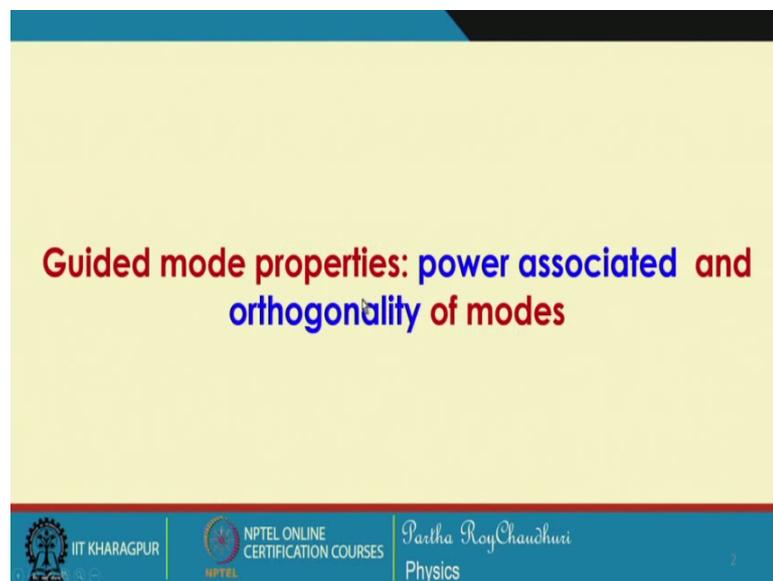


Modern Optics
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Lecture - 22
Waves in guided structures and modes (Contd.)

We were discussing the Guiding Properties of the Basic Wave guide that is a Dielectric Planar Slab.

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In terms of the TE TM mode, the field distribution across the wave guide, the eigen value equation and the related parameter. And now we will discuss the some more guided more properties which are very important and relevant particularly, in terms of device applications like; power associated, orthogonality of modes, real eigen value.

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Contents

- ✓ Some key properties of guided modes: power carried by guided modes: **TE-** and **TM** modes
- ✓ Real eigen value of mode and orthogonality of guided modes
- ✓ Radiation modes and fields

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So, from that point of view we organize this discussion under the following points, some key properties of the guided modes; power carried by the mode; TE and TM modes and we will see the eigen values of the wave equation are real. And we will also as a consequence see that the guided modes are orthogonal to each other. And then, another very relevant discussion is the radiation modes and corresponding fields. So, we will see that it forms a continuum of radiation mode for certain condition of the propagation constant.

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Power flow associated with a mode

The power flow is given by $\langle S \rangle = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*)$

Planar dielectric slab waveguide

For **TE mode**: $E_y = E_{0y}(x) e^{i(\omega t - \beta z)}$

$$H_x = -\frac{\beta}{\mu_0 \omega} E_y = -\frac{\beta}{\mu_0 \omega} E_{0y} e^{i(\omega t - \beta z)}$$
$$H_z = \frac{i}{\mu_0 \omega} \frac{\partial E_y}{\partial x} = \frac{i}{\mu_0 \omega} \frac{\partial E_{0y}}{\partial x} e^{i(\omega t - \beta z)}$$

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So, power flow: power flow associated with a mode. Now that the power associated with a mode can be determined from the calculation of pointing vectors time average of S is equal to half the real part of this E cross H star. Where H star is the complex conjugate of the magnetic field vector. First we will be considered the case of a planar a dielectric slab waveguide and the TE TM modes and we will see that this understanding this calculation is also valid for the other modes and approximately valid for almost all are such waveguides.

So, for TM TE modes we represent the electric field in this way, E_y equal to the amplitude $E_0 y$ is a function of x which varies across the waveguide dimension S_x and this phase factor into the power of $i \omega t$ minus βz . Now H_x and E_y they are connected we have seen with through this expression which is a consequence of the Maxwell's equation and writing explicitly the electric field we can write in this form.

Now, $E_z H_z$ it is also connected to you why through this equation and these two equations we will be used to calculate the pointing vector, because for a TE mode you have $E_y H_x$ and H_z we will translate H_x and H_z in terms of E_y using this equation.

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Using these relations:

$$\langle S_x \rangle = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*)_x = \frac{1}{2} \text{Re}(E_y H_z^* - E_z H_y^*) = \frac{1}{2} \text{Re}(E_y H_z^*) \quad \left\{ E_z = 0 \right.$$

$$= \frac{1}{2} \text{Re} \left\{ \frac{i}{\mu_0 \omega} E_y \frac{dE_y^*}{dx} \right\} = 0$$

time average of $\sin kx$ and $\cos kx$ over a period

$$\langle S_y \rangle = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*)_y = \frac{1}{2} \text{Re}(E_z H_x^* - E_x H_z^*) = 0 \quad \left\{ E_z, E_x = 0 \right.$$






And then we can evaluate so, using these relations is connection relations we can we write. So, for the S_x component of the pointing vector that is the pointing vector which is which is to determine the any energy flow along the x direction we read to write this equation in this form half real part of E cross H star and the x component of that.

So, $\mathbf{E} \times \mathbf{H}^*$ can be written in this form $E_y H_z^* - E_z H_y^*$, but E_z for a TE mode E_z equal to 0. So, this second quantity does not appear only this $E_y H_z^*$ will have to calculate. And then this S_x becomes half of the real part of this quantity because H_z^* we can translate into E_y using this equation H_z will be will be expressed using this equation.

So, this equation, but this is equal to 0, because if I substitute for E_y for the symmetric TE mode it is a cosine $K_x x$. The derivative of this will give you $\sin K_x x$ the product of this and time average to over this will become always 0. Time average of $\sin x$ and cosine x over a period is 0 and vice versa if you take the antisymmetric mode. Then \sin function after the derivative it will become a cosine and this will lead to the same consequence that is this quantity will become equal to 0.

That means there is no power flow along the x direction, there is no power flow along the x direction of the waveguide. Now if we calculate the y component of the pointing vector then, in the same way we can we can represent that half of real part of $\mathbf{E} \times \mathbf{H}^*$ star y component of that. This will lead to half of real part of $E_z H_x^* - E_x H_z^*$ and that is equal to 0 because, E_z and E_x both are 0. So, E_z and E_x both are appearing here. So, that is why this quantity equal to 0, it tells you that there is no power flow along the y direction as well.

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$$\begin{aligned} \langle S_z \rangle &= \frac{1}{2} \operatorname{Re}(\vec{E} \times \vec{H}^*)_z = \frac{1}{2} \operatorname{Re}(E_x H_y^* - E_y H_x^*) \\ &= -\frac{1}{2} \operatorname{Re}(E_y H_x^*) \quad \left\{ H_y = 0 \right. \\ &= \frac{\beta}{2\mu_0\omega} |E_y|^2 \end{aligned}$$

Therefore power flows only along z -direction

Also, this relation is approximately valid for all waveguides in weakly guiding case



Now let us check the power flow in the z direction and for that we calculate the time average of the z component of the pointing vector, in the same way we express this quantity in terms of E and z E and H star. And when you expand this take the x z component of E cross H we can write this E x H y star minus E y H x star and because again H y is equal to 0 for TE mode.

So, we end up with this quantity and then, we can write this equation as if we if you translate H x using that relation then, you can write that this equal to beta by twice mu naught omega E y mode square. So, this is the power flow around E z. This also tells you that along the direction of propagation that is along z direction power only flows through the wave guide. And as a matter of fact to do this calculation and you see that this relation is approximately valid for all waveguides under the weakly guiding approximation ok.

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$$\begin{aligned}
 P &= \frac{\beta}{2\mu_0\omega} \int_{-\infty}^{\infty} |E_y|^2 dx \\
 &= \frac{\beta}{2\mu_0\omega} \int_{-\infty}^{\infty} (A^2 \cos^2 \kappa x + C^2 e^{-2\gamma x}) dx \quad \text{Consider symmetric mode} \\
 &= \frac{\beta}{2\mu_0\omega} 2 \times \left\{ A^2 \int_0^{\frac{d}{2}} \frac{1}{2} (1 + \cos 2\kappa x) dx + C^2 \int_{\frac{d}{2}}^{\infty} e^{-2\gamma x} dx \right\}
 \end{aligned}$$

$E_y(x) = A \cos \kappa x + B \sin \kappa x$ for $|x| < \frac{d}{2}$
 $E_y(x) = C e^{-\gamma x}$ for $|x| > \frac{d}{2}$

Now, how to evaluate this power in terms of the known quantities, we write this expression and we will have to integrate it from minus infinity to plus infinity. And this E y square will be replaced by this expression which is valid for the core that is one mode x you less than d by 2, remember the wave guide structure. And this is the evanescently decaying part of the field which is there outside the core region that is in the cladding region for x mode of x greater than d by 2.

So, if you substitute the value of E mode square we can write in this form A square cosine square Kappa x plus C square E to the power minus twice gamma x d x if we considered the symmetric mode. And then if you do this algebra; you can write this cosine square Kappa x as 1 plus cosine twice Kappa x d x. And this remains as it is and it follows that we can after if you do this integration if you do this integration this quantity at this quantity.

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The slide contains the following mathematical content:

$$P = \frac{\beta}{2\mu_0\omega} \left[A^2 \left(\frac{d}{2} + \frac{\sin 2\kappa \frac{d}{2}}{2\kappa} \right) + C^2 \frac{e^{-2\gamma \frac{d}{2}}}{\gamma} \right]$$

$$= \frac{\beta}{4\mu_0\omega} A^2 \left[\left(d + \frac{2 \sin \kappa \frac{d}{2} \cos \kappa \frac{d}{2}}{\kappa} \right) + 2 \frac{C^2 e^{-2\gamma \frac{d}{2}}}{A^2 \gamma} \right]$$

Use continuity of field: $E_y \left(\frac{d}{2} \right) = A \cos \kappa \frac{d}{2} = C e^{-\gamma \frac{d}{2}}$

$$\Rightarrow \frac{C^2}{A^2} e^{-2\gamma \frac{d}{2}} = \cos^2 \kappa \frac{d}{2}$$

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Then we can write that P equal to this A square d by 2 sin of 2 k d by 2 twice k plus C square E to the power of this twice gamma d by twice written this d by 2 because, we can use this k d by 2 in this factor for utilizing this eigen value equations.

So, if you get to outside then it reduces to this form. Now we will use the continuity of the field that d by 2. So, E y or d by 2 is equal to this and at for the cladding part that is outside the core at the interface actually we are matching this field. So, we can write these two are equal. So from here we can write C square by A square times this will be equal to, so that we have an intention to replace this C square by A square in terms of cosine Kappa d by 2.

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$$\begin{aligned}
 P &= \frac{\beta}{4\mu_0\omega} A^2 \left[d + \frac{2 \sin \kappa \frac{d}{2} \cos \kappa \frac{d}{2}}{\kappa} + \frac{2}{\gamma} \left(1 - \sin^2 \kappa \frac{d}{2} \right) \right] \\
 &= \frac{\beta}{2\mu_0\omega} A^2 \left[d + \frac{2}{\gamma} + \frac{2 \sin \kappa \frac{d}{2} \cos \kappa \frac{d}{2}}{\kappa} - \frac{2}{\gamma} \sin^2 \kappa \frac{d}{2} \right] \\
 &= \frac{\beta}{2\mu_0\omega} A^2 \left[d + \frac{2}{\gamma} + \frac{2 \sin \kappa \frac{d}{2} \cos \kappa \frac{d}{2}}{\gamma \kappa} \left(\gamma - \kappa \tan \kappa \frac{d}{2} \right) \right] \\
 &= \frac{\beta}{2\mu_0\omega} A^2 \left[d + \frac{2}{\gamma} \right] \quad \text{Eigenvalue equation: } \gamma = \kappa \tan \kappa \frac{d}{2}
 \end{aligned}$$

And after doing that we can rewrite this equation in this form. So, beta by twice mu naught omega A square d plus twice by gamma this is the. Now if you take twice Kappa d twice sin Kappa d by 2 cosine Kappa d by 2 and gamma k outside. Then we can write gamma minus k tan Kappa d by 2 and this quantity this expression is well known because it is the eigen value equation for the symmetric TE mode gamma equal to Kappa tan Kappa d by 2. So, this is this quantity becomes 0.

Therefore, we end up with only this quantity and we can write the power flow the content of the power per unit length of the wave guide by this expression beta by twice mu naught omega A square d plus 2 by gamma. And gamma is it also represents the depth of penetration of the field in the one upon gamma in the cladding region. So, this expression gives you the total power flowing with the along the wave guide per unit length of the wave guide.

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Planar dielectric slab waveguide

Power carried by symmetric TE mode: $P = \frac{A^2 \beta}{2\mu_0 \omega} \left[d + \frac{2}{\gamma} \right]$

..... following a similar calculation

Power carried by antisymmetric TE mode is same as TE mode

Power carried by TM mode both symmetric and antisymmetric:

$$P = \frac{A^2 \beta}{2\epsilon_0 \omega n_1^2} \left[\frac{d}{2} + \frac{(n_1 n_2)^2 k_0^2 (n_1^2 - n_2^2)}{\gamma n_2^4 \kappa^2 + n_1^4 \gamma^2} \right]$$






Now, for a planar dielectric slab we calculated the power carried by a symmetric TE mode which is equal to this. And if you follow the same calculation then, we can also calculate we can also determine the power carried by the antisymmetric mode. Because in that case in place of the sin function we can use the in place of cosine function you can use the sin function to represent the antisymmetric mode. The derivative of that will be cosine and again it will be only the z component of power and this will turn out to be the same as the symmetric TE mode the same value we will have.

Now, this power carried by the symmetric and antisymmetric modes for both TE and for the TE mode of the waveguide will a result the same power flow expression. And for the power carried by TM modes both for symmetric and antisymmetric modes, we can in the same way we can show following the same calculation. You can show that this also can be represented by this expression where additionally, you have n_1 into square n_1 square n_2 square and behave usual, because disappears n_1 and n_2 square appears as the continuity of the magnetic field across the across the interface ok.

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Orthogonality of modes of waveguides

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Mode properties of waveguides

The wave equation:

$$\frac{\partial^2 E_y}{\partial x^2} + (k_0^2 n^2 - \beta^2) E_y = 0$$
$$\frac{\partial^2 \psi_m}{\partial x^2} + k_0^2 n^2(x) \psi_m = \lambda_m \psi_m$$

$\beta^2 = \lambda_m$ denote the eigenvalue of the operator $\frac{\partial^2}{\partial x^2} + k_0^2 n^2(x)$

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Now, we will look at another important aspect of the waveguide modes which is the orthogonality of mode and very interesting to know that all the all the modes which are supported by the waveguide will be orthogonal to each other. For that we write the wave equation for the electric field, for this particular case that is for the TE mode and the electric field of that is E_y .

So, we can write this expression k_0^2 this is n^2 of x to represent both core and cladding and so, if we considered the m -th mode. And we can write the mode by I

writing this ψ_m as the no designation. Then $\nabla^2 \psi_m$ and $k_0^2 n^2 \psi_m$ will be equal to $\lambda_m \psi_m$. This λ_m which is to represent this beta square is the eigen value of this eigen value equation.

So, this wave equation and eigen value equation will result the eigen value λ_m . And so, that is what? That beta square equal to λ_m denote the eigen value of the operator this. So, you can this quantity as an operator which is operating on ψ_m will give ψ_m back with the eigen value λ_m .

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Eigenvalue equation for modes

$$\frac{\partial^2 \psi_m}{\partial x^2} + k_0^2 n^2(x) \psi_m = \lambda_m \psi_m$$

the complex conjugate of the above equation

$$\frac{\partial^2 \psi_k^*}{\partial x^2} + k_0^2 n^2(x) \psi_k^* = \lambda_k^* \psi_k^*$$

multiply these equations ψ_k^ and ψ_m respectively and subtract*

$$\psi_k^* \frac{\partial^2 \psi_m}{\partial x^2} - \psi_m \frac{\partial^2 \psi_k^*}{\partial x^2} = (\lambda_m - \lambda_k^*) \psi_k^* \psi_m$$

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Now, for ψ_m this is the equation and if you take the complex conjugate of the above equation for a different mode, that is $\nabla^2 \psi_k$ and $k_0^2 n^2 \psi_k$ equal to $\lambda_k \psi_k$. And then we should write $\lambda_k^* \psi_k^*$. And, now if we multiply these two equations, assuming that ψ_m and ψ_k they are different. Then respectively and then if you subtract we can write this equation in this form multiply this equation by ψ_k^* and this equation by ψ_m so, we get this equation.

This is very interesting to note that from here this quantity the left hand side is 0 why because, if you look at the left hand side this quantity can be expressed as $\psi_k^* \nabla^2 \psi_m - \psi_m \nabla^2 \psi_k^*$. And this involves ψ_k and ψ_m which so, we can write in this form and $\nabla \cdot (\psi_k^* \nabla \psi_m - \psi_m \nabla \psi_k^*)$ which if you integrate over the whole space both sides.

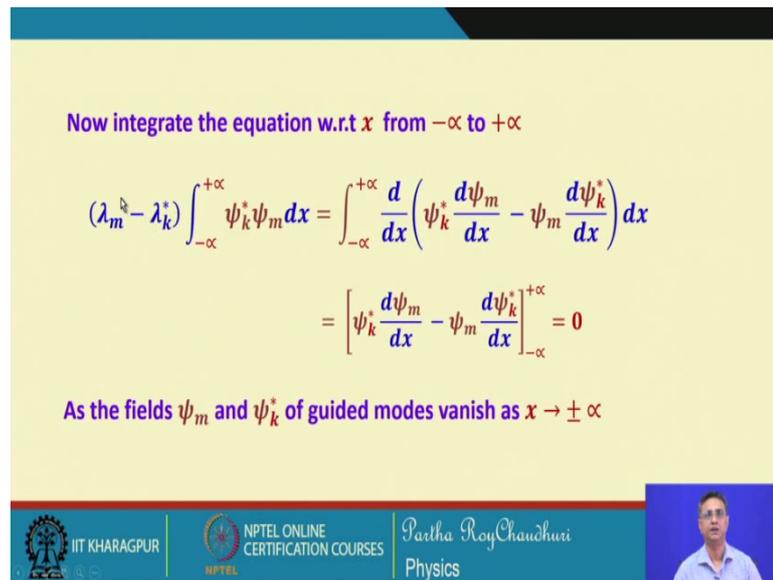
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Now integrate the equation w.r.t x from $-\infty$ to $+\infty$

$$(\lambda_m - \lambda_k) \int_{-\infty}^{+\infty} \psi_k^* \psi_m dx = \int_{-\infty}^{+\infty} \frac{d}{dx} \left(\psi_k^* \frac{d\psi_m}{dx} - \psi_m \frac{d\psi_k^*}{dx} \right) dx$$

$$= \left[\psi_k^* \frac{d\psi_m}{dx} - \psi_m \frac{d\psi_k^*}{dx} \right]_{-\infty}^{+\infty} = 0$$

As the fields ψ_m and ψ_k^* of guided modes vanish as $x \rightarrow \pm \infty$



Then you can write this equation as minus infinity to plus infinity integration of this will give you this quantity which is equal to $\psi_k^* \frac{d\psi_m}{dx} - \psi_m \frac{d\psi_k^*}{dx}$. So, this will be the result of the integration $\psi_k^* \psi_m$ and, but this is equal to 0 because the ψ_m and ψ_k for guided modes. So, vanish as x tends to plus infinity and minus infinity the fields are the fields will become 0 far away from the waveguide across the along the side line.

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Real eigen value

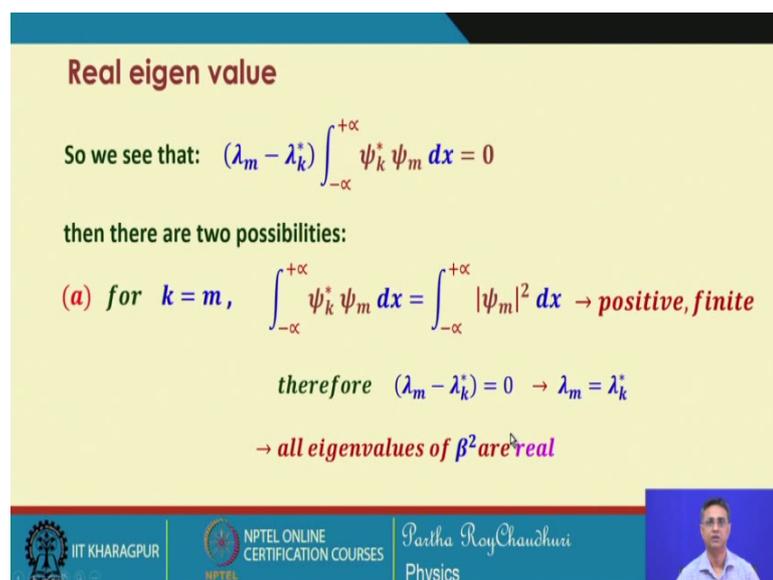
So we see that: $(\lambda_m - \lambda_k) \int_{-\infty}^{+\infty} \psi_k^* \psi_m dx = 0$

then there are two possibilities:

(a) for $k = m$, $\int_{-\infty}^{+\infty} \psi_k^* \psi_m dx = \int_{-\infty}^{+\infty} |\psi_m|^2 dx \rightarrow \text{positive, finite}$

therefore $(\lambda_m - \lambda_k) = 0 \rightarrow \lambda_m = \lambda_k$

\rightarrow all eigenvalues of β^2 are real



So, we see that this quantity $\lambda_m - \lambda_k$ and integration from minus infinity to plus infinity $\psi_k^* \psi_m dx$ equal to 0 is a very well known form of this

integral. So, this tells you that there are two possibilities; one is that k and m they are equal, that is the two modes k equal to m they are the same modes that is; that means, then this integration if this is equal to 0.

So, then this is non 0 this is non 0 so, $\int \psi_m^2 dx$ will be always positive, because the for a for one mode k equal to m this becomes this $\int \psi_m^* \psi_m$ will be equal to mode $\int \psi_m^2$, which is the probability of this function. So, it is always positive and finite so, it cannot be 0.

In that case, the $\lambda_m - \lambda_k^*$ that is eigen values difference of the eigen values are 0. Which immediately tells you that λ_m is equal λ_k^* ; that means, the for the modes the eigen values all eigen values that is the λ square which was represented by λ_m and λ_k are real. This is a very important constituents of this understanding.

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Orthogonality condition

So we see that: $(\lambda_m - \lambda_k^*) \int_{-\alpha}^{+\alpha} \psi_k^* \psi_m dx = 0$

then there are two possibilities:

(b) for $k \neq m$, so $(\lambda_m - \lambda_k^*) \neq 0$

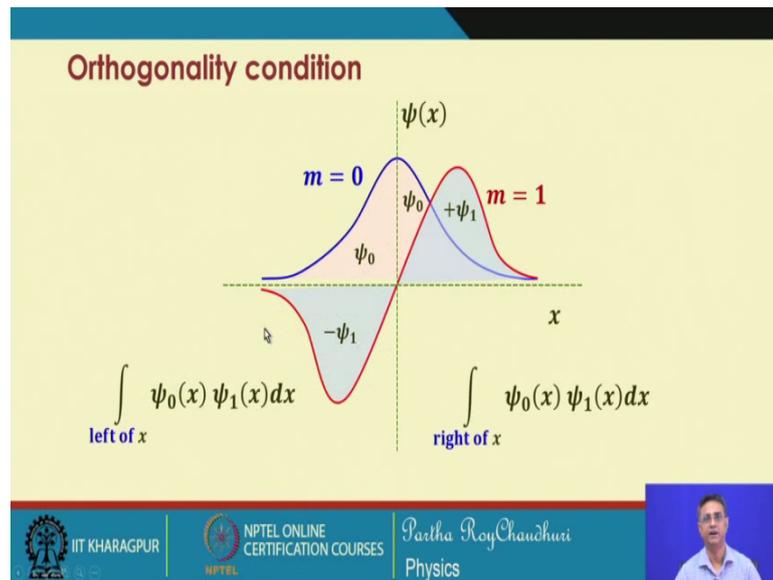
therefore $\int_{-\alpha}^{+\alpha} \psi_k^* \psi_m dx = 0$

→ modes are orthogonal

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So, we have this expression and if you look at the second possibility that is for k not equal to m , in that case $\lambda_m - \lambda_k$ not equal to 0. It means that this must be equal to 0 this integration this integrant will be equal to 0 and this integration 0 means this is the orthogonality condition that the product of these two the inner product of these two wave functions will be always equal to 0. So, these are two very useful outcome of this of this is more properties guided modes of the wave guides.

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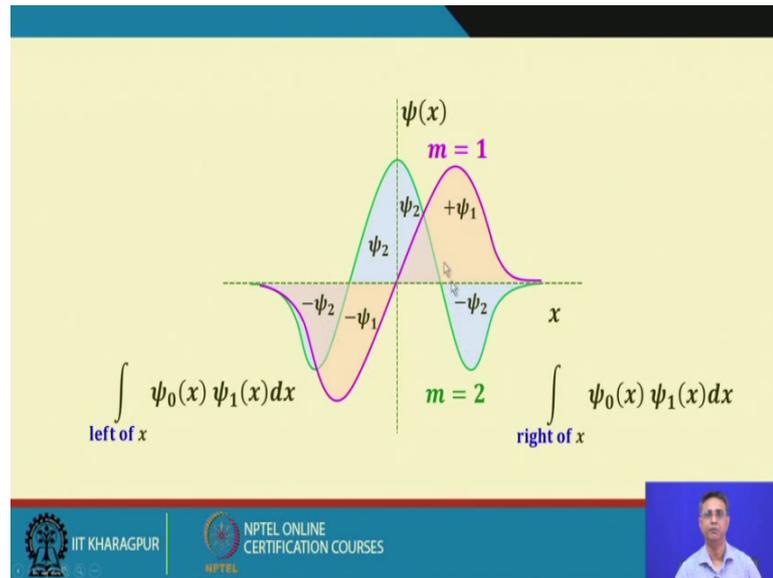


And that is true for all wave guides, now look at this orthogonality condition when m equal to 0, you have this mode which is the fundamental mode for a symmetric planar dielectric structure. And when m equal to 1 that is the first order mode, then we have an antisymmetric distribution of the field so that has this shape. Now if you take the product of these two wave functions that is ψ_0 and ψ_1 if you take the product of these two wave function ψ_0 and ψ_1 .

So, this product that is I look at the product only at the left hand side that is ψ_0 and ψ_1 and I integrate. So, this quantity this quantity multiplied by this quantity, the area under this will be will be same as this quantity multiplied by this because, across this across this line the left hand side area and the right hand side areas are same. So, this integral and this integral both of them are equal, but they are having opposite sign because the product ψ_0 and minus ψ_1 here the product is ψ_0 and plus ψ_1 .

So, they will cancel each other and the result is that the this product summed over, the total $\psi_0 \psi_1$ over the entire wave guide will be equal to 0; that is what is the orthogonality and this is true for every mode to every other mode.

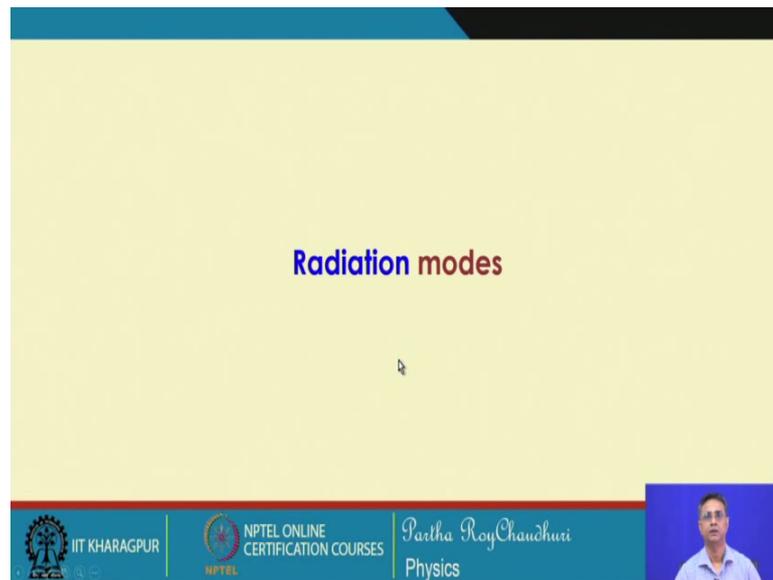
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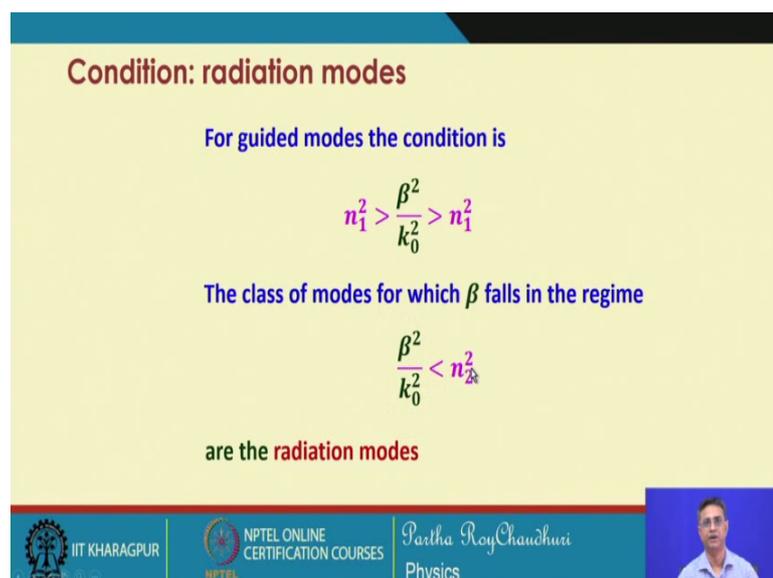
For example, if we take m equal to 1 that is the first antisymmetric mode and the second symmetric mode that is m equal to 2 which has a field distribution like this. And again if we compute the overlap so, this ψ_1 is here and this part so, this part multiplied entire the ψ_2 and ψ_1 will give you the left hand side. And similarly $\psi_2 \psi_1$ plus and ψ_2 on the right hand side they will give you the same value. And with the minus time for this part and this part and for plus sin for this part and this part which will be put to when if you put equal to will be equal to 0.

So that means that for every mode to every other mode we can do the same calculation and see that modes are orthogonal. This is a very interesting property and seen in a quantum mechanics also.

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Now radiation mode for the modes guided modes the condition is this, that this beta by k_0 square, but that is what we call the ineffective square we will be always lying between n_1 square and n_2 square, this should be this should be n_2 this is by mistake this should be n_2 . So, will be between n_1 and n_2 square the class of modes for which beta falls in this regime that is beta square by k_0 square this an effective value an effective will be less than n_2 square in that case the modes will be called the radiation mode.

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Wave equation for radiation modes

Recall the wave equation (say TE mode)

$$\frac{\partial^2 E_y}{\partial x^2} + (k_0^2 n^2(x) - \beta^2) E_y = 0$$

In the region: $|x| > \frac{d}{2}$ the wave equation is

$$\frac{\partial^2 E_y}{\partial x^2} + (k_0^2 n_2^2 - \beta^2) E_y = 0$$

$$\frac{\partial^2 E_y}{\partial x^2} + \delta^2 E_y = 0 \quad \text{where } \delta^2 = k_0^2 n_2^2 - \beta^2$$



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And why it is? So, we can see that let us recall any of the wave equations for examples the TE mode, then we can write down this equation which is well known again. And in the region that is mode x greater than d by 2 the wave equation that in the cladding region, the wave equation can be written in this form, because this n square of x will be a constant value n^2 square. And this equation to be satisfied we can write this expression earlier we call this is the mode parameter or the transverse component of the propagation vector here we write this equation as δ square.

And so, that where δ square is equal to k_0 square n^2 square minus β square. And this equation is also well known in physics and we can write the solution as E_y is equal to e to the power plus minus $i \delta x$.

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Oscillatory solution

Now for β satisfying the condition: $\frac{\beta^2}{k_0^2} < n_2^2$

The quantity $\delta^2 = k_0^2 n_2^2 - \beta^2$ is now positive

Therefore the solution to the wave equation in the region: $|x| > \frac{d}{2}$

$$E_y \approx e^{\pm i\delta x}$$

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So, that is what so, now, for beta satisfying the condition beta square by k 0 square less than this we have this quantity, which is now positive, because of the difference of this quantity is now positive. Therefore, the solution to the wave equation in this region that is in the cladding region can be expressed in terms of this that is what I just told that the solution of this equation will be because delta is now positive. So, we can write this equation E y is equal to approximately equal to E plus minus e to the power of i the solution is in this form ok.

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Oscillatory solution

$$E_y \approx e^{\pm i\delta x}$$

- ✓ Evidently radiation modes are **oscillatory** unlike the fields of **guided modes** which are **exponentially decaying**
- ✓ δ can assume any continuous value satisfying the condition above radiation modes are **continuous** unlike the **discrete guided modes**

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So, E_y is $e^{\pm i\delta x}$; that means, evidently these modes are oscillatory. For a mode to be guided the condition is that it should be an oscillatory function in the core, but an exponentially decaying function, even asymptotically a decaying function in the cladding. But it is seen from this condition that if we assume that an effective n^2 is less than n^2 then the modes are the fields are oscillatory even in the cladding region.

And which is which is different from the guided mode which are exponentially decaying. And also we can see δ can assume any continuous value, because there is no restriction on δ now so the value which condition above the radiation modes are continuous. And which is different from the discrete guided modes of the wave guide.

So, we have seen that for this condition you have a set of continuum modes which are the radiation mode, but actually they are from the ray theory you can see that these waves the ray is corresponding to these waves do not satisfy the critical angle condition. And that is why they are affected out from the interface and it forms a continuum of radiation mode.

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-----Summary of discussions-----

- ✓ Some key properties of guided modes: power carried by a guided mode: expressions for TE- and TM modes
- ✓ Real eigen value of modes and orthogonality of guided modes
- ✓ Radiation modes and fields

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So, in these discussion we summarize that we have we have discussed the power carried by the guided modes we calculated for the TE modes of the slab wave guide for symmetric field distribution. And also for antisymmetric field this distribution we have

seen that if we carry out the similar calculation for TM modes you can also determine the power flow with the waveguide for TM mode case also.

Then we look at the look at the orthogonality condition as a consequence found that eigen values of the modes are real. And the modes are orthogonal every mode to every other mode is an orthogonal we try to understand in terms of the overlap of the modes. The two modes we under consideration that the product of the wave functions integrated over the whole space will be equal to 0 which is the orthogonality condition.

Then the another important aspect that the radiation mode I just to ensure that for guided modes beta should be between n_1 and n_2 the an effective value $\beta^2 = k_0^2 n_{\text{eff}}^2$ should be within n_1 and n_2 or and we find their data continuous mode, continuum field distribution representing the radiation modes.

Thank you.