Modern Optics
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Lecture – 23
Waves in guided structures and modes (Contd.)

So far we have discussed simple optical waveguides, basic structures like symmetric planar dielectric slab waveguides, asymmetric planar dielectric slab waveguides, and wave guides of step index profile, so that is one of the simple most structures in optical waveguides, which has very simple analytical formulation. But there are various optical wave guides with complex refractive index profiles.

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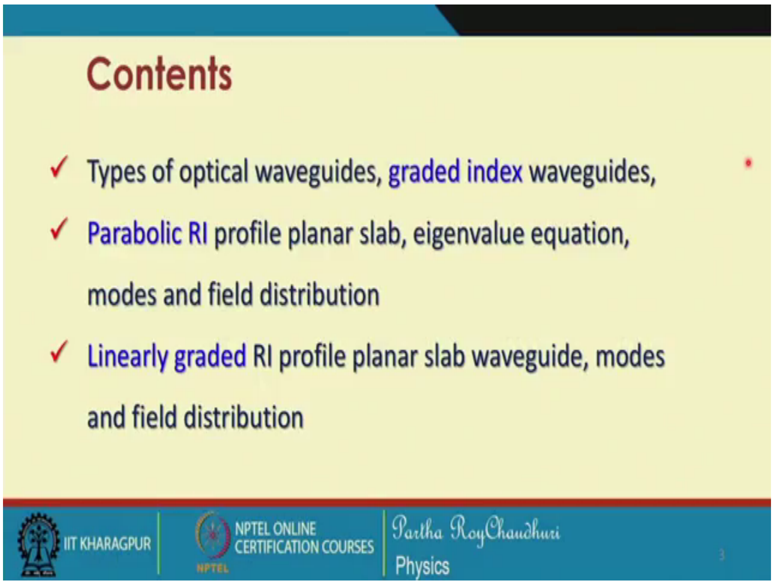

So, in this occasion, we will discuss another variety of optical wave guides, which are graded index profile, which will be representative of the class of graded profile. We will talk about the propagation characteristics. So, in this discussion, we will take two examples of graded index optical waveguides. And we will see the propagation characteristics the modes, the eigenvalue equation and the field distributions. So, these two are linearly gradient and parabolically graded refractive index profile.

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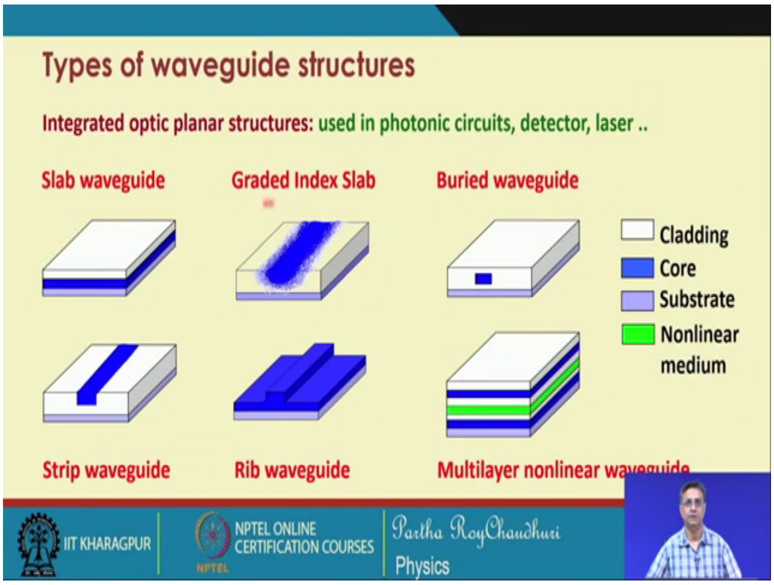

But, actually there are several different kinds of optical waveguides in practice, which are used in integrated optics, optical communication, photonics, photonic sensing, signal processing. The one, which we have discussed so far is the simple slab structure, it could be symmetric or it could be asymmetric as well.

The refractive index profile, as you can see is a simple step profile such that the refractive index remains same over a certain region, and it becomes different over a certain homogenous region. It could as well be an wave guide, which has a graded index, refractive index profile. For example, titanium diffused, integrated optic waveguides, there are several examples depending on that type of application, where the profile is linear exponential, it could be a complex varying function as well.

Then there could be wave guides, which is called buried wave guides. Where the guiding region is embedded inside, and there is a substrate, there is a film. So, these are the colors used to represent the different layers of this strip waveguide, then you have rib wave guide, you have a you have an elevated structure, which is surrounded, which is which has two symmetrical wave guide structures on either side.

And then, you have an very useful and very interesting optical waveguides multilayer non-linear structures, which are very common in electro up to electronic devices, gallium arsenide, gallium indium, phosphide. So, these are of very particular importance in up to electronic devices, but this is a step index profile, these are all step index profile, this is the one, which is graded index profile. And these are all most of these wave guides are very often used in integrated optics and photonics.

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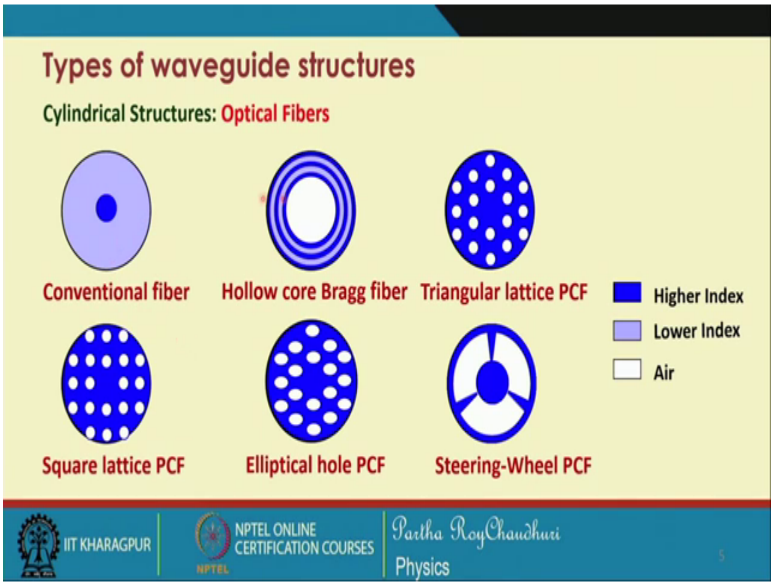

There is another class of optical waveguides; we have mentioned about this that cylindrical structure, which are optical fibers. And now a days, this is one very important optical waveguide for telecommunication, connecting cities, countries lot of optical fibers are led underground and also under the sea for telecommunication. There are a variety of optical waveguides, this is the one, which is the simplest that is the conventional structure, core cladding structure, which has a core and there is a cladding surrounding it.

There is a hollow core Bragg fiber this is also called omni guide structure that is it guides light from all directions that is annular rings, and there is a hollow. It is the hollow core that that guides the light. There is a new class of optical waveguides, which are very let research importance. These are photonic crystal fiber, this one is the again the basic photonic crystal structure, which is called the triangular lattice photonic crystal. The white part is to represent the air that is lower refractive index.

Then, you have square lattice photonic crystal fiber. You have this waveguide fiber guide, which is made up of elliptical, air holes running along the length of the optical fiber. There are variety of optical fibers of this kind, which are in general called microstructure optical fibers. You have a steer wheel type of most part of the cladding region is air only a standard silica core is suspended as a inside the fiber. So, there are variety of optical waveguides, this could be planar structure, this could be cylindrically symmetric structure.

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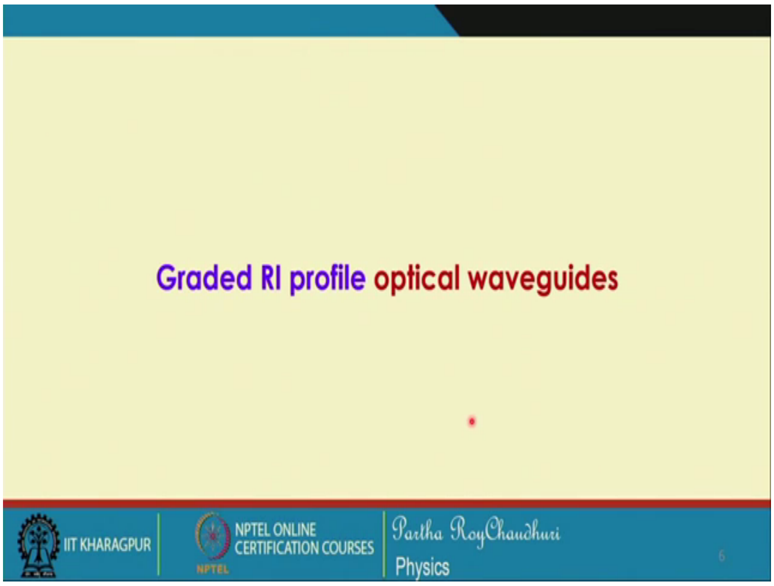

And then, we will now discuss the graded index profile. In cylindrical structure also there are graded refractive index profile optical fibers, which are also very important in local area network for dispersion compensation. And there are many other interesting applications.

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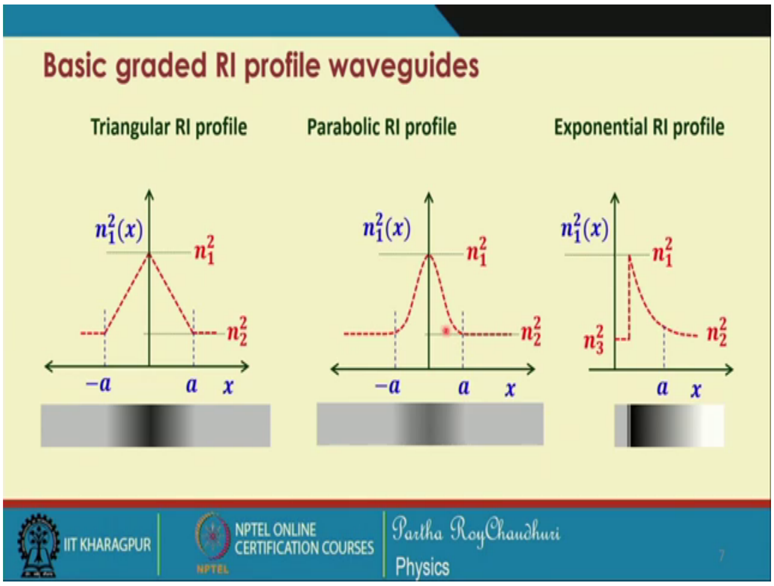

So, we will discuss a very basic type of graded index profile. And before we do that, we will first have a look at the kind of refractive index profile that this optical waveguides do have. For example, a triangular profile is basically a symmetric linear profile that is the refractive index falls linearly, and it has a maximum value at the center. So, between minus a to plus a, which we will treat as the core region.

You have a falling refractive index starting from the maximum at the center, which is n 1 square. And it assumes, a constant value at the edges that is at the point x equal to a or minus a. So, over this region because of this symmetric linear profile, we call this is a triangular refractive index profile, as very interesting, analytically, and from mathematical point of view also.

Then we have a very common refractive index profile wave guide, which is parabolic refractive index profile. And at the center, you have the maximum value n 1 square, and then it falls parabolically as a function or like a parabola. And it assumes, a constant refractive index n 2 on either side that is away from the core.

Then, you have an exponential profile. So, it has them highest refractive index profile n 1 square. Then in is asymmetric profile in one side, it falls suddenly is a step profile and it falls to n 3. On the other side, it has an exponential drop of the refractive index. And this is a very useful structure in integrated optics. There are examples, where we people have solved this problem for one side dividing into 3 distinct regions and solving the more problems.

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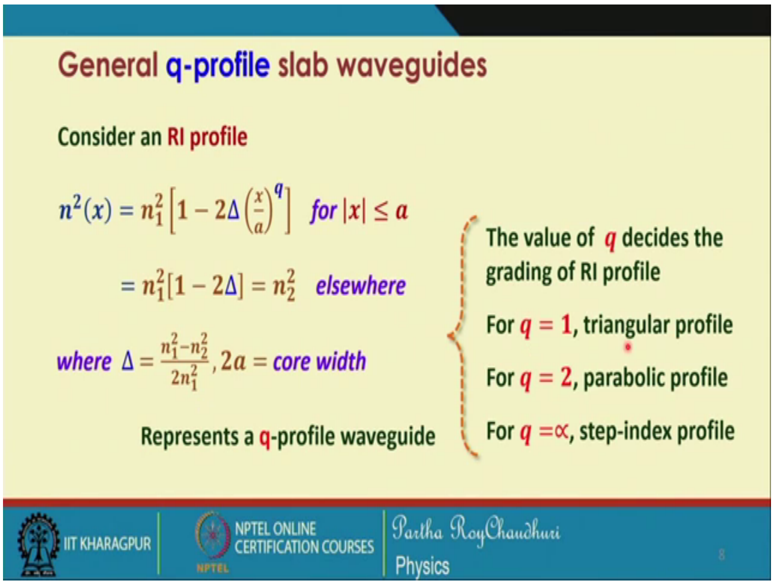

So, there is a definition for general q-profile waveguides, where you can represent, all these various types of refractive index profiles by one single mathematical expression, where we write this n square of x. The refractive index profile as n 1 square 1 minus twice delta x by a to the power of q for the core region defined by this quantity x mod of x less than a that is x between plus a and minus a.

So, this becomes equal to n 2 square elsewhere that is outside the core region. So, this is a general expression for any graded profile slab waveguide. Where this delta is the relative core cladding refractive index difference, which is defined by this n 1 square minus n 2 square by so if you expand this, this will become n 1 plus n 2 into n 1 minus n 2 divided by 2 n 1 square. For a very small difference in the refractive index, this becomes twice n 1 into n 1 minus n 2 by 2 n, so that is another way of defining this thing, and this twice a is the core width.

So, this represents a general graded index profile for slab waveguides. We can we can reduce this general mathematical expression to any particular profile, which you want by simply putting. For example, q equal to 1 will give you a linear profile, and we haves called this triangular profile. If you put q equal to 1, it simply becomes a linear function. And if you put q equal to 2, it becomes a parabolic function. It can assume, any value between 1 and 2 as well, so which is not exactly parabolic, and also not linear, but somewhere in between the variation.

For q equal to infinity if you put, then it becomes a step index profile, because this quantity to the power of infinity will you represent a you will get simply n 1 square and you will get simply n 2 square. So, this expression is very useful while doing analysis for the modes of a given graded refractive index profile of the wave guide.

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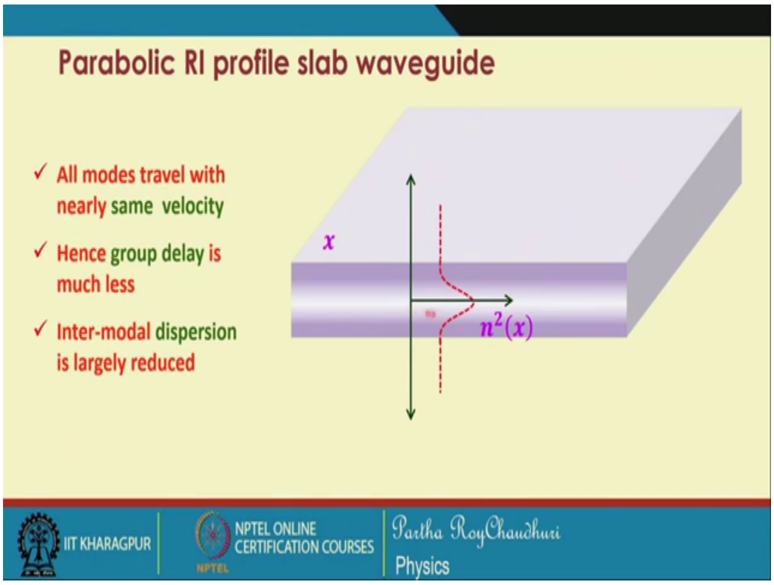

So, we will first consider the parabolic index slab waveguide and example of a parabolic waveguide schematically, it looks like this. You have the highest refractive index profile at the center and as you move away towards on either side, then the refractive index falls. And it assumes a constant value n 2 square. So, the beauty of such a waveguide is that all modes travel nearly with same velocity. We can so analytically, we can so with simple description of the various modes that they will travel with almost nearly same velocity.

So, therefore there is a very less group delay, delay that is the phase lag between the individual modes of the waveguide. And therefore, inter-modal dispersion is largely reduced. And inter-modal dispersion is a very useful and very important part in optical communication, which controls the you know the capacity of the data or information transmission through optical waveguides particularly in optical fibers.

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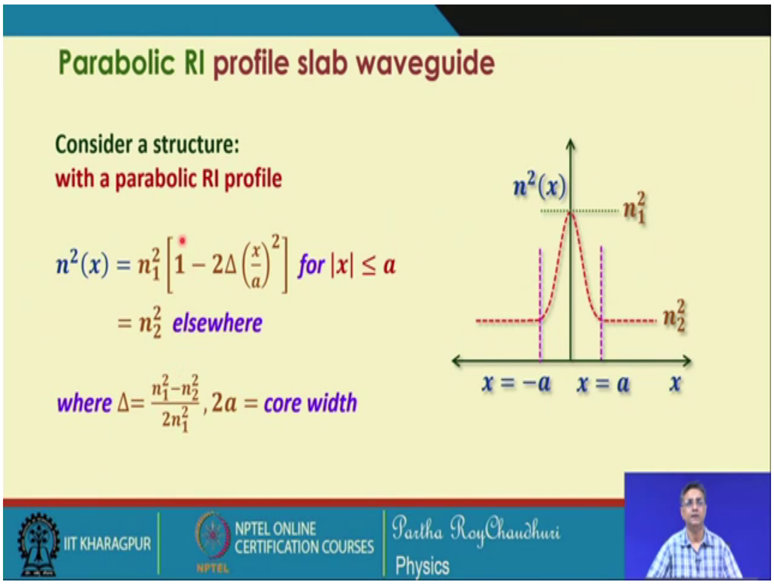

Consider a structure, which is given by this. For a for a parabolic index refractive index profile, we have seen we have to replace this q by 2. And it gives you n square x equal to n 1 square and this to the power of 2, which is for the co region that is for the region, which is defined by plus minus a x less than plus minus a, and otherwise it is n 2 square elsewhere. And again, we continue with the same definition of delta and this core width like this. So, you have a waveguide, which has a core between minus a to plus a. And the maximum refractive index is n 1 square here, the constant minimum value that it assumes at the cladding is n 2 square right.

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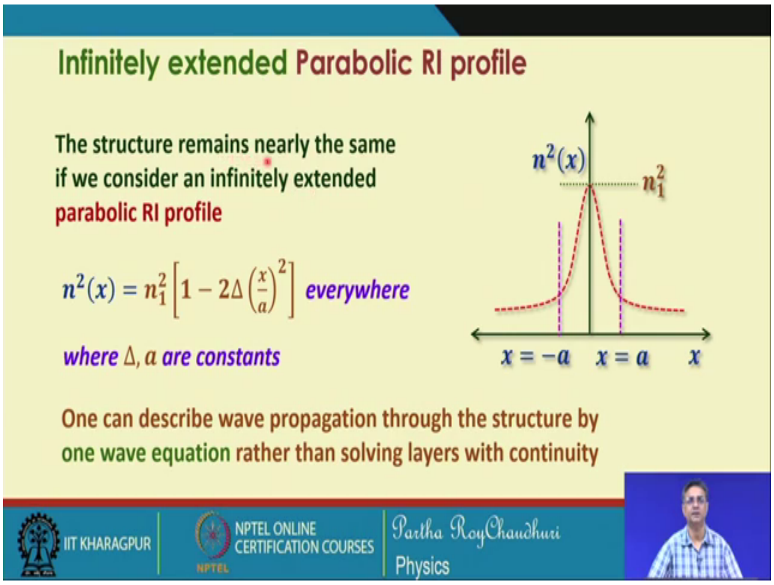

The structure remains nearly same if I if I apply this one equation that is I represent this refractive index profile by one equation that is this n square equal to n 1 square, then x by a square it is everywhere. In that case, this refractive index is no longer n 2 square, but it still falls. But, this variation is so small that it actually does not become, it does not become abruptly different. One interesting point to note here is that, because the field outside the core is very less is almost negligible.

So, this does not affect the mode the model field distribution only a very small very infinitesimal change in the model profile will be observed. So, we can but the advantage of that we can very nicely represent this profile, which is representative of this profile by a single equation. And there is not much great difference, because this assumes an asymptote, whereas it also becomes a constant. And the field is very less as we have mentioned.

So, we can represent, the waveguide by using one equation delta and a are constants for this, which follow the same definition. So, one wave equation rather than solving layers with continuity. If we have to solve this equation, we have to solve for this layer and also for this layer, then we have to use the continuity condition at the interfaces in order to get the modes. But by using this equation, we will have to solve only one equation, because we assume the refractive index profile to be represented by one equation.

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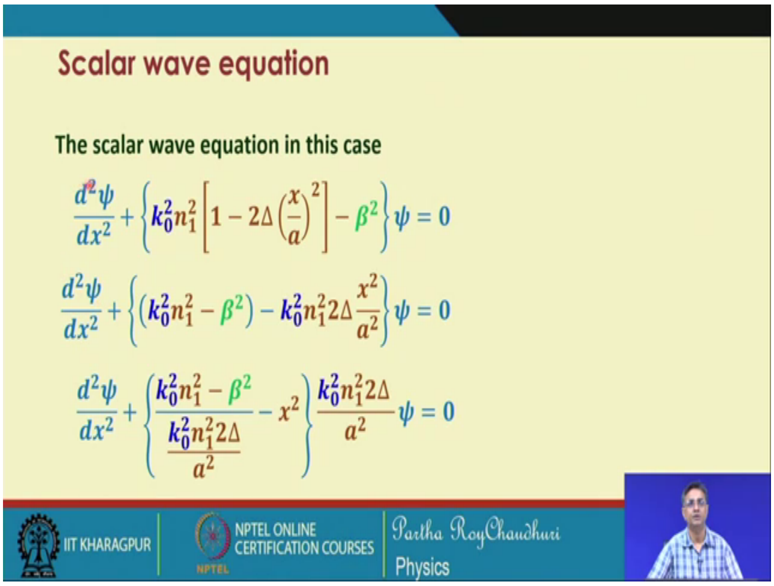

Therefore, the scalar wave equation that represents the wave, which is traveling through this structure, can be given by this equation, del square psi del x square this is one-dimensional wave equation. And we have used the refractive index profile n square of x replaced by this quantity n 1 square. So, this is the one expression for n 1 square, x n we do not have two layers.

And if we simplify this, we can take this k 0 n 1 square and beta square together the remaining part we segregate, we keep it separately, then this quantity we divide by this one. Need simple algebraic manipulation, in order to in order to get any known form of the equation. So, by doing this, this is the quantity where beta we are looking for x is the position in the waveguide direction and these are all constants.

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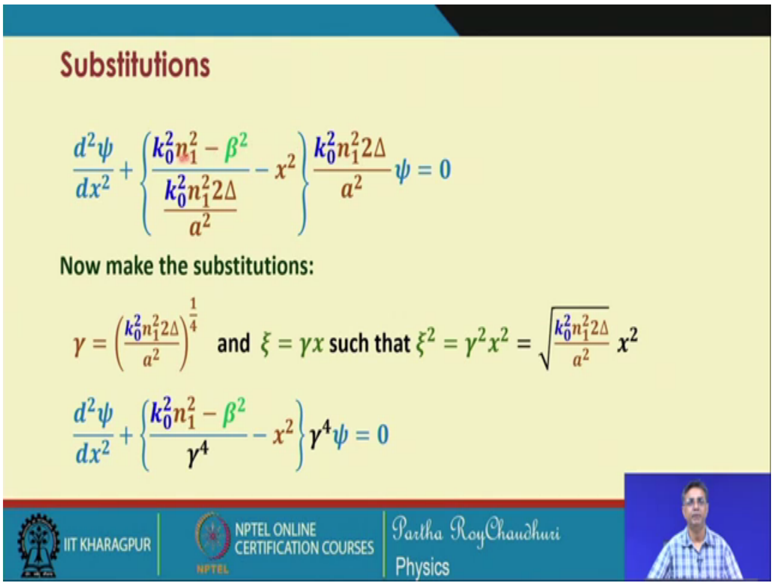

Therefore, this equation we can make simple substitution for this put gamma for this one that is k 0 n 1 square twice delta by a square to the power of 1 by 4 equal to gamma. Then put another constant, another variable is psi, which will be equal to gamma into x such that psi square will be equal to gamma square x square. And gamma square x square you can see, because this is now gamma square, I have defined gamma in this way.

Therefore, gamma square x square will be this quantity multiplied by x square. So, this equation wave equation, now reduces to this form k 0 n 1 square beta square, which remains this as it is and divided by gamma to the power of 4 minus x square. And outside, you have one more gamma to the power of 4 into psi equal to 0. So, this is the reduced form of the equation. Now, oh we because we have introduced gamma and psi, so you would like to change the variable from x to psi.

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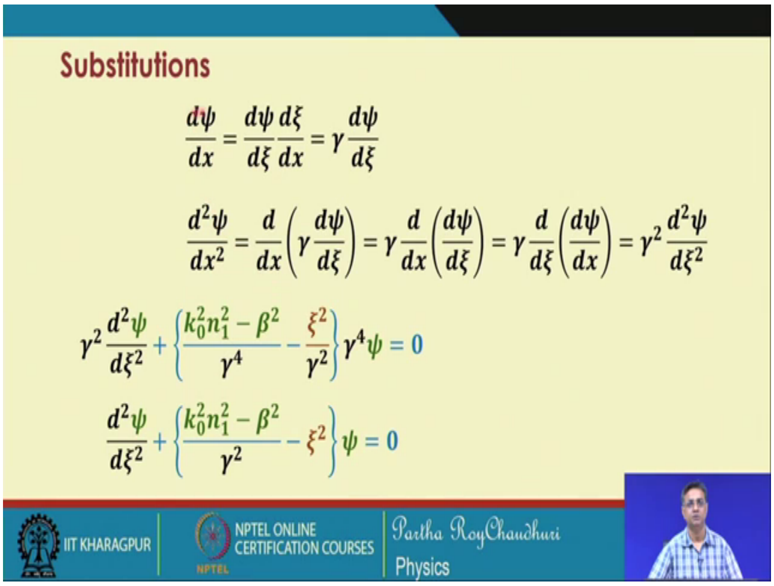

To do that, we did this simple differential calculus del psi del psi del x equal to del psi del psi del psi d x, which will give you gamma times del psi del psi. If you do it twice the differentiation, then this will leave. This gamma twice, detached from this equation. Therefore, del 2 psi del x square will be equal to gamma square del 2 psi del psi square.

So, using this definition for the new variable for x as psi, we can write this equation gamma square del 2 psi del 2 psi del psi square plus this quantity minus psi square by gamma square gamma to the power of 4. Now, you can remove gamma square all throughout, then you get this equation. And this equation will remind you or some known form.

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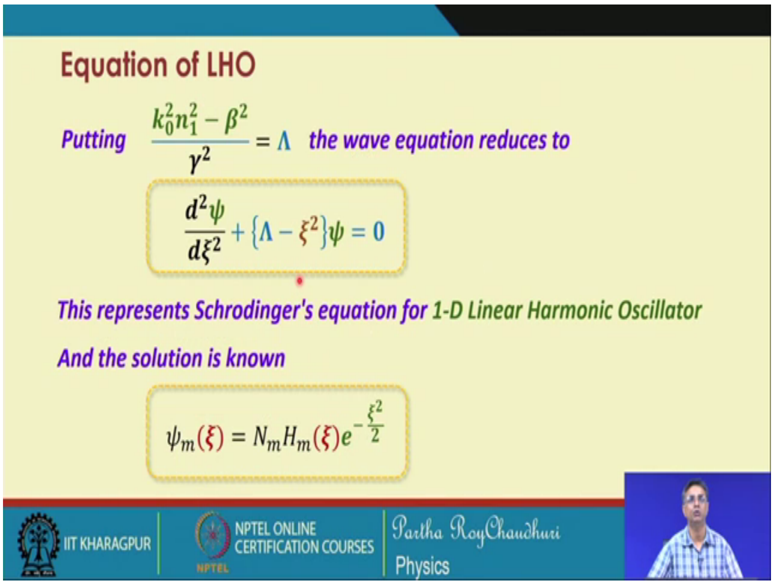

Now, if you put n is a k 0 square n 1 square minus beta square by gamma square equal to lambda, then this wave equation this equation, I put this quantity equal to capital lambda, then this equation reduces to this form. And this form is very well known. We have solved this problem in one-dimensional linear harmonic oscillator. So, we do not need to solve it further using the various techniques.

So, the solution of this equation that is one-dimensional linear harmonic oscillator problem in quantum mechanics. The solutions are known that psi m of psi is m N. This is Hermite gauss function, this is the Hermite polynomial and e to the power of psi square by x square, psi is the representative for x. So, this is a Gaussian function, there is an Hermite polynomial. So, the wave functions are Hermite gauss function with the normalization constant N m.

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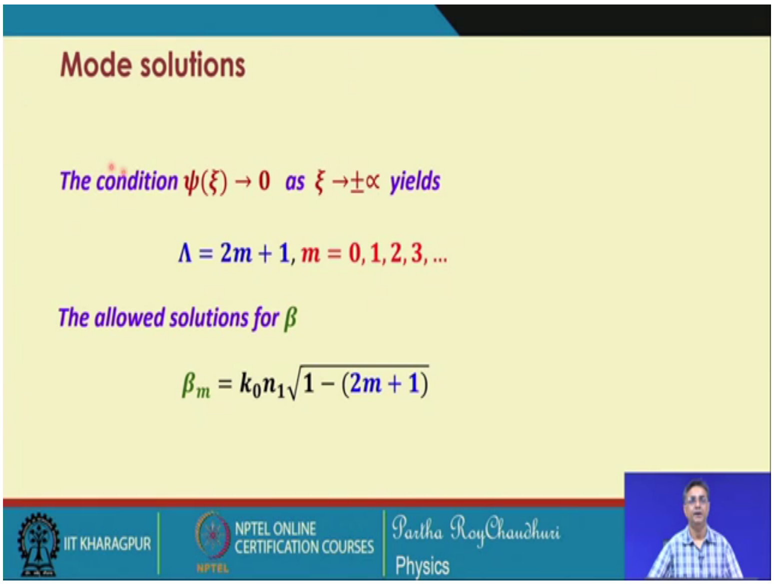

The condition psi of psi tends to 0 as psi tends to plus minus infinity that is that as x tends to plus minus infinity will give you this condition. This is also well known that lambda equal to twice m plus 1 will be for the various values of m including 0, 1, 2, 3 etcetera. Therefore, from here because this lambda includes this beta, you can remember the beta the definition of this.

We have used this as lambda, so this lambda can be will give you that beta m equal to k 0 n 1 under root of 1 minus square. So, this m will assume different values of 1, 2, 3. And we will get the various modes propagation constants, m equal to 1; m equal to 2 etcetera. m equal to 0 will be the fundamental mode. And in that case, we will put m equal to 0 and so on.

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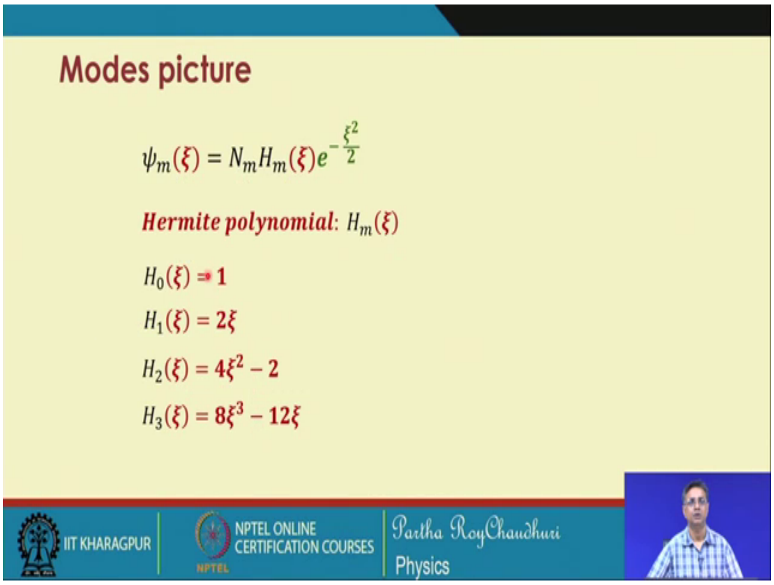

Now, we will look at the modes picture, how the modes look like. Hermite polynomial we know, and these are first few orders of the Hermite polynomials H 0 equal to 1; H 1 equal to this is all known; H 2 equal to 4 psi square minus 2 and so on. So, this will help us to put the values of psi 0; psi 1; psi 2 etcetera. And what will be the modes shape.

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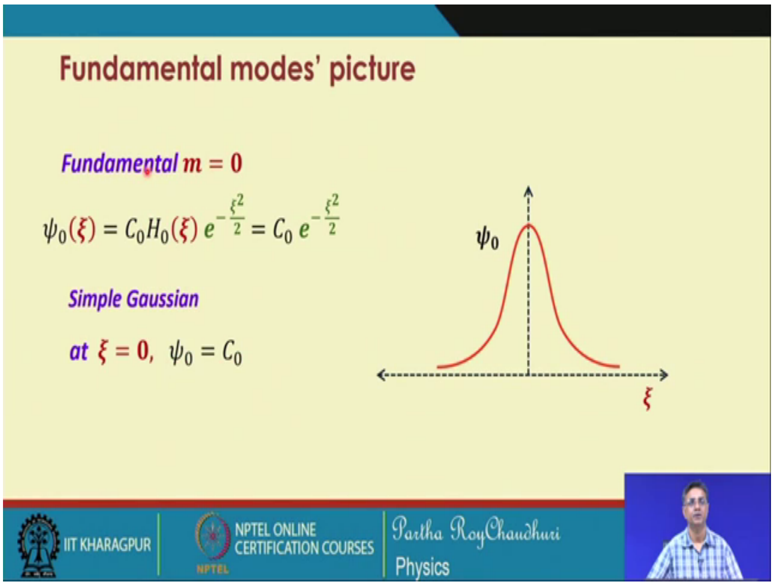

So, to do that, let us see that fundamental mode will be represented by m equal to 0. Therefore, psi m becomes psi 0, C 0 is the constant, this is Hermite polynomial of H 0, and this is Gaussian function. And if you try to plot this equation, this is a constant, because H 0 equal to 1, C 0 1. So, put together will give you the normalization constant. And it is simply C 0 e to the power of psi square by 2. So, this is a sample Gaussian function.

Even though, it looks complicated that the wave function is a Hermite Gauss value function, but it reduces to a simple Gaussian as far as the fundamental mode is concerned. And this, we will see that for all optical waveguides. The modes are more or less similarly of the same nature, same kind of profile, but there would be difference in the sharpness at the interfaces.

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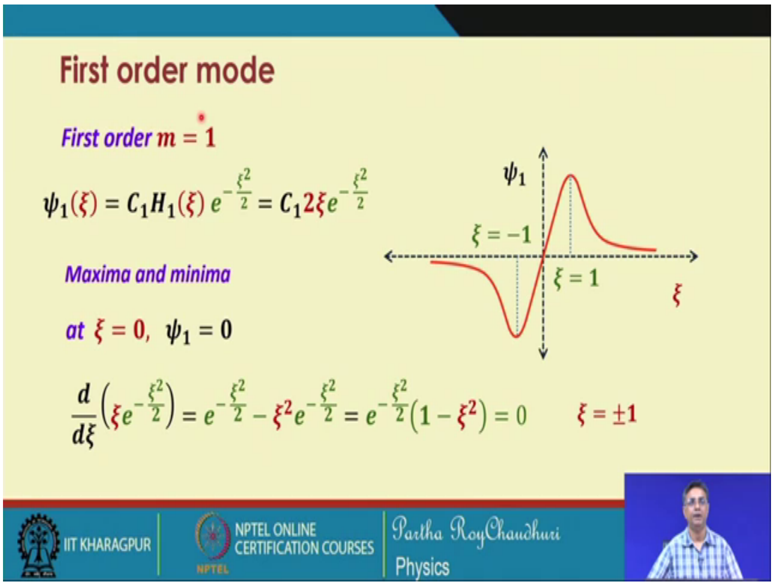

Now, look at m equal to 1, then this polynomial takes this form. So, the wave function psi 1 will be will be represented by C 1 2 psi, because psi 1 of psi equal to 2 psi now. Now, we look for the maximum, minimum of this at psi equal to 0 at psi equal to 0, this function will become 0 psi 1. So, this must be one point.

When I take the derivative of this function of this function I take the derivative of this function, then it gives you 1 minus psi square equal to 0, e to the power of psi square by 2 cannot be 0. So, this gives you psi equal to plus minus 1. So, these are the two points, where the wave function will have maxima and minima. And that is what is indicated by this that psi equal to plus 1; that psi equal to minus 1. So, this is also a known profile. Whether it is step index or a graded index, it has more or less the same structure, same kind of a mode profile.

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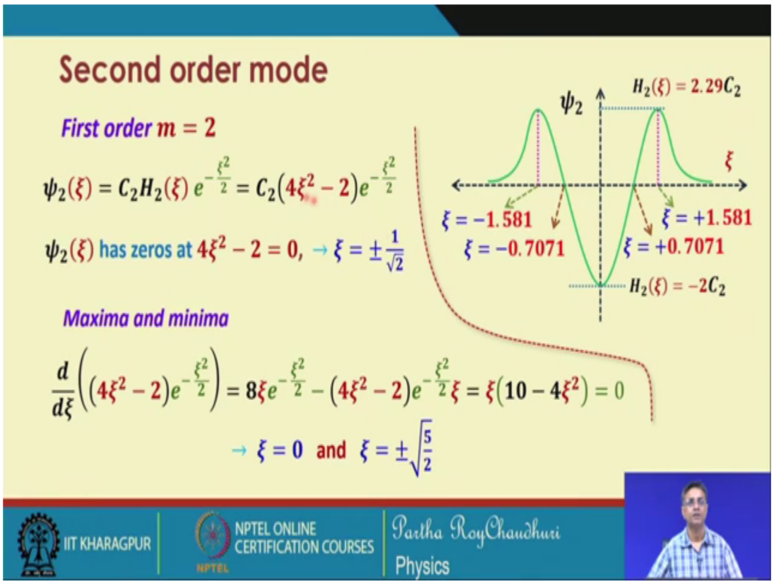

For first order mode similarly, if you put m equal to 2, then we can write this equation like this. The for the wave function, we can write this expression. Now, psi 2 has 0's at this quantity equal to 0, 4 psi square minus 2 equal to 0 that gives you psi equal to plus minus 1 by under root 2. So, these are the points, where psi 2 will be will be 0, this point, and this point. And rest other points, we will have to do this look for the maxima and minima.

So, we differentiate this expression, which will give you this possibility that psi into 10 minus 4 psi square equal to 0. So, either psi equal to 0, will give you this condition, which is a minima. If you put it back into the main equation, you will get the condition for minima. And if you take 10 minus 4 psi square equal to z 0, you will get this condition, because in that case psi equal to plus minus under root 5 by 2. So, you get these two conditions. So, you can reproduce, the complete mode picture for all other values of the m, for m equal to 3; m equal to 4 just by following this approach. So, you have a complete description of the various orders of modes.

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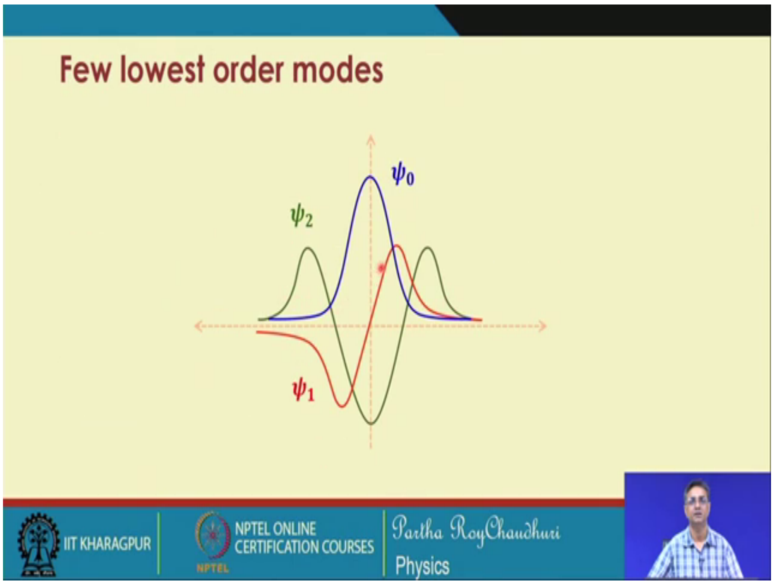

And this first few order of modes for this parabolic index profiles are represented. By this, the blue curve is the fundamental, then you have psi 1 red, and then you have green, which psi 2 and so on and so forth.

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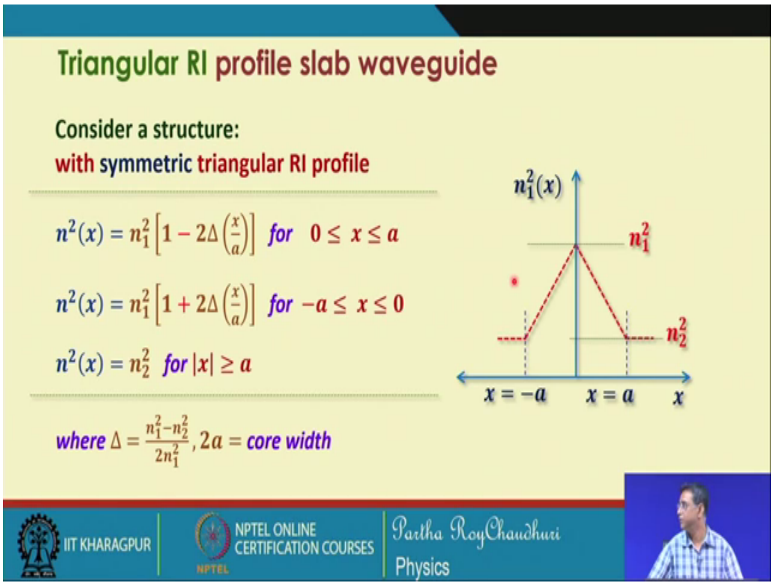

Now, we will consider the other graded index profile, which is a symmetric linear profile and we call the triangular profile. So, for triangular profile, we write n 1 square of x. Now, this q has become one, this is for the core, and this is for the cladding, and you have this. So, this is for the other side of the core, this is for the first side may be right hand side of the core, this is for the left side of the core. So, you have 3 regions 1, 2, and 3 actually 4 region, so you have this side also. And you will have to use the continuity condition for this. And these are the same definition, as we have followed earlier right.

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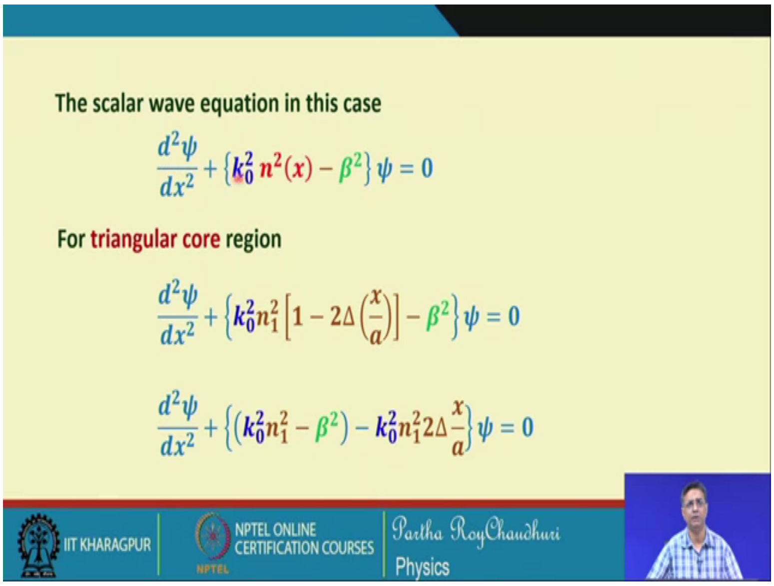

So, the scalar wave equation representing the modes for this structure will be given by this. Then we do some algebraic manipulation, we put k 0 square n 1 square minus beta square in one place, and leave all others at the other place.

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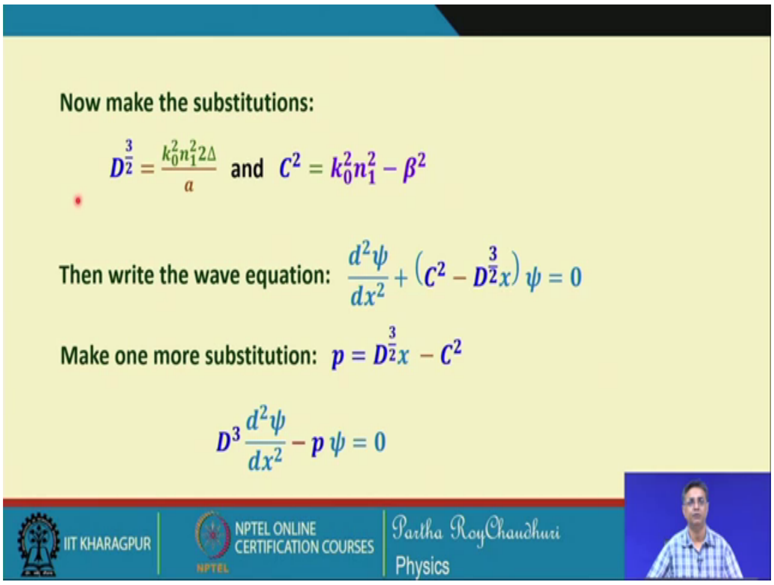

Then, we can identify the form of the equation represented by this color wave equation for this structure. Put D 2, D to the power of 3 by 2 equal to this. And C square is equal to this quantity. Then, we can write the wave equation like this. Now, one more substitution, because D and C these quantities can be represented by one parameter p. So, this gives you, this very reduced form of the wave equation right.

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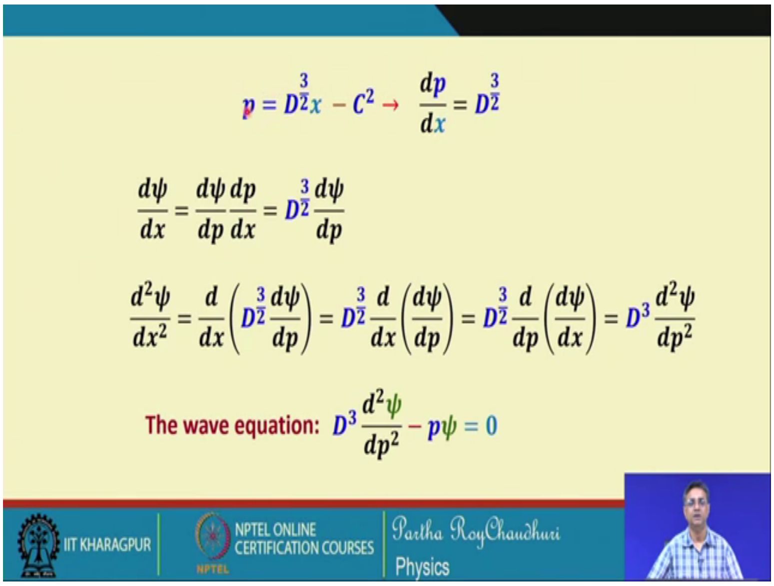

Now, because you have changed the variable. So, you also have to change the differential equation in terms of this p. So, delta 2 psi del x square can be represented by this separa[tion]- by this variable change in the variable in terms of dp. For every time you get for one differentiation, you get this quantity D 3 by 2. If you do it twice, you get D cube. And therefore, the equation becomes this.

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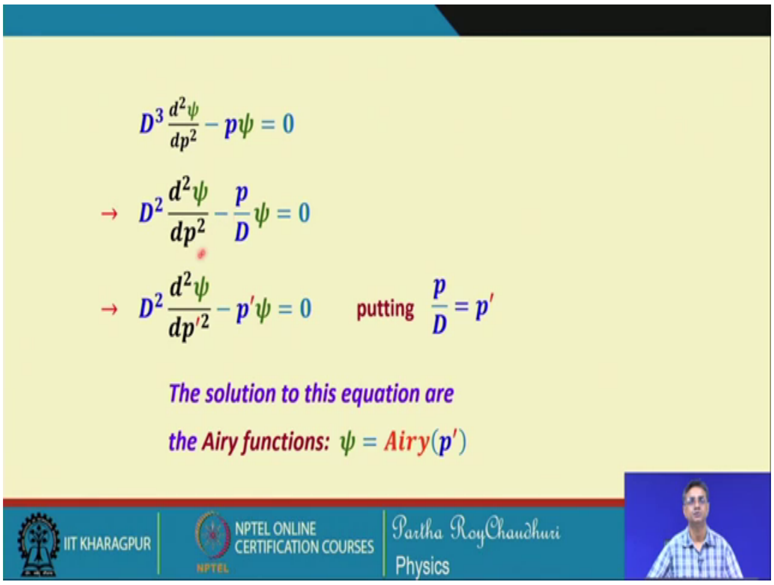

Now, if you divide this equation by D 3 cube D 3 cube, then you can you divide by this by D only, then D square into p by D or this will give you the known form. And this equation will represent, because you have again put p, D is a constant. So, p by D equal to p dash, the equation becomes this form.

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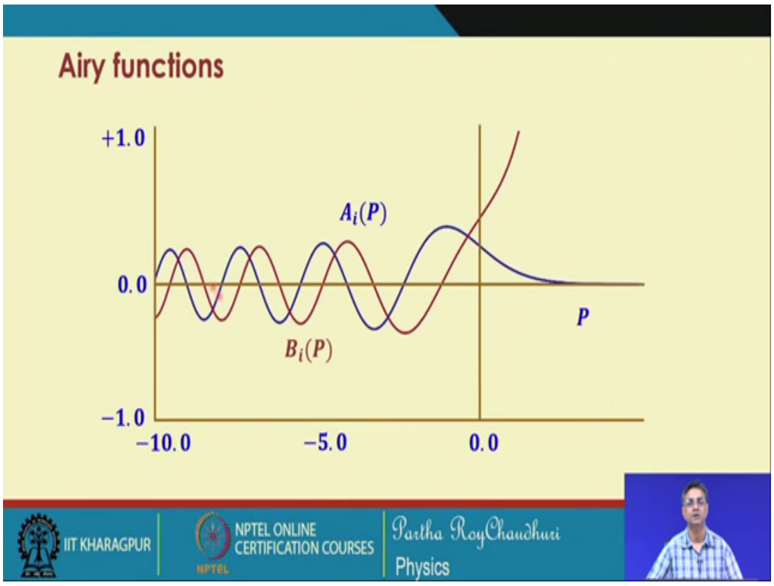

The solution of this equation is very well known, because this represents the airy equation and the solutions are airy functions. So, these are the two airy functions, A i and

B i. One has to choose the exact function depending on the physical conditions that is represented by the waves, electromagnetic waves propagating in the structure.

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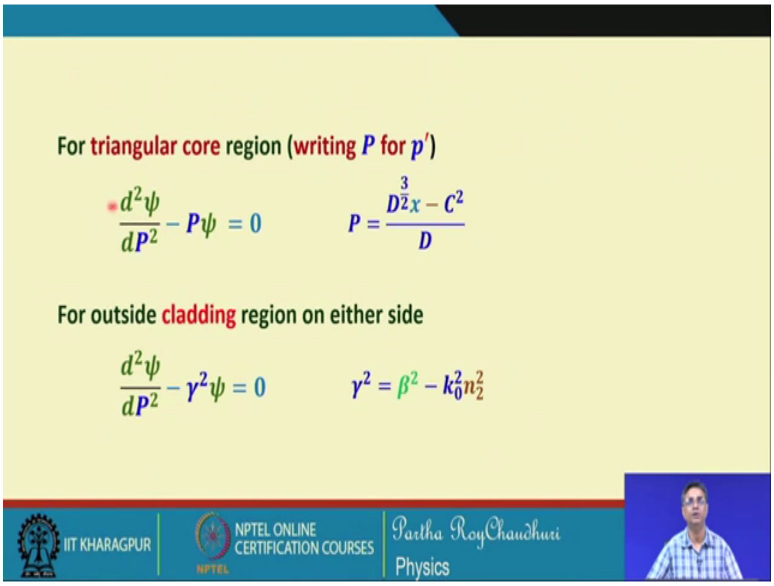

For the triangular core structure. This is the wave equation now, and for outside the region you have that already known decaying function, which is given by this equation. So, we can define the wave equation for all the four regions this will be represented, this equation will represent the cladding region, this equation will represent both the core regions right.

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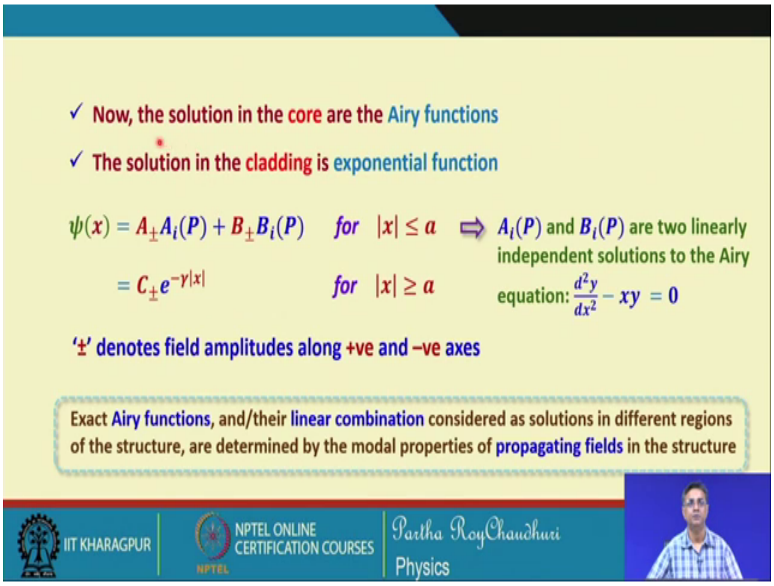

So, the solution of this equation can be put together in this form. For the core region it is this, and for the cladding region it is decaying function. So, this airy function, this airy equation this will give you this the solutions A i and B i.

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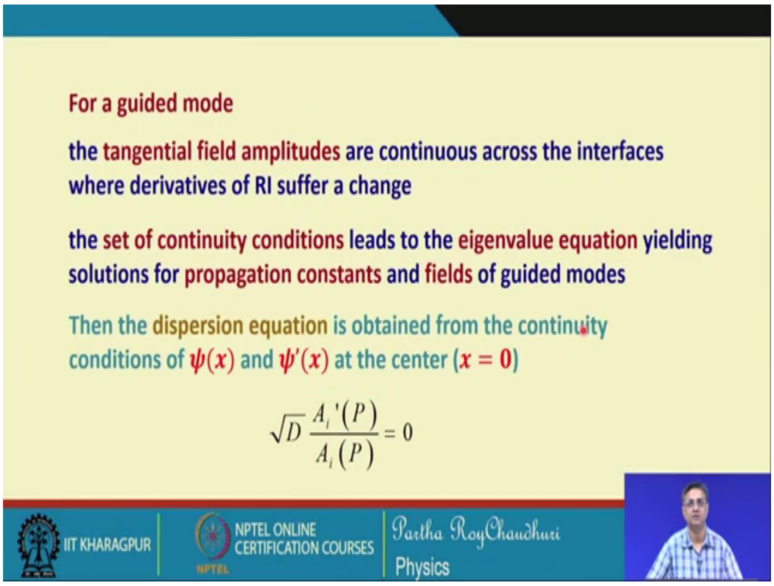

Then, we will have to use the continuity condition at the interfaces for this. And the continuity condition will come out to be like this under root D A i dash that. If derivative of this divide by this equal to 0, then we will get the dispersion equation here.

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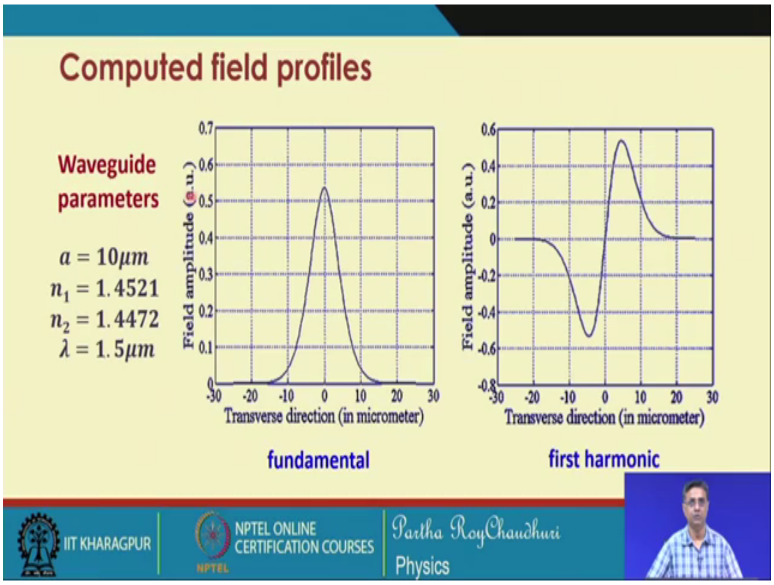

If you now plot this using this, we can identify the various order modes of the structure for the for the waveguide with linear profile. These are the parameters that that if you use, we will get a graph. We will get a plot of the model field fundamental model field, which will be like this. This is a computer generated plot for these waveguide parameters. This is for the first harmonic that first order mode, which we seen.

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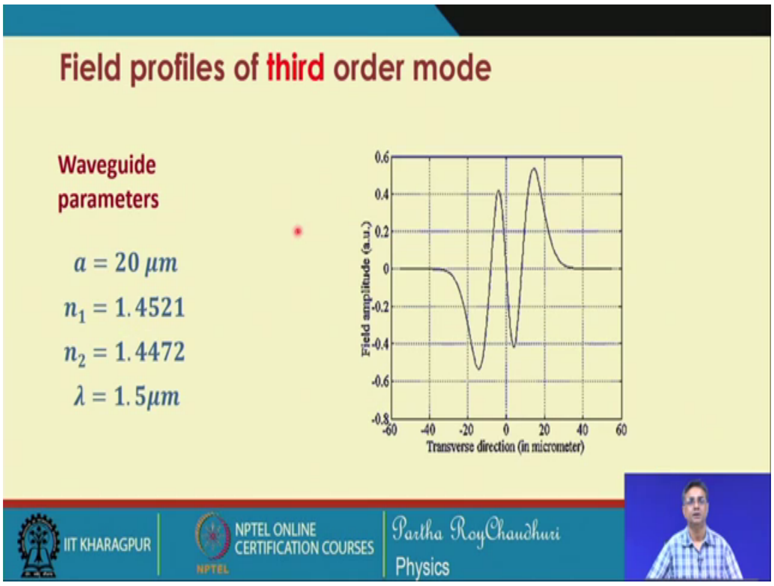

And then, if you take a waveguide parameters like a equal to 20, if you increase the width of the waveguide, the other conditions remaining the same, then you will get higher order modes like this for the structure that we have used for the this particular wave guide.

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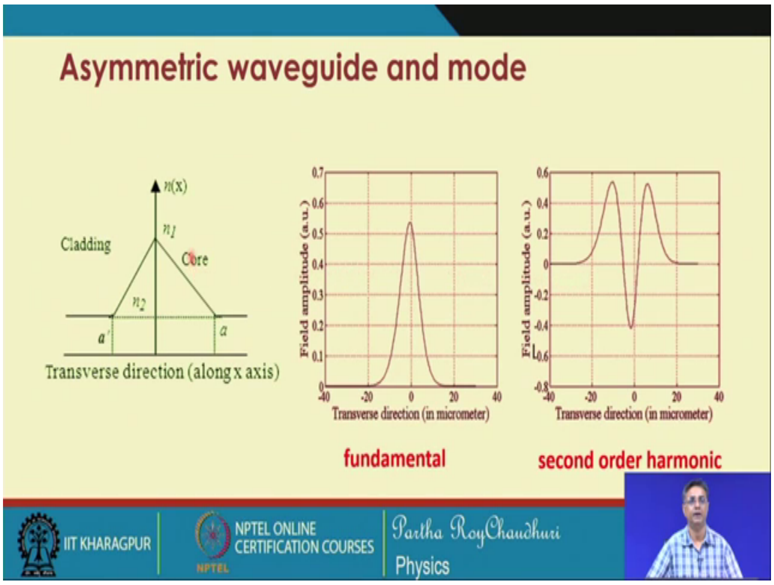

If you have a waveguide structure, which is asymmetric, but still linear. This is again a triangular profile, but this is not a regular not an equilateral, not a regular triangle asymmetric triangle. So, this side is falling faster, this is falling slower. And these toward this is a dash, this is a, these two are different values. The width the in that case, you can see that the mode is slightly extended on the left hand side, whereas this is the field decay is first.

So, this is how you can the electromagnetic waves react to the change in the refractive index or the permittivity profile of the medium. Go for the first order mode, second order mode, so it looks like this. We have you can again see the asymmetry looking at the 0 position. This side, and this side they are not symmetric, because the waveguide structure is not symmetric.

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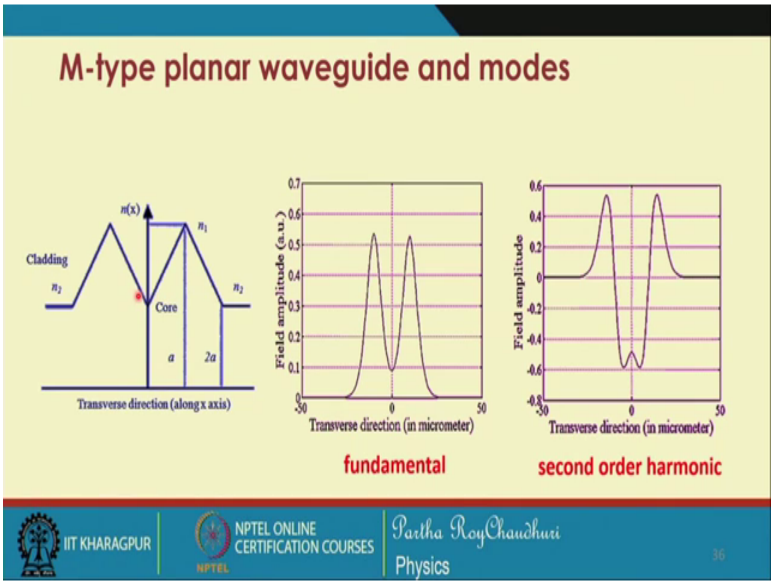

So, these are the various order modes of this. There are other modes that we have seen we have solved that the cladding structure, you have a dual triangular profile sort of structure where the modes will look like this. These are a particular importance in quantum well lasers and up to electronic devices, where various specifications are required for the propagation of the electromagnetic waves in the structure. You have the fundamental mode, which will look like this two lobe structure. You have a 0 because of the very nature of the waveguide structure.

You have the second order mode, which will look like this. So, we can see from here that, the modes of the structure for various kind of optical waveguides. They follow the similar route only difference is that the profile besides how sharp or how slowly the waveguide the fields are falling across the core cladding interface.

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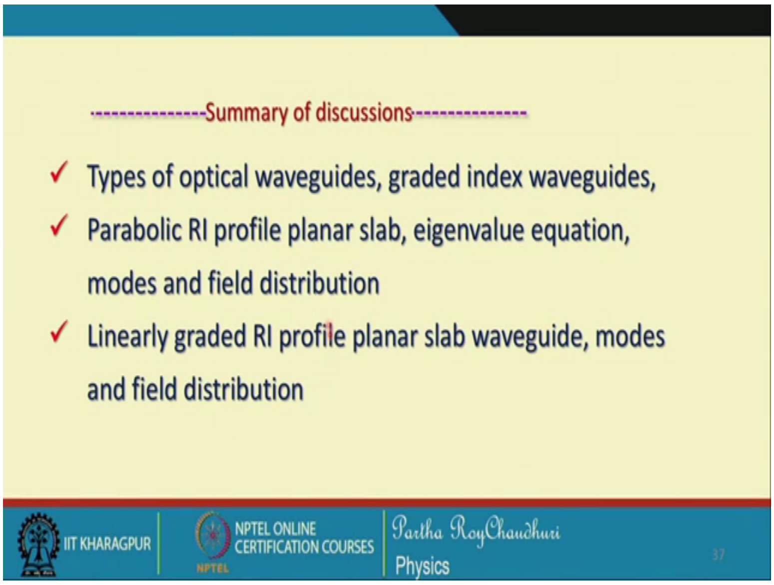

So, we have discussed in this context graded index profile. And for that, we considered two very important and very useful class of wave guides, one is parabolic index profile. We have parabolic index fibers also we will discuss that problem.

Then, we have optical waveguide, slab wave guides, which are linear and we called it is a triangular profile structure. We looked at the modes and the field distribution, then we talked about the various other profiles of the wave guides, which are of the linear refractive index profile symmetric, asymmetric structures, and two lobe structure and so on.

Thank you.