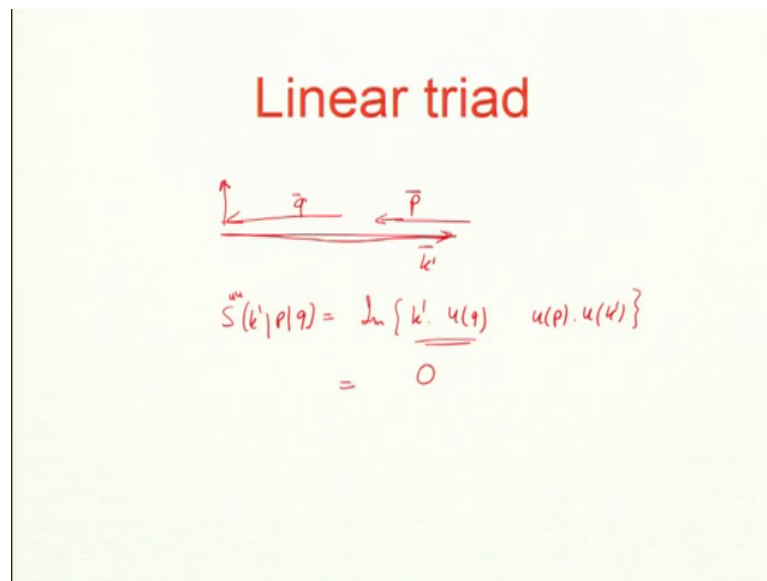


Physics of Turbulence
Prof. Mahendra K. Verma
Department of Physics
Indian Institute of Technology, Kanpur

Lecture – 23
Energy Transfers
Examples

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So, let us look at some examples now for energy transfers. So, linear triad so, imagine that we have \mathbf{k}' , \mathbf{p} and \mathbf{q} triangle you can form a triangle like this (see the slide above). Now, what about energy going to \mathbf{k}' from \mathbf{p} with \mathbf{q} as a helper.

0. So, energy transfers in a co-linear triad is 0. So, this is nice result. In fact, it is a meaningful result. This say another way to say, this formula is a good formula without delta, because I mean if you do this fluid mechanics you say that if the ways are if the triad is along same line, you should get 0. So, it, it comes from here.

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Example 2

$$\mathbf{u} = \hat{x}2B\cos y + \hat{y}2C\cos x + (\hat{x} - \hat{y})2A\sin(x + y)$$

mode	u_x	u_y
(1,0)	0	C
(-1,0)	0	C
(0,1)	B	0
(0,-1)	B	0
(1,1)	$\frac{\Delta}{i}$	$-\frac{\Delta}{i}$
(-1,-1)	$-\frac{\Delta}{i}$	$\frac{\Delta}{i}$

$\vec{k} = \vec{p} + \vec{q}$

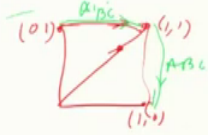
Now, let us do the example which we have been working on for quite a while. Now, this example you seen it again and again. So, third time you are seeing it. So, how many modes are there? So, $\cos(y)$ and $\cos(x)$ and this one; so, we have this I mean, I did this before, I do not want to repeat it. So, there are 2 triads I mean \mathbf{k} and $-\mathbf{k}$ triad, and one triad I have drawn it here, see the slide above.

So, here it is $\mathbf{k} = \mathbf{p} + \mathbf{q}$, \mathbf{q} , \mathbf{p} and \mathbf{k} in that direction. And what is the magnitude of this Fourier modes or not magnitude the Fourier amplitudes? Please see the slide for details.

Both the components $1i$ and A/i and $-A/i$, it comes from here. So, it is in that direction $\sqrt{2}A/i$, because you just magnitude is $\sqrt{2}A$ and that direction will come from here A/i , and it is a complex vector. So, it is a really schematic ok, I mean I cannot really draw the real vector in this, but like that. So, you can compute now $S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})$. So, let us compute it. So, we use our formula, the second formula.

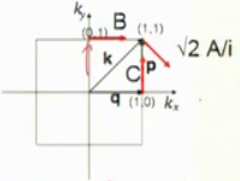
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Example 2



$$\mathbf{u} = \hat{x}2B\cos y + \hat{y}2C\cos x + (\hat{x} - \hat{y})2A\sin(x + y)$$

mode	u_x	u_y
(1,0)	0	C
(-1,0)	0	C
(0,1)	B	0
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(1,1)	$\frac{\Delta}{i}$	$-\frac{\Delta}{i}$
(-1,-1)	$-\frac{\Delta}{i}$	$\frac{\Delta}{i}$



$$S^{uu}(k|p|q) = \int_m \{ k \cdot u(q) \} \frac{u(p) \cdot u'(k)}{u(k)} = \int_m \{ C (B^2) \cdot \frac{(B \cdot A)(\Delta)}{i} \}$$

$$S^{uu}(p|q|u) = - \int_m \{ p \cdot u(k) \} \frac{u(p) \cdot u(q)}{u(k)} = 0$$

So, see the slide above for $S^{uu}(k|p|q)$, and $S^{uu}(p|q|k')$.

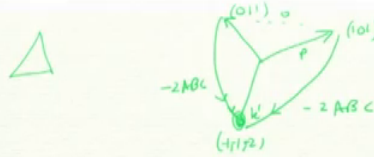
So, if you look at the one of the pictures, which I have in my in the book this energy transfer ABC is also oscillating. So, in fact, this is same as what similar ok, it is not, this is not what is for asymmetric square pattern, but equations are not the same, but the basically the dynamics is the same, energy goes from one mode to another mode and then it back. So, it is oscillating ok. So, it is connected with that and that is why I am interested in pattern for which is blinking pattern, and this is capturing that ok. So, is it a clear? So, this is how we compute. Now, slightly more complicated problem is 3D problem.

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Example 3

$$\mathbf{u} = 4C(\hat{x}\sin x \cos z - \hat{z}\cos x \sin z) + 4B(\hat{y}\sin y \cos z - \hat{z}\cos y \sin z) \\ + 8A(-\hat{x}\sin x \cos y \cos 2z - \hat{y}\cos x \sin y \cos 2z + \hat{z}\cos x \cos y \sin 2z)$$

mode	$\mathbf{u}(\mathbf{k})$	$\boldsymbol{\omega}(\mathbf{k})$	$E_u(\mathbf{k})$	$H_K(\mathbf{k})$
(1,1,2)	$(-\frac{A}{i}, -\frac{A}{i}, \frac{A}{i})$	$(3A, -3A, 0)$	$3A^2/2$	0
(1,0,1)	$(\frac{C}{i}, 0, -\frac{C}{i})$	$(0, 2C, 0)$	C^2	0
(0,1,1)	$(0, \frac{B}{i}, -\frac{B}{i})$	$(-2B, 0, 0)$	B^2	0



So, Now the example 3, we have done in the class, but energy transfer we need to compute.
Solve this example at home.