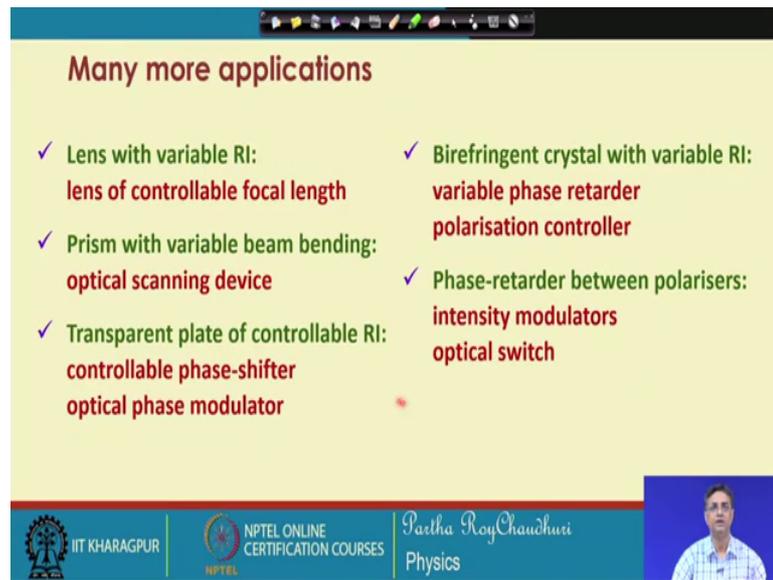
Many more applications in devices like directional coupler, optical switches in integrated optics

Widely used in signal processing, phase, amplitude and frequency modulation of light

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So, thus electro-optic effect provides a means to encode information and data in the signal, the light modulation can occur at very high frequencies that is the beauty of this electro-optic effect it can be in the in the range of megahertz to gigahertz. Hence the modulators are of great use in fact product commercially available and use electro-optic devices in the optical network and optical communication system. Many more applications in the devices like directional coupler, optical switches are also commercially available in the integrated optics. And this electro-optic system is used in signal processing phase amplitude and frequency modulation of light we will study some very interesting example cases in the following sections.

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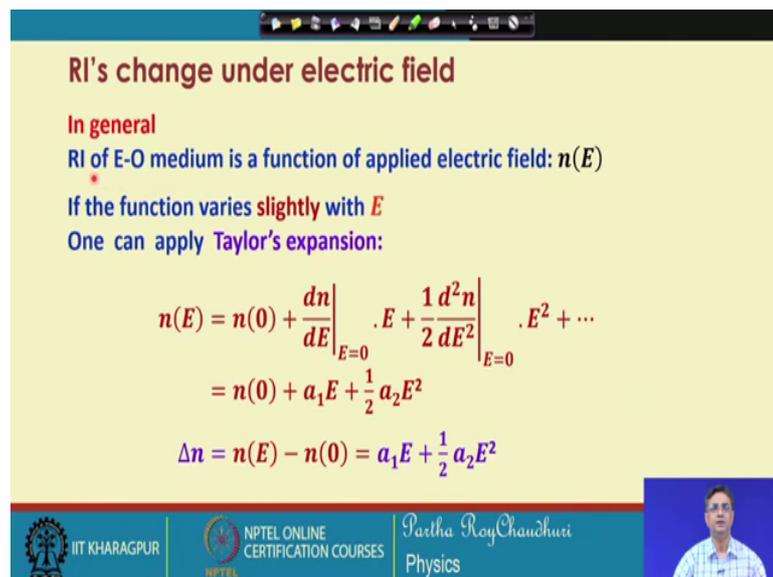
Many more applications

- ✓ Lens with variable RI: lens of controllable focal length
- ✓ Prism with variable beam bending: optical scanning device
- ✓ Transparent plate of controllable RI: controllable phase-shifter, optical phase modulator
- ✓ Birefringent crystal with variable RI: variable phase retarder, polarisation controller
- ✓ Phase-retarder between polarisers: intensity modulators, optical switch

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Additional there are many more applications like lens with variable refractive index with a controllable focal length, prism with variable beam bending optical scanning devices, controllable phase shifter, optical phase modulator birefringent crystal with variable refractive indices, then intensity modulators optical switches and many more applications.

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RI's change under electric field

In general
RI of E-O medium is a function of applied electric field: $n(E)$

If the function varies slightly with E
One can apply Taylor's expansion:

$$n(E) = n(0) + \left. \frac{dn}{dE} \right|_{E=0} \cdot E + \frac{1}{2} \left. \frac{d^2n}{dE^2} \right|_{E=0} \cdot E^2 + \dots$$
$$= n(0) + a_1 E + \frac{1}{2} a_2 E^2$$
$$\Delta n = n(E) - n(0) = a_1 E + \frac{1}{2} a_2 E^2$$

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So, in general the electro-optic medium is a function of the medium is a function of the refractive index is a function of the electric field and one can apply a Taylor of expansion to look at how the electric field is causing the changes in the refractive index property.

So, if you apply a Taylor expansion and written only the first 3 terms then you can see that changed delta n can be written as n E minus n o which is equal to a 1 written a 1 for this and a 2 for this del 2 n del E square at E equal to 0. So, we can represent this change in the refractive index by writing this E and E square dependent on (Refer Time: 21:18).

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Impermeability change under electric field

Define $\eta = \frac{1}{n^2}$ impermeability tensor

so that $\Delta\eta = \Delta\left(\frac{1}{n^2}\right) = -\frac{2}{n^3}\Delta n = -\frac{2}{n^3}a_1E - \frac{1}{n^3}a_2E^2$

Take $r = -\frac{2}{n^3}a_1$ and $s = -\frac{a_2}{n^3}$

Therefore: $\Delta n = -\frac{n^3}{2}r \cdot E - \frac{n^3}{2}s \cdot E$

And then: $\Delta\eta = \Delta\left(\frac{1}{n^2}\right) = r \cdot E + s \cdot E^2$

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So now, let us now defined another quantity that is impermeability which is one upon n square and evidently this is a tensor and this so the of the n impermeability change in the impermeability is delta n is equal to 1 by n square which can be written as if you take this derivative of this 1 by n square this gives you this, and there will be two terms. As a result this because delta n value I will pick up from here to pick up from here and put it here, then if I substitute for this quantity twice n cube a 1 and this as r equal to this and s equal to this then you can write delta n is equal to this, and delta eta that is changed in the impermeability equal to r into E and s into E square.

So, you see that the impermeability change is a function of is a combination superposition of the field dependent and the square of the field dependent terms.

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In a direct way

$$\Delta n = n(E) - n(0) = \left. \frac{dn}{dE} \right|_{E=0} \cdot E + \left. \frac{1}{2} \frac{d^2n}{dE^2} \right|_{E=0} \cdot E^2$$

Since $\Delta \left(\frac{1}{n^2} \right) = -\frac{2}{n^3} \Delta n = -\frac{2}{n^3} \left. \frac{dn}{dE} \right|_{E=0} \cdot E - \frac{1}{n^3} \left. \frac{d^2n}{dE^2} \right|_{E=0} \cdot E^2$

Also $\Delta \left(\frac{1}{n^2} \right) = r \cdot E + s \cdot E^2$

where $r = -\frac{2}{n^3} \left. \frac{dn}{dE} \right|_{E=0}$ and $s = -\frac{1}{n^3} \left. \frac{d^2n}{dE^2} \right|_{E=0}$



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You can look at the same equation in a in a direct way that if I put this equations in this form, then I take 1 by n square which can be represented in this form, and if I directly substitute this values for as writing as r and s equal to this and this then also we end up with this equation. So, looking at the same expression from two different waves basically they have the same origin.

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The two effects

Pockels Effect:

In many materials the s term is negligible

$$\Rightarrow \Delta \left(\frac{1}{n^2} \right) = r \cdot E \quad \text{and} \quad \Delta n = -\frac{1}{2} r n^3 E \quad \text{where, } r \text{ is Pockels coefficient}$$

Typical values of r is: $10^{-12} - 10^{-10} \text{ m/V}$ or $1 - 100 \text{ pm/V}$

Example:

For $E = 10^6 \text{ V/m} \Rightarrow \Delta n = r \frac{n^3}{2} E = 10^{-6} \text{ to } 10^{-4}$ (very small)



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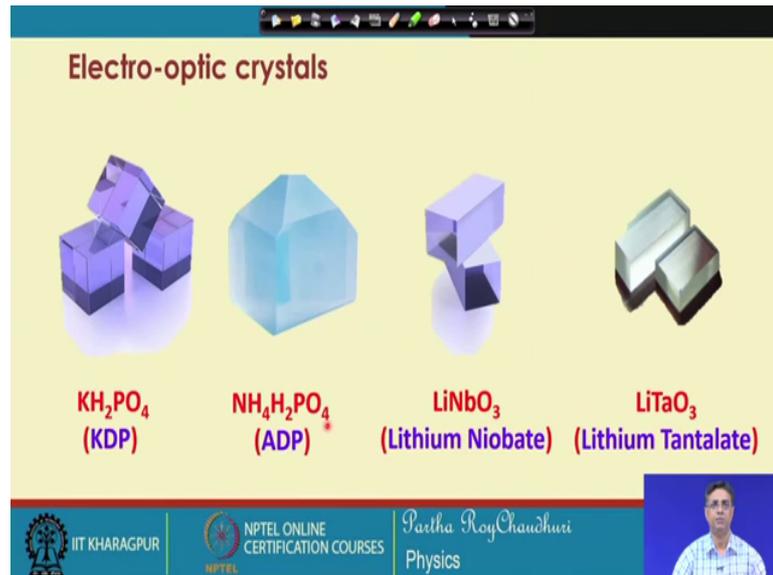
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So, Pockels effect in many materials this s term is negligible that is discard effect is negligible, but this r term is quite prominent. So, in that case this r is the Pockels

coefficient typical values of r is in the range of 10^{-12} to 10^{-10} or 1 to 100 picometre volt. Example if you take one that for an electric field of the order of 10^6 volt per meter then change in the refractive index is only 10^{-6} to 10^{-4} which is very small but the phase change associated with this may be very large and appreciable. We will see that example in future.

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So, there are any important electro-optic materials crystals; KDP potassium dihydrogen phosphate, then ADP ammonium dihydrogen phosphate lithium niobate lithium tantalite. These are very useful and commercially used electro-optic material crystals.

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The two effects

Kerr Effect:

If the material is centro-symmetric
 $n(E)$ is an even function and $r = 0$

$\Rightarrow \Delta\left(\frac{1}{n^2}\right) = s.E^2$ and $\Delta n = -\frac{1}{2}sn^3E^2$ where, s is Kerr coefficient

Typical values of s is: $10^{-18} - 10^{-14} m^2/V^2$

Example:

For $E = 10^6 V/m \Rightarrow \Delta n = -s \frac{n^3}{2} E = 10^{-6} \text{ to } 10^{-2}$

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Then the second effect is the Kerr effect if the material is centrosymmetric then even and even function n of E is an even function then r equal to 0 in that case this pockel effect is absent, but Kerr effect is present and Δn will be represented by this equation. The typical values of s will be of the order of 10 power minus 18 to 10 power minus 14 and electric field of the order of 10 power of minus 6 will create a change in the refractive index which is a order of 10 power minus 6 to minus 2 which is appreciable.

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The two effects

Kerr Effect:

Some polar liquids, such as
nitrotoluene ($C_7H_7NO_2$) and nitrobenzene ($C_6H_5NO_2$)
exhibit very large Kerr constants

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So, some polar liquids examples nitro toluene, nitrobenzene they exhibit very large Kerr constants.

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Centrosymmetric crystal

- ✓ Crystal exhibits inversion symmetry
- ✓ All r_{ij} coefficients are identically zero
- ✓ For inversion symmetry, the directions (x, y, z) and $(-x, -y, -z)$ are indistinguishable
- ✓ An applied field E and $-E$ must induce the same electro-optic response

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Centrosymmetric there is crystal there is one class of crystal which is very interesting it has some inversion symmetry and all the r_{ij} coefficients associated with the electro-optic tensors are identically 0. And for inversion symmetry the direction $x y z$ and the reverse direction that is minus x minus y minus z are indistinguishable. So, there is no change with respect to the crystal. So, as a result and applied field E and minus E must induce the same electro-optic response.

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Centrosymmetric crystal

$$\Delta\left(\frac{1}{n^2}\right) = r_{ij}E_j = r_{ij}(-E_j) = -r_{ij}E_j$$

$$\Rightarrow (r_{ij} + r_{ij})E_j = 0$$

$$\Rightarrow 2r_{ij}E_j = 0$$

$$\Rightarrow r_{ij} = 0$$

$$r_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

\Rightarrow no Pockel effect

Centro-symmetric crystal exhibit only Kerr effect




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To look at this if I write r_{ij} and then I reverse the field minus i then I will get minus r_{ij} E_j put together this will be equal to 0 that means, r_{ij} this. So, for centrosymmetric crystal all the coefficients r_{ij} will be equal 0 and there is no Pockels effect. So, if there is an s missing here, so there is no Pockels effect. And centrosymmetric crystal even though it does not so any Pockels effect, but it may exhibit Kerr effect we will see example at the lateritic.

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General index ellipsoid

Index ellipsoid in the principal axis system:

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad \text{or} \quad \frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1 \quad x, y, z = 1, 2, 3$$

In general, with other coordinate system:

$$\frac{x'^2}{n_1^2} + \frac{y'^2}{n_2^2} + \frac{z'^2}{n_3^2} + \frac{2y'z'}{n_4^2} + \frac{2z'x'}{n_5^2} + \frac{2x'y'}{n_6^2} = 1$$

In compact notation, one can write: $\sum_{i,j=x,y,z} \frac{x_i y_j}{n_{ij}^2} = 1$




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So, general index ellipsoid for an anisotropic material in this and in other coordinate system we can represent this like this in the compact form it comes out like this. We may write this general index ellipsoid in this form, ok.

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Electro-optic tensor

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 2yz + \left(\frac{1}{n^2}\right)_5 2zx + \left(\frac{1}{n^2}\right)_6 2xy = 1$$

- ✓ When a steady electric field $\mathbf{E}(x, y, z) = (E_1, E_2, E_3)$ is applied
- ✓ a change in $\left(\frac{1}{n^2}\right)$ i.e., $\Delta\left(\frac{1}{n^2}\right) = \Delta\eta$ occurs for all **six components**
- ✓ the elements of η are altered as a function of \mathbf{E} as: $\eta_{ij} = \eta_{ij}(\mathbf{E})$

$$\eta_{ij}(\mathbf{E}) = \eta_{ij}(0) + \sum_k \frac{\partial \eta_{ij}}{\partial E_k} E_k + \sum_k \frac{1}{2} \frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l} E_k E_l$$

$$\eta_{ij}(\mathbf{E}) = \eta_{ij}(0) + \sum_k r_{ijk} E_k + \sum_k s_{ijkl} E_k E_l$$

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Now, with this electro when a steady electric field; E_1, E_2, E_3 are applied a change in this that is this Δn occurs for all 6 components and then we can write this impermeability as the impermeability with no field. And then sum of the additional terms and we can write this equation in this form because we have represented this as the Pockels coefficient and this quantity as the Kerr coefficient. So, you can write this change in the impermeability in presence of electric field by this equation. So, this minus this will give you both the effects of by represented by in these two terms.

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Electro-optic tensor

$$\eta_{ij}(E) = \eta_{ij}(0) + \sum_k r_{ijk} E_k + \sum_k s_{ijkl} E_k E_k$$

27 coefficients for r_{ijk} : Pockels coefficients

81 coefficients for s_{ijkl} : Kerr coefficients

η is symmetric: $\eta_{ij} = \eta_{ji}$

r and s are invariant to permutations of i and $j \rightarrow r_{ijk} = r_{jik}$ and $s_{ijkl} = s_{jikl}$

s is also invariant under permutations of k and $l \rightarrow s_{ijkl} = s_{ijlk}$

$\rightarrow r_{ijk} = 18$ coefficients and $s_{ijkl} = 36$ coefficients

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So, these are the compact equation and you can see that this will give you twenty 7 coefficients Pockels coefficients and this will give you 81 coefficients E_k and E_k . So, n is symmetric n_{ij} equal η_{ij} equal to η_{ji} , s and r are invariant to permutations of i and j . As a result r_{ijk} equal to r_{jik} and similarly s_{ijkl} equal to s_{jikl} . And in the same way s is also invariant under permutations of k and l . So, you can write in this form k and l are reverse.

Therefore because there invariant two permutations of i and j and k and l . So, you can shrink these indices and actually they will represent the same coefficient. So, the number of coefficients reduces to 18 and 36 for the Kerr coefficient or pockel it is 18.

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Electro-optic tensor

$$\eta_{ij}(E) = \eta_{ij}(0) + \sum_k r_{ijk} E_k + \sum_k s_{ijkl} E_k E_k$$

For simplicity replace (i, j) with a single index m

$$\begin{aligned} (1, 1) &\rightarrow 1 & (2, 3) &= (3, 2) = 4 \\ (2, 2) &\rightarrow 2 & (1, 3) &= (3, 1) = 5 \\ (3, 3) &\rightarrow 3 & (1, 2) &= (2, 1) = 6 \end{aligned}$$

And similarly replace (k, l) with a single index n

$$r_{ijk} = r_{mk} \quad s_{ijkl} = s_{mn}$$

$$r_{112} = r_{12}, \quad r_{12k} = r_{6k}$$

$$s_{1112} = s_{16}, \quad s_{1231} = s_{65}$$

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So, just to understand how it shrinks to 1 because 1 will be represented by 1, 2 2 will be represented by 2 and so on. 2 3 will be the 2 3 and 3 2 there again same, so you write this equal to 4 and similarly 3 1, 5 and 2 1, 6 and this is have we have seen this in the index ellipsoid matrix the same notation and similarly replacing k l with single index n we can write this k and l will be represented by this. So, s k l m n, so there can be an again shrink to this. So, r 112 will be represented by r 12, s 11 12 will be represented by 1, 6 1 and 2 gives you 6 you can see that; 1 and 2 gives 6 and 1 1 gives you 1. So, it becomes s 16 and similarly 12 31 will become s 65. So, effectively we have 18 and 36 coefficients.

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Electro-optic tensor

Pockels coefficients

.....without field.....

$$x^2 \left(\frac{1}{n_x^2}\right) + y^2 \left(\frac{1}{n_y^2}\right) + z^2 \left(\frac{1}{n_z^2}\right) = 1 \quad (\text{principal axes system})$$

.....with the field.....

$$x^2 \left\{ \frac{1}{n_x^2} + \Delta \left(\frac{1}{n^2}\right)_x \right\} + y^2 \left\{ \frac{1}{n_y^2} + \Delta \left(\frac{1}{n^2}\right)_y \right\} + z^2 \left\{ \frac{1}{n_z^2} + \Delta \left(\frac{1}{n^2}\right)_z \right\} + \Delta \left(\frac{1}{n^2}\right)_{yz} 2yz + \Delta \left(\frac{1}{n^2}\right)_{zx} 2zx + \Delta \left(\frac{1}{n^2}\right)_{xy} 2xy = 1$$

.....change in $\left(\frac{1}{n^2}\right)$ i.e., $\Delta \left(\frac{1}{n^2}\right) = \Delta\eta(E)$

$$\Delta\eta(E) = \Delta \left(\frac{1}{n^2}\right) = \sum_{j=1}^3 r_{ij} E_j \quad i = 1, 2, 3 \dots 6$$

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So, with without field you have this equation in the principal axes system, and with the field you have this equation in the principal axes system. And the changes are in the impermeability is represented by this equation. So, you have delta n square which is this is for the Pockel coefficient. Only if you pick up the Pockel term then you have this Pockel coefficient which is represented by this.

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Electro-optic tensor

Pockels coefficients

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^3 r_{ij} E_j \quad i = 1, 2, 3 \dots 6$$

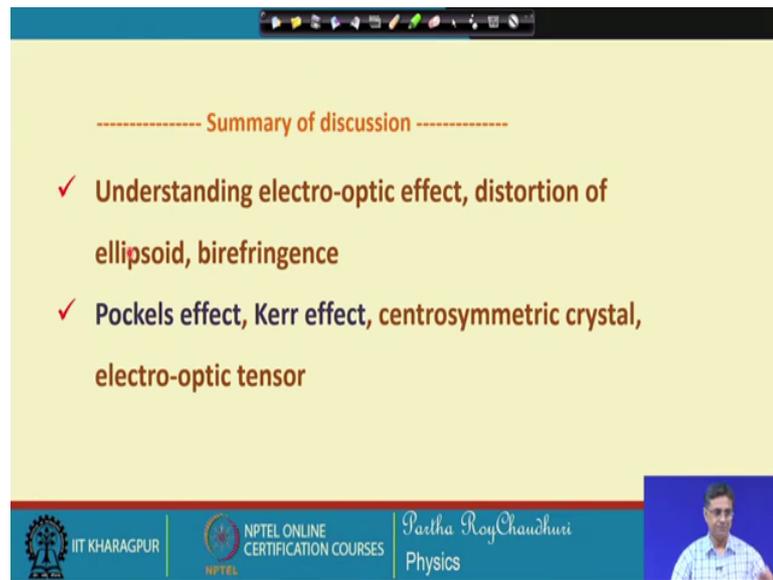
r_{ij} : electro-optic tensor (18 terms)

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

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An example Pockels coefficients you have this delta n 1 delta n square 1, 2, 3 etcetera, and you have this tensor which looks like this and this is the electric field. So, this compact equation represents this matrix equation, and you have 18 terms in this Pockels coefficient tensor.

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----- Summary of discussion -----

- ✓ Understanding electro-optic effect, distortion of ellipsoid, birefringence
- ✓ Pockels effect, Kerr effect, centrosymmetric crystal, electro-optic tensor

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So, in this discussion will continue will continue with this tensor for various examples like isotropic medium, gallium arsenide, anisotropic medium like KDP, etcetera and then will continue our discussion.

Thank you.