Mathematical Methods in Physics-I
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Lecture – 46
Series and Sequence

So, welcome back student to the next class of complex analysis. So, today we will be going to start something called series and sequence of complex number.

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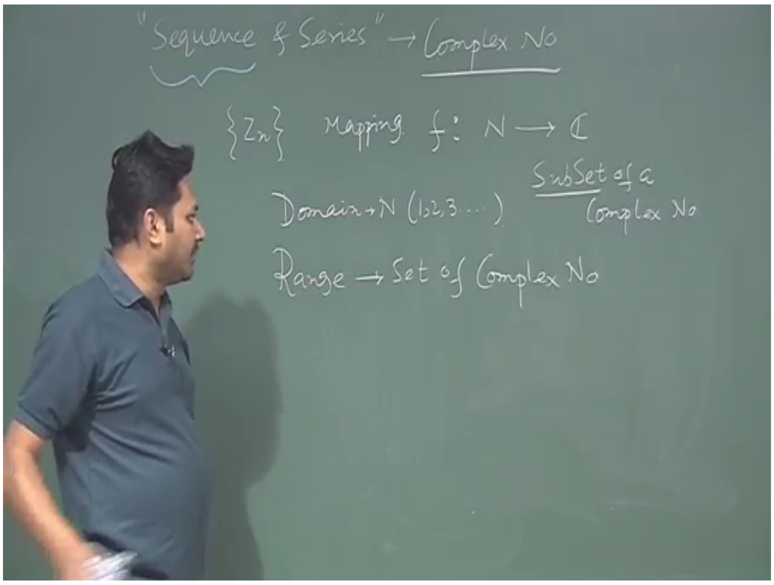

So, first let us start with this sequence what is the meaning of sequence in complex number. So, it is sequence is something say I represent something like Z to the power n is my sequence defined like this; what is the meaning? It is a mapping where the map is something like that I will have some integer real integer positive real integer say N and it will give me some set of complex number c.

So, it is mapping where I will have some integer I put some integer and as a result I am getting set of set of a complex number; that means, my domain here is integer N 1 2 3 and my range here set of complex number. So, I have something called sequence this is Z n means this is some complex number having some index N; that means, it is a function of index n where n is some integer. So, I will change this number 1, 2, 3, 4, which is related to this N and then as a result what happened that every time when I change this number, I will get generate; I will generate new numbers, new complex numbers, it may be repetitive, it may be non repetitive and this new numbers are the range which is in the set of complex number, this is subset rather, this is subset of complex number is not set, this is a some subset of the subset of some set of complex number.

Whatever I am getting a sum; subset some amount of complex number I am getting with this very brief definition things will be let me erase this.

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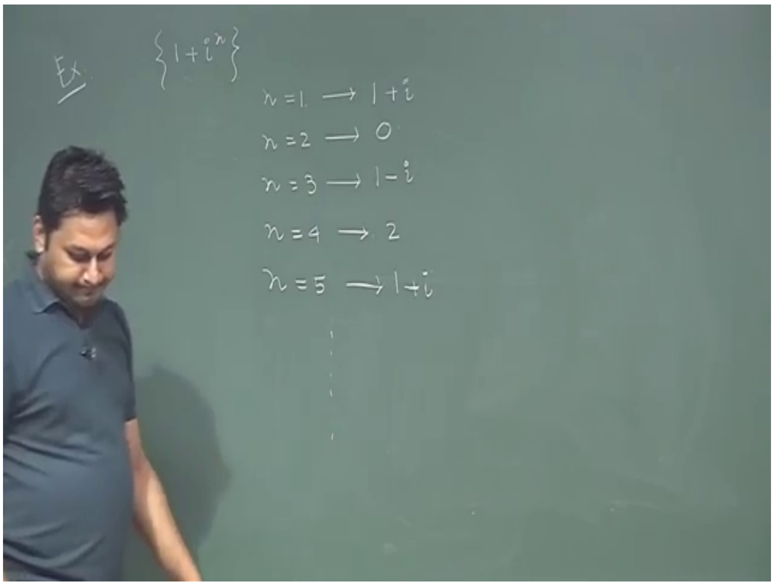

So, let me give you one example that maybe make like easier 1 plus i to the power n this is; obviously, complex number and i to the power n means this complex number is depends on the value of n where n 1, 2, 3, 4, 5 and I will have different complex number and if I put this different complex number then they will form a sequence that is all. So, when n is equal to 1, I will have my first number this, when n is equal to 2, I have second number when n is equal to 3, I have number i minus 1, minus i, n is equal to 4, I have 2, n is equal to 5, I have and so on.

So; that means, I am changing my n 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and as a result; what I am getting is this numbers which I called the sequence. Now the obvious questions that should come to our mind is how many numbers I am getting; here if I go; n tends to infinity and is this number is going on or it will be converging to some points.

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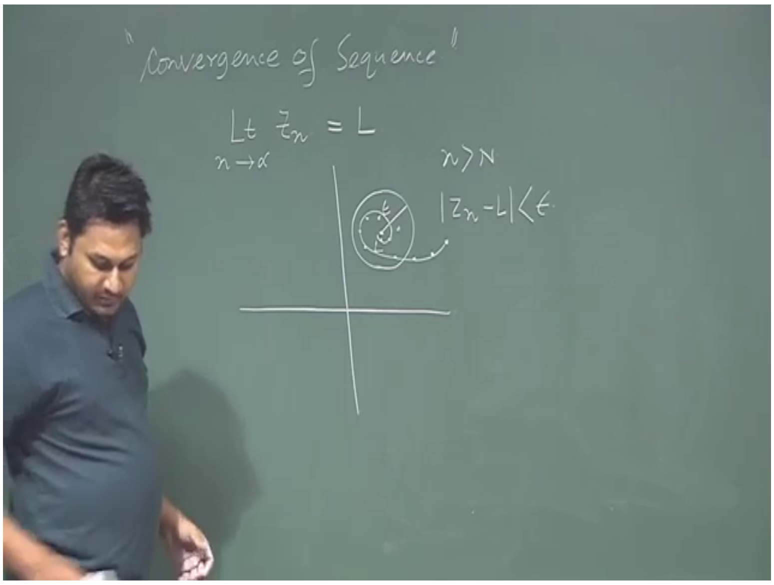

So, next thing is convergence of sequence convergence of sequence con convergence as I mentioned if I go. So, in mathematical form if n tends to infinity if I put n tends to infinity, let me change the chock here if I put n tends to infinity the n th; the n th number n th sequence number, I want to find out the limit for n tends to infinity say it goes to some value l.

It goes to some value L this is the mathematical so; that means, when I change the value the number of value when I am changing. So, it will try to go to some try to locate some particular point which I called L in another way if n tends to n is always if I put n some I change the in such a way I always it is always greater than some value n begin then I will have this relation this relation. So, let me visualize these things. So, this is the domain I have this is the complex number that is changing. So, in the complex plane the numbers are changing and it is changing in such a way this is the domain here the value is L and this is epsilon and the values I am changing the values through this through this n.

So, when I am changing say some point is here some point is here some point is here some point is here some point is here some point is here, here, here, here, here. So, as I increase the point is as I increase the value of n the sequence of the point is tend go to some specific value L if that is the case; that means, the sequence is converging and converging to the value l. So, let me give you another example let me give you the another example; example gives life makes life simpler.

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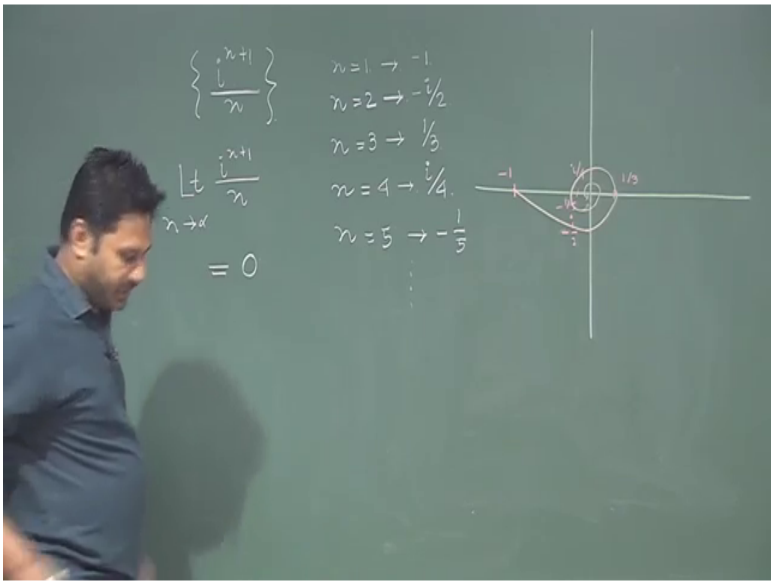

So, this is another this is a sequence the sequence is represented by this; this second bracket inside that I have some complex quantity which is related to the value n now I change n. So, every time when I change n I will start getting new values.

So, when n is equal to 1, I have minus 1 when n equal to 2, I have n equal to 2, then it is 3 n equal to 1 is n equal to 2 it is 3. So, minus i by 2 n equal to 3 when n equal to 3, I have this 3 plus 1; 4 i to the power 4 is 1 and it is 3. So, 1 by 3 n equal to 4 when n equal to 4 I have i by 4 n equal to 5, I have minus 1 by 5 and so on. This is the pattern the way this term is changing. Now if I want to put that let me put the coordinate. Now I want to put this number here in this jet plane and if I do that; then I find n equal to 1 is minus 1. So, first point is say this is minus 1 here.

Second point minus i by 2; so, somewhere here; this is minus 1 by 2 with i; third point is 1 by 3 somewhere here is 1 by 3 fourth point is i by 4 somewhere here i by 4 fifth point is minus i by 5. So, somewhere here i by minus i by 5 and so on. So, if I now draw these things, then I can find this function is changing like this and seems to it is changing like this and seems to go to some value like 0 and obviously, if you take the limit of this things limit n tends to infinity i n plus 1 n the result is 0 so; that means, this sequence is converging and converging to a value 0.

We will obviously, show more examples so, that understanding will be clear, but before that.

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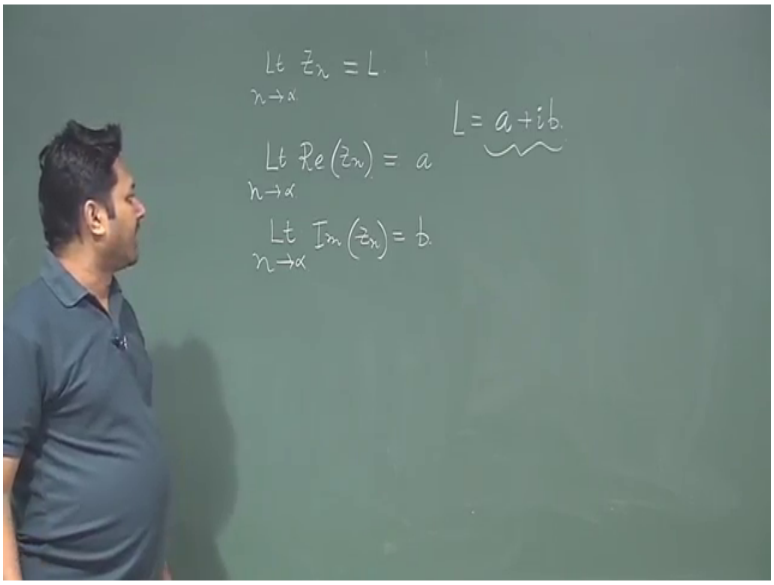

Let me; so, when limit n tends to infinity Z n is L that eventually means I mean when this is the case, you should realize one fact that L is a complex number because this is the complex number and I put this complex number with the limit n tends to infinity. So, whatever the result, I am getting should be a complex number. So, L is a complex number. So, I can write this as a plus i b and this limit goes to this point that is eventually means limit n tends to infinity real part of whatever Z n, I have should go to the point a limit n tends to infinity imaginary part of Z n and also go to some point b.

So; that means, this complex number containing 2 quantity one is a and b both are real and when n tends to infinity goes to n tends to infinity Z n goes to some value L which is a plus i be a complex number the real numb real part of this goes to the real part of this things and the imaginary part of this Z n goes to the imaginary part of this things that is essential so that we can have the limit in our hand.

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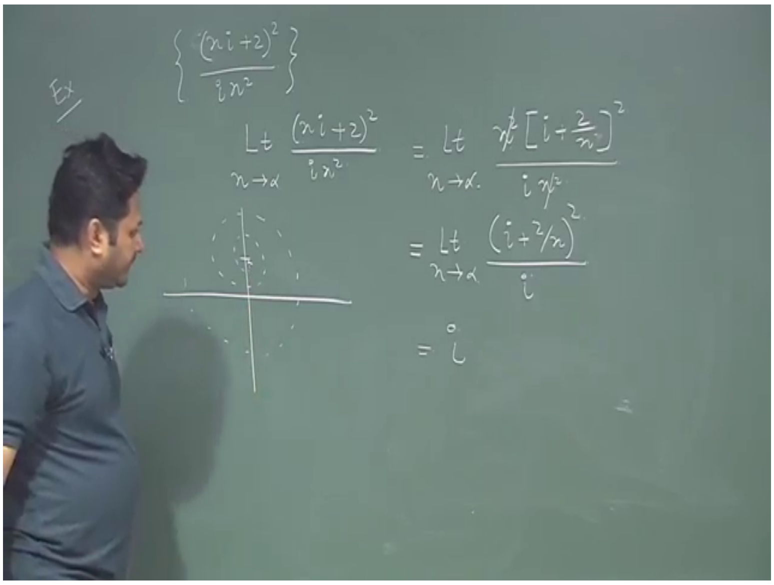

Now, we go to few more examples and then things may be clear. So, next is this example, n i n i plus 2 whole square divided by i n square n i plus 2 whole square; let me check it is correct or not and then n square i.

So, I need to find out we need to find out whether this is this sequence is converging or not this is a converging sequence or not so; that means, I need to find out this quantity limit n tends to infinity whatever the quantity I have here n i plus 2 whole square divided by i n square this limit just all. So, if I evaluate this limit and if this limit gives me some value other than infinity then I can say that this is converging by the way I need to mention one thing also. So, limit n tends to infinity Z n if it goes to infinity then; obviously, it is divergent it is diverging is nature; that means, it goes to some infinite even though I am moving my n tends to infinity this is not going or tending to some specific point rather is goes to infinity .

So if I try to find out the limit n tends to infinity. So, what I will do that I will take n common here. So, it will be n square i plus 2 by n square 2 by n; if I take n common outside divided by i n square n square n square can be cancel out and now we are in a position to put. So, limit n tends to infinity i plus 2 n square divided by i is nothing, but I so; that means, this sequence if you try to put 1, 2, 3 of the value of n; you will find that in the previous case, what are the function is whatever the sequence is given, it is; it was going to the value 0 here we find it is going to the value i so; that means, if this is my point. So, i is here. So, the point will gradually go to this point i.

So; that means, it is converging to that particular point which we called the converging point here in case in this case here it is i; you can also verify by the way you can also verify that this quantities real and imaginary part at instant say infinity goes to let me.

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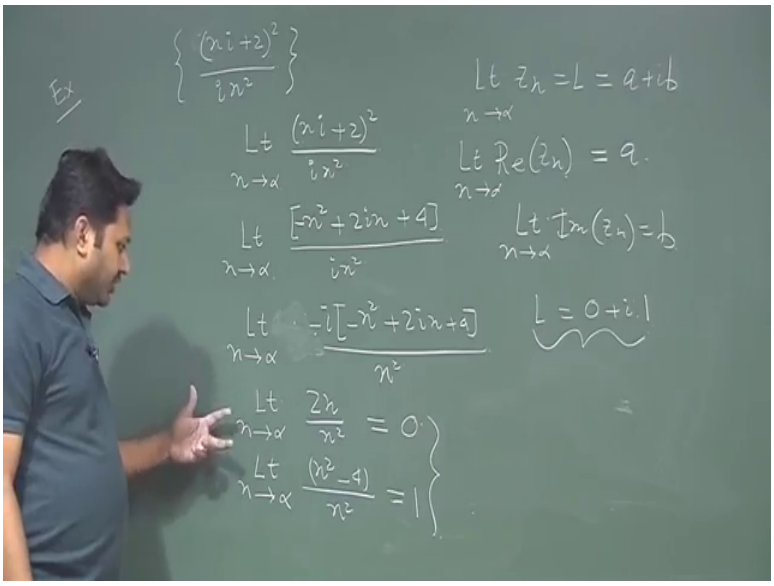

So, I need to find out this limit right. So, I can separate it out to real and imaginary part. So, if I do that then it will be if I make the square. So, it will be minus of n square plus 2 i n plus 4 divided by i of n square.

So, limit n tends to infinity if I multiply with i, then it will be minus of I multiply minus i. So, that the minus i into minus of n square plus 2 i n plus 4 divided by n square. So, real and imaginary part, I can separate out. So, one limit n tends to infinity 1; the real part is how much with the real part is when I multiply, it will be 2 n divided by n square this is my; the real part that I should have this is the real part that I should have what will be the imaginary part here the imaginary part, if I put the n tends to infinity what value, I will get 0 what will be the imaginary part of this quantity.

Imaginary part is this one. So, it will be n square minus 4 with I divided by n square now if I take n square common and if put n tends to infinity then this term will not be there only I will remain. So, this value will be i. So, as I mentioned that when limit n tends to infinity Z n goes to L which is a plus i b, then the real part of Z n with the limit n tends to infinity should be a and limit n tends to infinity imaginary part of Z n should goes to b in this case you can find that the real part of goes to 0 and imaginary part goes to i; that means, my L is equal to 0 plus I in by the way here the imaginary part I am talking about not the imaginary part; I am taking also I here.

So, in principle when I am talking about imaginary of these things this I should not be there. So, then the limit should be 0 and one and then the value will be something like this my a is 0 and b is 1 in that sense. So, there is a slight I should say it is a mistake because when I am taking the imaginary part of Z, I should take only the imaginary part without taking i.

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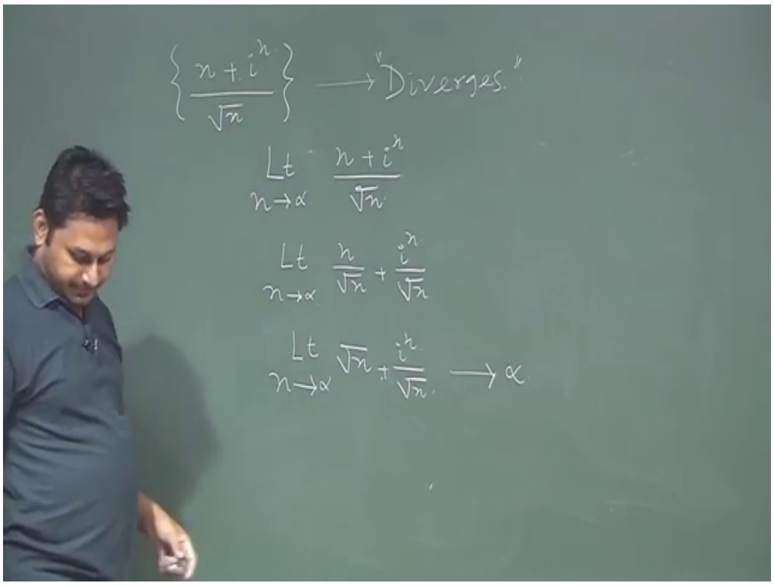

So, that is why this is will be the case and I will get one here I believe you understand how this things are working. So, let me go to another example my next example is something like n plus i to the power n divided by root over of n and I need to find out whether this is converging to some point or not. So, again I need to do the same procedure limit n tends to infinity quantity is n plus I to the power n divided by root over of n. So, limit n tends to infinity I can separate out into 2 part n plus i to the power n root over of n which is n tends to infinity root over of n plus i to the power n i root over of n.

So, when intensive infinity this quantity can go to 0, but this quantity is infinity. So, that value is infinity; that means, this sequence is diverging is diverges this sequence is diverges next; another example and its gives me something like this.

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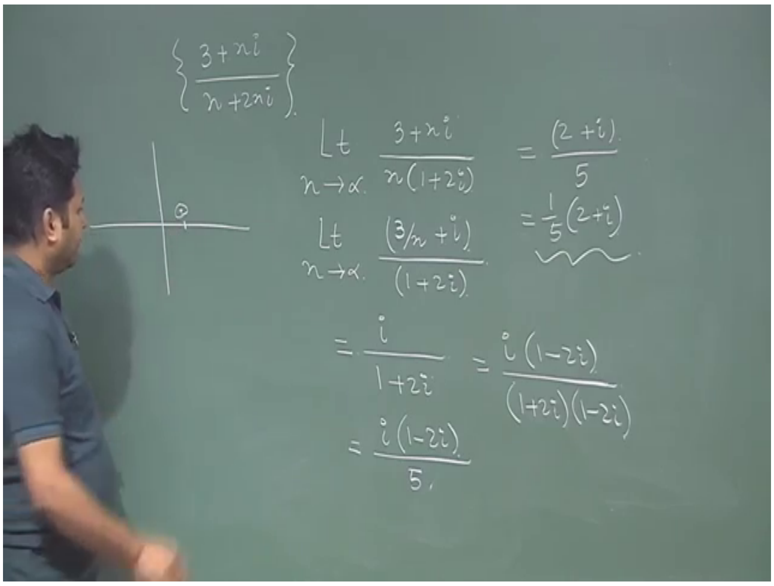

3 plus n i divided by n plus 2 n i 3 plus n i n plus 2 n i. So, now, I need to find out whether this sequence is converging or not if it is converging then again we need to find out these things. So, it will be 3 plus n i divided by n 1 plus 2 I right. So, when n tends to infinity. So, my first target will to remove this n to make it one by n and remove this n.

So, if I do that then if I divide these things. So, it will where will be if I take n common out of there. So, it will be 3 divided by n plus I divided by one plus 2 i; this now n tends to infinity I can put because this quantity will going to vanish. So, by in result here after putting the limit is something like 1 divided by 1 plus 2 i; again I need to put this as a plus i b form. So, that I can find what is the value where it is converging what is the exact value. So, I should multiply with the complex conjugate here I will multiply a plus one plus 2 i. So, one minus 2 i we can multiply. So, I will have here 1 plus 2 i 1 minus 2 i.

And these things is how much i 1 minus 2 i divided by a plus b into a minus b a square minus b square here minus b square means 4 minus four. So, it will be just 5 one plus 4 5 a plus b into a minus b a square minus b square. So, i is there. So, minus will be plus. So, 1 plus 4 it will be 5. So, then the limit is something like if I multiply i, then it should be 2

1 into 2; 2 plus i by 5. So, it is just 1 by 5; 2 plus i, this is the point where this sequence will going to converge that is the point 1 by 5 2 plus i.

Where the sequence is going to converge after having lot about the; so, now, I believe whatever the sequence is given to you; you can really understand whether this is a converging or not it is not a very big deal to find out whether it is a converging or not you just put the limit n tends to infinity and when you put n tends to infinity then you will find that this quantity which is the n th term of this under the condition n tends to infinity, you will find that if this is going to a specific value of specific point here in this case; this point is 2 by 5 plus 2 by 5 plus 1 by I 1 by. So, 2 by 5 is somewhere here and 1 by this is somewhere here. So, some point here it is converging this is converging somewhere here in this point.

So, I mean it depends on how this; the sequence is given and every time the procedure is same and you will find a specific point if it is converging you will find a specific point where it is converging after having that.

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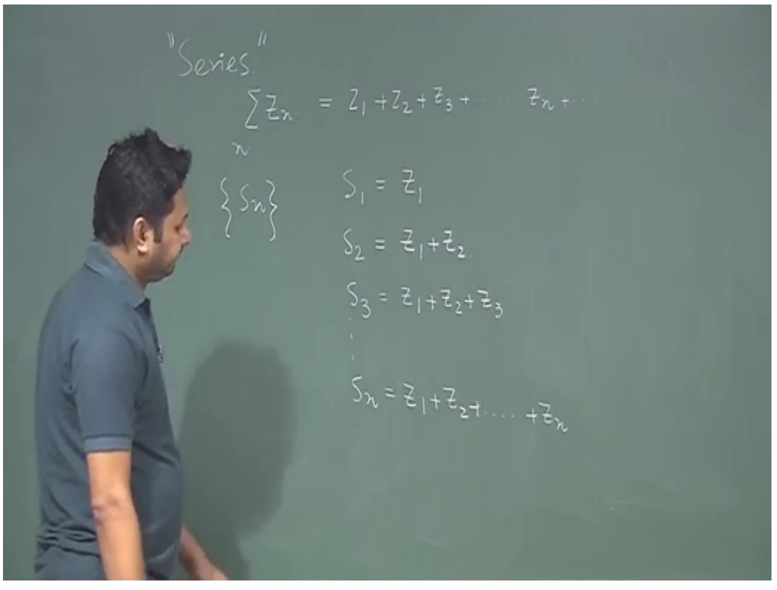

So, today I like to introduce another thing before completing the class it is called series sequence we learn now we need to learn series. So, serious is something where I have say this quantity Z n some quantity I will going to add say first one is Z 1 first second one is Z 2, third one is Z 3 and Z n and So on and it is basically the; we called it as partial sum and then if I put the partial per sequence of the partial sum, then I will have this. So, what is the sequence of the partial sum? So, this is the sequence of the partial sum.

So, what is the meaning of partial sum say S one is Z 1 I S 2 is Z 1 plus Z 2 S 3 is Z one plus Z 2 plus Z 3 and. So, on so; that means, when I put this entire thing I am basically putting S n S n is nothing, but Z 1 plus Z 2 plus this quantity. So, it is the series is it is called the partial sum because I have one quantity then I have 2 quantity which is sum over this then I have 3 quantity then I have n quantity and this all quantity I defined by S 1, S 2, S 3, S n and this I put here. So, this is the partial sum this is some sort of sequence I have. So, every time I am adding I am getting the new terms. So, when I put the S n th term then it is basically gives me the series of all quantities.

Again if it is if this entire sum limit n tends to infinity Z n n th term of this quantity.

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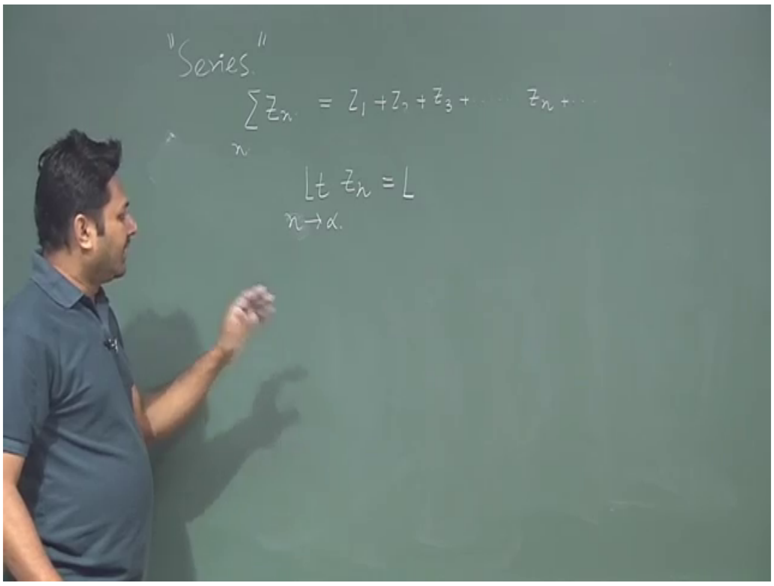

If it goes to some value say L, then we can say that this sum is convergent; that means, I have different kind of terms in my hand which is which I am going to adding and then try to find out the n th term whatever I am going to add if n tends to infinity it is some finite value it goes to some finite value. So, I am adding the term, but this goes to some finite value then we called this is convergent. So, today we do not have much time to discuss it more. So, in the next class we will start here, I will conclude the class here in the next class, we will start from this point and try to find out how the series; we will going to evolve and how to find out the convergence of series and all this things. So, with this note let me conclude the class here. So, see you in the next class where we will going to study more in detail what is the meaning of series and all this things. So, see you in the next class.

Thanks for your attention.